EL-MOASSER

By a group of supervisors

THE MAIN BOOK

2nd PREP.
2025
FIRST TERM

Maths

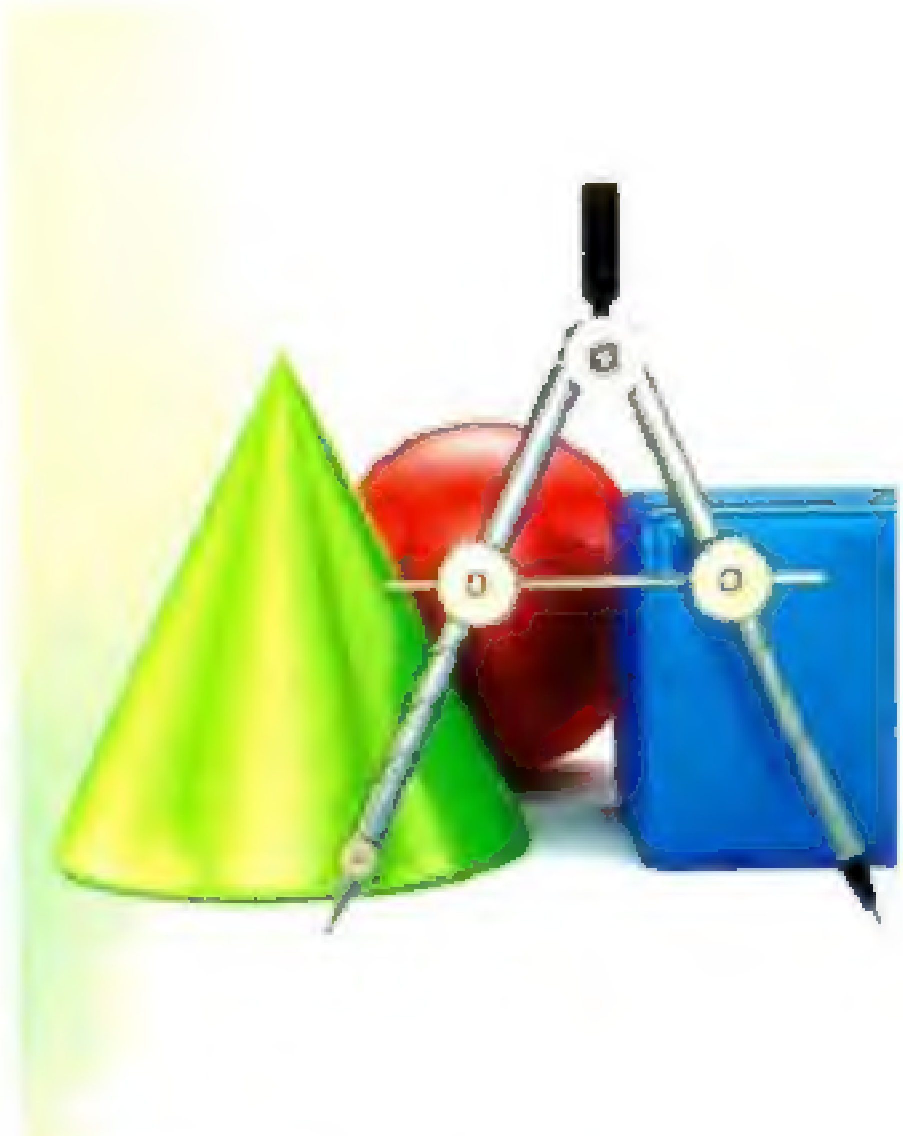
GPS

Interactive E-learning Application

Contents

First — Algebra and Statistics

Second — Geometry



Notes

The notes found at the margin of some pages in **geometry** and referred to by (*) are theorems and corollaries have been studied before

First

Algebra and Statistics

UNIT ONE

Real Numbers

Lessons of the unit :

1. The cube root of a rational number.
2. The set of irrational numbers $\acute{\mathbb{Q}}$.
3. The set of real numbers $\mathbb{R}$ – Ordering numbers in $\mathbb{R}$.
4. Intervals.
5. Operations on the real numbers.
6. Operations on the square roots.
7. The two conjugate numbers.
8. Operations on the cube roots.
9. Applications on the real numbers.
10. Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$.

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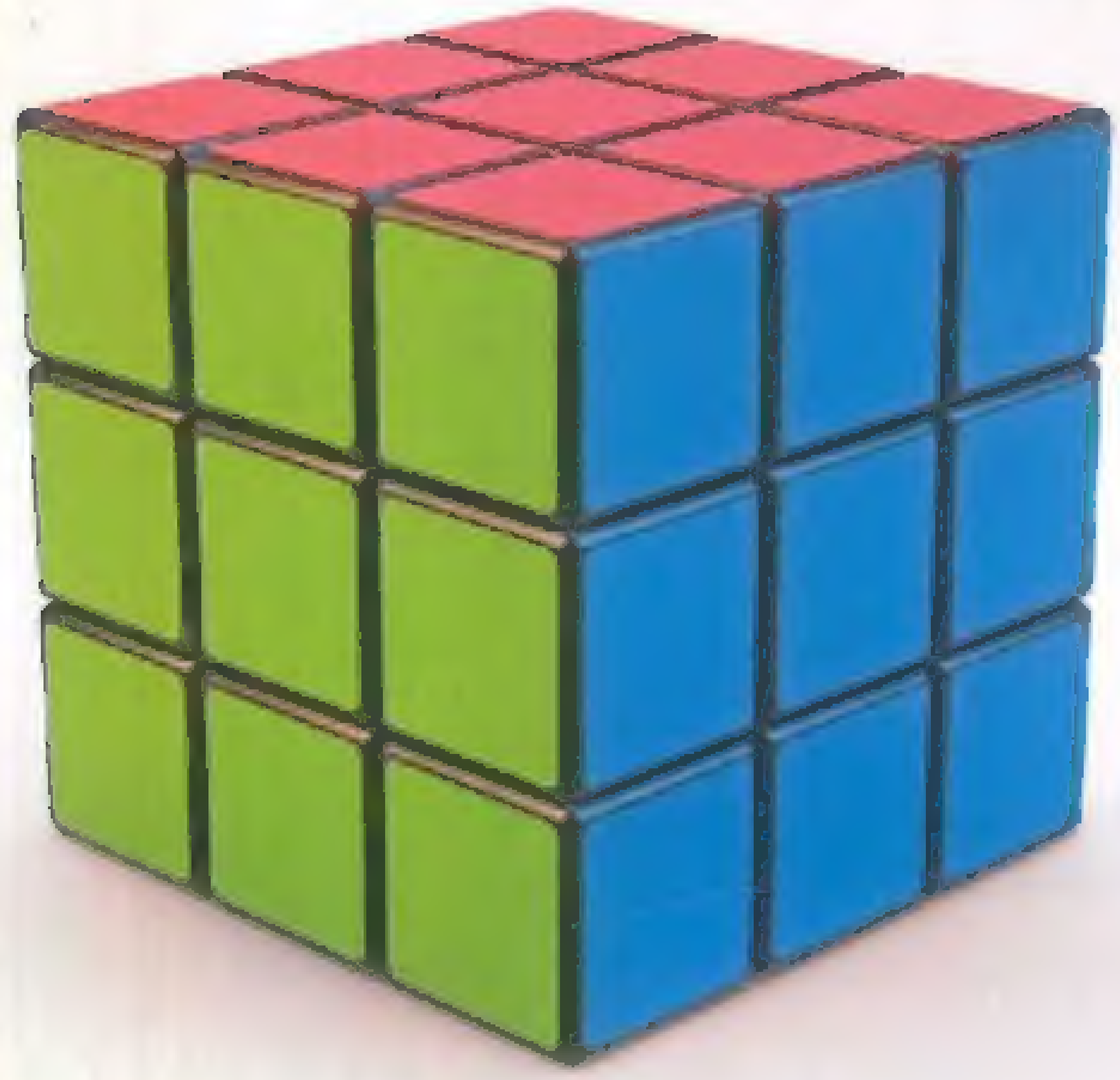
Unit Objectives : By the end of this unit, student should be able to :

- recognize the cube root of a rational number.
- find the cube root of a rational number.
- recognize the set of irrational numbers.
- represent the irrational number on the number line.
- recognize the set of real numbers.
- perform the operations on the intervals.
- perform the arithmetic operations on the real numbers.
- perform the operations on the square roots and the cube roots.
- recognize two conjugate numbers.
- apply what he studied in the real numbers to find the volumes and the areas of some of the solids.
- solve equations and inequalities of the first degree in one variable in $\mathbb{R}$



Lesson 1 The cube root of a rational number

Remember the square root of the perfect square rational number

The square root of the perfect square rational number (a) is the rational number whose square equals (a)

- The symbol $\sqrt{\ }$ means the positive square root of a number.

For example:

25 has two square roots which are 5 and – 5

Because : $(5)^2 = 25$, $(-5)^2 = 25$

and we write $\sqrt{25} = 5$, $-\sqrt{25} = -5$, $\pm\sqrt{25} = \pm 5$

Notice that :
The two square roots of the rational number , each of them is the additive inverse of the other and their sum = zero.

- $\sqrt{16} = 4$, $-\sqrt{16} = -4$, $\pm\sqrt{16} = \pm 4$
- $\sqrt{0} = 0$
- $\sqrt{\text{negative number}}$ is meaningless.
- $\sqrt{a^2} = |a|$

For example:

$\sqrt{3^2} = |3| = 3$, $\sqrt{(-6)^2} = |-6| = 6$

- Sometimes , you need to factorize a number to its prime factors to facilitate finding its square root , then you take a factor from each two equal factors , then the product of these taken factors is the square root of this number.

For example:

$\because 441 = \underbrace{3 \times 3}_{3} \times \underbrace{7 \times 7}_{7}$

$\therefore \sqrt{441} = 3 \times 7$

$= 21$

441	3
147	3
49	7
7	7
1	

You can use your calculator to check your answer.

The cube root of a rational number

- The product of a number by itself three times is the cube of that number.

  For example: 64 is the cube of 4 because $4 \times 4 \times 4 = 64$

- The reverse of finding the cube is finding the cube root.

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- Finding the cube root of a number is finding another number if multiplied by itself three times , we get the first number.

  For example: 4 is the cube root of 64 because $64 = 4 \times 4 \times 4$

Definition

The cube root of the number "a" is the number whose cube equals a

- The symbol $\sqrt[3]{\ }$ (read as "the cube root of ") is used to designate the cube root.

  For example: $\sqrt[3]{64}$ designates the cube root of 64

- The cube root of a positive number is positive and the cube root of a negative number is negative.

  For example: $\sqrt[3]{64} = 4$ and $\sqrt[3]{-64} = -4$

  i.e. The cube root of any number has the same sign of this number.

Finding the cube root of a rational number (representing a perfect cube)

- The perfect cube rational number is the number which can be written as a cube of a rational number i.e. (rational number)^3 as the numbers : $8 = 2^3$, $-27 = (-3)^3$
- The cube root of a perfect cube rational number is also a rational number.

  For example: $\sqrt[3]{8} = 2$, $\sqrt[3]{-27} = -3$

- If a number is not a perfect cube , then you indicate its cube root by using the cube root symbol.

  For example: The cube root of 4 is $\sqrt[3]{4}$ because 4 is not a perfect cube.

- $\sqrt[3]{a^3} = a$ — For example: $\sqrt[3]{5^3} = 5$, $\sqrt[3]{(-5)^3} = -5$
- $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$ — For example: $\sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2$
- You can use factorization to find the cube root of a perfect cube number , as in the following example.

Example 1 Find each of the following :

1. $\sqrt[3]{216}$ 2. $\sqrt[3]{\frac{-8}{125}}$ 3. $\sqrt[3]{0.064}$

Solution

1. $\sqrt[3]{216} = 2 \times 3 = 6$

$$\begin{array}{r|l} 216 & 2 \\ 108 & 2 \\ 54 & 2 \\ 27 & 3 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

2. $\sqrt[3]{-\frac{8}{125}} = -\frac{2}{5}$

$$\begin{array}{r|l} 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \qquad \begin{array}{r|l} 125 & 5 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array}$$

3. $\sqrt[3]{0.064} = \sqrt[3]{\frac{64}{1000}} = \frac{2 \times 2}{2 \times 5}$

$= \frac{4}{10} = 0.4$

$$\begin{array}{r|l} 64 & 2 \\ 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1000 & 2 \\ 500 & 2 \\ 250 & 2 \\ 125 & 5 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array}$$

TRY by yourself 1

Complete the following :

1. $\sqrt[3]{27} =$ .... 2. $\sqrt[3]{-125} =$ ....

3. $\sqrt[3]{512} =$ .... 4. $\sqrt[3]{\frac{27}{8}} =$ ....

Example 2 Choose the correct answer from those given :

1. $\sqrt[3]{\text{.........}} = -5$
   (a) 125 (b) 25 (c) – 25 (d) – 125

2. $\sqrt{4} - \sqrt[3]{-8} =$ ....
   (a) – 4 (b) – 2 (c) 4 (d) 8

3. $\sqrt{(-7)^2} - \sqrt[3]{(-7)^3} =$ ....
   (a) – 14 (b) zero (c) 7 (d) 14

4. If $\sqrt[3]{x} = \sqrt{4}$, then $x =$ ....
   (a) 2 (b) 4 (c) 8 (d) ± 8

5. $\sqrt{x^4} = \sqrt[3]{\text{.........}}$
   (a) x (b) x^2 (c) x^4 (d) x^6

Solution

1 (d) The reason : $(-5)^3 = -125$

2 (c) The reason : $\sqrt{4} - \sqrt[3]{-8} = 2 - (-2) = 2 + 2 = 4$

3 (d) The reason : $\sqrt{(-7)^2} \quad \sqrt[3]{(-7)^3} = 7 - (-7) = 7 + 7 = 14$

4 (c) The reason : $\because \sqrt[3]{x} = \sqrt{4}$ $\quad \therefore \sqrt[3]{x} = 2$ $\quad \therefore x = 2^3 = 8$

5 (d) The reason : $\because \sqrt{x^4} = x^2$ $\quad \therefore (x^2)^3 = x^6$ $\quad \therefore \sqrt{x^4} = \sqrt[3]{x^6}$

TRY by yourself 2

Complete the following :

1 $\sqrt{16} + \sqrt[3]{-64} = \dots\dots\dots$

2 $\sqrt{64} - \sqrt[3]{64} = \dots\dots\dots$

3 $\sqrt[3]{27} = \sqrt{\dots\dots\dots}$

4 If $\sqrt{9} = \sqrt[3]{x}$, then $x = \dots\dots\dots$

Solving equations in $\mathbb{Q}$ using the cube root

- If "a" is a perfect cube number , then the equation : $x^3 = a$ has a unique solution in $\mathbb{Q}$, which is $\sqrt[3]{a}$

For example :

- The equation : $x^3 = 8$ has a unique solution in $\mathbb{Q}$ which is $\sqrt[3]{8} = 2$
- The equation : $x^3 = 9$ has no solution in $\mathbb{Q}$ because 9 is not a perfect cube.

Example 3 **Solve each of the following equations in $\mathbb{Q}$:**

1 $40x^3 - 1 = -136$

2 $(y-2)^3 = -343$

Solution

1 $\because 40x^3 - 1 = -136$ $\quad \therefore 40x^3 = -136 + 1$

$\therefore 40x^3 = -135$ $\quad \therefore x^3 = -\frac{135}{40}$

$\therefore x^3 = -\frac{27}{8}$ $\quad \therefore x = \sqrt[3]{-\frac{27}{8}}$ $\quad \therefore x = -\frac{3}{2}$

2 $\because (y-2)^3 = -343$

Taking the cube root of each side : $\therefore \sqrt[3]{(y-2)^3} = \sqrt[3]{-343}$

$\therefore y - 2 = -7$ $\quad \therefore y = -7 + 2$ $\quad \therefore y = -5$

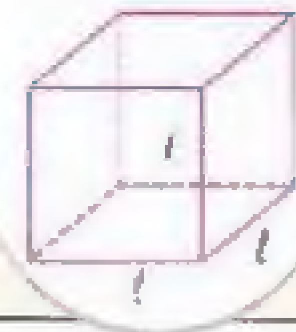
TRY by yourself 3

Find in $\mathbb{Q}$ the S.S. of each of the following equations :

1 $27x^3 - 2 = 62$

2 $(5x-3)^3 - 2 = 6$

Applications on the cube root of a rational number



Remember	For example
If the edge length of a cube is ℓ cm. , then :	If the volume of a cube is 8 cm^3 , then :
• The volume of the cube = ℓ^3 cm^3	• The edge length = $\sqrt[3]{8} = 2$ cm.
• The area of one face = ℓ^2 cm^2	• The area of one face = $2^2 = 4$ cm^2
• The lateral area= $4\,\ell^2$ cm^2	• The lateral area = $4 \times 2^2 = 16$ cm^2
• The total area = $6\,\ell^2$ cm^2	• The total area = $6 \times 2^2 = 24$ cm^2

Example 4

1 Find the length of the inner edge of a vessel in the shape of a cube if its capacity = 8 litres.

2 Find the radius length of a sphere of volume $\frac{36}{125}\pi$ cm^3

(Knowing that : The volume of the sphere = $\frac{4}{3}\pi r^3$).

where r is the radius length of the sphere , π is the ratio between the circumference of the circle and its diameter length.

3 Find the diameter length of a sphere of volume equals 38808 cm^3 $(\pi \approx \frac{22}{7})$

Solution

1 $\because$ The capacity of the vessel = 8 litres = $8 \times 1000 = 8000$ cm^3

$\therefore$ The inner edge length = $\sqrt[3]{8000}$

= 20 cm.

Remember that

1 litre = 1000 cm^3

2 $\because$ The volume of the sphere = $\frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi r^3 = \frac{36}{125}\pi$ $\quad\quad$ $\therefore \frac{4}{3} r^3 = \frac{36}{125}$

$\therefore r^3 = \frac{36}{125} \times \frac{3}{4}$ $\quad\quad$ $\therefore r^3 = \frac{27}{125}$

$\therefore r = \sqrt[3]{\frac{27}{125}} = \frac{3}{5}$ cm. $\quad\quad$ $\therefore$ The radius length of the sphere = $\frac{3}{5}$ cm.

3 $\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi r^3 = 38808$ $\qquad\therefore \frac{4}{3} \times \frac{22}{7} r^3 = 38808$

$\therefore \frac{88}{21} r^3 = 38808$ $\qquad\therefore r^3 = 38808 \times \frac{21}{88}$

$\therefore r^3 = 9261$ $\qquad\therefore r = \sqrt[3]{9261}$

$$\begin{array}{r|l} 9261 & 3 \\ 3087 & 3 \\ 1029 & 3 \\ 343 & 7 \\ 49 & 7 \\ 7 & 7 \\ 1 & \end{array} \quad \begin{array}{l} 3 \\ \\ 7 \end{array}$$

$\therefore r = 3 \times 7 = 21$ cm.

$\therefore$ The diameter length $= 21 \times 2 = 42$ cm.

Notice that : You can use the calculator to find $\sqrt[3]{9261}$ directly.

TRY by yourself 4

1. Find the length of the inner edge of a vessel in the shape of a cube with capacity equals 27 litres.
2. Find the length of the diameter of a sphere of volume 36π cm^3.
   (Knowing that : the volume of the sphere $= \frac{4}{3}\pi r^3$)

Lesson 2 The set of irrational Numbers $\mathbb{Q}'$

The sets of numbers

You had studied before the following sets of numbers :

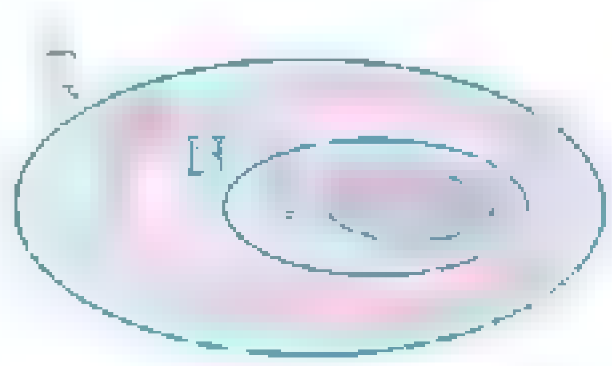
- The set of counting numbers : $\mathbb{C} = \{1, 2, 3, 4, ...\}$
- The set of natural numbers : $\mathbb{N} = \{0, 1, 2, 3, ...\} = \mathbb{C} \cup \{0\}$
- The set of integers : $\mathbb{Z} = \{..., 3, 2, 1, 0, -1, -2, -3, ...\}$
- The set of positive integers : $\mathbb{Z}_+ = \{1, 2, 3, ...\} = \mathbb{C}$
- The set of negative integers : $\mathbb{Z}_- = \{-1, -2, -3, ...\}$
- The set of rational numbers : $\mathbb{Q} = \{\frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0\}$

Examples of rational numbers : $\frac{2}{3}$, $-\frac{1}{2}$, zero , 3 , -5 , 0.2 , 25% , ...

Notice that :

$\mathbb{C} \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$

The opposite figure shows that.



Irrational numbers

The square root of a rational number which is not a perfect square is not a rational number	The cube root of a rational number which is not a perfect cube is not a rational number
For example: $\sqrt{2} \notin \mathbb{Q}$ because there is no rational number whose square is 2 , so $\sqrt{2}$ cannot be written as $\frac{a}{b}$ where a and b are integers , $b \neq 0$	For example: $\sqrt[3]{4} \notin \mathbb{Q}$ because there is no rational number whose cube is 4 , so $\sqrt[3]{4}$ cannot be written as $\frac{a}{b}$ where a and b are integers , $b \neq 0$
π is not a rational number (However $\frac{22}{7}$, 3.14 and 3.142 are rational numbers , each of them represents an approximating value of π)	Other examples of numbers not rational: $\sqrt{5}+1$, $1-\sqrt[3]{7}$, $2\sqrt{7}$, $-\frac{\sqrt[3]{9}}{5}$

The set of irrational numbers is denoted by $\mathbb{Q}'$

Notice that : $\mathbb{Q}$ and $\mathbb{Q}'$ are disjoint sets. i.e. $\mathbb{Q} \cap \mathbb{Q}' = \varnothing$

! Remarks

- $(\sqrt{a})^2 = \sqrt{a} \times \sqrt{a} = a$, where $a \geq 0$ For example: $(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = 2$
- $(\sqrt[3]{a})^3 = \sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$, where $a \in \mathbb{Q}$ For example: $(\sqrt[3]{-7})^3 = \sqrt[3]{-7} \times \sqrt[3]{-7} \times \sqrt[3]{-7} = -7$

Example Show which of the following numbers belongs to $\mathbb{Q}$ and which of them belongs to $\mathbb{Q}'$:

1. $\sqrt{0.49}$
2. $\sqrt[3]{-0.064}$
3. $\sqrt{\frac{25}{49}}$
4. $\sqrt[3]{\frac{25}{49}}$
5. $\sqrt{25}+\sqrt[3]{16}$

Solution

1. $\because \sqrt{0.49} = 0.7 = \frac{7}{10}$ $\therefore \sqrt{0.49} \in \mathbb{Q}$
2. $\because \sqrt[3]{-0.064} = -0.4 = -\frac{4}{10}$ $\therefore \sqrt[3]{-0.064} \in \mathbb{Q}$
3. $\because \sqrt{\frac{25}{49}} = \sqrt{\left(\frac{5}{7}\right)^2} = \frac{5}{7}$ $\therefore \sqrt{\frac{25}{49}} \in \mathbb{Q}$

- $\because \sqrt[3]{\frac{25}{49}} \notin \mathbb{Q}$ because there is no rational number whose cube is $\frac{25}{49}$

  $\therefore \sqrt[3]{\frac{25}{49}} \in \mathbb{Q}'$

- $\because \sqrt{25} + \sqrt[3]{16} = 5 + \sqrt[3]{16}$ $\because$ There is no rational number whose cube is 16

  $\therefore \sqrt[3]{16} \notin \mathbb{Q}$ $\quad \therefore (5 + \sqrt[3]{16}) \notin \mathbb{Q}$ $\quad \therefore (\sqrt{25} + \sqrt[3]{16}) \in \mathbb{Q}'$

Try 1

Complete using one of the symbols $\mathbb{Q}$ or $\mathbb{Q}'$:

1 $3 \in$	2 $\sqrt{3} \in$	3 $\sqrt{9} \in$
4 $\sqrt[3]{-8} \in$	5 $\sqrt[3]{5} \in$	6 $\sqrt[3]{-9} \in$

Solving equations in $\mathbb{Q}'$

Example 2 **If $x \in \mathbb{Q}'$, find the S.S. of each of the following equations :**

1 $x^2 - 2 = 3$	2 $\frac{1}{2} x^3 = 2$
3 $64 x^3 - 2 = -29$	4 $(x^2 - 10)(x^3 - 4) = 0$

Solution

1 $\because x^2 - 2 = 3$ $\quad \therefore x^2 = 3 + 2$

$\therefore x^2 = 5$ $\quad \therefore x = \pm\sqrt{5}$

$\therefore$ The S.S. $= \{\sqrt{5} , -\sqrt{5}\}$

2 $\because \frac{1}{2} x^3 = 2$ $\quad \therefore x^3 = 2 \times 2$

$\therefore x^3 = 4$ $\quad \therefore x = \sqrt[3]{4}$

$\therefore$ The S.S. $= \{\sqrt[3]{4}\}$

Notice that : –
We used the concept of the square root to find the value of x according to the following remark :
If $x^2 = a$, then $x = \pm\sqrt{a}$

3 $\because 64 x^3 - 2 = -29$ $\quad \therefore 64 x^3 = -29 + 2$ $\quad \therefore 64 x^3 = -27$

$\therefore x^3 = -\frac{27}{64}$ $\quad \therefore x = \sqrt[3]{-\frac{27}{64}}$ $\quad \therefore x = -\frac{3}{4}$

$\because -\frac{3}{4} \in \mathbb{Q}$ $\quad \therefore -\frac{3}{4} \notin \mathbb{Q}'$ $\quad \therefore$ The S.S. $= \varnothing$

4 $\because (x^2 - 10)(x^3 - 4) = 0$

$\therefore x^2 - 10 = 0$ or $x^3 - 4 = 0$

$\therefore x^2 = 10$ $\quad \therefore x^3 = 4$

$\therefore x = \pm\sqrt{10}$ $\quad \therefore x = \sqrt[3]{4}$

$\therefore$ The S.S. $= \{\sqrt{10} , -\sqrt{10} , \sqrt[3]{4}\}$

Remember that
For any two numbers x , y :
If x y = zero , then
x = zero or y = zero

TRY 2 *Find the S.S. in $\bar{\mathbb{Q}}$ for each of the following :*

(1) $2x^3 - 7 = 3$ (2) $\frac{1}{2}x^2 - 5 = 3$

[illegible]

If you use the calculator to find the values of some irrational numbers , you will find that .

$\sqrt{2} \simeq 1.4142 \ldots$, $\sqrt{3} \simeq 1.73205 \ldots$, $\sqrt{5} \simeq 2.236 \ldots$

i.e. The irrational number is represented by an infinite decimal and not recurring

And you can deduce an approximated value of the irrational number without using the calculator.

For example:

You can deduce an approximated value of the irrational number $\sqrt{5}$ as follows :

$\because 4 < 5 < 9$ (notice that we chose 4 and 9 because each of them is a perfect square , and the number 5 includes between them) and by taking the square root for all the terms.

$\therefore \sqrt{4} < \sqrt{5} < \sqrt{9}$ $\therefore 2 < \sqrt{5} < 3$

i.e. $\sqrt{5} = 2 +$ decimal less than 1

To find an approximated value of the number $\sqrt{5}$, you search for the values of the following numbers : $(2.1)^2$, $(2.2)^2$ and $(2.3)^2$

, then you find that $(2.1)^2 = 4.41$, $(2.2)^2 = 4.84$, $(2.3)^2 = 5.29$

$\because 4.84 < 5 < 5.29$ $\therefore \sqrt{4.84} < \sqrt{5} < \sqrt{5.29}$ $\therefore 2.2 < \sqrt{5} < 2.3$

We can say that 2.2 and 2.3 are approximated values of $\sqrt{5}$ and thus we can get more accurate values for the irrational number $\sqrt{5}$ and we can use the calculator to check the approximated value of the number $\sqrt{5}$

Remark

Each irrational number lies between two rational numbers.

Example 3 **Prove that :**

1. $\sqrt{3}$ lies between 1.7 and 1.8
2. $\sqrt[3]{12}$ lies between 2.2 and 2.3

Solution

1. $\because (\sqrt{3})^2 = \sqrt{3} \times \sqrt{3} = 3$, $(1.7)^2 = 2.89$, $(1.8)^2 = 3.24$

$\because 2.89 < 3 < 3.24$ $\quad \therefore \sqrt{2.89} < \sqrt{3} < \sqrt{3.24}$ $\quad \therefore 1.7 < \sqrt{3} < 1.8$

i.e. $\sqrt{3}$ lies between 1.7 and 1.8

You can solve the problem using the calculator as follows :

$\because \sqrt{3} \simeq 1.73$ $\quad$, $\because 1.7 < 1.73 < 1.8$

$\therefore 1.7 < \sqrt{3} < 1.8$ $\quad \therefore \sqrt{3}$ lies between 1.7 and 1.8

2. $\because (\sqrt[3]{12})^3 = \sqrt[3]{12} \times \sqrt[3]{12} \times \sqrt[3]{12} = 12$, $(2.2)^3 = 10.648$, $(2.3)^3 = 12.167$

, $\because 10.648 < 12 < 12.167$ $\quad \therefore \sqrt[3]{10.648} < \sqrt[3]{12} < \sqrt[3]{12.167}$

$\therefore 2.2 < \sqrt[3]{12} < 2.3$

i.e. $\sqrt[3]{12}$ lies between 2.2 and 2.3

You can solve the problem using the calculator as follows :

$\sqrt[3]{12} \simeq 2.289$ $\quad \because 2.2 < 2.289 < 2.3$

$\therefore 2.2 < \sqrt[3]{12} < 2.3$ $\quad \therefore \sqrt[3]{12}$ lies between 2.2 and 2.3

3

1. Find two consecutive integers such that $\sqrt{13}$ lies between them.
2. Prove that : $\sqrt{7}$ lies between 2.6 and 2.7

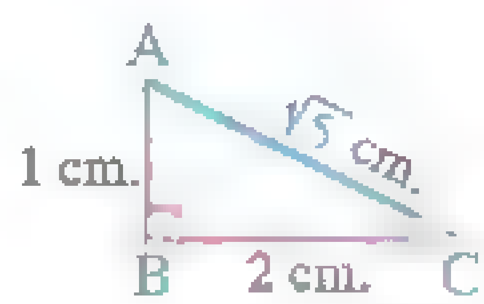
[illegible]

- If you draw the right-angled triangle ABC at B such that :

AB = 1 cm. , BC = 2 cm. , then according to Pythagoras' theorem you find :

$(AC)^2 = (AB)^2 + (BC)^2 = (1)^2 + (2)^2 = 1 + 4 = 5$

$\therefore AC = \sqrt{5}$ cm.



i.e. The length of $\overline{AC}$ represents the irrational number $\sqrt{5}$

- If you draw the number line where the distance between each two consecutive numbers is 1 cm. and you open the compasses with a distance equal to the length of $\overline{AC}$ and using O which represents zero as a centre and draw an arc cutting the number line at the point X

on the right of the point O , then the point X represents the number $\sqrt{5}$ on the number line.

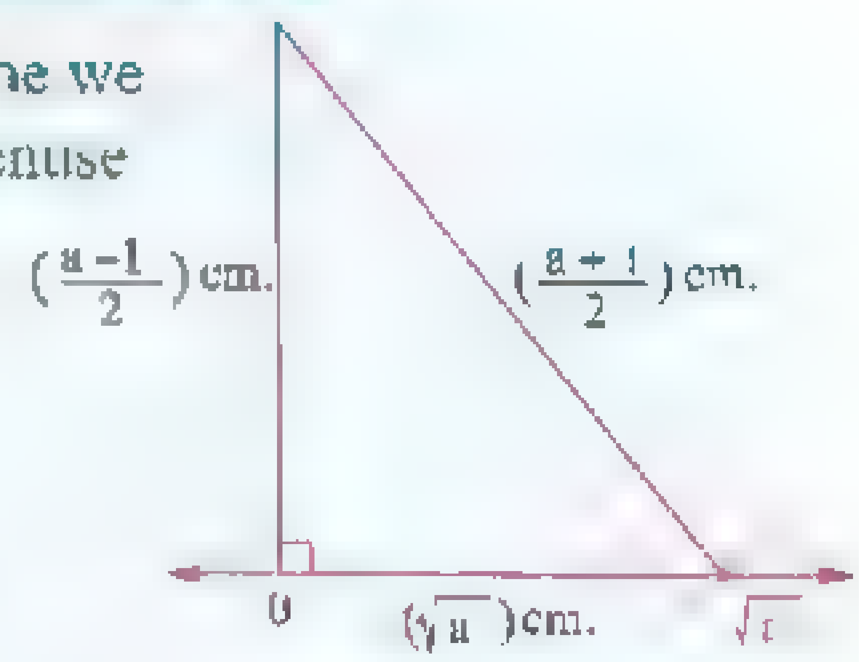
- And with the same length of $\overline{AC}$, if you use O as a centre and draw an arc cutting the number line at the point Y on the left side of O , then the point Y represents the number $-\sqrt{5}$ on the number line.

Generally

Each irrational number can be represented by a point on the number line.

How to represent an irrational number on the number line

To represent the number $\sqrt{a}$ (where $a > 1$) on the number line we find the lengths of the two sides which represent the hypotenuse and one side in a right-angled triangle.



Draw the triangle on the number line , where :

- $\frac{a+1}{2}$ represents the length of the hypotenuse.
- $\frac{a-1}{2}$ represents the length of the side of the right angle.

which is drawn perpendicular to the number line at zero so , the point of intersection of the hypotenuse with the number line is the point which represents the number $\sqrt{a}$

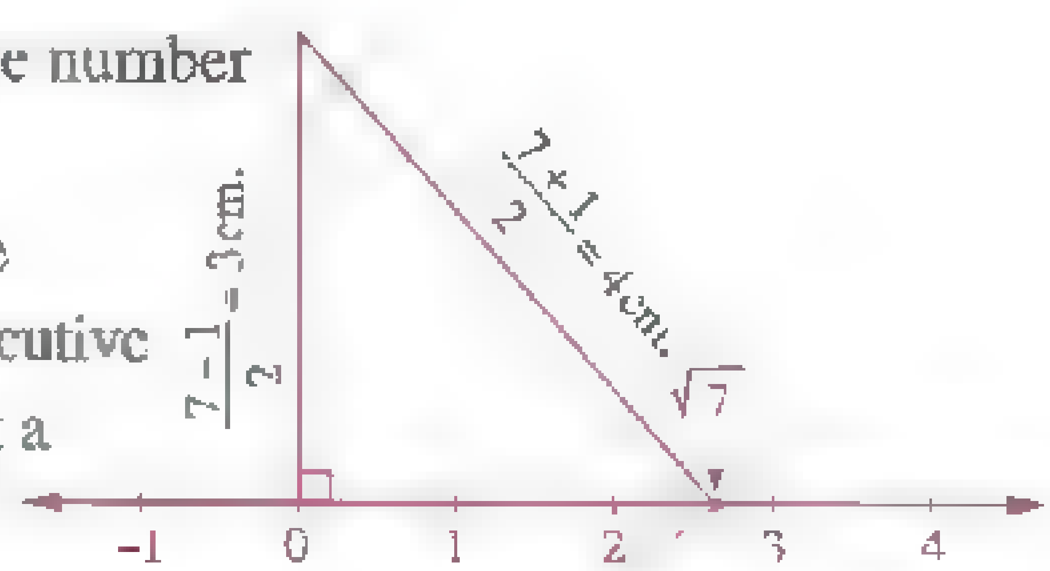
Example 1 **Represent the following numbers on the number line :**

1 $\sqrt{7}$	2 $-\sqrt{7}$	3 $1+\sqrt{7}$
4 $2 \quad \sqrt{7}$	5 $2\sqrt{7}$	

Solution

1 To represent the number $\sqrt{7}$ on the number line do as follows

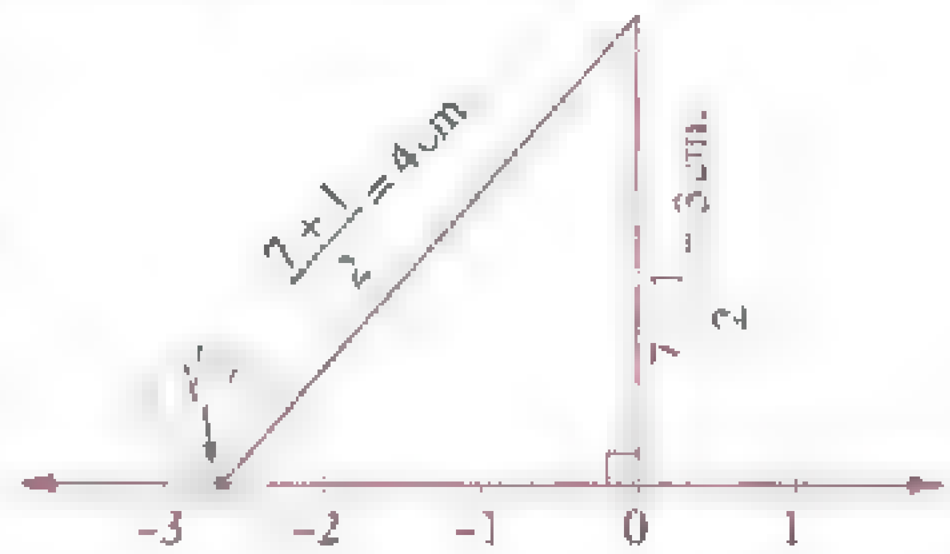
- Draw the number line where the distance between each two consecutive numbers is 1 cm. and draw on it a right-angled triangle in which.



- The length of the side of the right angle $= \frac{7-1}{2} = 3$ cm.
- The length of the hypotenuse $= \frac{7+1}{2} = 4$ cm.

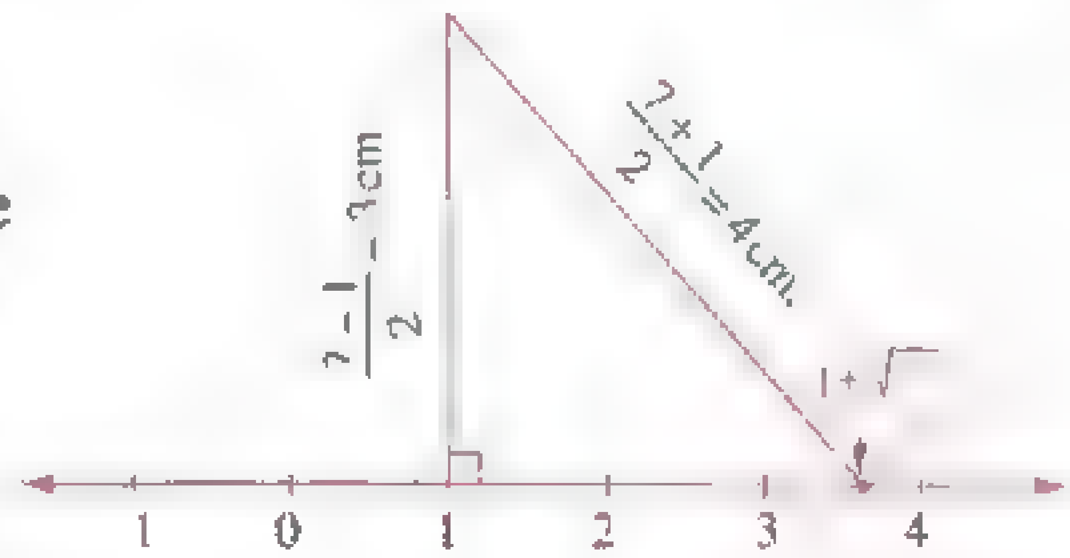
- From zero draw the side of the right angle of length 3 cm. perpendicular to the number line.
- Use the compasses and open it to 4 cm. , fix the tip on the top of the side of the right angle and draw an arc to cut the number line at a point on the right of zero (Because : $\sqrt{7}$ is a positive number).
- So the point of intersection of the arc with the number line is the point that represents the number $\sqrt{7}$

2 To represent the number $-\sqrt{7}$ on the number line we follow the same steps , but we draw the arc to the left (Because : $-\sqrt{7}$ is a negative number)



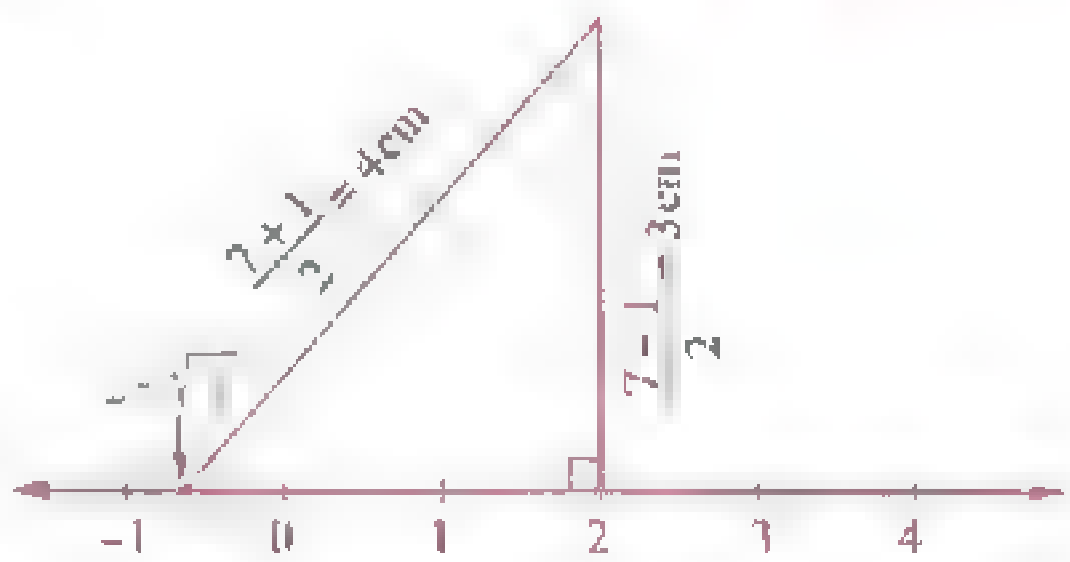
3 To represent the number $1+\sqrt{7}$ on the number line.

- We start at 1 on the number line and raw the side of the right angle of length 3 cm. perpendicular to the number line and follow the previous steps.

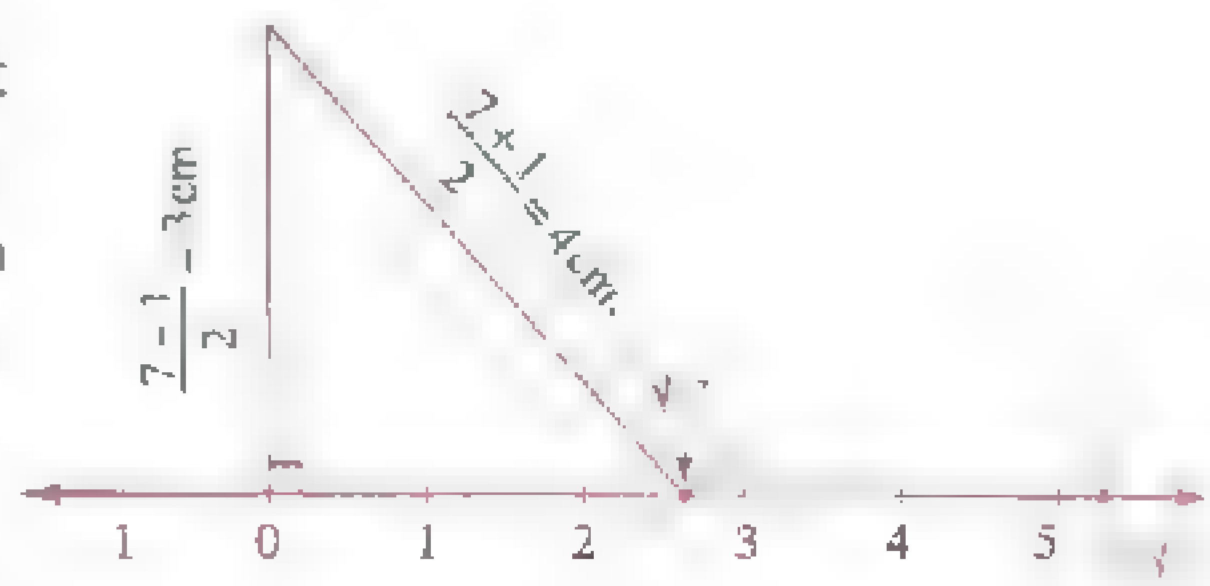


4 To represent the number $2-\sqrt{7}$ on the number line.

- We start at 2 on the number line and draw the side of the right angle of length 3 cm. perpendicular to the number line and follow the previous steps.



5 To represent the number $2\sqrt{7}$ on the number line we follow the previous steps to represent the number $\sqrt{7}$ and by using the compasses we open it by a distance equal to the distance between zero and $\sqrt{7}$, we fix the tip of the compasses at $\sqrt{7}$ and draw an arc cuts the number line at the point which represents the number $2\sqrt{7}$



TRY 4 Represent the following numbers on the number line :

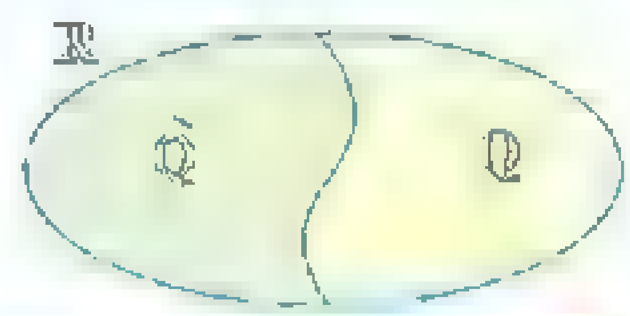
1 $\sqrt{10}$ 2 $-\sqrt{10}$ 3 $2+\sqrt{10}$ 4 $3-\sqrt{10}$

Lesson 3 The set of real numbers $\mathbb{R}$ and ordering numbers in $\mathbb{R}$

The set of real numbers

It is the set obtained from the union of the set of rational numbers and the set of irrational numbers. It is denoted by $\mathbb{R}$

i.e. $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$ (as shown in the opposite figure)

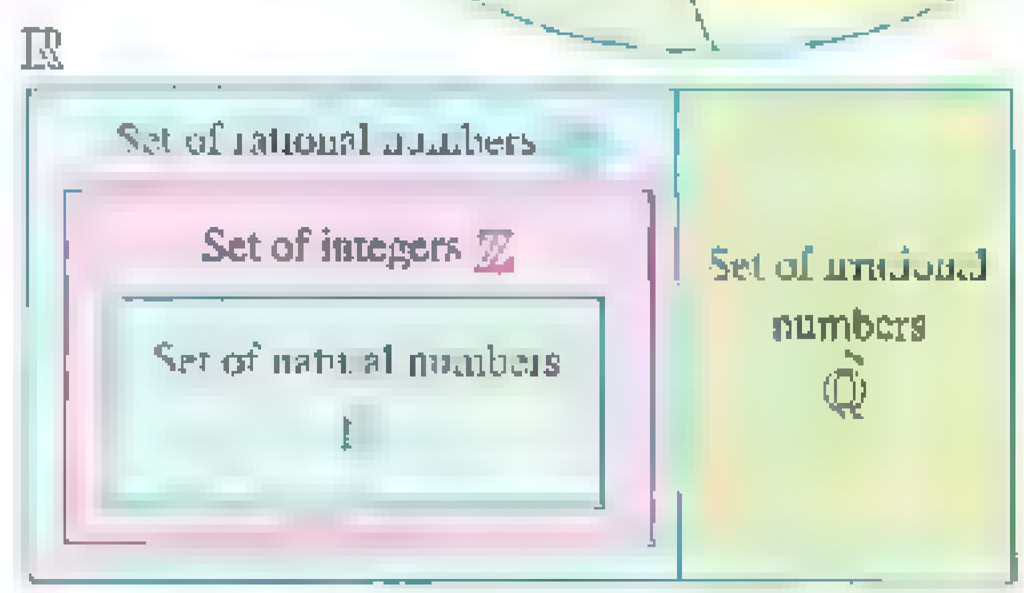


Noticing that : $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

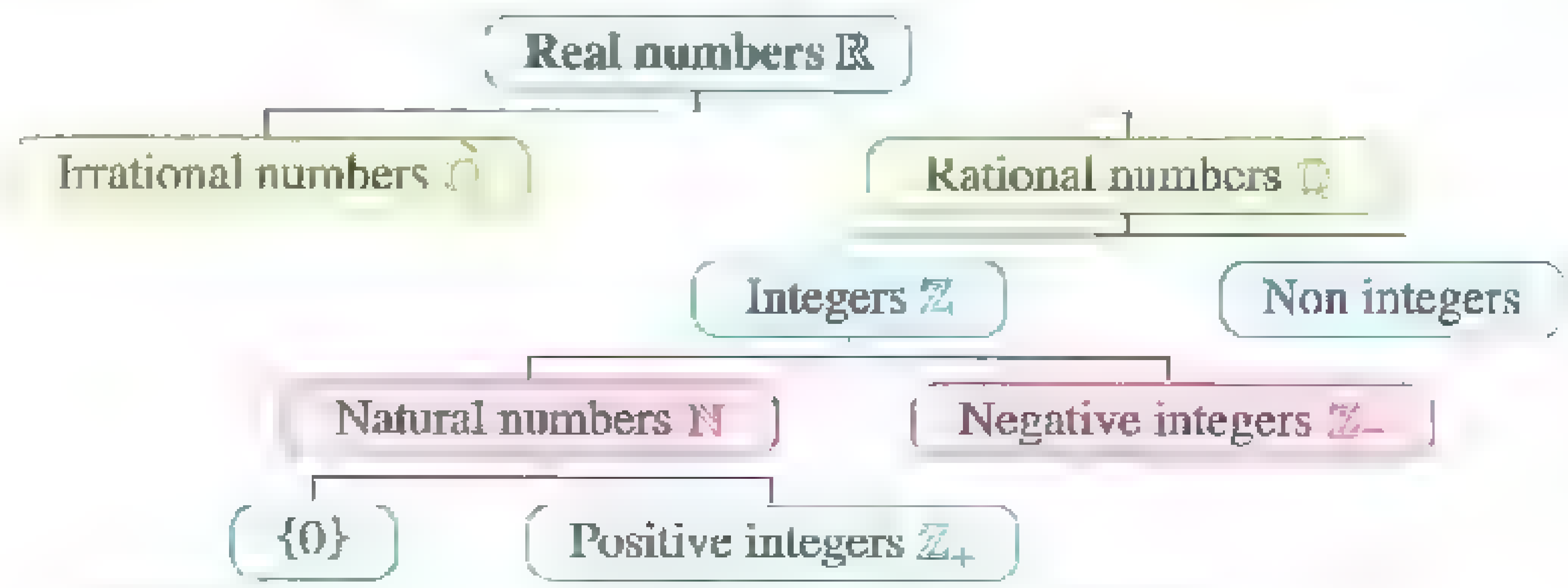
- **The opposite Venn diagram shows that :**

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

and $\grave{\mathbb{Q}} \subset \mathbb{R}$

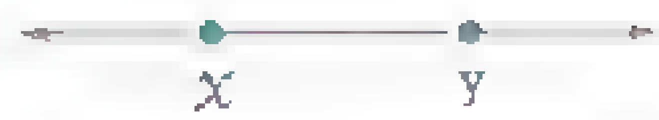


The following diagram shows the relation among the sets of numbers that we studied till now :



Ordering numbers in $\mathbb{R}$

- Each real number is represented by a unique point on the number line.
- The set of real numbers is an ordered set.
- If the point representing the number x on the number line lies on the left of the point representing the number y as shown in the figure , then $x < y$ or $y > x$

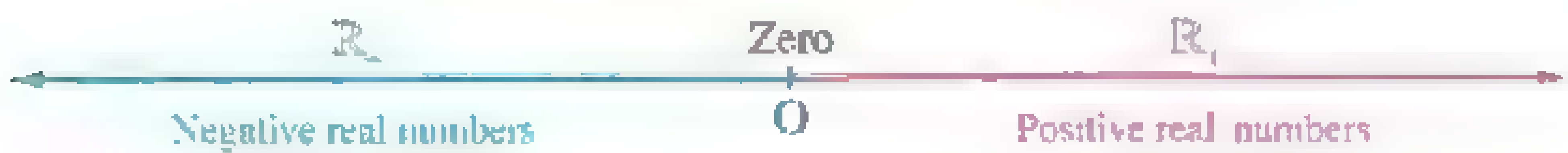


- Each real number represented by a point lying on the right side of the origin O is greater than zero , and all these numbers form a set called "the set of the positive real numbers" denoted by $\mathbb{R}_+$

$$\mathbb{R}_+ = \{x : x \in \mathbb{R} , x > \text{zero}\}$$

- Each real number represented by a point lying on the left side of the origin O is less than zero and all these numbers form a set called "the set of the negative real numbers" denoted by $\mathbb{R}_-$

$$\mathbb{R}_- = \{x : x \in \mathbb{R} , x < \text{zero}\}$$



Remarks

- $\mathbb{R}_+ \cap \mathbb{R}_- = \varnothing$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_-$
- The number zero is neither positive nor negative.
- $\mathbb{R}_+ \cup \{0\} = \{x : x \in \mathbb{R} , x \geq 0\}$
  and it is called the set of the non-negative real numbers.
- $\mathbb{R}_- \cup \{0\} = \{x : x \in \mathbb{R} , x \leq 0\}$
  and it is called the set of the non-positive real numbers.
- The set of real numbers without zero (The non zero real numbers) is denoted by $\mathbb{R}^*$
  i.e. $\mathbb{R}^* = \mathbb{R} - \{0\} = \mathbb{R}_+ \cup \mathbb{R}_-$

Example Arrange the following numbers ascendingly :

$\sqrt{75}$, $\sqrt{68}$, $\sqrt{45}$, 8 , 7 and $\sqrt{32}$

Solution

- Arrange the positive numbers which are $\sqrt{75}$, $\sqrt{68}$ and 7

  $\because 7 = \sqrt{49}$

  , $\because 49 < 68 < 75$ $\qquad\qquad$ $\therefore \sqrt{49} < \sqrt{68} < \sqrt{75}$

  i.e. $7 < \sqrt{68} < \sqrt{75}$

- Arrange the negative numbers which are $-\sqrt{45}$, -8 and $\sqrt{32}$

  $\because 8 = \sqrt{64}$

  , $\because 64 > 45 > 32$ $\qquad\qquad$ $\therefore \sqrt{64} > \sqrt{45} > \sqrt{32}$

  $\therefore -\sqrt{64} < -\sqrt{45} < -\sqrt{32}$

  i.e. $-8 < -\sqrt{45} < -\sqrt{32}$

  $\therefore$ The ascending order is : -8 , $-\sqrt{45}$, $-\sqrt{32}$, 7 , $\sqrt{68}$ and $\sqrt{75}$

! Remark

You can use the calculator to get the solution by finding approximated values of the roots.

TRY 1 *Complete each of the following using one of the symbols > or < :*

1 $\sqrt{2}$ 1	2 $-\sqrt{3}$ .... -1	3 $\sqrt[3]{9}$ 3
4 $-\sqrt[3]{7}$ .... -2	5 $\sqrt{7}$ 2.6	6 $-\sqrt[3]{16}$ .... -2.52

Example Write three irrational numbers included between 11 and 12

Solution

$\because (11)^2 = 121$, $(12)^2 = 144$

$\therefore$ 125 , 126 and 130 are three integers included between 121 and 144

$\therefore 121 < 125 < 126 < 130 < 144$

$\therefore \sqrt{121} < \sqrt{125} < \sqrt{126} < \sqrt{130} < \sqrt{144}$

$\therefore$ The required irrational numbers are : $\sqrt{125}$, $\sqrt{126}$ and $\sqrt{130}$

(Notice that : There are other irrational numbers included between 11 and 12)

TRY 2 *Write three irrational numbers included between 7 and 8*

Example Find the S.S. in $\mathbb{R}$ for each of the following equations :

1 $3x^2 + 125 = 221$ 2 $\frac{1}{6}x^3 - 8 = 28$ 3 $2x^2 + 6 = 4$

Solution

1 $\because 3x^2 + 125 = 221$ $\therefore 3x^2 = 221 - 125$ $\therefore 3x^2 = 96$

$\therefore x^2 = \frac{96}{3}$ $\therefore x^2 = 32$ $\therefore x = \pm\sqrt{32}$

$\therefore$ The S.S. $= \{\sqrt{32}, -\sqrt{32}\}$

2 $\because \frac{1}{6}x^3 - 8 = 28$ $\therefore \frac{1}{6}x^3 = 36$ $\therefore x^3 = 6 \times 36$

$\therefore x^3 = 216$ $\therefore x = \sqrt[3]{216}$ $\therefore x = 6$

$\therefore$ The S.S. $= \{6\}$

3 $\because 2x^2 + 6 = 4$ $\therefore 2x^2 = 4 - 6$ $\therefore 2x^2 = -2$

$\therefore x^2 = -\frac{2}{2}$ $\therefore x^2 = -1$ $\therefore x = \pm\sqrt{-1}$

, $\because \sqrt{-1} \notin \mathbb{R}, -\sqrt{-1} \notin \mathbb{R}$ $\therefore$ The S.S. $= \varnothing$

TRY 3 *Find in $\mathbb{R}$ the S.S. of the following two equations :*

1 $5x^3 + 3 = 28$

2 $\frac{3}{4}x^2 + 2 = -10$

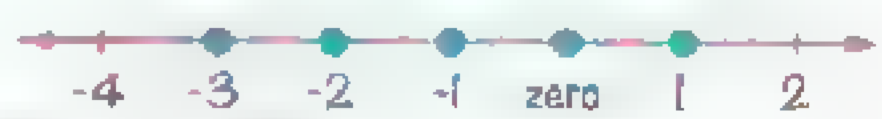
Lesson 4 Intervals

Prelude Through your previous study , you knew different methods to express a subset of the set of natural numbers and a subset of the set of integers and you learnt how to represent them on the number line.

For example:

If X = the set of integers which are greater than or equal to – 3 and less than 2

* Then you can express the set X by the description method as follows : ⟶ $X = \{a : a \in \mathbb{Z} , -3 \le a < 2\}$
* You can also express it by listing method as follows : ⟶ $X = \{-3 , -2 , -1 , 0 , 1\}$
* The set X is represented on the number line as shown in the figure : ⟶



• **And now the question is : Is it possible to use the same previous methods to express a subset of the set of real numbers and represent it on the number line ?**

Assuming that : K = the set of real numbers that are greater than or equal to – 3 and less than 2

* You can express the set K by the description method as follows : ⟶ $K = \{a : a \in \mathbb{R} , -3 \le a < 2\}$
* But it is impossible to express the set K by listing method because there are an infinity of real numbers existing between – 3 and 2

For the same reason , it is impossible to represent this set K by separate points on the number line as shown in the previous figure therefore we use another method to express a subset of the set of real numbers , which is the intervals.

• **In the following , we will show the types of intervals :**

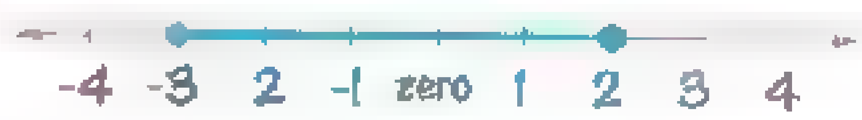
First Limited intervals

A Closed interval

Notice that : The smaller number must be written first when you write the interval.

- The set $\{x : x \in \mathbb{R} , -3 \le x \le 2\}$ expresses the set of real numbers which consists of the two numbers -3 and 2 and all the real numbers included between them.

  We denote it by $[-3 , 2]$ and it is called a «closed interval».
- It is represented on the number line as shown in the figure :



Notice that : $-3 \in [-3 , 2]$, $2 \in [-3 , 2]$

We express this by drawing two shaded circles at the two points representing the two numbers -3 and 2

B Opened interval

- The set $\{x : x \in \mathbb{R} , -3 < x < 2\}$ expresses the set of real numbers included between the two numbers -3 and 2 such that the two numbers -3 and 2 are not contained in this set.

  We denote this set by $]-3 , 2[$ and it is called an «opened interval».
- It is represented on the number line as in the figure :

Notice that : $-3 \notin]-3 , 2[$ and $2 \notin]-3 , 2[$

We express this by drawing two unshaded circles at the two points representing the two numbers -3 and 2

C Half opened interval (Half closed interval)

1. - The set $\{x : x \in \mathbb{R} , -3 \le x < 2\}$ expresses the number -3 and all the real numbers included between -3 and 2 without the number 2 , we denote it by $[-3 , 2[$ and it is called a «half opened interval» or «half closed interval».
   - It is represented on the number line as in the figure :

   Notice that : $-3 \in [-3 , 2[$, $2 \notin [-3 , 2[$

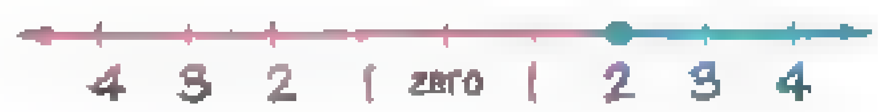
2. - The set $\{x : x \in \mathbb{R} , -3 < x \le 2\}$ expresses the number 2 and all the real numbers included between -3 and 2 without the number -3 , we denote it by $]-3 , 2]$ and it is called a «half opened interval» or «half closed interval».
   - It is represented on the number line as in the figure :

   Notice that : $-3 \notin]-3 , 2]$, $2 \in]-3 , 2]$

- The set $\{x : x \in \mathbb{R} , x \geq 2\}$ expresses the set of real numbers which consists of the number 2 and all the real numbers which are greater than 2 with no end.

  It is denoted by $[2 , \infty[$ where the symbol «∞» is read as positive infinity and it doesn't represent a real number

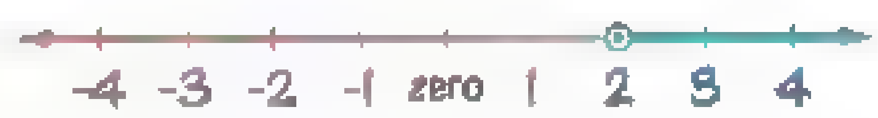
- It is represented on the number line as shown in the figure :



Notice that : $2 \in [2 , \infty[$

- The set $\{x : x \in \mathbb{R} , x > 2\}$ expresses the set of all real numbers which are greater than the number 2 with no end. It is denoted by $]2 , \infty[$

- It is represented on the number line as shown in the figure :



Notice that : $2 \notin]2 , \infty[$

- The set $\{x : x \in \mathbb{R} , x \leq 2\}$ expresses the set of real numbers which consists of the number 2 and all the real numbers which are smaller than the number 2 with no end.

  It is denoted by $]-\infty , 2]$ where the symbol «$-\infty$» is read as negative infinity and it doesn't represent a real number.

- It is represented on the number line as shown in the figure :

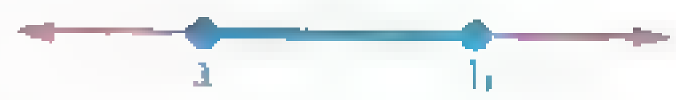
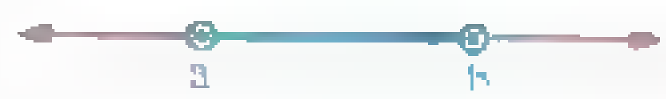
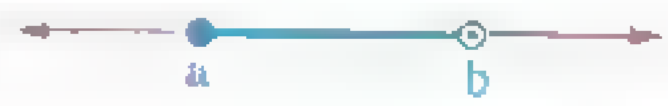
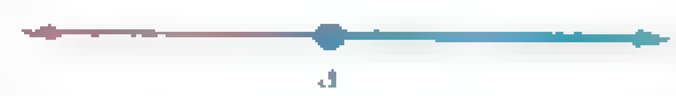
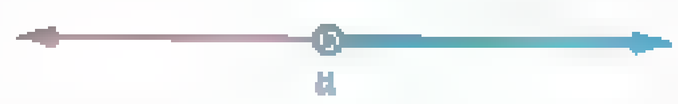
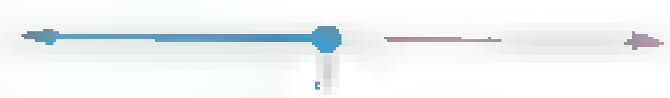
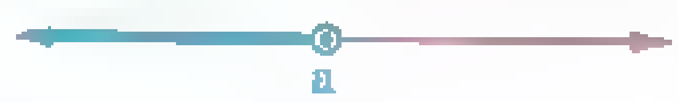
Notice that : $2 \in]-\infty , 2]$

- The set $\{x : x \in \mathbb{R} , x < 2\}$ expresses the set of all real numbers which are smaller than the number 2 with no end. It is denoted by $]-\infty , 2[$

- It is represented on the number line as shown in the figure :

Notice that : $2 \notin]-\infty , 2[$

- We can express the previous symbolically in the following table assuming that : $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $a < b$

Types of intervals		The interval	Expression by distinguished property	Representation on the number line	Notice that
The limited intervals	Closed	$[a , b]$	$\{x : x \in \mathbb{R} , a \le x \le b\}$		• $a \in [a , b]$ • $b \in [a , b]$
	Opened	$]a , b[$	$\{x : x \in \mathbb{R} , a < x < b\}$		• $a \notin]a , b[$ • $b \notin]a , b[$
	half opened (half closed)	$[a , b[$	$\{x : x \in \mathbb{R} , a \le x < b\}$		• $a \in [a , b[$ • $b \notin [a , b[$
		$]a , b]$	$\{x : x \in \mathbb{R} , a < x \le b\}$		• $a \notin]a , b]$ • $b \in]a , b]$
The unlimited intervals		$[a , \infty[$	$\{x : x \in \mathbb{R} , x \ge a\}$		$a \in [a , \infty[$
		$]a , \infty[$	$\{x : x \in \mathbb{R} , x > a\}$		$a \notin]a , \infty[$
		$]-\infty , a]$	$\{x : x \in \mathbb{R} , x \le a\}$		$a \in]-\infty , a]$
		$]-\infty , a[$	$\{x : x \in \mathbb{R} , x < a\}$		$a \notin]-\infty , a[$

! Remarks

- $\mathbb{R} =]-\infty , \infty[$ • $\mathbb{R}_+ =]0 , \infty[$ • $\mathbb{R}_- =]-\infty , 0[$
- The set of non-negative real numbers $= \mathbb{R}_+ \cup \{0\} = [0 , \infty[$
- The set of non-positive real numbers $= \mathbb{R}_- \cup \{0\} =]-\infty , 0]$

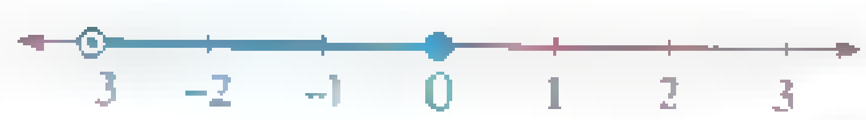
Example 1 Write each of the following sets in the form of an interval , then represent it on the number line :

1 $\{x : x \in \mathbb{R} , -3 < x \le 0\}$ 2 $\{a : a \in \mathbb{R} , 1 \ge a \ge -2\}$

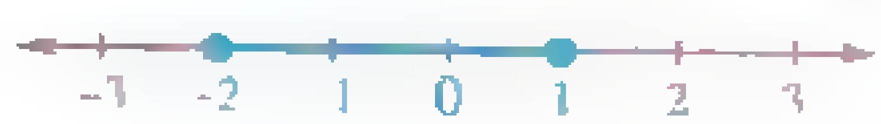
3 $\{x : x \in \mathbb{R} , x > 0\}$ 4 $\{y : y \in \mathbb{R} , -1 \ge y\}$

Solution

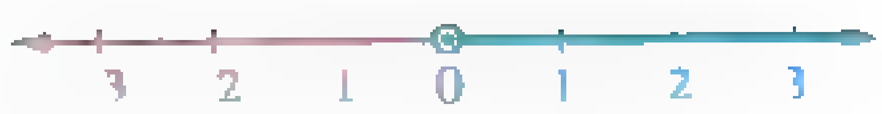
1 $]-3 , 0]$



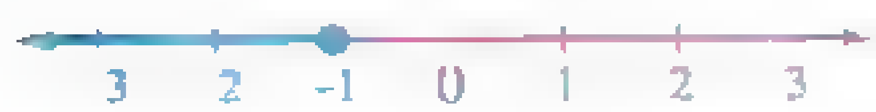
2 $[-2 , 1]$



3 $]0, \infty[$



4 $]-\infty, -1]$



Example 1 Choose the correct answer from those given :

1 $4 \in$ ....

ⓐ $]4, 7[$ ⓑ $[-4, 4[$ ⓒ $]2, 5[$ ⓓ $[-11, -4]$

2 $\sqrt[3]{-8}$ .... $[-8, -2[$

ⓐ $\in$ ⓑ $\notin$ ⓒ $\subset$ ⓓ $\not\subset$

3 $\{1, 6\}$ .... $]1, 6]$

ⓐ $\in$ ⓑ $\notin$ ⓒ $\subset$ ⓓ $\not\subset$

4 If $x \in [-5, \infty[$, then ....

ⓐ $x > -5$ ⓑ $x \geq -5$ ⓒ $x < -5$ ⓓ $x \leq -5$

5 The sum of the real numbers in the interval $[-3, 3[$ is ....

ⓐ -6 ⓑ -3 ⓒ zero ⓓ 6

Solution

1 (c)

2 (b) The reason : $\because \sqrt[3]{-8} = -2$, $[-8, -2[$ is open at -2

$\therefore -2 \notin [-8, -2[$

3 (d) The reason : $1 \notin]1, 6]$ because the interval is open at 1

4 (b)

5 (b) The reason : Each number belongs to the interval has its additive inverse except -3 because $3 \notin [-3, 3[$

TRY 1

Complete using one of the symbols $\in$, $\notin$, $\subset$ or $\not\subset$:

1 -1 .... $[-4, -1[$

2 $\frac{1}{2}$ .... $]0, 1[$

3 $\{2, 4\}$ .... $[2, 5[$

4 $\sqrt{7}$ .... $]2, 3]$

5 $\{-1, 0, 1\}$ .... $[0, 1]$

6 $|-5|$ .... $]-\infty, 0]$

Operations on intervals

You studied before the sets and how to carry out the operations of intersection , union , difference and complement on them.

For example:

If $X = \{1 , 2 , 3 , 4\}$, $Y = \{3 , 4 , 5 , 6\}$, then :

- $X \cap Y$ = the set of elements which are common in X and $Y = \{3 , 4\}$
- $X \cup Y$ = the set of all elements in X or Y without repeating $= \{1 , 2 , 3 , 4 , 5 , 6\}$
- $X - Y$ = the set of elements which are in X and not in $Y = \{1 , 2\}$
- $Y - X$ = the set of elements which are in Y and not in $X = \{5 , 6\}$
- If the universal set $U = \{1 , 2 , 3 , 4 , 5 , 6 , 7\}$,

  then the complement of X which is denoted by $\acute{X} = U - X$

  i.e $\acute{X}$ = the set of elements which are in U and not in $X = \{5 , 6 , 7\}$

The following examples show how to carry out the operations of intersection , union and difference on intervals :

Example 1 **If $X = [-3 , 3]$ and $Y = [-1 , 5[$, find using the number line :**

1 $X \cup Y$

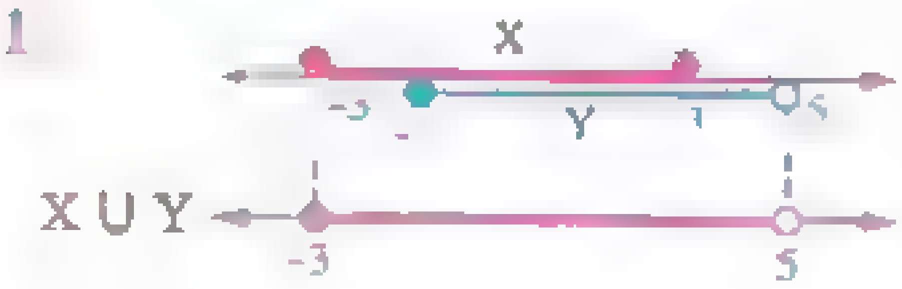
2 $X \cap Y$

3 $X - Y$

4 $Y - X$

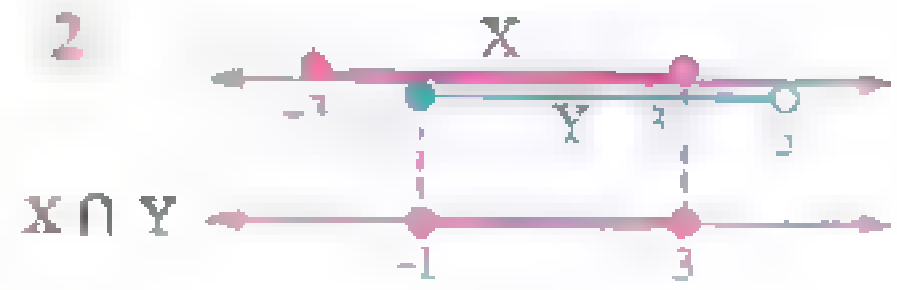
Solution

1



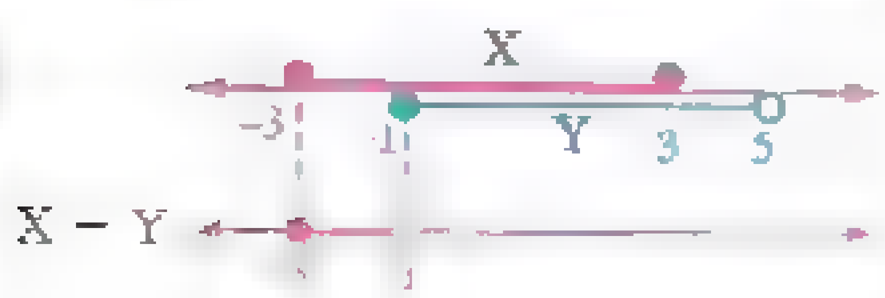
$X \cup Y = [-3 , 5[$

2



$X \cap Y = [-1 , 3]$

3



$X - Y = [-3 , -1[$

4

$Y - X =]3 , 5[$

Example Find each of the following :

1 $]-\infty , 2] \cap]-3 , \infty[$

2 $]-\infty , 3] \cup [-2 , 5[$

3 $[5 , \infty[-]5 , \infty[$

4 $[2 , \infty[\cap]-\infty , 2[$

Solution

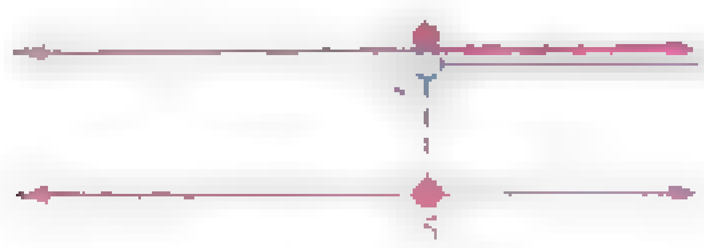
1

$]-\infty , 2] \cap]-3 , \infty[=]-3 , 2]$

2

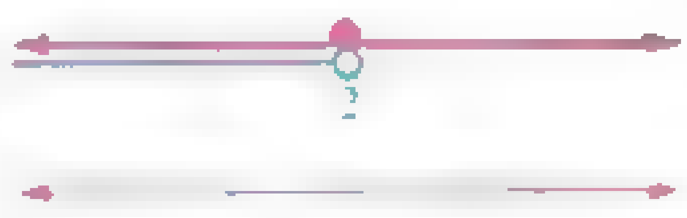
$]-\infty , 3] \cup [-2 , 5[=]-\infty , 5[$

3



$[5 , \infty[-]5 , \infty[= \{5\}$

4



$[2 , \infty[\cap]-\infty , 2[= \varnothing$

Example If $X =]-\infty , 2[$ and $Y = [-1 , 5]$, find using the number line :

1 $X \cup Y$

2 $X \cap Y$

3 $X - Y$

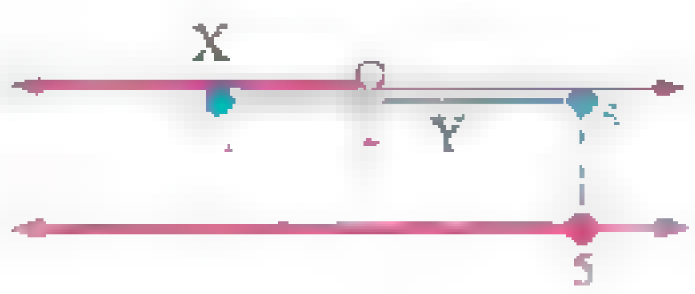
4 $Y - X$

5 $\grave{X}$

6 $\grave{Y}$

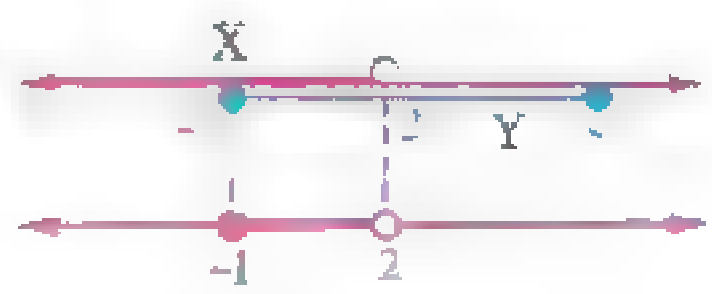
Solution

1



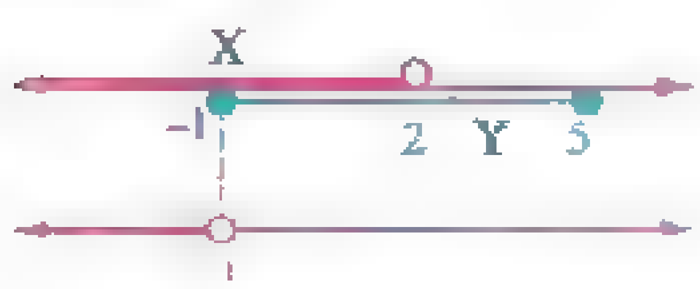
$X \cup Y =]-\infty , 5]$

2



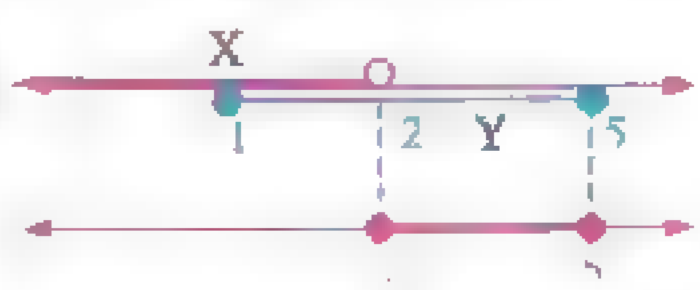
$X \cap Y = [-1 , 2[$

3



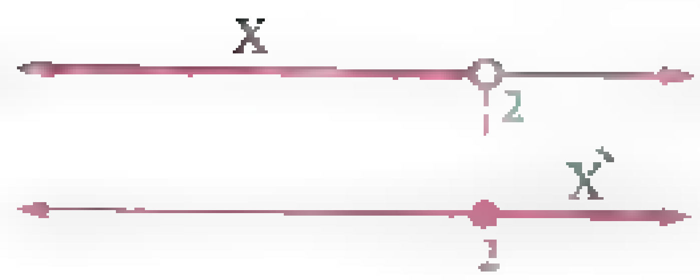
$X - Y =]-\infty , -1[$

4



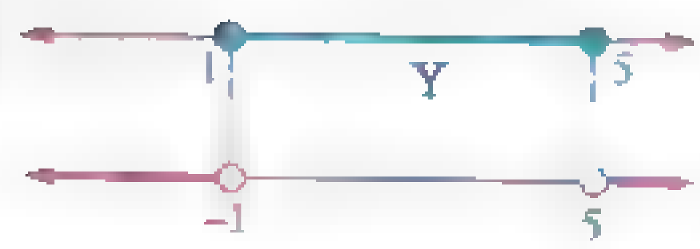
$Y - X = [2 , 5]$

5



$\grave{X} = [2 , \infty[$

6



$\grave{Y} =]-\infty , -1[\cup]5 , \infty[$
$= \mathbb{R} - [-1 , 5]$

Example 8 If $X = [1 , 4[$ and $Y = \{1 , 4\}$, find :

1 $X \cap Y$
2 $X \cup Y$
3 $X - Y$
4 $Y - X$

Solution

1 $X \cap Y = \{1\}$
2 $X \cup Y = [1 , 4]$
3 $X - Y =]1 , 4[$
4 $Y - X = \{4\}$

TRY 2 *If $X = [-1 , 3[$ and $Y =]0 , 4]$, find using the number line :*

1 $X \cap Y$
2 $X \cup Y$
3 $X - [0 , \infty[$
4 $]0 , \infty[- Y$
5 $\grave{X}$
6 $X \cap \{-2 , -1 , 0 , 1 , 2 , 3\}$

Lesson 5 Operations on the real numbers

First Addition

- We know that $2X$ and $3X$ are two like algebraic terms and their sum is an algebraic term like them.

  Where $2X + 3X = (2 + 3)X = 5X$

  Then we deduce that : $2\sqrt{5} + 3\sqrt{5} = (2 + 3)\sqrt{5}$

  $= 5\sqrt{5}$

Remember that

The real number $2\sqrt{5}$ is produced by multiplying the rational number 2 by the irrational number $\sqrt{5}$

- We know that $2X$ and $3y$ are two unlike algebraic terms and we express their sum by an algebraic expression whose simplest form is $2X + 3y$

  Therefore we deduce that :

  The two real numbers $2\sqrt{3}$ and $3\sqrt{2}$, their sum is expressed by a real number whose simplest form is $2\sqrt{3} + 3\sqrt{2}$

Properties of addition of real numbers

Closure

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ we find that $(a + b) \in \mathbb{R}$

i.e. The sum of any two real numbers is a real number, therefore we say $\mathbb{R}$ is closed under addition.

For example: $\sqrt{5} \in \mathbb{R}$ and $2\sqrt{5} \in \mathbb{R}$, we find that : $\sqrt{5} + 2\sqrt{5} = 3\sqrt{5} \in \mathbb{R}$

Commutative property

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a + b = b + a$

For example: $5\sqrt[3]{2} + 4\sqrt[3]{2} = 9\sqrt[3]{2}$, $4\sqrt[3]{2} + 5\sqrt[3]{2} = 9\sqrt[3]{2}$

i.e. $5\sqrt[3]{2} + 4\sqrt[3]{2} = 4\sqrt[3]{2} + 5\sqrt[3]{2}$

Associative property

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a + b) + c = a + (b + c) = a + b + c$

For example: $\left(\sqrt{3} + 2\sqrt{3}\right) + 5\sqrt{3} = 3\sqrt{3} + 5\sqrt{3} = 8\sqrt{3}$,

$\sqrt{3} + \left(2\sqrt{3} + 5\sqrt{3}\right) = \sqrt{3} + 7\sqrt{3} = 8\sqrt{3}$

i.e. $\left(\sqrt{3} + 2\sqrt{3}\right) + 5\sqrt{3} = \sqrt{3} + \left(2\sqrt{3} + 5\sqrt{3}\right)$

The additive neutral

For every $a \in \mathbb{R}$ it will be $a + 0 = 0 + a = a$

i.e. Zero is the additive neutral.

For example: $\sqrt{2} + 0 = 0 + \sqrt{2} = \sqrt{2}$, $-\sqrt[3]{5} + 0 = 0 + \left(-\sqrt[3]{5}\right) = -\sqrt[3]{5}$

The additive inverse of every real number

For every $a \in \mathbb{R}$ there is $(-a) \in \mathbb{R}$ where $a + (-a) =$ zero (the additive neutral)

For example:
- The additive inverse of $\sqrt{3}$ is $-\sqrt{3}$ and vice versa because $\sqrt{3} + \left(-\sqrt{3}\right) = 0$
- The additive inverse of $2 + \sqrt{5}$ is $-\left(2 + \sqrt{5}\right)$ and equals $-2 - \sqrt{5}$
- The additive inverse of $3 - \sqrt{2}$ is $-\left(3 - \sqrt{2}\right)$ and equals $\sqrt{2} - 3$
- The additive inverse of zero is itself.

! Remark

Since every real number has an additive inverse , then the subtraction operation is possible entirely in $\mathbb{R}$, and it is defined as follows :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a - b = a + (-b)$

i.e. The subtraction operation $(a - b)$ means adding the number a to the additive inverse of the number b

And we can deduce that :

Subtraction operation in $\mathbb{R}$ is not commutative and it is not associative.

Example

Choose the correct answer from those given :

1 $\sqrt{7}+\sqrt{7}=$ ....

(a) $\sqrt{14}$ (b) $2\sqrt{7}$ (c) 7 (d) 14

2 $2\sqrt{2}-3\sqrt{2}=$ ....

(a) 1 (b) $-\sqrt{2}$ (c) $\sqrt{2}$ (d) $5\sqrt{2}$

3 $4+\sqrt{3}-7-\sqrt{3}=$ ....

(a) $3-2\sqrt{3}$ (b) $-3+2\sqrt{3}$ (c) -3 (d) 3

4 If $X=9\sqrt{5}$, $y=5\sqrt{3}$, then $X-y=$

(a) $4\sqrt{2}$ (b) $4\sqrt{5}$ (c) $4\sqrt{3}$ (d) $9\sqrt{5}-5\sqrt{3}$

5 The additive inverse of $\sqrt{7}-\sqrt{5}$ is ....

(a) $-\sqrt{7}-\sqrt{5}$ (b) $\sqrt{7}+\sqrt{5}$ (c) $\sqrt{5}-\sqrt{7}$ (d) $\sqrt{7}-\sqrt{5}$

6 If $\sqrt{2}+X=0$, then $X-\sqrt{2}=$ ....

(a) zero (b) $-\sqrt{2}$ (c) $-2\sqrt{2}$ (d) $2\sqrt{2}$

Solution

1 (b)

2 (b) **The reason :** $2\sqrt{2}-3\sqrt{2}=(2-3)\sqrt{2}=-\sqrt{2}$

3 (c) **The reason :** $4+\sqrt{3}-7-\sqrt{3}=(4-7)+\left(\sqrt{3}-\sqrt{3}\right)=-3+0=-3$

4 (d) **The reason :** $X-y=9\sqrt{5}-5\sqrt{3}$ and this is the simplest form of the difference.

5 (c) **The reason :** The additive inverse of $\sqrt{7}-\sqrt{5}$ is $-\left(\sqrt{7}-\sqrt{5}\right)$

which is $-\sqrt{7}+\sqrt{5}$ or $\sqrt{5}-\sqrt{7}$

6 (c) **The reason :** X is the additive inverse of $\sqrt{2}$ which is $-\sqrt{2}$

$\therefore X-\sqrt{2}=-\sqrt{2}-\sqrt{2}=-2\sqrt{2}$

Try 1

1 *Write the additive inverse for each of the following numbers :*

$\sqrt{2}$, $-\sqrt[3]{5}$, $\sqrt{2}+\sqrt{7}$, $\sqrt[3]{5}-3$, $-\sqrt{6}-\sqrt[3]{7}$

2 *Simplify to the simplest form :*

1 $2+2\sqrt{7}-1-5\sqrt{7}$

2 $3\sqrt{5}+\sqrt{3}-3\sqrt{5}+5\sqrt{3}$

Second Multiplication

- We know that : $3 \times 2X = (3 \times 2)X = 6X$

  Therefore we find that : $3 \times 2\sqrt{3} = (3 \times 2)\sqrt{3} = 6\sqrt{3}$

- We know also $2X \times 5X = (2 \times 5)(X \times X) = 10X^2$

  Therefore we find that : $2\sqrt{3} \times 5\sqrt{3} = (2 \times 5) \times \left(\sqrt{3} \times \sqrt{3}\right) = 10\left(\sqrt{3}\right)^2 = 10 \times 3 = 30$

As example:
- $-2 \times 3\sqrt{5} = (-2 \times 3)\sqrt{5} = -6\sqrt{5}$
- $4\sqrt{2} \times \sqrt{2} = 4\left(\sqrt{2}\right)^2 = 4 \times 2 = 8$
- $-2\sqrt{7} \times 4\sqrt{7} = (-2 \times 4) \times \left(\sqrt{7}\right)^2 = -8 \times 7 = -56$

Properties of multiplication of real numbers

Closure

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b \in \mathbb{R}$

i.e. The product of any two real numbers is a real number therefore we say :

$\mathbb{R}$ is closed under multiplication.

For example: $\sqrt{3} \in \mathbb{R}$ and $2\sqrt{3} \in \mathbb{R}$ We find that : $\sqrt{3} \times 2\sqrt{3} = 2 \times 3 = 6 \in \mathbb{R}$

Commutative property

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b = b \times a$

For example: $2\sqrt{5} \times 3\sqrt{5} = 6 \times 5 = 30$, $3\sqrt{5} \times 2\sqrt{5} = 6 \times 5 = 30$

i.e. $2\sqrt{5} \times 3\sqrt{5} = 3\sqrt{5} \times 2\sqrt{5}$

Associative property

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a \times b) \times c = a \times (b \times c) = a \times b \times c$

For example: $\left(2\sqrt{7} \times 4\sqrt{7}\right) \times \sqrt{7} = 56 \times \sqrt{7} = 56\sqrt{7}$,

$2\sqrt{7} \times \left(4\sqrt{7} \times \sqrt{7}\right) = 2\sqrt{7} \times 28 = 56\sqrt{7}$

i.e. $\left(2\sqrt{7} \times 4\sqrt{7}\right) \times \sqrt{7} = 2\sqrt{7} \times \left(4\sqrt{7} \times \sqrt{7}\right)$

The multiplicative neutral

For every $a \in \mathbb{R}$ it will be $a \times 1 = 1 \times a = a$ i.e. One is the multiplicative neutral in $\mathbb{R}$

For example: $\sqrt[3]{5} \times 1 = 1 \times \sqrt[3]{5} = \sqrt[3]{5}$

The multiplicative inverse of any non-zero real number

For every real number $a \neq 0$, there is a real number $\frac{1}{a}$ where $a \times \frac{1}{a} = 1$ which is the multiplicative neutral.

For example:

- The multiplicative inverse of $\sqrt{3}$ is $\frac{1}{\sqrt{3}}$ because $\sqrt{3} \times \frac{1}{\sqrt{3}} = 1$
- The multiplicative inverse of $\frac{\sqrt{2}}{5}$ is $\frac{5}{\sqrt{2}}$
- The multiplicative inverse of 1 is itself and also the multiplicative inverse of -1 is itself.

Notice that :

- Both the number and its multiplicative inverse have the same sign.
- There is no multiplicative inverse for zero because $\frac{1}{\text{zero}}$ is meaningless (i.e. Undefined)

Remark

Since each non-zero real number has a multiplicative inverse , then the division operation by any real number does not equal zero is possible in $\mathbb{R}$ and it is defined as follows :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}^*$ it will be $a \div b = a \times \frac{1}{b}$

i.e. The division operation $(a \div b)$ means multiplying the number a by the multiplicative inverse of the number b such that $b \neq 0$

And we can deduce that :

Division operation in $\mathbb{R}$ is not commutative and it is not associative.

Example Find the result of : $\frac{\sqrt{5}}{5} \times \frac{4\sqrt{5}}{12\sqrt{2}} \div \frac{1}{3\sqrt{2}}$

Solution

$$\left(\frac{\sqrt{5}}{5} \times \frac{4\sqrt{5}}{12\sqrt{2}}\right) \div \frac{1}{3\sqrt{2}} = \frac{5}{15\sqrt{2}} \div \frac{1}{3\sqrt{2}} = \frac{1}{3\sqrt{2}} \times 3\sqrt{2} = 1$$

Example Write each of the following such that the denominator is an integer :

1 $\frac{9}{\sqrt{3}}$ **2** $-\frac{3}{\sqrt{2}}$ **3** $\frac{5}{3\sqrt{5}}$

Solution

1 Multiplying the two terms of $\frac{9}{\sqrt{3}}$ by $\sqrt{3}$

, we get $\frac{9}{\sqrt{3}} = \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$ **Notice that :** $\frac{\sqrt{3}}{\sqrt{3}} = 1$

[illegible] $\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

3 $\frac{5}{3\sqrt{5}} = \frac{5}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{3 \times 5} = \frac{\sqrt{5}}{3}$

Another solution : $\because \sqrt{5} \times \sqrt{5} = 5$ $\quad \therefore \frac{5}{3\sqrt{5}} = \frac{\sqrt{5} \times \sqrt{5}}{3\sqrt{5}} = \frac{\sqrt{5}}{3}$

Example [illegible]

Choose the correct answer from those given :

1 The multiplicative inverse of $\frac{\sqrt{5}}{10}$ is ....

(a) $\sqrt{10}$ (b) $\sqrt{5}$ (c) $2\sqrt{5}$ (d) $-2\sqrt{5}$

2 The additive inverse of $\frac{7}{\sqrt{7}}$ is ....

(a) $\frac{\sqrt{7}}{7}$ (b) 7 (c) $-\sqrt{7}$ (d) -7

3 The multiplicative inverse of $\frac{3\sqrt{2}}{4}$ equals $\frac{\cdots\cdots}{3}$

(a) $4\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{2}$ (d) $\frac{4}{3\sqrt{2}}$

Solution

1 **(c) The reason :** The multiplicative inverse of $\frac{\sqrt{5}}{10}$

is $\frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$

2 **(c) The reason :** The additive inverse of $\frac{7}{\sqrt{7}}$

is $-\frac{7}{\sqrt{7}} = -\frac{7}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = -\frac{7\sqrt{7}}{7} = -\sqrt{7}$

3 **(b) The reason :** The multiplicative inverse of $\frac{3\sqrt{2}}{4}$

is $\frac{4}{3\sqrt{2}} = \frac{4}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$

TRY 2

1 *Find each of the following :*

1. $\sqrt{5} \times \frac{1}{\sqrt{5}} \times \sqrt{5}$
2. $\frac{\sqrt{3}}{3} \times \frac{4\sqrt{5}}{20} \times \frac{5\sqrt{3}}{\sqrt{5}}$

---

2 *Make the denominator an integer :*

1. $\frac{3}{\sqrt{7}}$
2. $\frac{9}{2\sqrt{6}}$

[illegible]

For any three real numbers a , b and c it will be :

- $a(b \pm c) = ab \pm ac$
- $(b \pm c)\,a = ba \pm ca$

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Example 1 Find each of the following :

1. $2\sqrt{3}\left(5\sqrt{3}-4\right)$
2. $\left(2+\sqrt{3}\right)\left(\sqrt{3}+7\right)$
3. $\left(7\sqrt{2}-5\right)\left(7\sqrt{2}+5\right)$
4. $\left(5\sqrt{3}-2\right)^2$

Solution

1. $2\sqrt{3}\left(5\sqrt{3}-4\right) = 2\sqrt{3} \times 5\sqrt{3} + 2\sqrt{3} \times (-4)$

$$= 10 \times 3 - 8 \times \sqrt{3} = 30 - 8\sqrt{3}$$

2. $\left(2+\sqrt{3}\right)\left(\sqrt{3}+7\right) = 2\sqrt{3} + 14 + 3 + 7\sqrt{3}$

$$= \left(2\sqrt{3} + 7\sqrt{3}\right) + (14 + 3) = 9\sqrt{3} + 17$$

3. By multiplying by inspection :

$$\left(7\sqrt{2}-5\right)\left(7\sqrt{2}+5\right) = \left(7\sqrt{2}\right)^2 - (5)^2$$

$$= 7^2 \times \left(\sqrt{2}\right)^2 - 5^2$$

$$= 49 \times 2 - 25 = 98 - 25 = 73$$

Notice that :
$(a+b)(a-b) = a^2 - b^2$

4 Multiplying by inspection

$$\therefore \left(5\sqrt{3}-2\right)^2 = \left(5\sqrt{3}\right)^2 - 2\times 5\sqrt{3}\times 2 + (-2)^2$$
$$= 5^2 \times \left(\sqrt{3}\right)^2 - 20\sqrt{3} + 4$$
$$= 25 \times 3 - 20\sqrt{3} + 4$$
$$= 75 - 20\sqrt{3} + 4 = 79 - 20\sqrt{3}$$

Notice that :
- $(a+b)^2 = a^2 + 2\,a\,b + b^2$
- $(a-b)^2 = a^2 - 2\,a\,b + b^2$

Example If $X = 5\sqrt{3} - 2$, $y = 5\sqrt{3} + 2$

, find the value of the expression : $X^2 + 2\,X\,y + y^2$

Solution From multiplying by inspection , we find that :

$(X + y)^2 = X^2 + 2\,X\,y + y^2$

$$\therefore X^2 + 2\,X\,y + y^2 = \left(5\sqrt{3} - 2 + 5\sqrt{3} + 2\right)^2$$
$$= \left(10\sqrt{3}\right)^2 = (10)^2 \times \left(\sqrt{3}\right)^2 = 100 \times 3 = 300$$

Example **Give an estimation for the result of :**

$\left(5 + \sqrt{10}\right)\left(3 - \sqrt[3]{7}\right)$, then check your answer using the calculator.

Solution First : The estimation of $\sqrt{10}$ is 3 $\left(\text{because } \sqrt{9} = 3\right)$

$\therefore$ The estimation of $\left(5 + \sqrt{10}\right)$ is $5 + 3 = 8$

, the estimation of $\sqrt[3]{7}$ is 2 $\left(\text{because } \sqrt[3]{8} = 2\right)$

$\therefore$ The estimation of $\left(3 - \sqrt[3]{7}\right)$ is $3 - 2 = 1$

$\therefore$ The estimation of $\left(5 + \sqrt{10}\right)\left(3 - \sqrt[3]{7}\right)$ is $8 \times 1 = 8$

Second : By using the calculator , we find that the result approximated to the nearest thousandths is 8.873

i.e. The estimation is accepted.

TRY 3

1 *Find the result of each of the following in the simplest form :*

1 $5\sqrt{2}\left(3\sqrt{2} - 2\right)$ 2 $\left(2\sqrt{3} - 3\right)\left(2\sqrt{3} + 3\right)$

2 If $X = 2\sqrt{3} - 1$ and $y = 2\sqrt{3} + 1$

, find the value of the expression : $X^2 - 2\,X\,y + y^2$

Lesson 6 Operations on the square roots

If a and b are two non negative real numbers , then :

1. $\sqrt{a} \times \sqrt{b} = \sqrt{a\,b}$

For example: • $\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$ • $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

2. $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ (where $b \neq 0$)

For example: • $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$ • $\sqrt{\frac{16}{49}} = \frac{\sqrt{16}}{\sqrt{49}} = \frac{4}{7}$

3. $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$ (where $b \neq 0$)

This operation is carried out to make the denominator an integer.

For example: • $\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$ • $\sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

! Remarks

1 $\sqrt{a^2+b^2} \neq a+b$, $\sqrt{a^2-b^2} \neq a - b$

For example:

- $\sqrt{6^2+8^2} \neq 6+8$ because $\sqrt{6^2+8^2} = \sqrt{100} = 10$
- $\sqrt{25-9} \neq 5-3$ because $\sqrt{25-9} = \sqrt{16} = 4$

2 $a\sqrt{b} = \sqrt{a^2 b}$

For example:

- $2\sqrt{\frac{1}{2}} = \sqrt{4 \times \frac{1}{2}} = \sqrt{2}$
- $15\sqrt{\frac{1}{3}} = 5 \times 3\sqrt{\frac{1}{3}} = 5\sqrt{9 \times \frac{1}{3}} = 5\sqrt{3}$

Example 1 Write each of the following in the form $a\sqrt{b}$ where a and b are two integers , b is the least possible value :

1 $\sqrt{27}$

2 $5\sqrt{54}$

3 $3\sqrt{\frac{2}{3}}$

4 $\frac{\sqrt{84}}{\sqrt{7}}$

Solution

1 $\sqrt{27} = \sqrt{9 \times 3}$

$= \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

2 $5\sqrt{54} = 5\sqrt{9 \times 6} = 5 \times \sqrt{9} \times \sqrt{6}$

$= 5 \times 3 \times \sqrt{6} = 15\sqrt{6}$

3 $3\sqrt{\frac{2}{3}} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 3 \times \frac{\sqrt{6}}{3} = \sqrt{6}$

Another solution :

$3\sqrt{\frac{2}{3}} = \sqrt{3^2 \times \frac{2}{3}} = \sqrt{3 \times 2} = \sqrt{6}$

4 $\frac{\sqrt{84}}{\sqrt{7}} = \sqrt{\frac{84}{7}} = \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Example Simplify to the simplest form :

1 $\sqrt{45}-2\sqrt{20}+2\sqrt{5}$ 2 $2\sqrt{18}+\sqrt{50}-42\sqrt{\frac{1}{2}}$

3 $2\sqrt{27}-3\sqrt{\frac{1}{3}}-\frac{6}{\sqrt{3}}$

Solution

1 $\sqrt{45}-2\sqrt{20}+2\sqrt{5}=\sqrt{9\times5}-2\sqrt{4\times5}+2\sqrt{5}$

$=\sqrt{9}\times\sqrt{5}-2\times\sqrt{4}\times\sqrt{5}+2\sqrt{5}$

$=3\sqrt{5}-2\times2\sqrt{5}+2\sqrt{5}$

$=3\sqrt{5}-4\sqrt{5}+2\sqrt{5}=\sqrt{5}$

2 $2\sqrt{18}+\sqrt{50}-42\sqrt{\frac{1}{2}}=2\sqrt{9\times2}+\sqrt{25\times2}-42\times\frac{\sqrt{1}}{\sqrt{2}}$

$=2\times\sqrt{9}\times\sqrt{2}+\sqrt{25}\times\sqrt{2}-42\times\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$=2\times3\sqrt{2}+5\sqrt{2}-21\sqrt{2}=-10\sqrt{2}$

3 $2\sqrt{27}-3\sqrt{\frac{1}{3}}-\frac{6}{\sqrt{3}}=2\sqrt{9\times3}-3\times\frac{\sqrt{1}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}-\frac{6}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=6\sqrt{3}-\sqrt{3}-\frac{6\sqrt{3}}{3}$

$=6\sqrt{3}-\sqrt{3}-2\sqrt{3}=3\sqrt{3}$

Example Find the result of each of the following :

1 $2\sqrt{3}\left(\sqrt{6}+5\right)$ 2 $\left(3\sqrt{2}-5\right)\left(3\sqrt{2}+5\right)$ 3 $\left(\sqrt{2}+\sqrt{6}\right)^2$

Solution

1 $2\sqrt{3}\left(\sqrt{6}+5\right)=2\sqrt{3}\times\sqrt{6}+2\sqrt{3}\times5$

$=2\sqrt{18}+10\sqrt{3}$

$=2\sqrt{9\times2}+10\sqrt{3}$

$=6\sqrt{2}+10\sqrt{3}$

2 $\left(3\sqrt{2}-5\right)\left(3\sqrt{2}+5\right)=\left(3\sqrt{2}\right)^2-(5)^2$

$= 3^2 \times \left(\sqrt{2}\right)^2-(5)^2$

$= 9 \times 2 - 25$

$= 18 - 25 = -7$

Remember that

$(a-b)(a+b) = a^2 - b^2$

3 $\left(\sqrt{2}+\sqrt{6}\right)^2=\left(\sqrt{2}\right)^2+2\times\sqrt{2}\times\sqrt{6}+\left(\sqrt{6}\right)^2$

$= 2+2\sqrt{12}+6$

$= 8+2\sqrt{4\times 3}=8+4\sqrt{3}$

Remember that

- $(a+b)^2 = a^2 + 2\,ab + b^2$
- $(a-b)^2 = a^2 - 2\,ab + b^2$

Example 4 If $a = \dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{2}}$, find the value of $a^2 + 2\sqrt{3}$

Solution To facilitate the solution, we will make the denominator an integer by multiplying both the numerator and the denominator by $\sqrt{2}$

$$\therefore a = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}\times\sqrt{2}-\sqrt{2}\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{12}-2}{2}$$

$$= \frac{\sqrt{4\times 3}-2}{2} = \frac{2\sqrt{3}-2}{2} = \frac{2\left(\sqrt{3}-1\right)}{2} = \sqrt{3}-1$$

$$\therefore a^2 = \left(\sqrt{3}-1\right)^2 = \left(\sqrt{3}\right)^2 - 2\times\sqrt{3}\times 1 + 1 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$$

$$\therefore a^2 + 2\sqrt{3} = 4 - 2\sqrt{3} + 2\sqrt{3} = 4$$

Another method to simplify a :

$\because a = \dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{2}}$ $\qquad \therefore a = \dfrac{\sqrt{6}}{\sqrt{2}} - \dfrac{\sqrt{2}}{\sqrt{2}} = \sqrt{\dfrac{6}{2}} - 1 = \sqrt{3}-1$

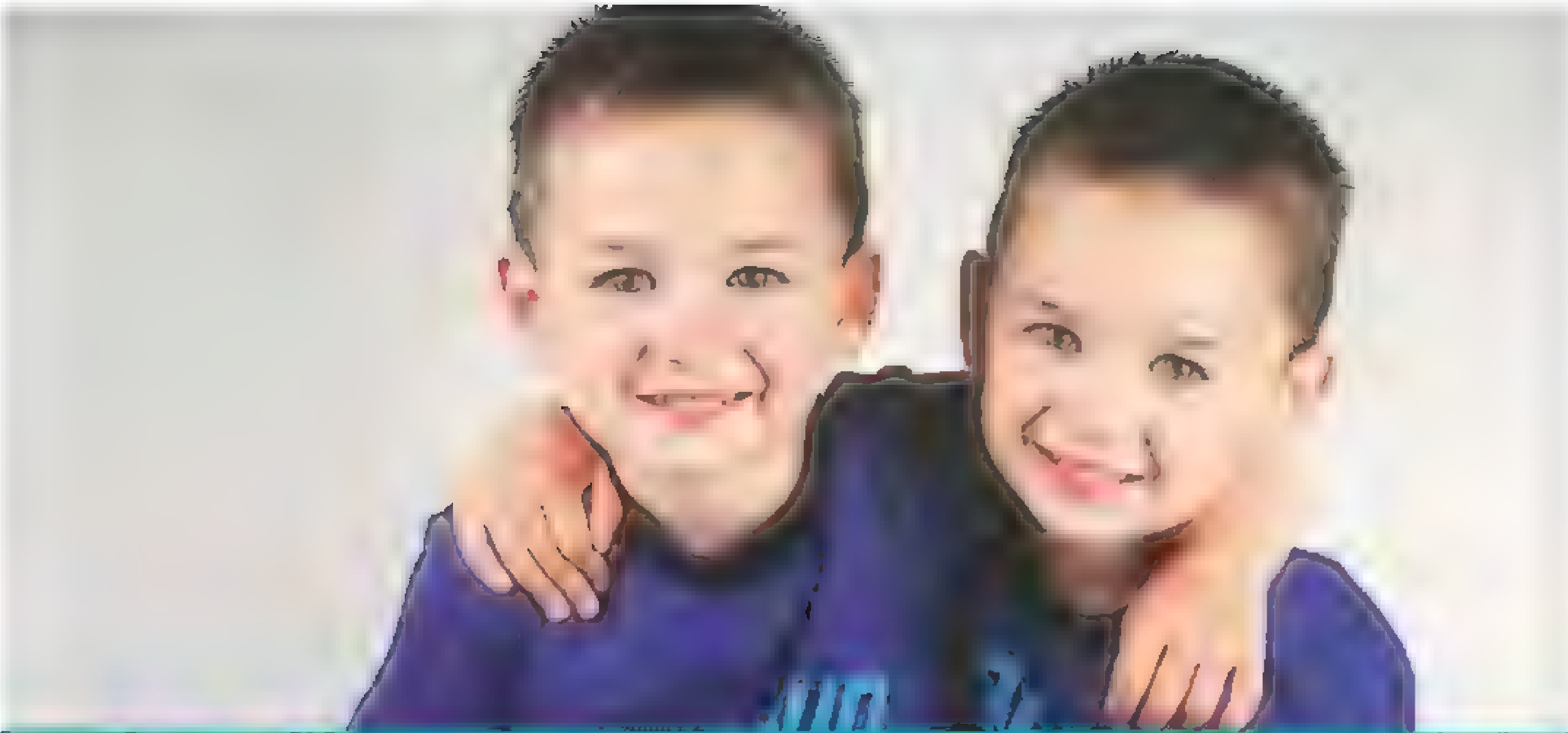
TRY

1 ***Simplify to the simplest form :***

1 $\sqrt{75}-2\sqrt{27}+\sqrt{3}$ $\qquad$ 2 $2\sqrt{50}-3\sqrt{2}-4\sqrt{\dfrac{1}{8}}$

2 ***Write each of the following such that the denominator is an integer :***

1 $\dfrac{5\sqrt{3}}{2\sqrt{5}}$ $\qquad$ 2 $\dfrac{1+\sqrt{3}}{3\sqrt{3}}$



Lesson 7 The two conjugate numbers

If a and b are two positive rational numbers

Then each of the two numbers $\left(\sqrt{a}+\sqrt{b}\right)$ and $\left(\sqrt{a}-\sqrt{b}\right)$ is conjugate to the other one and we find that :

- Their sum $\left(\sqrt{a}+\sqrt{b}\right)+\left(\sqrt{a}-\sqrt{b}\right)=2$ [illegible]

- Their product $= \left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=\underbrace{\left(\sqrt{a}\right)^2}_{\text{the square of 1st term}} - \underbrace{\left(\sqrt{b}\right)^2}_{\text{the square of 2nd term}} =$ [illegible] b

For example: $\left(\sqrt{3}-\sqrt{2}\right)$ its conjugate is $\left(\sqrt{3}+\sqrt{2}\right)$, then we find that

- Their sum $=2\sqrt{3}$
- Their product $=3-2=1$

! Remark

The product of the two conjugate numbers is always a rational number.

! Remark

If we have a real number whose denominator is written in the form $\left(\sqrt{a}+\sqrt{b}\right)$ or $\left(\sqrt{a}-\sqrt{b}\right)$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

Example 11 **Choose the correct answer from those given :**

1 The number $\dfrac{4}{\sqrt{7}-\sqrt{3}}$ in the simplest form is ....

ⓐ $\sqrt{7}-\sqrt{3}$ ⓑ $\sqrt{7}+\sqrt{3}$
ⓒ $4\sqrt{7}-4\sqrt{3}$ ⓓ $4\sqrt{7}+4\sqrt{3}$

2 The conjugate of $\dfrac{1}{\sqrt{3}-\sqrt{2}}$ is ....

ⓐ $\sqrt{3}-\sqrt{2}$ ⓑ $\sqrt{3}-2$ ⓒ $\sqrt{3}+\sqrt{2}$ ⓓ $\sqrt{3}+2$

3 The multiplicative inverse of $1-\sqrt{2}$ is ....

ⓐ $\sqrt{2}-1$ ⓑ $1-\sqrt{2}$ ⓒ $-1-\sqrt{2}$ ⓓ $1+\sqrt{2}$

4 If $\dfrac{1}{x}=\sqrt{10}-3$, then $x=$ ....

ⓐ $\sqrt{10}+3$ ⓑ $\sqrt{10}-3$ ⓒ $3-\sqrt{10}$ ⓓ $-3-\sqrt{10}$

Solution

1 **(b) The reason :** Multiplying the two terms of the number by the conjugate of the denominator which is $\left(\sqrt{7}+\sqrt{3}\right)$

$$\therefore \frac{4}{\sqrt{7}-\sqrt{3}}=\frac{4}{\sqrt{7}-\sqrt{3}}\times\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$$

$$=\frac{4\left(\sqrt{7}+\sqrt{3}\right)}{\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2}=\frac{4\left(\sqrt{7}+\sqrt{3}\right)}{7-3}$$

$$=\sqrt{7}+\sqrt{3}$$

2 **(a) The reason :** Multiplying the two terms of the number by the conjugate of the denominator which is $\left(\sqrt{3}+\sqrt{2}\right)$

$$\therefore \frac{1}{\sqrt{3}-\sqrt{2}}=\frac{1}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{\sqrt{3}+\sqrt{2}}{3-2}=\sqrt{3}+\sqrt{2}$$

$\therefore$ The conjugate of $\dfrac{1}{\sqrt{3}-\sqrt{2}}$ is $\sqrt{3}-\sqrt{2}$

3 (c) **The reason :** The multiplicative inverse of $1-\sqrt{2}$ is $\frac{1}{1-\sqrt{2}}$,

by multiplying the two terms of the number by the conjugate of the denominator which is $\left(1+\sqrt{2}\right)$

$$\therefore \frac{1}{1-\sqrt{2}} = \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{1+\sqrt{2}}{1-2}$$

$$= \frac{1+\sqrt{2}}{-1} = -1-\sqrt{2}$$

4 (a) **The reason :** $\because \frac{1}{x} = \sqrt{10}-3 \qquad \therefore x = \frac{1}{\sqrt{10}-3}$

$$\therefore x = \frac{1}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3} = \frac{\sqrt{10}+3}{10-9} = \sqrt{10}+3$$

Example

If $x = \frac{4}{2-\sqrt{2}}$ and $y = \frac{3-2\sqrt{2}}{3+2\sqrt{2}}$, write each of x and y such that its denominator is a rational number , then find $x+y$

Solution

$$\because x = \frac{4}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{4\left(2+\sqrt{2}\right)}{4-2} = \frac{4\left(2+\sqrt{2}\right)}{2}$$

$$= 2\left(2+\sqrt{2}\right) = 4+2\sqrt{2}$$

$$, y = \frac{3-2\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{\left(3-2\sqrt{2}\right)^2}{9-8} = \frac{9-12\sqrt{2}+8}{1} = 17-12\sqrt{2}$$

$$\therefore x+y = 4+2\sqrt{2}+17-12\sqrt{2} = 21-10\sqrt{2}$$

TRY

Write each of the following such that the denominator is a rational number :

(1) $\frac{12}{\sqrt{6}-\sqrt{2}}$ (2) $\frac{\sqrt{8}}{3+2\sqrt{2}}$

! Important remarks from direct product (multiplying by inspection)

- **We know that :** $(x - y)(x + y) = x^2 - y^2$
- **And we know also :**

$(x + y)^2 = x^2 + 2xy + y^2$

Then :

- $x^2 + xy + y^2 = (x + y)^2 - xy$
- $x^2 + y^2 = (x + y)^2 - 2xy$

$(x - y)^2 = x^2 - 2xy + y^2$

Then :

- $x^2 - xy + y^2 = (x - y)^2 + xy$
- $x^2 + y^2 = (x - y)^2 + 2xy$

Example If $x = \frac{2}{\sqrt{5} - \sqrt{3}}$ and $y = \sqrt{5} - \sqrt{3}$, prove that x and y are conjugate numbers , then find the value of each of :

1. $x^2 + 2xy + y^2$
2. $x^2 + xy + y^2$

Solution

$$\because x = \frac{2}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{2(\sqrt{5} + \sqrt{3})}{5 - 3}$$

$$= \frac{2(\sqrt{5} + \sqrt{3})}{2} = \sqrt{5} + \sqrt{3}$$

$, \because y = \sqrt{5} - \sqrt{3}$

$\therefore x$ and y are conjugate numbers.

1. $x^2 + 2xy + y^2 = (\sqrt{5} + \sqrt{3})^2 + 2(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) + (\sqrt{5} - \sqrt{3})^2$

$$= (5 + 2\sqrt{15} + 3) + 2(5 - 3) + (5 - 2\sqrt{15} + 3)$$

$$= 8 + 2\sqrt{15} + 4 + 8 - 2\sqrt{15} = 20$$

Another solution using the previous remarks :

Since $x^2 + 2xy + y^2 = (x + y)^2$

$$\therefore x^2 + 2xy + y^2 = \left[(\sqrt{5} + \sqrt{3}) + (\sqrt{5} - \sqrt{3})\right]^2$$

$$= (2\sqrt{5})^2 = 4 \times 5 = 20$$

$$\because x^2 + xy + y^2 = \left(\sqrt{5}+\sqrt{3}\right)^2 + \left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right) + \left(\sqrt{5}-\sqrt{3}\right)^2$$

$$= \left(5+3+2\sqrt{15}\right) + (2) + \left(5+3-2\sqrt{15}\right) = 18$$

Another solution using the previous remarks :

$$x^2 + xy + y^2 = (x+y)^2 - xy = \left(\sqrt{5}+\sqrt{3}+\sqrt{5}-\sqrt{3}\right)^2 - \left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$$

$$= \left(2\sqrt{5}\right)^2 - 2 = 20 - 2 = 18$$

2

If $x = \dfrac{3}{2\sqrt{2}-\sqrt{5}}$ and $y = 2\sqrt{2} - \sqrt{5}$, find the value of the expression :
$x^2 - y^2$

Lesson 8 Operations on the cube roots

If a and b are two real numbers , then :

1 $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab}$

For example:

- $\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt[3]{3 \times 9} = \sqrt[3]{27} = 3$
- $\sqrt[3]{2} \times \sqrt[3]{-4} = \sqrt[3]{2 \times -4} = \sqrt[3]{-8} = -2$
- $\sqrt[3]{16} = \sqrt[3]{8 \times 2} = \sqrt[3]{8} \times \sqrt[3]{2} = 2\sqrt[3]{2}$
- $\sqrt[3]{-54} = \sqrt[3]{-27 \times 2} = \sqrt[3]{-27} \times \sqrt[3]{2} = -3\sqrt[3]{2}$

2 $\dfrac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\dfrac{a}{b}}$ (where $b \neq 0$)

For example:

- $\dfrac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\dfrac{32}{4}} = \sqrt[3]{8} = 2$
- $\dfrac{\sqrt[3]{54}}{\sqrt[3]{-2}} = \sqrt[3]{-\dfrac{54}{2}} = \sqrt[3]{-27} = -3$
- $\sqrt[3]{\dfrac{8}{125}} = \dfrac{\sqrt[3]{8}}{\sqrt[3]{125}} = \dfrac{2}{5}$
- $\sqrt[3]{-\dfrac{27}{64}} = \dfrac{\sqrt[3]{-27}}{\sqrt[3]{64}} = \dfrac{-3}{4}$

Example Find the result of each of the following in its simplest form :

1 $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{4}{9}}$

2 $\sqrt[3]{\frac{5}{4}} \div \sqrt[3]{\frac{2}{25}}$

Solution

1 $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{4}{9}} = \sqrt[3]{\frac{2}{3} \times \frac{4}{9}} = \sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$

2 $\sqrt[3]{\frac{5}{4}} \div \sqrt[3]{\frac{2}{25}} = \sqrt[3]{\frac{5}{4} \div \frac{2}{25}} = \sqrt[3]{\frac{5}{4} \times \frac{25}{2}} = \sqrt[3]{\frac{125}{8}} = \frac{\sqrt[3]{125}}{\sqrt[3]{8}} = \frac{5}{2}$

Remarks

If a and b are two real numbers , then :

1 $\sqrt[3]{a^3 + b^3} \neq a + b$, $\sqrt[3]{a^3 - b^3} \neq a - b$

2 $\sqrt[3]{-a} = -\sqrt[3]{a}$

3 $a\sqrt[3]{b} = \sqrt[3]{a^3 b}$

For example: • $3\sqrt[3]{\frac{1}{9}} = \sqrt[3]{27 \times \frac{1}{9}} = \sqrt[3]{3}$

• $8\sqrt[3]{\frac{1}{4}} = 4 \times 2\sqrt[3]{\frac{1}{4}} = 4\sqrt[3]{8 \times \frac{1}{4}} = 4\sqrt[3]{2}$

4 $\sqrt[3]{\frac{a}{b}} = \sqrt[3]{\frac{a}{b} \times \frac{b^2}{b^2}} = \sqrt[3]{\frac{a b^2}{b^3}} = \frac{1}{b}\sqrt[3]{a b^2}$ (Where $b \neq 0$)

For example: $\sqrt[3]{\frac{1}{3}} = \sqrt[3]{\frac{1}{3} \times \frac{9}{9}} = \sqrt[3]{\frac{9}{27}} = \frac{1}{3}\sqrt[3]{9}$

Example Put each of the following in its simplest form :

1 $\sqrt[3]{24} + \sqrt[3]{3} - \sqrt[3]{81}$

2 $\sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}}$

3 $\sqrt[3]{81} + \sqrt{12} - 2\sqrt[3]{3} - 2\sqrt{3}$

Solution 1 $\sqrt[3]{24}+\sqrt[3]{3}-\sqrt[3]{81}=\sqrt[3]{8\times 3}+\sqrt[3]{3}-\sqrt[3]{27\times 3}$

$$=\sqrt[3]{8}\times\sqrt[3]{3}+\sqrt[3]{3}-\sqrt[3]{27}\times\sqrt[3]{3}$$

$$=2\sqrt[3]{3}+\sqrt[3]{3}-3\sqrt[3]{3}=\text{zero}$$

2 $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=\sqrt[3]{27\times 2}+6\sqrt[3]{8\times 2}-3\times 2\sqrt[3]{\frac{1}{4}}$

$$=\sqrt[3]{27}\times\sqrt[3]{2}+6\times\sqrt[3]{8}\times\sqrt[3]{2}-3\times\sqrt[3]{8\times\frac{1}{4}}$$

$$=3\times\sqrt[3]{2}+6\times 2\times\sqrt[3]{2}-3\times\sqrt[3]{2}$$

$$=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$$

Another solution :

$$\because \sqrt[3]{\frac{1}{4}}=\sqrt[3]{\frac{1}{4}\times\frac{16}{16}}=\sqrt[3]{\frac{16}{64}}=\frac{\sqrt[3]{16}}{\sqrt[3]{64}}=\frac{1}{4}\sqrt[3]{16}$$

$$=\frac{1}{4}\sqrt[3]{8\times 2}=\frac{1}{4}\times 2\sqrt[3]{2}=\frac{1}{2}\sqrt[3]{2}$$

$$\therefore \sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=3\sqrt[3]{2}+6\times 2\sqrt[3]{2}-6\times\frac{1}{2}\sqrt[3]{2}$$

$$=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$$

One more solution :

$$\because 6\sqrt[3]{\frac{1}{4}}=\sqrt[3]{6^3\times\frac{1}{4}}=\sqrt[3]{54}$$

$$\therefore \sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=\cancel{\sqrt[3]{54}}+6\sqrt[3]{16}-\cancel{\sqrt[3]{54}}$$

$$=6\sqrt[3]{16}=6\times 2\sqrt[3]{2}=12\sqrt[3]{2}$$

3 $\sqrt[3]{81}+\sqrt{12}-2\sqrt[3]{3}-2\sqrt{3}=\sqrt[3]{27\times 3}+\sqrt{4\times 3}-2\sqrt[3]{3}-2\sqrt{3}$

$$=\sqrt[3]{27}\times\sqrt[3]{3}+\sqrt{4}\times\sqrt{3}-2\sqrt[3]{3}-2\sqrt{3}$$

$$=3\sqrt[3]{3}+2\sqrt{3}-2\sqrt[3]{3}-2\sqrt{3}=\sqrt[3]{3}$$

Example Find in the simplest form : $2\sqrt[3]{4}\left(5\sqrt[3]{\frac{1}{2}}\ \ \sqrt[3]{32}\right)$

Solution $2\sqrt[3]{4}\left(5\sqrt[3]{\frac{1}{2}}-\sqrt[3]{32}\right)=2\times 5\sqrt[3]{4\times\frac{1}{2}}-2\times\sqrt[3]{4\times 32}$

$=10\sqrt[3]{2}-2\times\sqrt[3]{128}=10\sqrt[3]{2}-2\times\sqrt[3]{64\times 2}$

$=10\sqrt[3]{2}-2\times 4\sqrt[3]{2}=10\sqrt[3]{2}-8\sqrt[3]{2}=2\sqrt[3]{2}$

Example If $x=\sqrt[3]{5}+2$ and $y=\sqrt[3]{5}-2$, find the value of $(x+y)^3-(x-y)^3$

Solution $\because x+y=\sqrt[3]{5}+2+\sqrt[3]{5}-2=2\sqrt[3]{5}$

, $x-y=\sqrt[3]{5}+2-\left(\sqrt[3]{5}-2\right)=\sqrt[3]{5}+2-\sqrt[3]{5}+2=4$

$\therefore (x+y)^3-(x-y)^3=\left(2\sqrt[3]{5}\right)^3-(4)^3=2^3\times\left(\sqrt[3]{5}\right)^3-4^3$

$=8\times 5-64=40-64=-24$

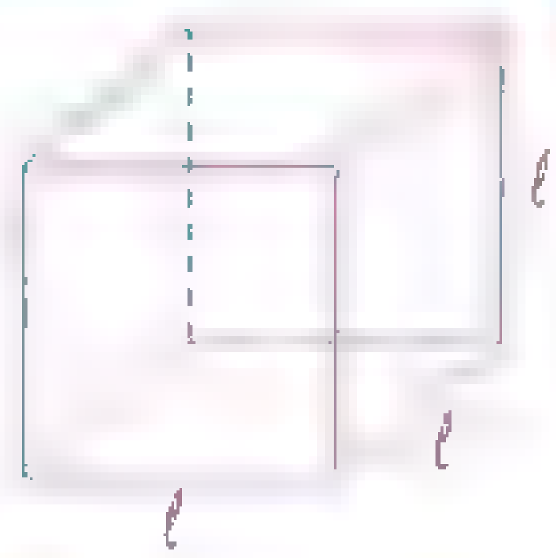
TRY

Simplify each of the following to the simplest form :

1. $5\sqrt[3]{2}-\sqrt[3]{16}+\sqrt[3]{-54}$
2. $\sqrt[3]{72}+\sqrt[3]{\frac{1}{3}}+\sqrt[3]{-9}$

Lesson 9 Applications on the real numbers

The cube



It is a solid whose six faces are congruent squares.

i.e. All its edges are equal in length.

Assuming that the edge length of the cube = ℓ length unit , then :

1. The area of each face = ℓ^2 square unit.
2. Its lateral area = $4\ \ell^2$ square unit.
3. Its total area (the area of its 6 faces) = $6\ \ell^2$ square unit.
4. Its volume = ℓ^3 cube unit.

Example 1 **Choose the correct answer from those given :**

1 A cube of volume 64 cm^3 , then the sum of its edge lengths is ....

(a) 16 cm. (b) 32 cm. (c) 48 cm. (d) 64 cm.

2 A cube of volume 125 cm^3 , then its total area = ...

(a) 200 cm^2 (b) 150 cm^2 (c) 125 cm^2 (d) 25 cm^2

3 A cube of volume 216 cm^3 , then its lateral area = ...

(a) 36 cm^2 (b) 72 cm^2 (c) 144 cm^2 (d) 216 cm^2

4 The lateral area of a cube is 4 cm^2, then its volume = ...

ⓐ 1 cm^3 ⓑ 2 cm^3 ⓒ 4 cm^3 ⓓ 16 cm^3

5 The total area of a cube is 294 cm^2, then its lateral area = ...

ⓐ 28 cm^2 ⓑ 49 cm^2 ⓒ 196 cm^2 ⓓ 343 cm^2

Solution

1 **(c) The reason :** $\because$ The volume of the cube $= \ell^3$ where ℓ is its edge length

$\therefore \ell^3 = 64$ $\therefore \ell = \sqrt[3]{64} = 4$ cm.

$\therefore$ The sum of the edge lengths $= 12\,\ell = 12 \times 4 = 48$ cm.

2 **(b) The reason :** $\because$ The volume of the cube $= \ell^3$ where ℓ is its edge length

$\therefore \ell^3 = 125$ $\therefore \ell = \sqrt[3]{125} = 5$ cm.

$\therefore$ The total area of the cube $= 6\,\ell^2 = 6 \times 5^2 = 150$ cm^2.

3 **(c) The reason :** $\because$ The volume of the cube $= \ell^3$ where ℓ is its edge length

$\therefore \ell^3 = 216$ $\therefore \ell = \sqrt[3]{216} = 6$ cm.

$\therefore$ The lateral area of the cube $= 4\,\ell^2 = 4 \times 6^2 = 144$ cm^2.

4 **(a) The reason :** $\because$ The lateral area of the cube $= 4\,\ell^2$ where ℓ is its edge length

$\therefore 4\,\ell^2 = 4$ $\therefore \ell^2 = 1$ $\therefore \ell = \sqrt{1} = 1$ cm.

$\therefore$ The volume of the cube $= \ell^3 = 1^3 = 1$ cm^3.

5 **(c) The reason :** $\because$ The total area of the cube $= 6\,\ell^2$ where ℓ is its edge length

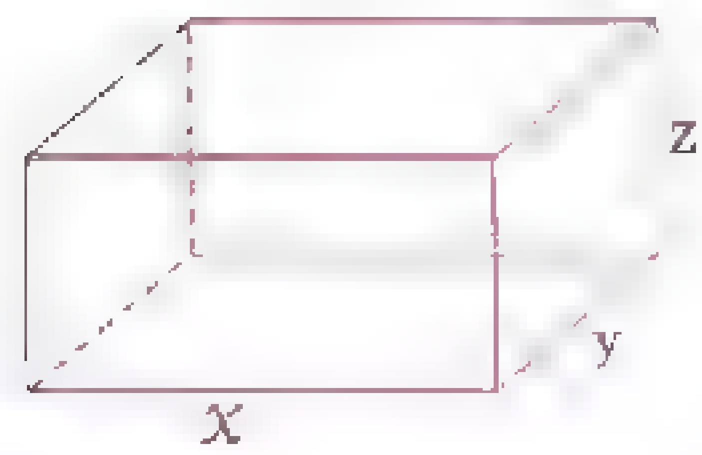
$\therefore 6\,\ell^2 = 294$ $\therefore \ell^2 = \frac{294}{6} = 49$

$\therefore$ The lateral area $= 4\,\ell^2 = 4 \times 49 = 196$ cm^2.

1 *Complete the following table :*

	Edge length of the cube	Area of one face	Lateral area	Total area	Volume
1	3 cm.	......	......	......	......
2	......	49 cm^2	......	......	......
3	......	......	144 cm^2	......	......
4	......	......	......	150 cm^2	......
5	......	......	......	......	64 cm^3

The cuboid



It is a solid that contains 6 faces , each of them is a rectangle and each two opposite faces are congruent. Assuming that the lengths of the edges of the cuboid are x , y and z length unit , then :

1. Its lateral area = the perimeter of the base × height = $2(x + y) \times z$ square unit.
2. Its total area (the area of its six faces) = the lateral area + twice the area of the base

$$= 2(x + y) \times z + 2xy$$

$$= 2(xy + yz + zx) \text{ square unit.}$$

3. Its volume = the area of the base × the height

$$= x \times y \times z \text{ cube unit.}$$

Remarks

- The cuboid may contain two opposite faces , each of them is a square.
- The cube is a special case of the cuboid.

  i.e. The cube is a cuboid with edges having the same length.

Example **The height of a cuboid is 4 cm. and its base is a square of side length 5 cm. Find :**

1 Its volume. 2 Its lateral area. 3 Its total area.

Solution

1 The volume of the cuboid = the area of the base × the height

$$= 5 \times 5 \times 4 = 100 \text{ cm}^3.$$

2 The lateral area of the cuboid = the perimeter of the base × the height

$$= 4 \times 5 \times 4 = 80 \text{ cm}^2.$$

3 The total area of the cuboid

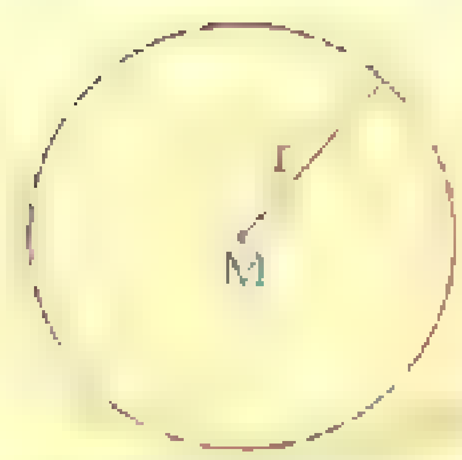
= the lateral area + twice the area of the base = $80 + 2 \times 25 = 130 \text{ cm}^2$.

2 The dimensions of a cuboid are 3 cm. , 4 cm. and 5 cm. Calculate its volume and its total area.

The circle

If M is a circle with radius length r , then :

1. The circumference of the circle = $2\pi r$ length unit.
2. The area of the circle = πr^2 square unit.



Example The area of a circle is 25π cm^2. Calculate its circumference in terms of π

Solution $\because$ The area of the circle $= \pi r^2$ $\qquad \therefore \pi r^2 = 25\pi$

$\therefore r^2 = 25$ $\qquad \therefore r = \sqrt{25} = 5$ cm.

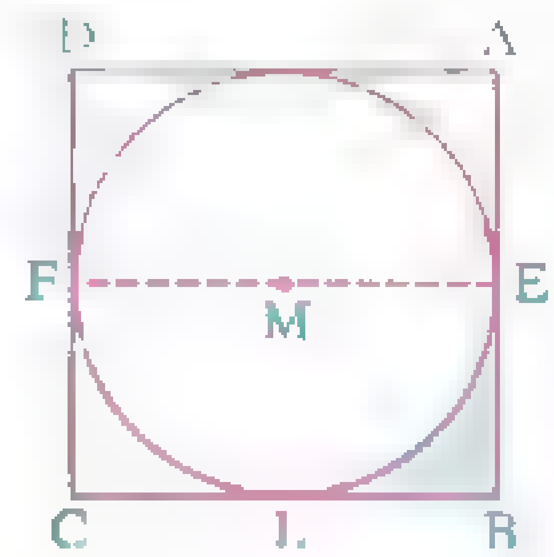
$\therefore$ The circumference of the circle $= 2\pi r$

$= 2 \times 5 \times \pi = 10\pi$ cm.

Example In the opposite figure :

A circle M is drawn inside a square (touching its sides).

If the area of the square = 196 cm^2 , find :



1. The area of the shaded part.
2. The perimeter of the shaded part.

Solution $\because$ The area of the square = 196 cm^2.

$\therefore$ The side length of the square $= \sqrt{196} = 14$ cm.

, $\because$ the side length of the square $= 2r$

$\therefore 14 = 2r$ $\qquad \therefore r = 7$ cm.

1. The area of the shaded part

$=$ (the area of the square – the area of the circle) $\div 4$

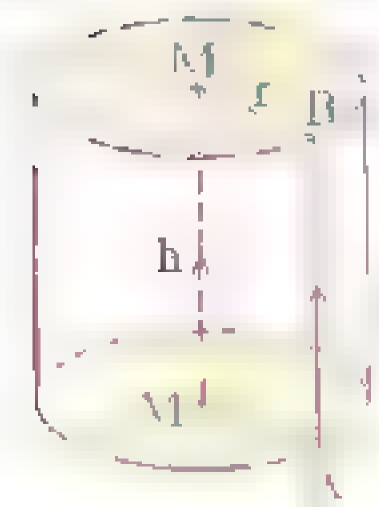
$= \left(196 - \frac{22}{\cancel{7}} \times \cancel{7} \times 7\right) \div 4 = 42 \div 4 = 10.5$ cm^2

2. The perimeter of the shaded part

$BE + BL + \frac{1}{4}$ circumference of the circle $= 7 + 7 + \left(\frac{1}{4} \times 2 \times \frac{22}{\cancel{7}} \times \cancel{7}\right)$

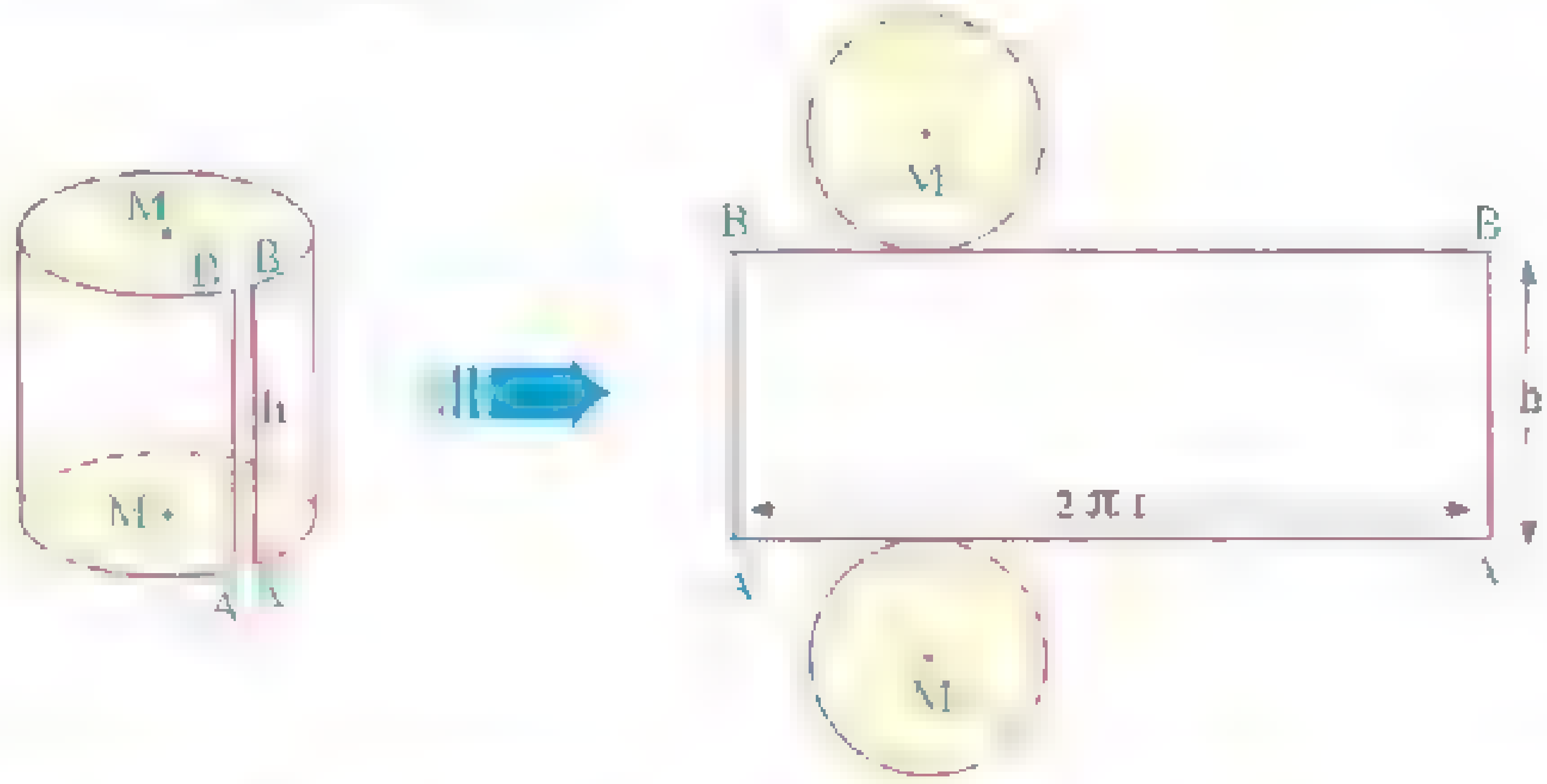
$= 14 + 11 = 25$ cm.

Try 3 The circumference of a circle is 88 cm. Find its area. $\left(\pi = \frac{22}{7}\right)$



The right circular cylinder

- It is a solid having two parallel congruent bases , each of them is a circular-shaped surface while its lateral surface is a curved surface which is called cylindrical surface.
- The line segment $\overline{M\grave{M}}$ drawn between the two centres of the two bases is perpendicular to each plane of the two bases and it is called the height of the cylinder.
- If we draw $\overline{AB}$ on the cylindrical surface such that $A \in$ the circle M , $B \in$ the circle $\grave{M}$, $\overline{AB} \parallel \overline{M\grave{M}}$ and if we cut the lateral surface of the cylinder at $\overline{AB}$ and flattened it out , then we will obtain the following figure :



This figure consists of the surface of the rectangle $AB\grave{B}\grave{A}$ and it is the same cylindrical surface of the cylinder in addition to the two surfaces of two circles which represent the two bases of the cylinder , then we find :

AB = the height of the cylinder.

$A\grave{A}$ = the circumference of the base of the cylinder.

$\therefore$ The lateral area of the cylinder = the area of the rectangle $AB\grave{B}\grave{A} = A\grave{A} \times AB$

= the circumference of the base of the cylinder × its height

and if we assume that the length of the radius of the base = r and its height = h , then :

1. The lateral area of the cylinder = $2\pi r h$ square unit.
2. The total area of the cylinder = the lateral area of the cylinder + twice the area of the base = $2\pi r h + 2\pi r^2$ square unit.
3. The volume of the cylinder = the area of the base × height = $\pi r^2 h$ cube unit.

Example 5 A right circular cylinder is of height 10 cm. and its volume is 1540 cm^3. Find its total area $\left(\pi = \frac{22}{7}\right)$

Solution $\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 1540 = \frac{22}{7} \times r^2 \times 10$ $\qquad\qquad \therefore 1540 = \frac{220}{7} r^2$

$\therefore r^2 = 1540 \times \frac{7}{220} = 49$ $\qquad\qquad \therefore r = \sqrt{49} = 7$ cm.

$\therefore$ The total area of the cylinder $= 2\pi r h + 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 7 \times 10 + 2 \times \frac{22}{7} \times 7^2$$

$$= 440 + 308 = 748 \text{ cm}^2.$$

Try 4

A right circular cylinder is of volume 90π cm^3 and its height is 10 cm. Find the diameter length of its base.

The sphere

- It is a solid with a curved surface whose all points are equidistant from a fixed point inside the sphere.
- The equal distances are called the radius length of the sphere.
- The fixed point is called the centre of the sphere.
- If we cut the sphere by a plane passing through its centre , then the resulted section is a circle having the same centre of the sphere and its radius length is the same of the sphere.

Assuming that the radius length of the sphere = r , then :

1. The area of the sphere $= 4\pi r^2$ square unit.
2. The volume of the sphere $= \frac{4}{3}\pi r^3$ cube unit.

Example 6 The volume of a sphere $= \frac{500}{3}\pi$ cm^3. Find the length of its diameter.

Solution $\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$ $\qquad\qquad \therefore \frac{500}{3}\pi = \frac{4}{3}\pi r^3$

$\therefore r^3 = \frac{500}{3} \times \frac{3}{4} = 125$ $\qquad\qquad \therefore r = \sqrt[3]{125} = 5$ cm.

$\therefore$ The diameter length of the sphere $= 2 \times 5 = 10$ cm.

Example 7 **A right circular cylinder is of height 6 cm. and its volume $= \frac{2}{3}$ the volume of a sphere whose radius length is 3 cm.**
Find the radius length of the base of the cylinder.

Solution Let the radius length of the sphere be r_1 cm. and the radius length of the base of the cylinder be r_2 cm.

$\because$ The volume of the sphere $= \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi (3)^3 = 36\pi$ cm^3

$\because$ The volume of the cylinder $= \frac{2}{3}$ the volume of the sphere.

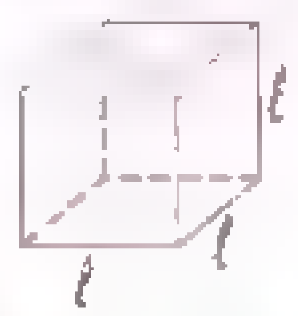
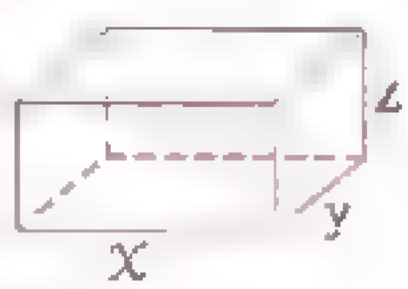
$\therefore \cancel{\pi} r_2^2 h = \frac{2}{3} \times 36 \cancel{\pi}$ $\qquad \therefore r_2^2 \times 6 = 24$

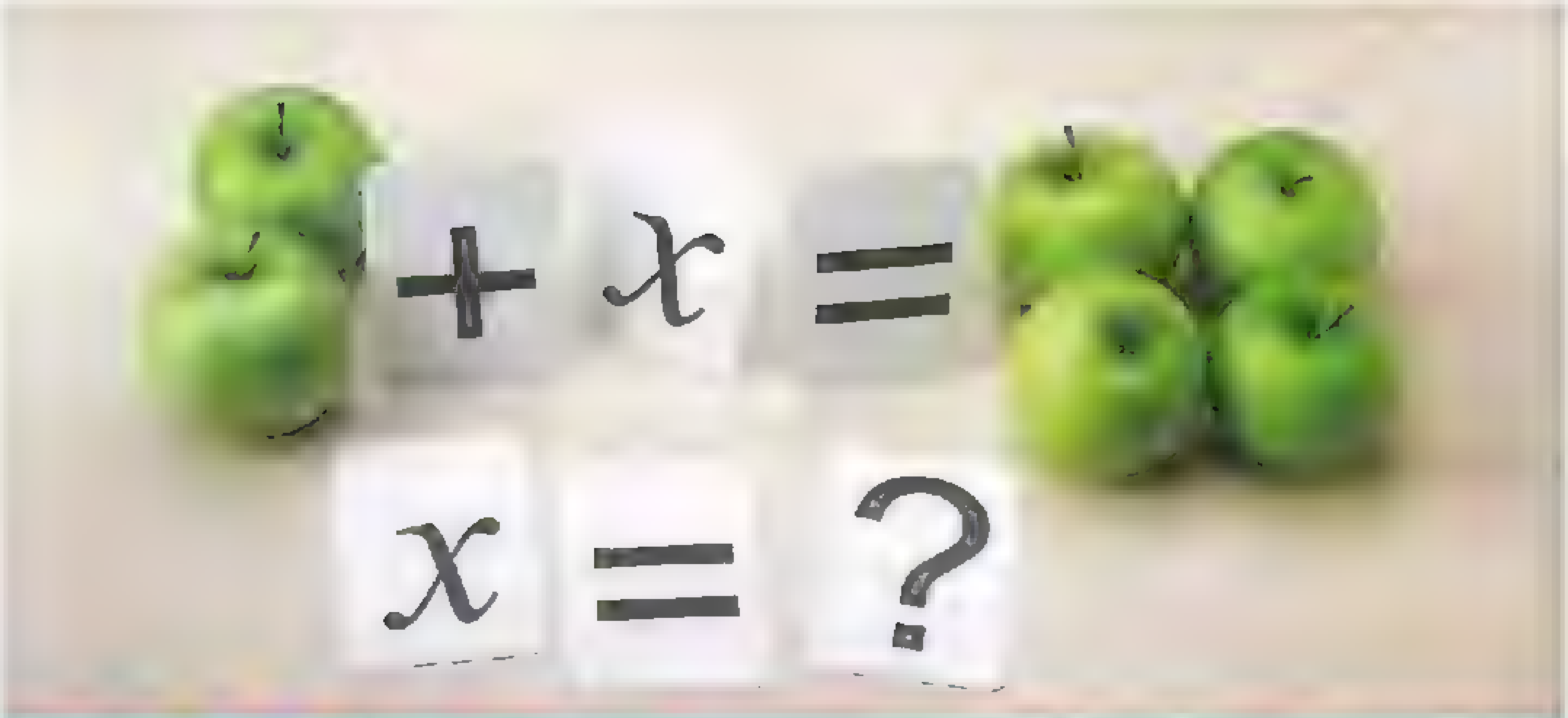
$\therefore r_2^2 = 4$ $\qquad \therefore r_2 = \sqrt{4} = 2$ cm.

$\therefore$ The radius length of the base of the cylinder $= 2$ cm.

5 The area of a sphere is 36π cm^2. Find its volume in terms of π

In the following , we will summarize the previous rules of areas and volumes of some solids :

The solid		The lateral area	The total area	The volume
The cube		$4\ell^2$	$6\ell^2$	ℓ^3
The cuboid		$2(x+y)\times z$	$2(xy+yz+zx)$	xyz
The cylinder		$2\pi r h$	$2\pi r h + 2\pi r^2$ $= 2\pi r(h+r)$	$\pi r^2 h$
The sphere		–	$4\pi r^2$	$\frac{4}{3}\pi r^3$



Lesson 10 Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$

First Solving equations of the first degree in one unknown in $\mathbb{R}$

* Each of the equations :
  - $2x - 5 = 3$
  - $\sqrt{3}\,x - 1 = 8$
  - $\frac{1}{2}x - \sqrt{5} = 0$

  is called an equation of the first degree in one variable (one unknown) which is x because the exponent of the variable x equals one.

* Solving the equation of the first degree in one variable means finding the real number which satisfies this equation.
* The following examples will show how to solve an equation of the first degree in one variable :

Example 1 **Find in $\mathbb{R}$ the S.S. of each of the following equations , then represent the solution on the number line :**

1 $3x + 2 = 1$

2 $\sqrt{3}\,x - 1 = 2$

3 $7x - \sqrt{7} = 6\sqrt{7}$

4 $x - \sqrt{5} = 1$

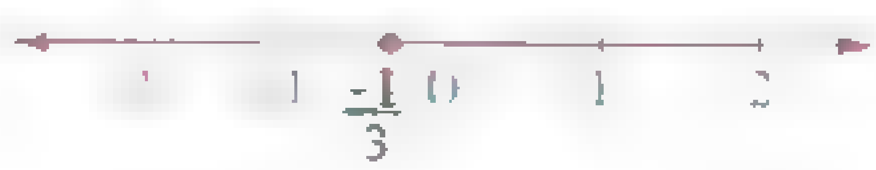
Solution 1 $\because 3x+2=1$ (adding -2 to both sides)

$\therefore 3x+2-2=1-2 \qquad \therefore 3x=-1$

(multiplying both sides by $\frac{1}{3}$ the multiplicative inverse of the coefficient of x)

$\therefore 3x \times \frac{1}{3} = -1 \times \frac{1}{3} \qquad \therefore x = -\frac{1}{3} \quad \therefore$ The S.S. $= \{-\frac{1}{3}\}$

- We can represent the number $-\frac{1}{3}$ on the number line as follows :



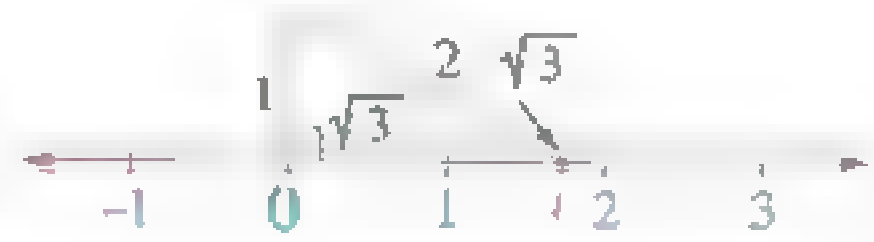
2 $\because \sqrt{3}\,x-1=2 \qquad \therefore \sqrt{3}\,x=2+1$

$\therefore \sqrt{3}\,x=3 \qquad \therefore x=\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\therefore x=\frac{3\sqrt{3}}{3} \qquad \therefore x=\sqrt{3}$

$\therefore$ The S.S. $=\{\sqrt{3}\}$

- We can represent the number $\sqrt{3}$ on the number line as follows :



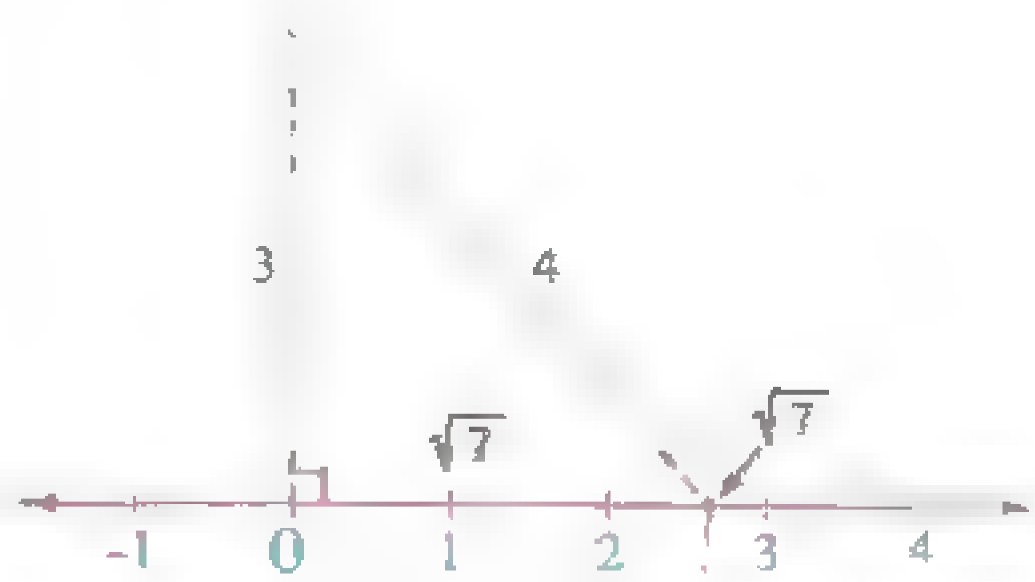
3 $\because 7x-\sqrt{7}=6\sqrt{7} \qquad \therefore 7x=6\sqrt{7}+\sqrt{7}$

$\therefore 7x=7\sqrt{7} \qquad \therefore x=\frac{7\sqrt{7}}{7}$

$\therefore x=\sqrt{7}$

$\therefore$ The S.S. $=\{\sqrt{7}\}$

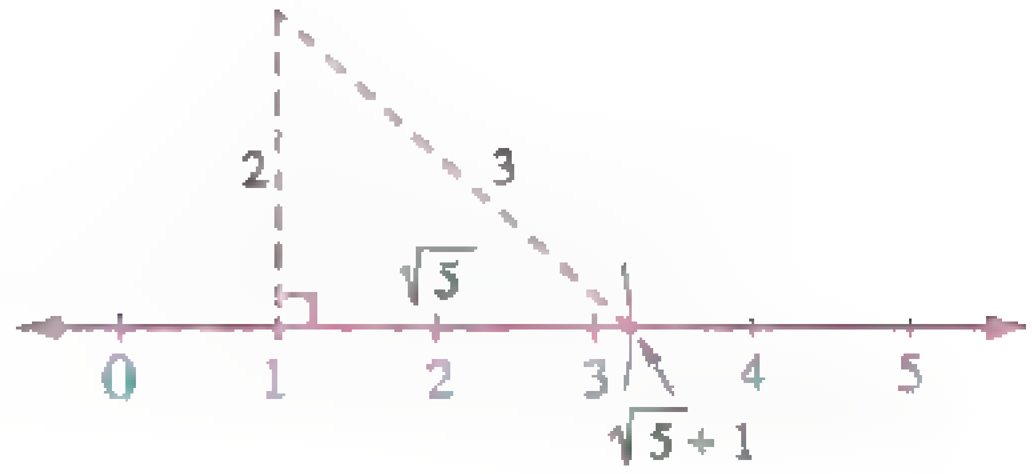
- We can represent the number $\sqrt{7}$ on the number line as follows :



4 $\because x-\sqrt{5}=1$ $\therefore x=1+\sqrt{5}$

$\therefore$ The S.S. $=\{1+\sqrt{5}\}$

- We can represent the number $(1+\sqrt{5})$ on the number line as follows :



1

Find in $\mathbb{R}$ the S.S. of each of the following equations , then represent the solution on the number line :

$\boxed{1}$ $2x+5=4$ $\boxed{2}$ $\sqrt{5}\,x-1=4$ $\boxed{3}$ $x-\sqrt{3}=2$

Solving inequalities of the first degree in one unknown in $\mathbb{R}$

- Each of the inequalities :
  - $2x<5$
  - $3x+2\leq 1$
  - $5+x>2x-1\geq 3+x$

  is called an inequality of the first degree in one unknown denoted by x

- Solving the inequality means finding all values of the unknown (x) which satisfy this inequality.
- The S.S. of the inequality in $\mathbb{R}$ will be written as an interval as will be shown later.

The methods of solving these inequalities in $\mathbb{R}$ depend on the properties of the inequality relation which will be summarized in the following :

Let a , b and c be three real numbers and assuming that $a<b$, then :

$a+c<b+c$ $\xrightarrow{\text{whether}}$ c is positive or negative (the addition property)

$ac<bc$ $\xrightarrow{\text{if}}$ c is positive (the property of multiplying by a positive real number)

$ac>bc$ $\xrightarrow{\text{if}}$ c is negative (the property of multiplying by a negative real number)

i.e. When we multiply (or divide) the two sides of an inequality by a negative number , we should change the symbol of the inequality.

Example **Find in $\mathbb{R}$ the S.S. of each of the following inequalities , then represent the solution on the number line :**

1 $2X + 6 < 2$ 2 $5 - 4X \leq -3$

Solution

1 $\because 2X + 6 < 2$ (adding the additive inverse of the number 6 (it is -6) to both sides)

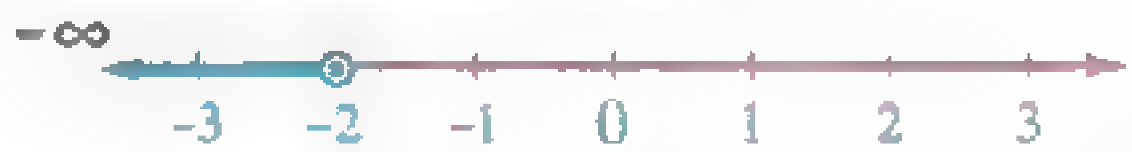
$\therefore 2X + 6 - 6 < 2 - 6$ $\therefore 2X < -4$

(multiplying both sides by the multiplicative inverse of the number 2 (it is $\frac{1}{2}$))

$\therefore 2X \times \frac{1}{2} < -4 \times \frac{1}{2}$ $\therefore X < -2$

$\therefore$ The S.S. is all the real numbers which are less than -2

i.e. The S.S. $=]-\infty , -2[$

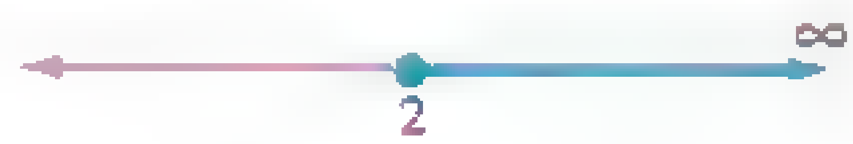


2 $\because 5 - 4X \leq -3$ (adding -5 to both sides)

$\therefore -4X \leq -8$ (dividing both sides by -4) $\therefore X \geq 2$

(Notice the change in the symbol of the inequality because we divided by a negative number)

$\therefore$ The S.S. $= [2 , \infty[$



Example **Find in $\mathbb{R}$ the S.S. of each of the following inequalities , then represent the solution on the number line :**

1 $-3 < 2X - 1 \leq 5$ 2 $3 < 3 - 5X < 13$

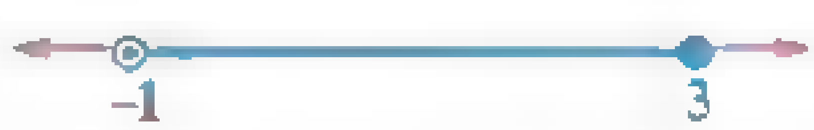
Solution

1 $\because -3 < 2X - 1 \leq 5$ (adding 1 to all sides)

$\therefore -2 < 2X \leq 6$ (dividing all sides by 2)

$\therefore -1 < X \leq 3$

$\therefore$ The S.S. $=]-1 , 3]$



2 $\because 3 < 3 - 5X < 13$ (subtracting 3 from all sides)

$\therefore 0 < -5X < 10$ (dividing all sides by -5)

$\therefore 0 > X > -2$

(Notice the change in the symbols of the inequality because we divided by a negative number)

$\therefore$ The S.S. $=]-2 , 0[$

Example 4 Find in $\mathbb{R}$ the S.S. of each of the following inequalities :

1 $x - 2 \geq 3x - 5$ 2 $x - 1 < 3x - 3 \leq x + 5$

Solution

1 $\because x - 2 \geq 3x - 5$ (adding 2 to both sides)

$\therefore x \geq 3x - 3$ (adding $-3x$ to both sides)

$\therefore -2x \geq -3$ $\left(\text{multiplying both sides by } -\frac{1}{2}\right)$

$\therefore x \leq \frac{3}{2}$ (Notice the change in the symbol of the inequality)

$\therefore$ The S.S. $= \left]-\infty , \frac{3}{2}\right]$

2 $\because x - 1 < 3x - 3 \leq x + 5$ (adding 3 to all sides)

$\therefore x + 2 < 3x \leq x + 8$ (adding $-x$ to all sides)

$\therefore 2 < 2x \leq 8$ $\left(\text{multiplying by } \frac{1}{2}\right)$

$\therefore 1 < x \leq 4$ $\therefore$ The S.S. $=]1 , 4]$

Try 2 *Find in $\mathbb{R}$ the S.S. of each of the following inequalities :*

1 $3x - 1 > 8$

2 $2 - 2x \geq -6$

3 $-16 < 5x + 4 \leq 9$

4 $2x + 1 > 4x - 3 > 2x - 11$

UNIT TWO

Relation between Two Variables

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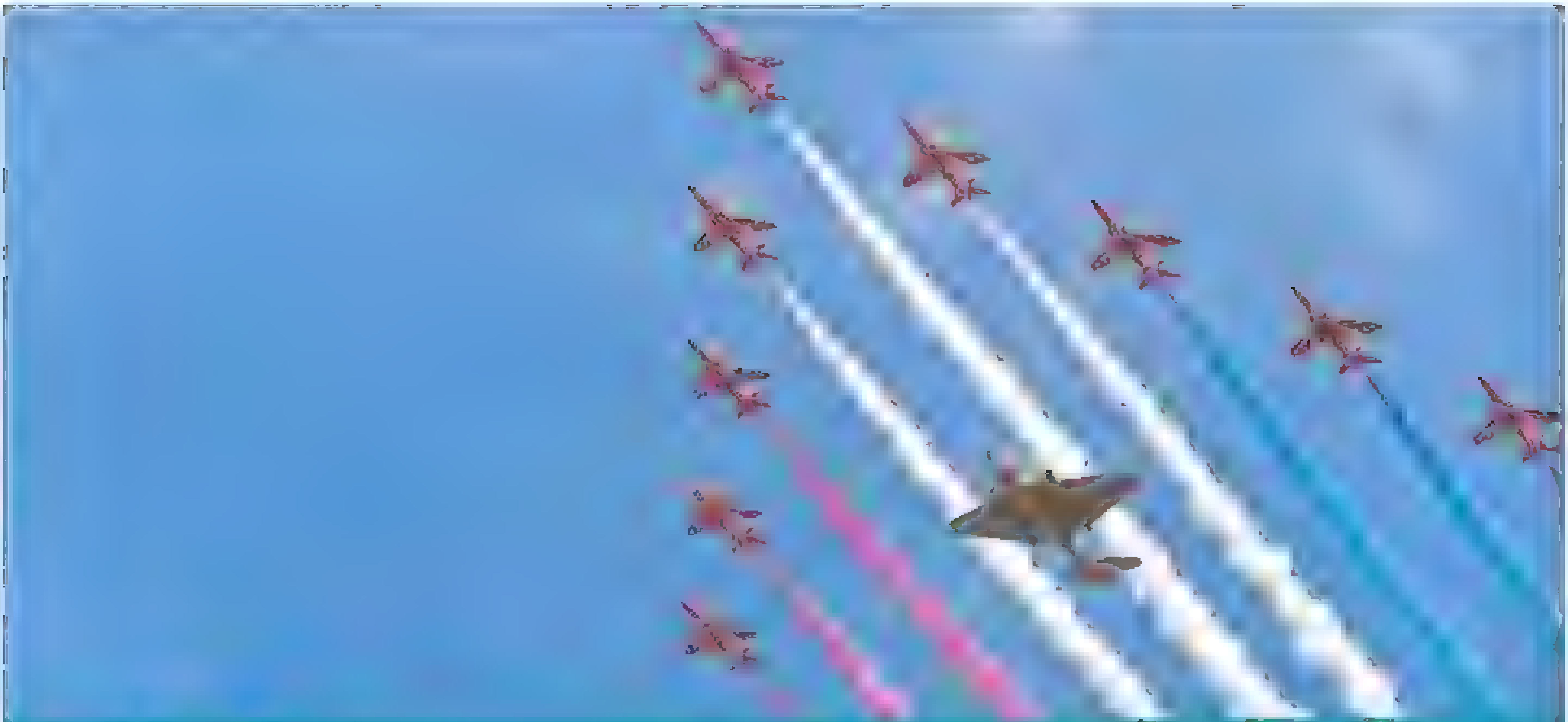
Lessons of the unit :

1. Relation between two variables.
2. Slope of straight line.
3. Real life applications on the slope.

Unit Objectives : By the end of this unit, student should be able to

- recognize the relation between two variables of first degree.
- represent the relation between two variables of first degree graphically.
- recognize the slope of the straight line.
- find the slope of the straight line passing through two given points.
- recognize the slope of the straight line parallel to x-axis and the slope of the straight line parallel to y-axis.
- verify using the slope of the straight line that the three points are collinear or not
- find the uniform velocity of a car by using the slope of the straight line.
- solve applications on the slope of the straight line.



Lesson 1 Relation between two variables

The concept of the relation between two variables

- **Islam has 50 pounds. If Islam went to the amusement park , he would find two kinds of favourite games :**

The first kind	The second kind
costs 5 pounds for playing one game.	costs 10 pounds for playing one game.

- What are the different possibilities for playing the two kinds such that he spends all his money ?
- **To find all the possibilities :**
  - Assume that he will play x games of the first kind and y games of the second kind.
  - Then the cost of playing the first kind is $5x$ pounds and the cost of playing the second kind is $10y$ pounds.
  - In order to spend all his money , it should be : $5x + 10y = 50$
  - This is an algebraic relation between the two variables x and y and it is called an equation of the first degree in two variables.
- We can simplify the previous relation by dividing all terms by 5 to get an equivalent equation which is : $x + 2y = 10$

  It can be written also in the form : $2y = 10 - x$

  i.e. $y = \frac{10 - x}{2}$

$$5x + 10y = 50 \quad \div 5$$
$$x + 2y = 10$$
$$2y = 10 - x \quad \div 2$$
$$y = \frac{10 - x}{2}$$

For example:

- If Islam decided that he will not play the first kind.

  i.e. $x = 0$, then $y = \frac{10-0}{2} = 5$

  i.e. He can spend all his money by playing 5 games of the second kind.

  We express that by the ordered pair $(0 , 5)$

- If he decided to play one game of the first kind.

  i.e. $x = 1$, then $y = \frac{10-1}{2} = 4\frac{1}{2}$

  but in this case , he cannot play $4\frac{1}{2}$ games of the second kind because the number of games must be a natural number.

- If he decided to play two games of the first kind

  i.e. $x = 2$, then $y = \frac{10-2}{2} = 4$

  i.e. He can spend all his money by playing 2 games of the first kind and 4 games of the second kind . We express that by the ordered pair $(2 , 4)$

Thus we can know the different possibilities and put them in a table such as the following :

Number of games of the 1st kind (x)	0	2	4	6	8	10
Number of games of the 2nd kind (y)	5	4	3	2	1	0

Remarks

- There is an infinite number of ordered pairs which satisfy the previous relation but some of them can't represent the possible numbers of each games because the number of games must be a natural number.
  - As we mentioned before $\left(1 , 4\frac{1}{2}\right)$ satisfies the relation but it is not possible to represent the number of games because $4\frac{1}{2} \notin \mathbb{N}$

    Similarly $(-2 , 6)$ satisfies the relation but it is not to be used because $-2 \notin \mathbb{N}$
- To find all the possibilities , we write the equation : $x + 2y = 10$ putting y in one hand side as : $y = \frac{10-x}{2}$

  We can also put x in one hand side as : $x = 10 - 2y$

  And the following example shows that.

The linear relation

- The linear relation is a relation of the first degree between two variables X and y , it is in the form
  $a\,X + b\,y = c$, where a , b and c are real numbers , a and b are not both equal to zero
- There is an infinite number of ordered pairs which satisfy this relation.
- If we represent it graphically , the graph will be a straight line therefore it is called a linear relation , this will be shown later when we study the graphic representation of the linear relation.

Example 1 Find three ordered pairs satisfying each of the following relations :

1 $3X + y = 5$

2 $3X - 2y = 6$

3 $2X = 3$

4 $y = -2$

Solution We can find these ordered pairs by setting a value for X and substituting in the relation to get its corresponding value of y or we do the converse :

1
- Set $X = 0$ $\therefore 3 \times 0 + y = 5$
  $\therefore y = 5$ $\therefore (0 , 5)$ satisfies the relation.
- Set $X = 1$ $\therefore 3 \times 1 + y = 5$
  $\therefore y = 5 - 3 = 2$ $\therefore (1 , 2)$ satisfies the relation.
- Set $X = -2$ $\therefore 3 \times (-2) + y = 5$
  $\therefore y = 5 + 6 = 11$ $\therefore (-2 , 11)$ satisfies the relation.

2 By substituting directly as we did in 1 we can get the ordered pairs but we will present another method of solution by putting one of the two variables in one hand side alone.

$\because 3X - 2y = 6$ $\therefore -2y = 6 - 3X$ (multiply by (-1))

$\therefore 2y = 3X - 6$ $\therefore y = \frac{3X - 6}{2}$

- Set $X = 0$ $\therefore y = \frac{3 \times 0 - 6}{2} = \frac{-6}{2} = -3$
  $\therefore (0 , -3)$ satisfies the relation.
- Set $X = 1$ $\therefore y = \frac{3 \times 1 - 6}{2} = -\frac{3}{2} = -1\frac{1}{2}$
  $\therefore \left(1 , -1\frac{1}{2}\right)$ satisfies the relation.
- Set $X = 2$ $\therefore y = \frac{3 \times 2 - 6}{2} = 0$
  $\therefore (2 , 0)$ satisfies the relation.

3 $\because 2X = 3$ $\therefore X = \frac{3}{2}$ $\therefore X = 1\frac{1}{2}$

This relation will be satisfied for all ordered pairs (X , y) where $X = 1\frac{1}{2}$ whatever the value of y such as $\left(1\frac{1}{2} , 0\right)$, $\left(1\frac{1}{2} , 1\right)$ and $\left(1\frac{1}{2} , 2\right)$

4 $y = -2$

This relation will be satisfied for all ordered pairs (X , y) , where $y = -2$, whatever the value of X such as $(0 , -2)$, $(1 , -2)$ and $(2 , -2)$

TRY 1 Find four ordered pairs satisfy the relation : $3X + y = 2$

Example

Choose the correct answer from those given :

1 Which of the following ordered pairs satisfies the relation : $2X - y = 1$?

(a) $(0 , 1)$ (b) $(5 , 3)$ (c) $(3 , 5)$ (d) $(-2 , 5)$

2 If $(2 , -3)$ satisfies the relation : $2X - y = c$, then c = ...

(a) -7 (b) -1 (c) 1 (d) 7

3 If $(-2 , 1)$ satisfies the relation : $3X + by = 1$, then b = ...

(a) -7 (b) -5 (c) 5 (d) 7

4 If $(k , 2k)$ satisfies the relation : $5X - y = 6$, then k = ...

(a) -18 (b) -2 (c) 2 (d) 18

5 If $(k , -2)$ satisfies the relation : $5X + 4y = 7$, then k = ...

(a) -3 (b) $-\frac{1}{5}$ (c) $\frac{1}{5}$ (d) 3

1 (c) **The reason :** By substituting each ordered pair in the given relation , we find that $(3 , 5)$ satisfies the relation as follows : putting : $X = 3$, $y = 5$

$\therefore 2X - y = 2(3) - 5 = 6 - 5 = 1$

$\therefore (3 , 5)$ satisfies the relation.

2 (d) **The reason :** $\because (2 , -3)$ satisfies the relation : $2X - y = c$

$\therefore 2(2) - (-3) = c$

$\therefore 4 + 3 = c$

$\therefore c = 7$

3 (d) **The reason :** $\because$ $(-2, 1)$ satisfies the relation : $3x + by = 1$

$\therefore 3(-2) + b \times 1 = 1$ $\qquad$ $\therefore -6 + b = 1$

$\therefore b = 1 + 6$ $\qquad$ $\therefore b = 7$

4 (c) **The reason :** $\because$ $(k, 2k)$ satisfies the relation : $5x - y = 6$

$\therefore 5k - 2k = 6$ $\qquad$ $\therefore 3k = 6$

$\therefore k = 2$

5 (d) **The reason :** $\because$ $(k, -2)$ satisfies the relation : $5x + 4y = 7$

$\therefore 5k + 4(-2) = 7$ $\qquad$ $\therefore 5k - 8 = 7$

$\therefore 5k = 15$ $\qquad$ $\therefore k = 3$

TEST 2 If $(3k, 2k)$ satisfies the relation : $x - 3y = 9$, find the value of k

The graphic representation of the linear relation

- We mentioned that linear relation between two variables x and y is usually written in the form : $ax + by = c$, where a, b and c are real numbers, a and b are not both equal to zero.
  This linear relation is represented graphically by a straight line (that is why it is called linear).
- To graph a linear relation, you need to graph at least two ordered pairs satisfying this relation. You can add a third ordered pair to check that the three points lie on the same straight line which is the graphic representation of the relation.

Example 1 **Represent the relation : $2x - y = 3$ graphically**

Solution

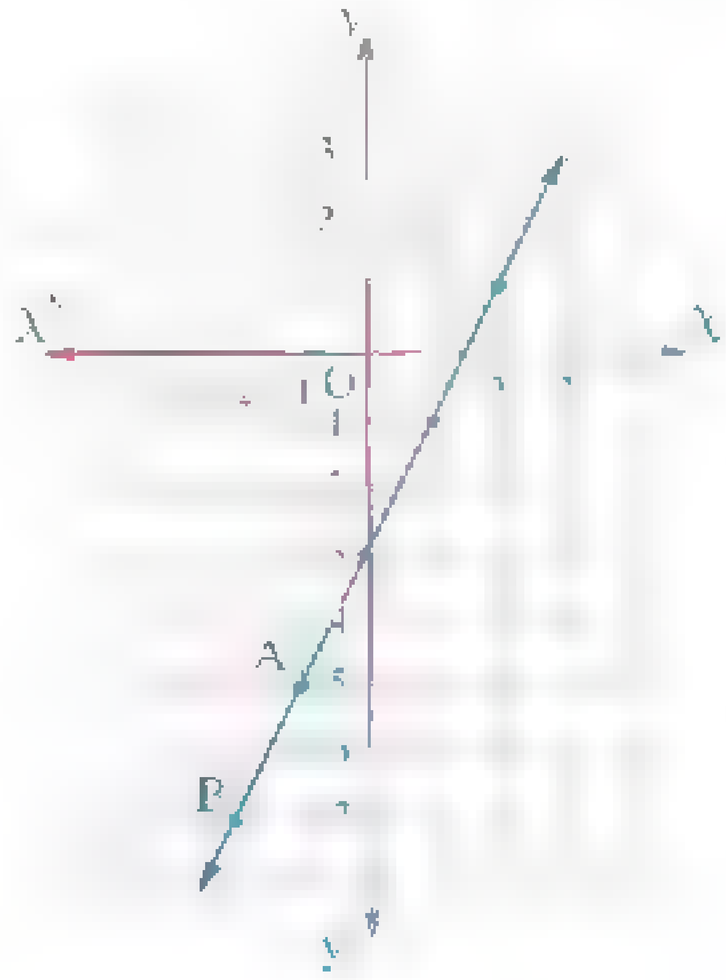
To represent this relation graphically, we should determine three ordered pairs satisfying the relation : $2x - y = 3$, as follows :

- Set $x = 0$ $\qquad$ $\therefore 2 \times 0 - y = 3$ $\qquad$ $\therefore -y = 3$ $\qquad$ $\therefore y = -3$
- Set $x = 1$ $\qquad$ $\therefore 2 \times 1 - y = 3$ $\qquad$ $\therefore -y = 1$ $\qquad$ $\therefore y = -1$
- Set $x = 2$ $\qquad$ $\therefore 2 \times 2 - y = 3$ $\qquad$ $\therefore -y = -1$ $\qquad$ $\therefore y = 1$

It is preferable to put the values of x and y in a table as the following :

x	0	1	2
y	-3	-1	1

Then we determine the points which represent these ordered pairs : $(0 , -3)$, $(1 , -1)$ and $(2 , 1)$ on orthogonal coordinates system , then we draw the straight line passing through these points , it will be the graphic representation of the relation : $2X - y = 3$



Remark

All the points of the straight line which represents the relation determine ordered pairs which satisfy the relation.

For example:

The point A determines the ordered pair $(-1 , -5)$ which satisfies the relation when we put $X = -1$ we find that $2 \times (-1) - y = 3$ i.e. $y = -5$ and also the point B $(-2 , -7)$

3 ***Represent the relation :*** $y - 2X = -1$ graphically.

Special cases

We studied before the relation : $aX + by = c$, where a , b are not both equal to zero and it is called a linear relation and it is represented graphically by a straight line and now we study the following cases :

1 If $a = 0$, $b \neq 0$

Then the relation becomes in the form :

$$by = c$$

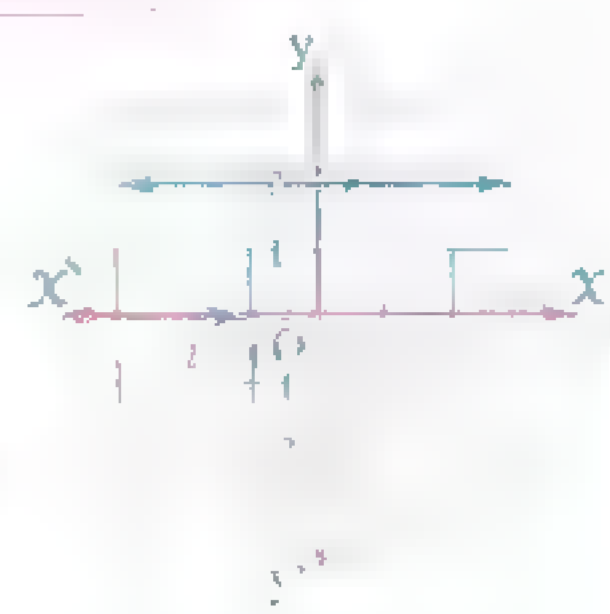
and it is represented graphically by a straight line parallel to X-axis and intersects y-axis at the point $\left(0 , \frac{c}{b}\right)$

Examples

For example :

The relation : $2y = 4$

i.e. $y = 2$ is represented by a straight line parallel to X-axis and intersects y-axis at the point $(0 , 2)$



Notice that :

The relation : $y = 0$ is represented by X-axis

2 If $b = 0$, $a \neq 0$

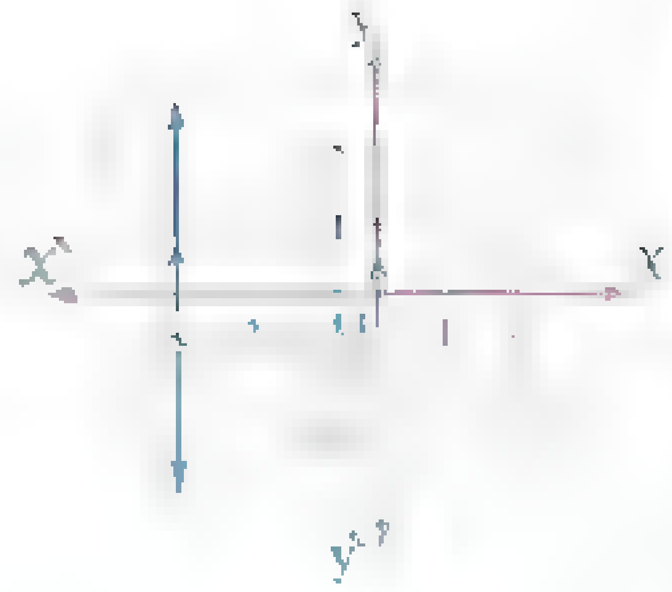
Then the relation becomes in the form :

$$a X = c$$

and it is represented graphically by a straight line parallel to y-axis and intersects X-axis at the point $\left(\frac{c}{a}, 0\right)$

For example :

The relation : $X = -3$ is represented by a straight line parallel to y-axis and intersects X-axis at the point $(-3 , 0)$



Notice that :

The relation : $X = 0$ is represented by y-axis

3 If $c = 0$

Then the relation becomes :

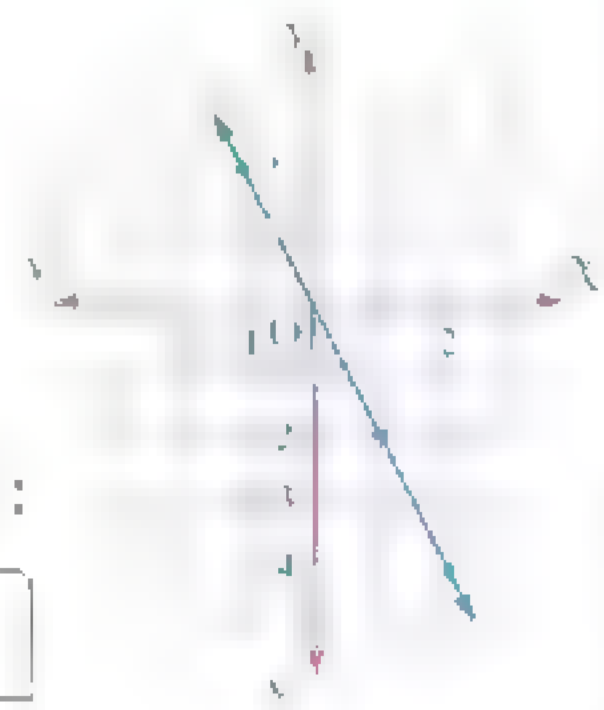
$$a X + b y = 0$$

and it is represented by a straight line passing through the origin point $(0 , 0)$

For example :

The relation : $2 X + y = 0$ is represented graphically by a straight line passing through the origin point as shown in the opposite graph :

X	1	−1	2
y	−2	2	−4



Example 1 **Graph the straight line which represents the relation : $2 X + 5 y = 10$ and if this straight line intersects X-axis at the point A and y-axis at the point B , find the area of Δ OAB where O is the origin point.**

Solution $\because 2 X + 5 y = 10$ $\qquad \therefore 2 X = 10 - 5 y$

$\therefore X = \frac{10 - 5 y}{2}$

- Set $y = 0$ $\qquad \therefore X = \frac{10 - 5 (0)}{2} = 5$

  $\therefore (5 , 0)$ satisfies the relation.

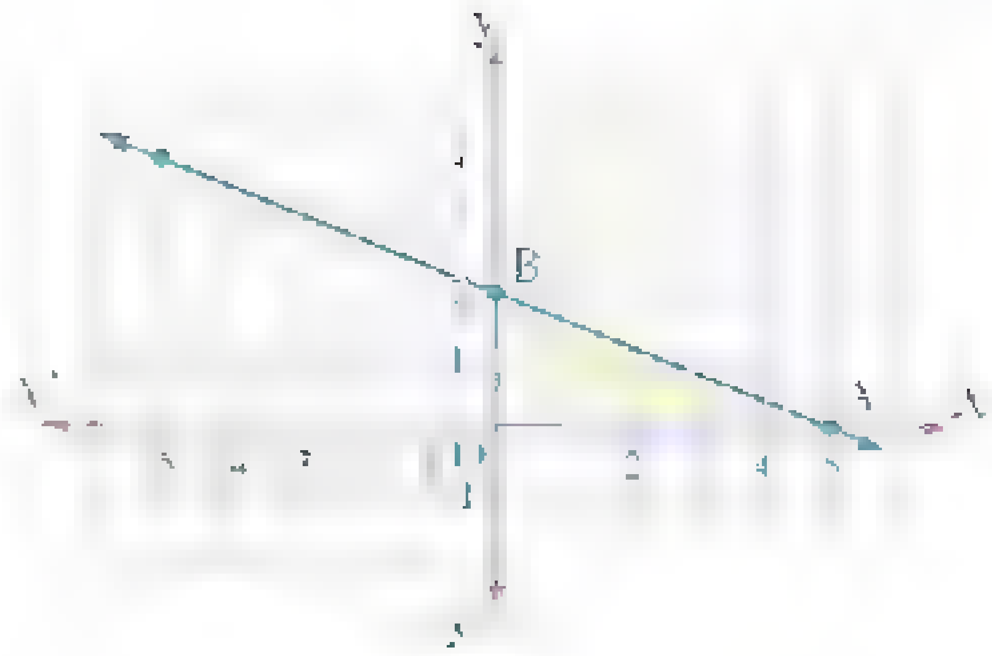
- Set $y = 2$ $\qquad \therefore X = \frac{10 - 5 (2)}{2} = 0$

  $\therefore (0 , 2)$ satisfies the relation.

- Set $y = 4$ $\qquad \therefore X = \frac{10 - 5 (4)}{2} = -5$

  $\therefore (-5 , 4)$ satisfies the relation

X	5	0	5
y	0	2	4



$\because$ The straight line intersects X-axis at the point (5 , 0)

$\therefore$ OA = 5 length units.

, $\because$ the straight line intersects y-axis at the point (0 , 2)

$\therefore$ OB = 2 length units.

$\therefore$ The area of Δ OAB = $\frac{1}{2}$ OA × OB = $\frac{1}{2}$ × 5 × 2 = 5 square units.

Remark

In the previous example , we can get the points of intersection of the straight line representing the relation : $2X + 5y = 10$ and the coordinate axes without using the graph as the following :

- Set $y = 0$ $\therefore 2X + 5 \times 0 = 10$

  $\therefore 2X = 10$ $\therefore X = 5$

  $\therefore$ The point of intersection with X-axis is (5 , 0)

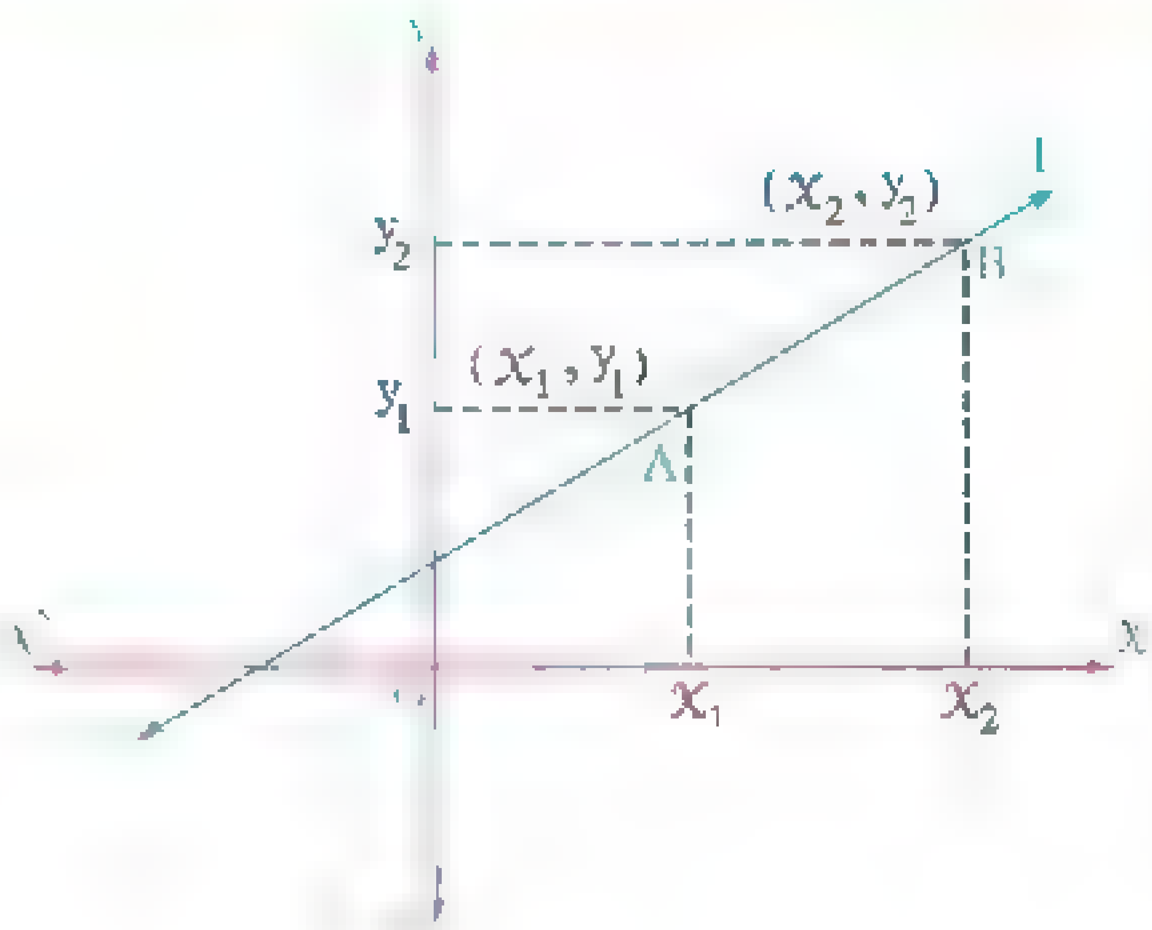
- Set $X = 0$ $\therefore 2(0) + 5y = 10$

  $\therefore 5y = 10$ $\therefore y = 2$

  $\therefore$ The point of intersection with y-axis is (0 , 2)

Lesson 2 Slope of straight line

If a point moves on a straight line L from the location A (X_1, y_1) to the location B (X_2, y_2), then :



– The change in the X-coordinates $= X_2 - X_1$

It is called (the horizontal change).

– The change in the y-coordinates $= y_2 - y_1$

It is called (the vertical change).

The ratio of the change in the y-coordinates to the change in the X-coordinates is called the slope of the straight line (S).

Definition

The slope of the straight line $= \dfrac{\text{the change in y-coordinates}}{\text{the change in } X\text{-coordinates}} = \dfrac{\text{the vertical change}}{\text{the horizontal change}}$

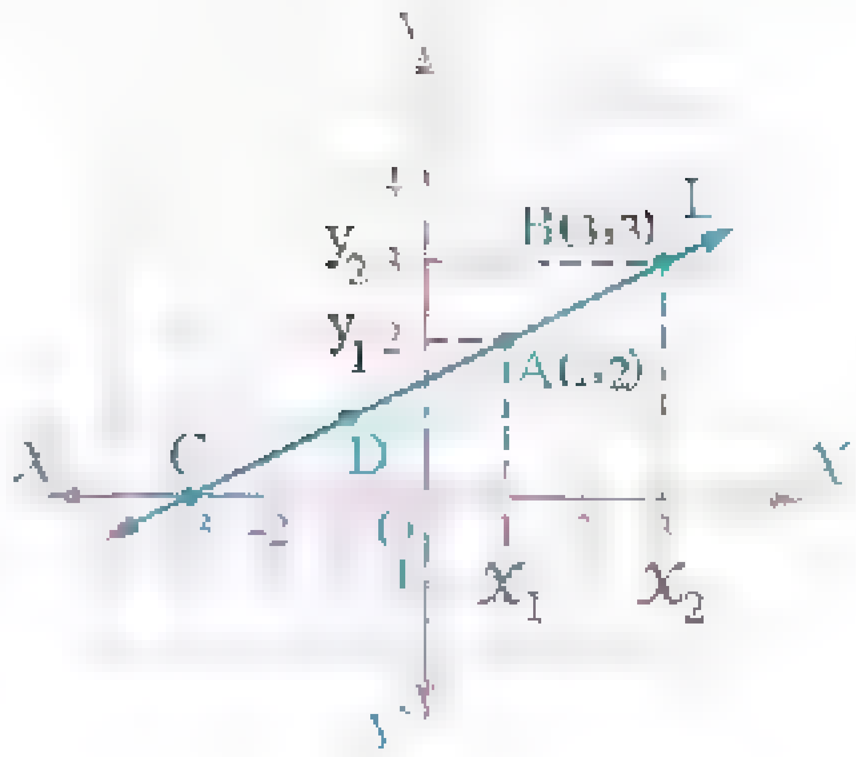
i.e. • $S = \dfrac{y_2 - y_1}{X_2 - X_1}$, where $X_1 \neq X_2$

• S is undefined if $X_1 = X_2$

Watch Video

Example In the opposite figure :

Find the slope of the straight line L



Solution We determine two points on the straight line such as $A = (1 , 2)$ and $B = (3 , 3)$

$$\because S = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore S = \frac{3-2}{3-1} = \frac{1}{2}$$

! Remark

In the previous example , notice that if we used another two points of the straight line to find its slope as the points C (– 3 , 0) and D (– 1 , 1) we find that :

$$S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{-1-(-3)} = \frac{1}{2} \text{ (the same result)}$$

i.e. The slope of the straight line is constant for any two selected points on it

Example Find the slope of the straight line passing through each pair of points in the following :

1 (2 , 4) , (4 , 5)

2 (1 , 3) , (4 , 2)

3 (– 2 , – 3) , (– 4 , 1)

4 (3 , 1) , (– 1 , 0)

Solution

1 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-4}{4-2} = \frac{1}{2}$

2 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{4-1} = \frac{-1}{3}$

3 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-3)}{-4-(-2)} = \frac{4}{-2} = -2$

4 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{-1-3} = \frac{-1}{-4} = \frac{1}{4}$

TRY yourself 1

Find the slope of the straight line passing through each pair of points in the following :

1 (2 , 1) , (3 , 4)

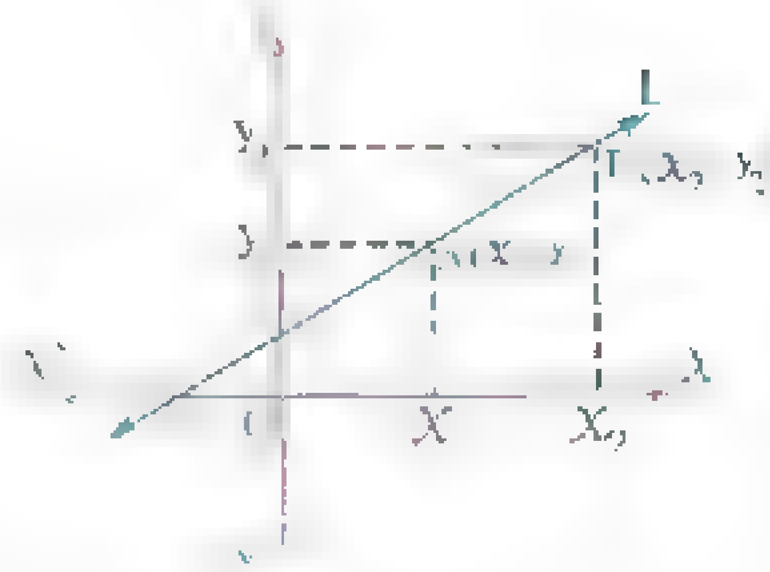
2 (3 , – 5) , (– 4 , 2)

3 (– 3 , – 1) , (1 , 0)

4 (– 6 , 3) , (– 4 , 2)

Remarks

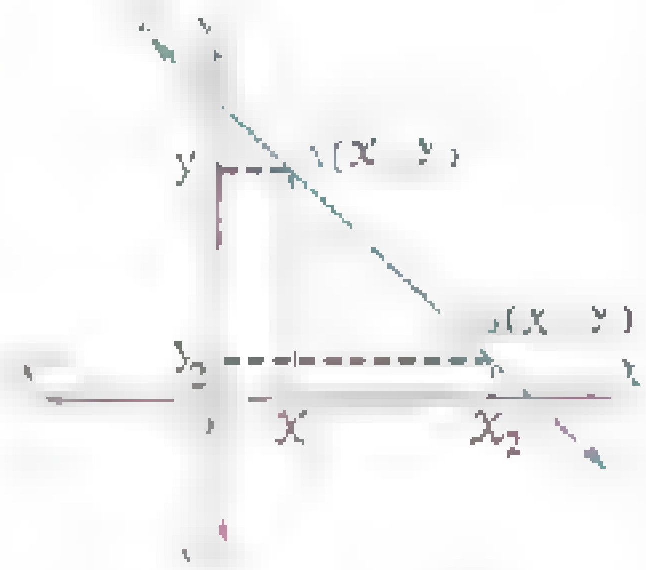
- If a point moves on a straight line from the location A (x_1, y_1) to the location B (x_2, y_2), where $x_2 > x_1$, then



1 If $y_2 > y_1$

i.e. y increases as x increases, then the slope of the straight line is a positive number.

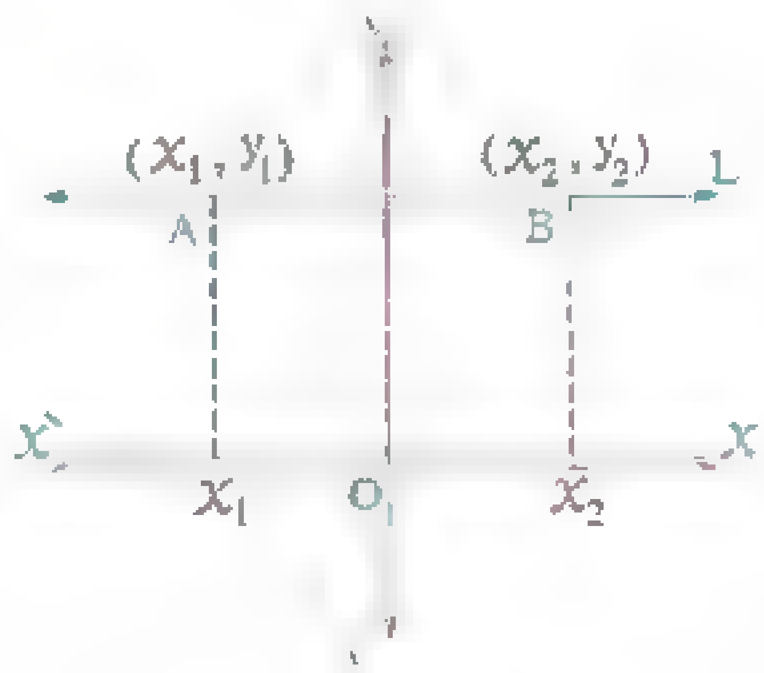
i.e. $S > 0$



2 If $y_2 < y_1$

i.e. y decreases as x increases, then the slope of the straight line is a negative number.

i.e. $S < 0$

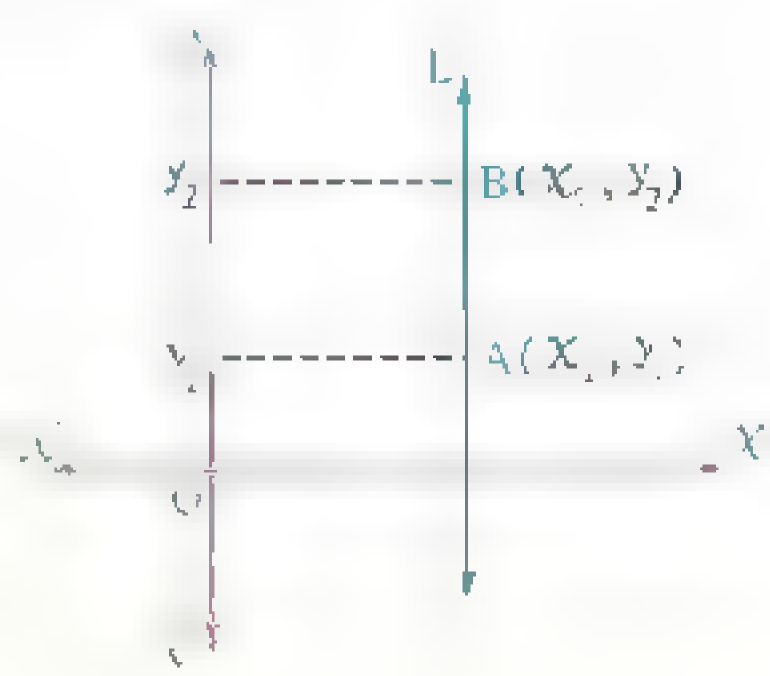


3 If $y_2 = y_1$

i.e. y is constant as x changes, then the slope of the straight line = zero

i.e. $S = 0$

i.e. The slope of the straight line parallel to x-axis = zero



4 If $x_2 = x_1$

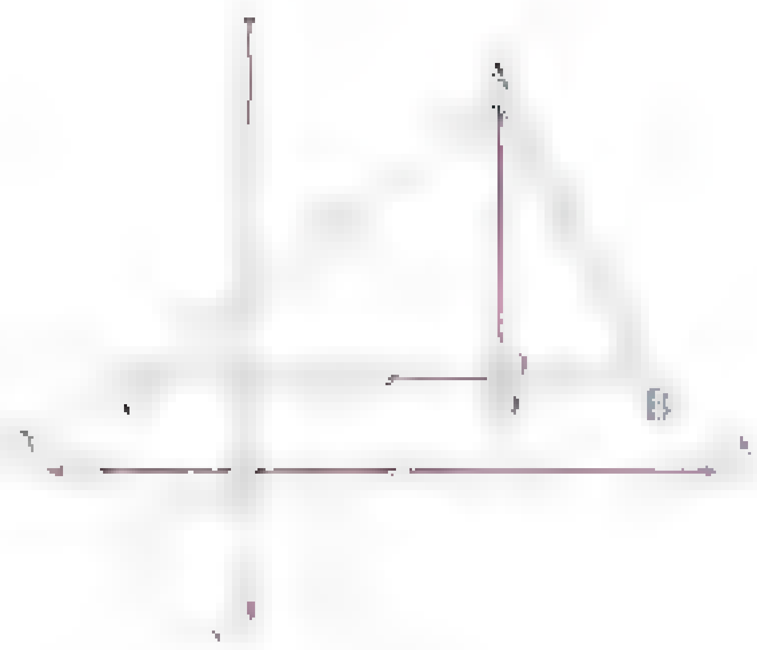
, then the slope of the straight line is undefined because there is no change in the x-axis.

i.e. $x_2 - x_1 = 0$

i.e. The slope of the straight line parallel to y-axis is undefined.

Example

In the opposite figure :



ABC is a triangle in which

$\overline{BC} \parallel \overleftrightarrow{XX'}$, $\overline{AD} \perp \overline{BC}$

Complete the following using one of the words (positive , negative , zero , undefined) in the spaces :

1 The slope of $\overleftrightarrow{AB}$ is ....

2 The slope of $\overleftrightarrow{BC}$ is ....

3 The slope of $\overleftrightarrow{AC}$ is ....

4 The slope of $\overleftrightarrow{AD}$ is ....

Solution

1 Negative

2 Zero

3 Positive

4 Undefined

Example

If the slope of the straight line passing through the two points $(-3 , 4)$ and $(1 , y)$ is 2 , find the value of y

Solution

$\because S = \frac{y_2 - y_1}{x_2 - x_1}$ $\therefore 2 = \frac{y-4}{1-(-3)}$ $\therefore 2 = \frac{y-4}{4}$

$\therefore y - 4 = 2 \times 4$ $\therefore y - 4 = 8$ $\therefore y = 12$

! An important remark

In the previous , we found that the slope of the straight line is constant and it does not change whatever the two selected points on the line , therefore to prove that the three points A , B and C are collinear , then we find the slope of $\overleftrightarrow{AB}$ and the slope of $\overleftrightarrow{BC}$

If the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$, then A , B and C are collinear.

Example

Prove that the points A (2 , 3) , B (4 , 2) and C (8 , 0) are collinear.

Solution

$\because S = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ The slope of $\overleftrightarrow{AB} = \frac{2-3}{4-2} = -\frac{1}{2}$, the slope of $\overleftrightarrow{BC} = \frac{0-2}{8-4} = \frac{-2}{4} = -\frac{1}{2}$

, $\because$ the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is common.

$\therefore$ The points A , B and C are collinear.

Example 6 **If the points A , B and C are collinear where A (3 , 2) , B (5 , – 1) and C (1 , k) , find the value of k**

Solution $\because S = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ The slope of $\overleftrightarrow{AB} = \frac{-1-2}{5-3} = \frac{-3}{2}$

, the slope of $\overleftrightarrow{BC} = \frac{k-(-1)}{1-5} = \frac{k+1}{-4}$

, $\because$ A , B and C are collinear , the slope of the straight line is constant for any two points on it.

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

$\therefore \frac{-3}{2} = \frac{k+1}{-4}$

$\therefore 2(k+1) = -3 \times (-4)$

$\therefore 2k + 2 = 12$

$\therefore 2k = 10$ $\therefore k = 5$

2

1. If the slope of the straight line passing through the two points (3 , 1) , (7 , a) is $\frac{3}{4}$, find the value of a

2. ***Prove that*** : C (– 1 , 2) $\in \overleftrightarrow{AB}$, where A (1 , 3) and B (3 , 4)

Lesson 3 Real life applications on the slope

- We studied before that if there is a linear relation between two variables x and y , then :

The slope of the straight line which represents this relation = $\frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}}$

i.e. The slope of the straight line (S) expresses the rate of change of y with respect to x

- In our life , there are many applications which we need to know the rate of change in dealing with them.

For example:

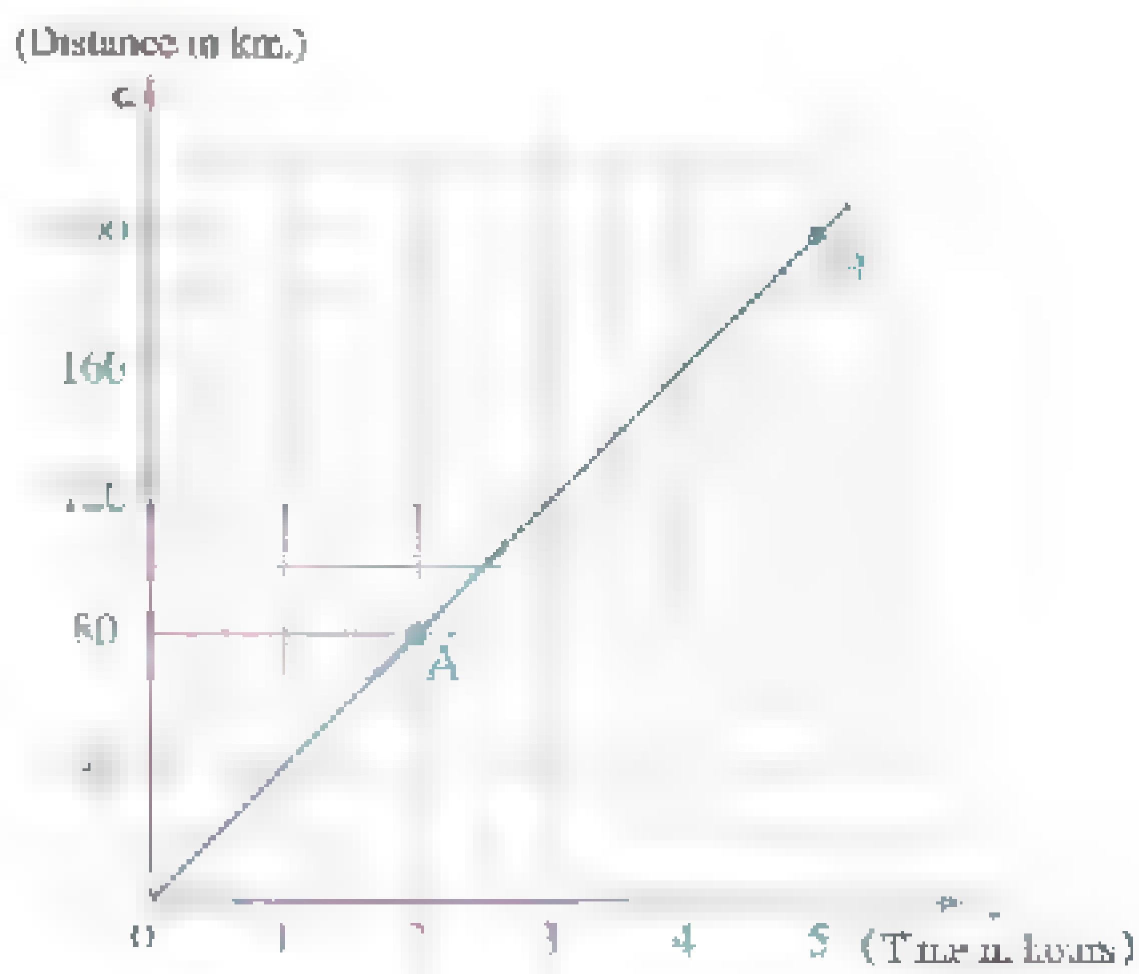
1 If the opposite graph represents the motion of a car , then :

The uniform velocity of the car (v)
= the rate of change of the distance (d) with respect to the time (t)

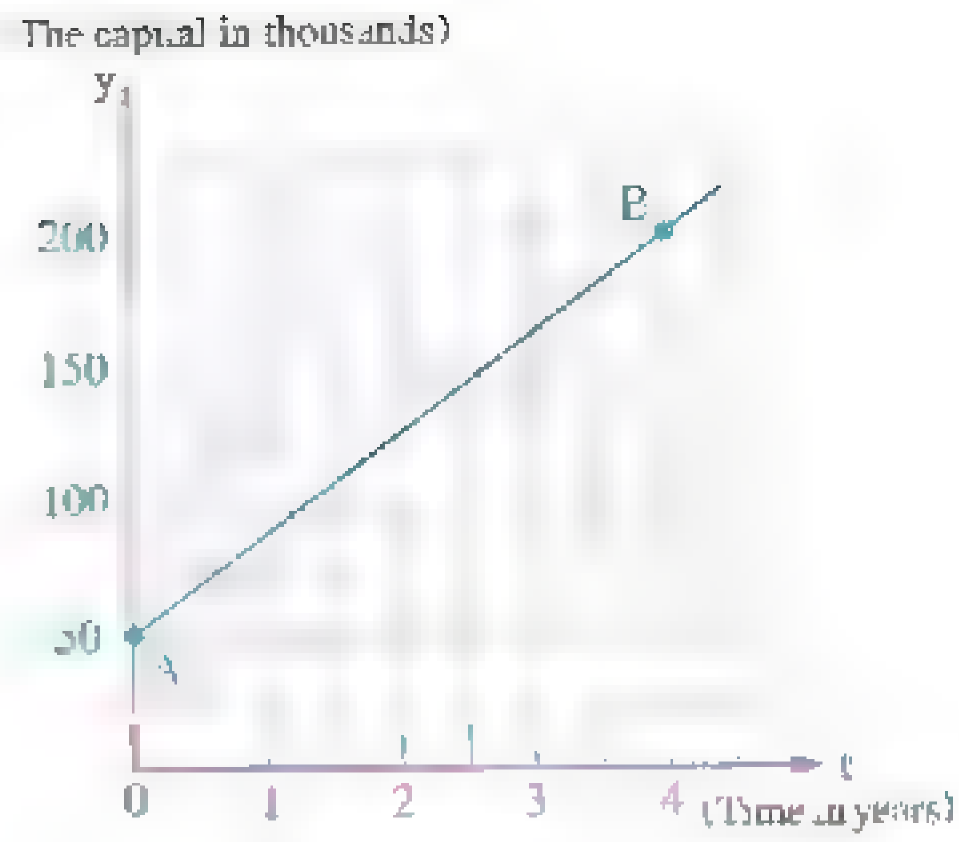
i.e. The uniform velocity of the car (v)
= the slope of the straight line (S)

and by selecting two points on the straight line as A (2 , 80) and B (5 , 200)

$\therefore v = \frac{d_2 - d_1}{t_2 - t_1} = \frac{200 - 80}{5 - 2} = \frac{120}{3} = 40$ km./hr.



If the opposite graph represents the change in the capital of a company (y) within the time (t) , then :



The rate of change in the capital of the company

= the slope of the straight line $\overleftrightarrow{AB}$

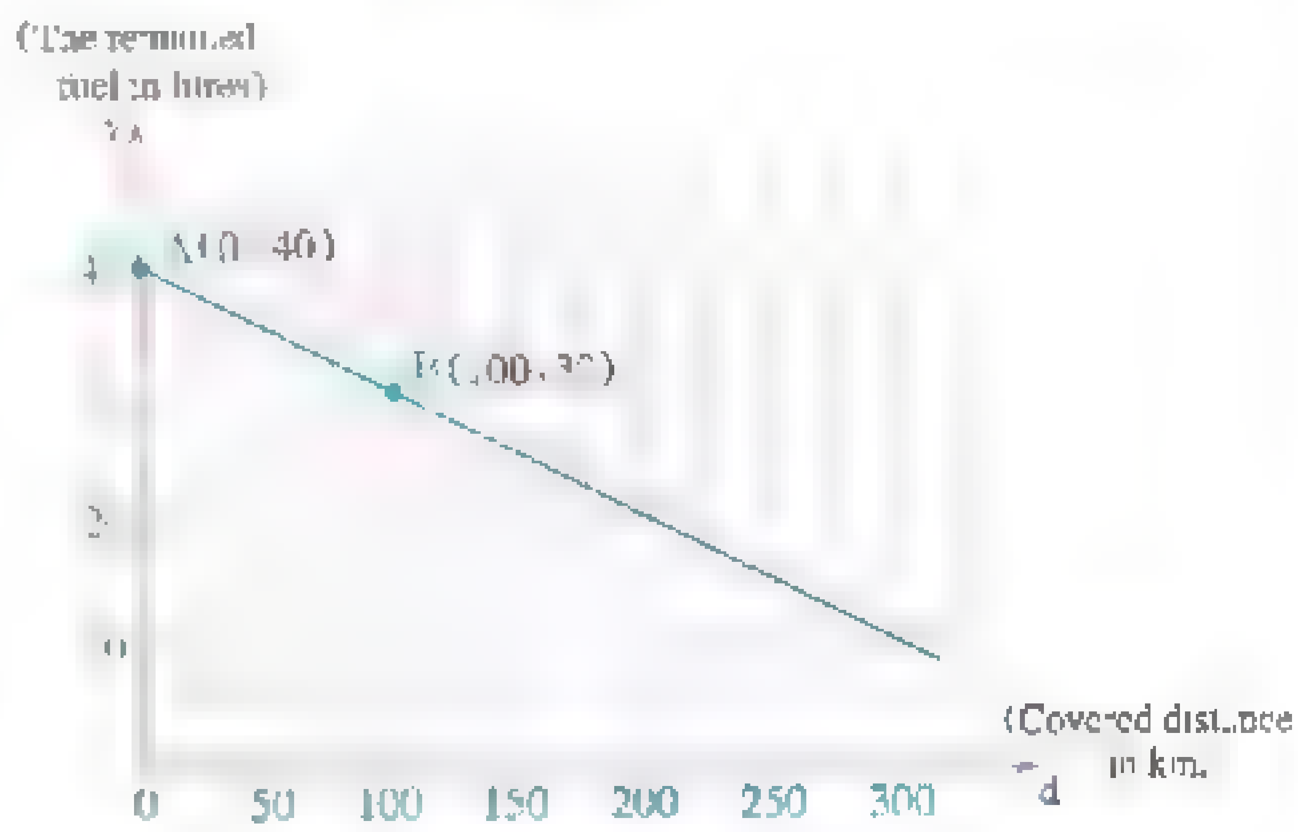
$\because$ The rate of change of the capital of the company

$$= \frac{y_2 - y_1}{t_2 - t_1} = \frac{200 - 50}{4 - 0}$$

$$= \frac{150}{4} = 37.5 \text{ thousand pounds / year.}$$

i.e. The capital of the company increases in the rate = 37.5 × 1000 = 37500 pounds/year

A person filled the tank of his car whose capacity is 40 litres with fuel. After he covered a distance 100 km. , he found that the remained fuel in the tank = 30 litres. The opposite figure shows the relation between the covered distance in km. (d) and the amount of the remained fuel in the tank in litres (y) , then :



The rate of consumption of fuel = the slope of $\overleftrightarrow{AB}$

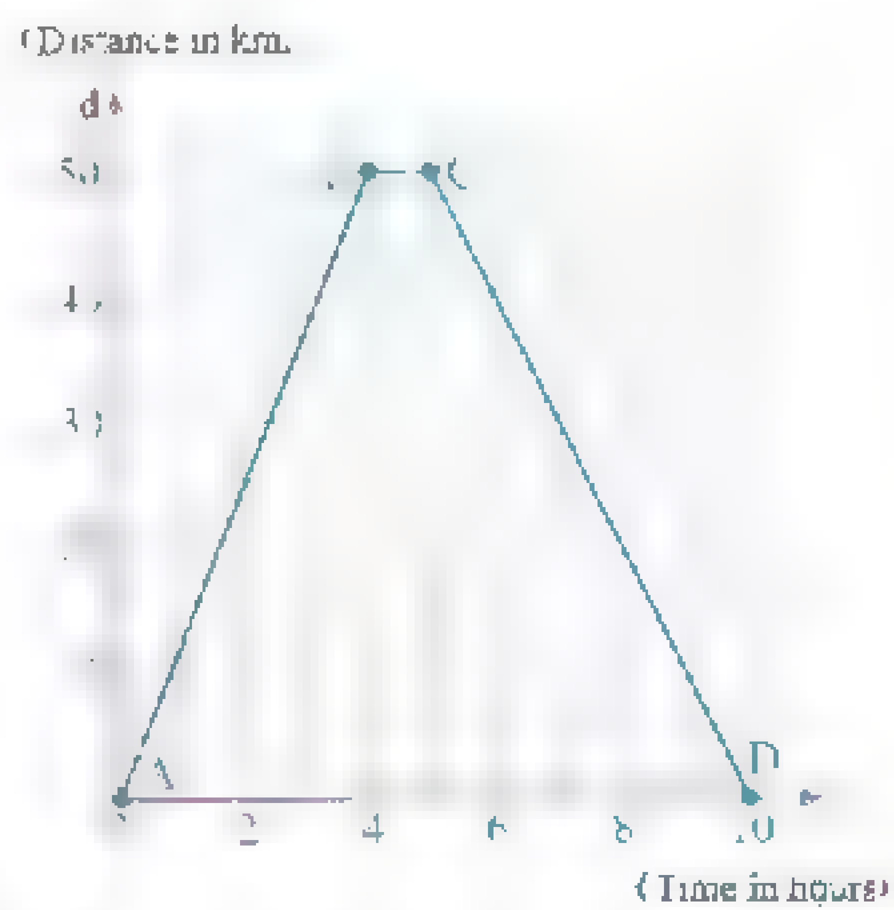
i.e. The rate of consumption of fuel $= \frac{y_2 - y_1}{d_2 - d_1} = \frac{30 - 40}{100 - 0} = \frac{-10}{100} = -\frac{1}{10}$ litre/km.

(The negative sign denotes the amount of fuel decreases in the rate of one litre for each 10 km.)

Example

Waleed rode his bicycle from Cairo to Benha , then he returned back to Cairo.

The opposite graph represents the bicycle motion during going and returning back :



1 Find his velocity in going trip.

2 Find his velocity in returning back trip.

3 Find the average velocity during all trips.

4 What do you say about the horizontal line segment in the graph ?

Solution

1 Taking the two points A $(0 , 0)$ and B $(4 , 50)$

$\therefore$ v (during going trip) $= \dfrac{50-0}{4-0} = 12.5$ km./hr.

2 Taking the two points C $(5 , 50)$ and D $(10 , 0)$

$\therefore$ v (during returning back trip) $= \dfrac{0-50}{10-5} = \dfrac{-50}{5} = -10$ km./hr.

(The negative sign means that Waleed moved in the opposite direction of his first motion returning back to Cairo with velocity 10 km./hr.)

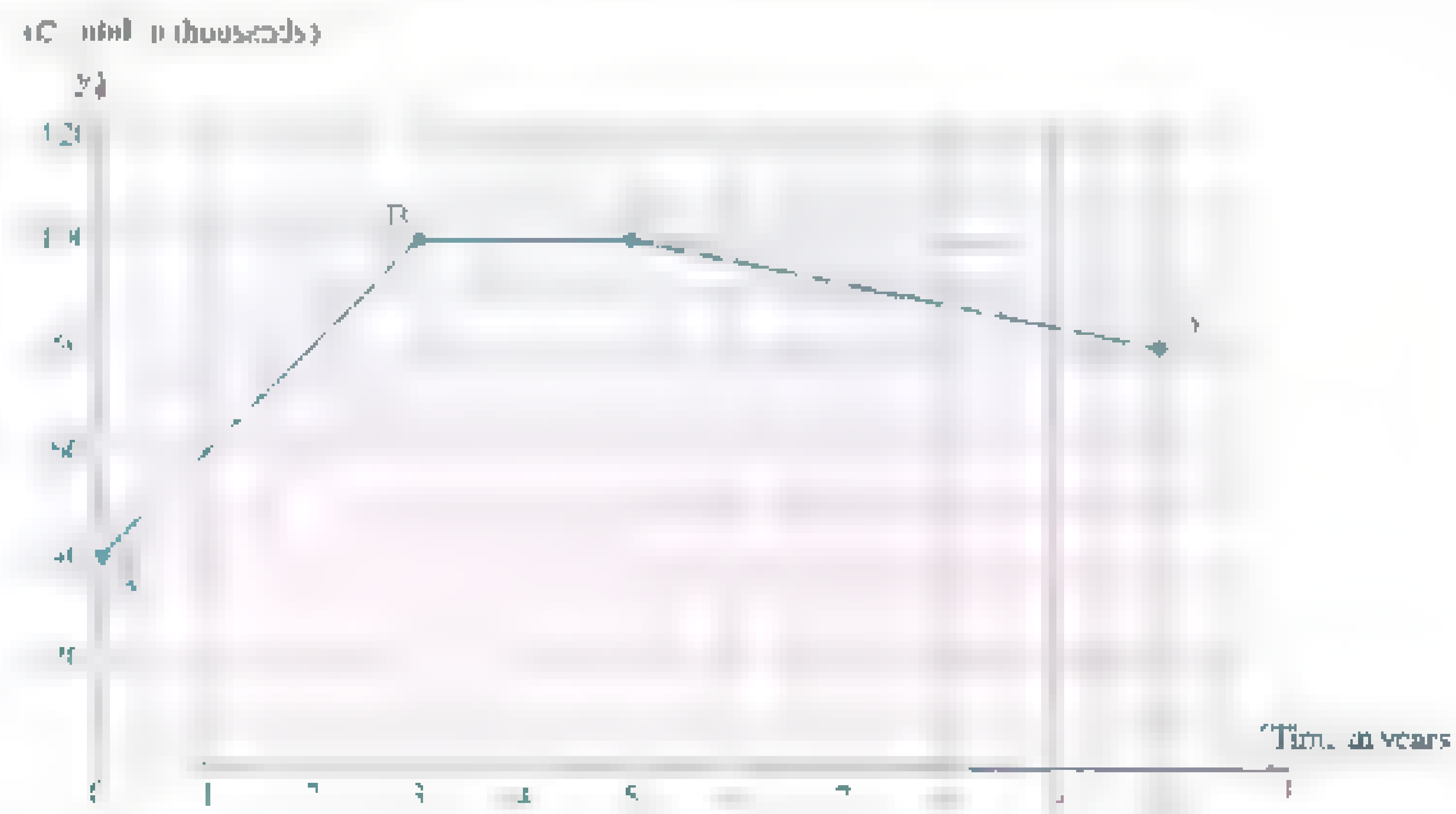
3 The average velocity $= \dfrac{\text{the total distance}}{\text{the total time}} = \dfrac{100}{10} = 10$ km./hr.

4 The horizontal line segment in the graph shows that Waleed stopped for an hour after he covered a distance equal to 50 km. , then he returned back to the start point.

Example 11 **The following graph shows the change of the capital of a company within 10 years :**

1 Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$ What is the meaning of each of them ?

2 Calculate the capital of the company at the beginning.



Solution $\because$ A $(0 , 40)$, B $(3 , 100)$, C $(5 , 100)$ and D $(10 , 80)$

1 • The slope of $\overleftrightarrow{AB} = \dfrac{100-40}{3-0} = \dfrac{60}{3} = 20$

It expresses the increase in the capital of the company within the first three years from the beginning in the rate of 20000 pounds/year

• The slope of $\overleftrightarrow{BC} = \dfrac{100-100}{5-3} = \dfrac{0}{2} = 0$

It expresses that the capital of the company is still constant without increasing or decreasing within the fourth and the fifth years from the beginning.

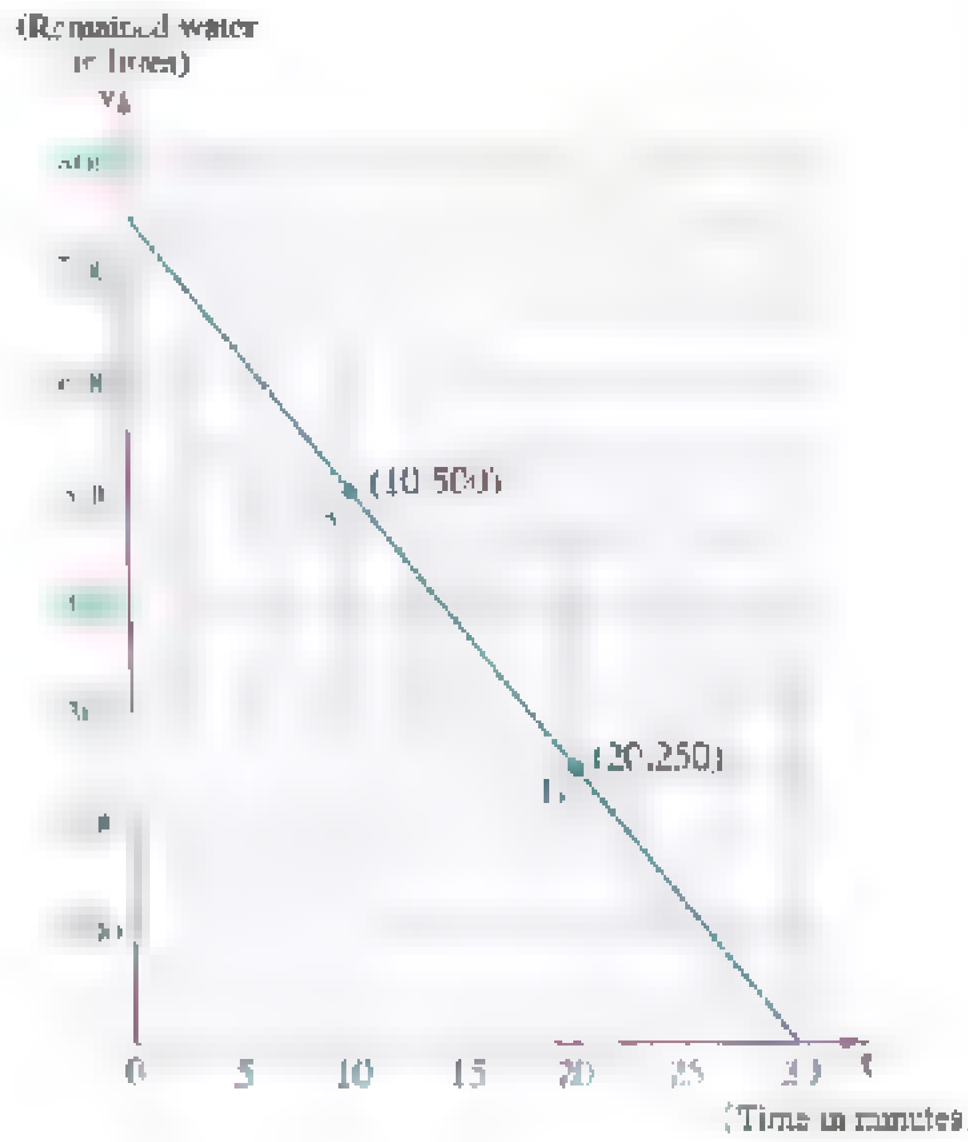
- The slope of $\overleftrightarrow{CD} = \frac{80-100}{10-5} = \frac{-20}{5} = -4$

  It expresses the decrease in the capital of the company within the last five years in the rate of 4000 pounds/year.

2 $\because$ A (0 , 40)

$\therefore$ The capital of the company in the beginning = 40000 pounds.

Example **A tank of water is filled with water completely.**
A tap is opened below the tank to empty it. The opposite graph represents the relation between the time (t) in minutes and the amount of water remained in the tank (v) in litres :



1 What is the greatest capacity of the tank ?

2 What is the time needed to empty the tank ?

3 What is the amount remained in the tank after 20 minutes ?

4 What is the rate of emptying the tank ?

Solution

1 From the graph , we find that $\overleftrightarrow{AB}$ intersects the axis which represents the amount of remained water (v) at the point (0 , 750)

$\therefore$ The greatest capacity of the tank = 750 litres.

2 From the graph , we find that $\overline{AB}$ intersects the axis which represents the time (t) at the point (30 , 0)

$\therefore$ The needed time for emptying the tank is 30 minutes.

3 $\because$ The point (20 , 250) $\in \overleftrightarrow{AB}$

$\therefore$ After 20 minutes , the remained amount of water in the tank is 250 litres.

4 The rate of emptying the tank = the slope of $\overleftrightarrow{AB}$

$$= \frac{v_2 - v_1}{t_2 - t_1} = \frac{250-500}{20-10} = \frac{-250}{10} = -25$$

$\therefore$ The tank is emptied by the rate 25 litres/minute.

UNIT THREE

Statistics

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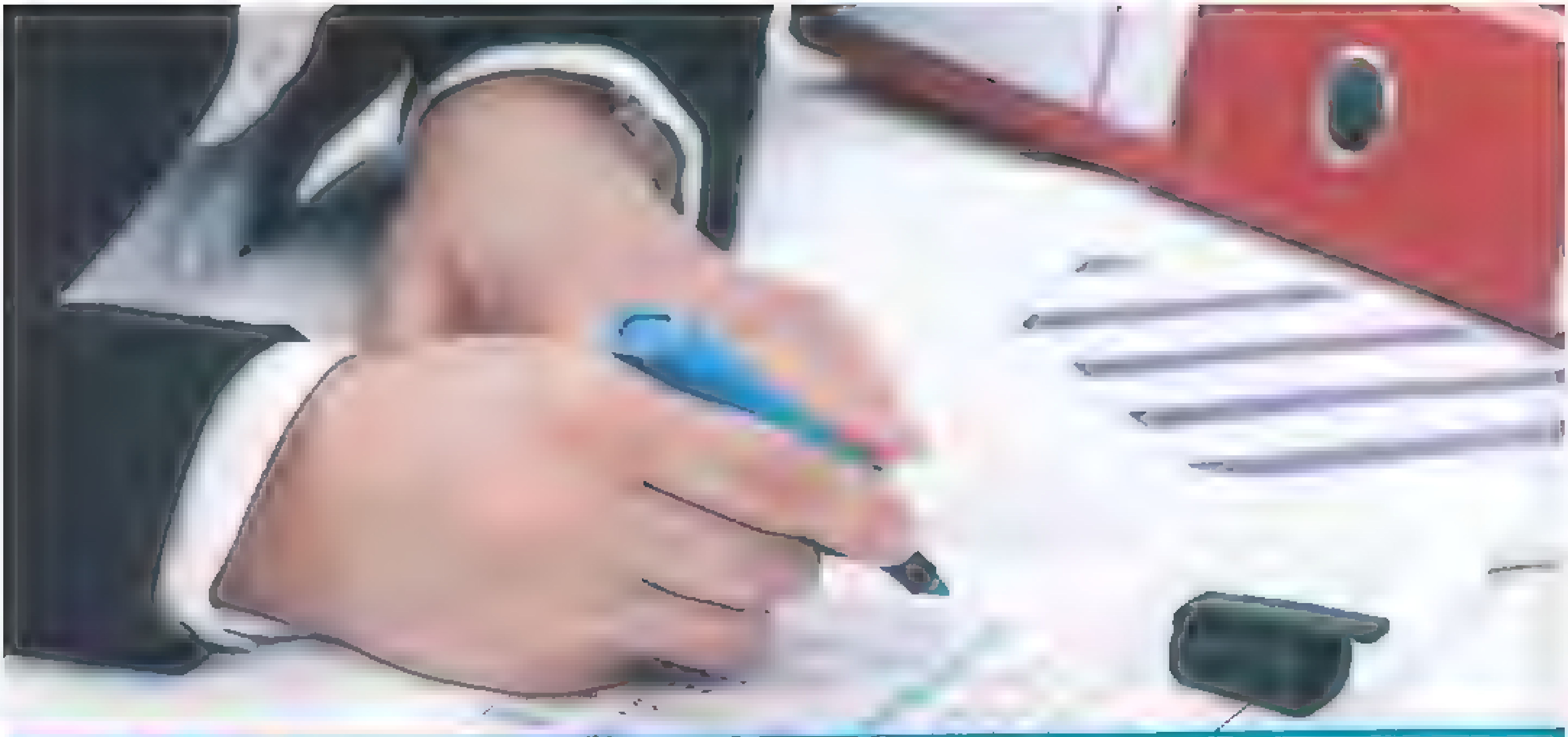
Lessons of the unit

1. Collecting and organizing data.
2. The ascending and descending cumulative frequency tables and their graphical representation.
3. Mean.
4. Median.
5. Mode.

Unit Objectives : By the end of this unit, student should be able to

- organize data in frequency tables with sets.
- form each of the ascending and descending cumulative frequency tables
- graph each of the ascending and descending cumulative frequency tables
- find the mean of a set of data organized in a frequency table with sets.
- find the median of a frequency distribution with sets
- calculate the mode from a frequency table with sets



Lesson 1 Collecting and organizing data

In the last year , you knew how to organize data and put them in a simple frequency table , but when summarizing large masses of data , it is useful to distribute them into sets , and determine the number of individuals belonging to each set.

The table consisting of sets and their corresponding frequencies is called "frequency table with sets"

The following example shows how to organize data into a frequency table with sets.

Example **In the following table , these are the marks of 54 students in one of the classes in grade two preparatory in a school , which they took in an exam in mathematics where the full mark is 60**

42	54	36	46	34	45	51	40	48
48	40	47	25	48	45	36	56	44
38	47	30	37.5	40	(20)	42	28	50
47	55	27	45	30	42	51	43	46
29	43	(59)	35	44.5	32	24	39	54
41	36	45	39	42	58	35	50	45

The required is forming the frequency table with sets.

1 Determine the range

(it is the difference between the greatest mark and the smallest mark)

$\because$ The smallest mark is 20 and the greatest mark is 59

$\therefore$ The range = 59 − 20 = 39

2 Divide these data into a suitable number of sets of marks , say 10 disjoint sets , the length of each of them is 4 , then you obtain the following sets :

- **The first set :**
  The students who obtain 20 marks till less than 24 marks , which is written as (20 –)

- **The second set :**
  The students who obtain 24 marks till less than 28 marks , it is written as (24 –)

- **The third set :**
  The students who obtain 28 marks till less than 32 marks , it is written as (28 –) and so on till you reach the tenth set.

- **The tenth set :**
  The students who obtain 56 marks till less than 60 , it is written as (56 –)

3 **Form the tally table as follows :**

Sets	Tallies	Frequency
20 –	/	1
24 –	///	3
28 –	////	4
32 –	////	4
36 –	~~////~~ //	7
40 –	~~////~~ ~~////~~	10
44 –	~~////~~ ~~////~~ //	12
48 –	~~////~~ //	7
52 –	///	3
56 –	///	3
Total		54

(The tally table)

Omit the tallies column from the table to get the final form of the frequency table with sets. It can be written vertically or horizontally.

The following is the horizontal form of the frequency table :

[illegible]	20-	24-	28-	32-	36-	40-	44-	48-	52-	56-	Total
[illegible]	1	3	4	4	7	10	12	7	3	3	54

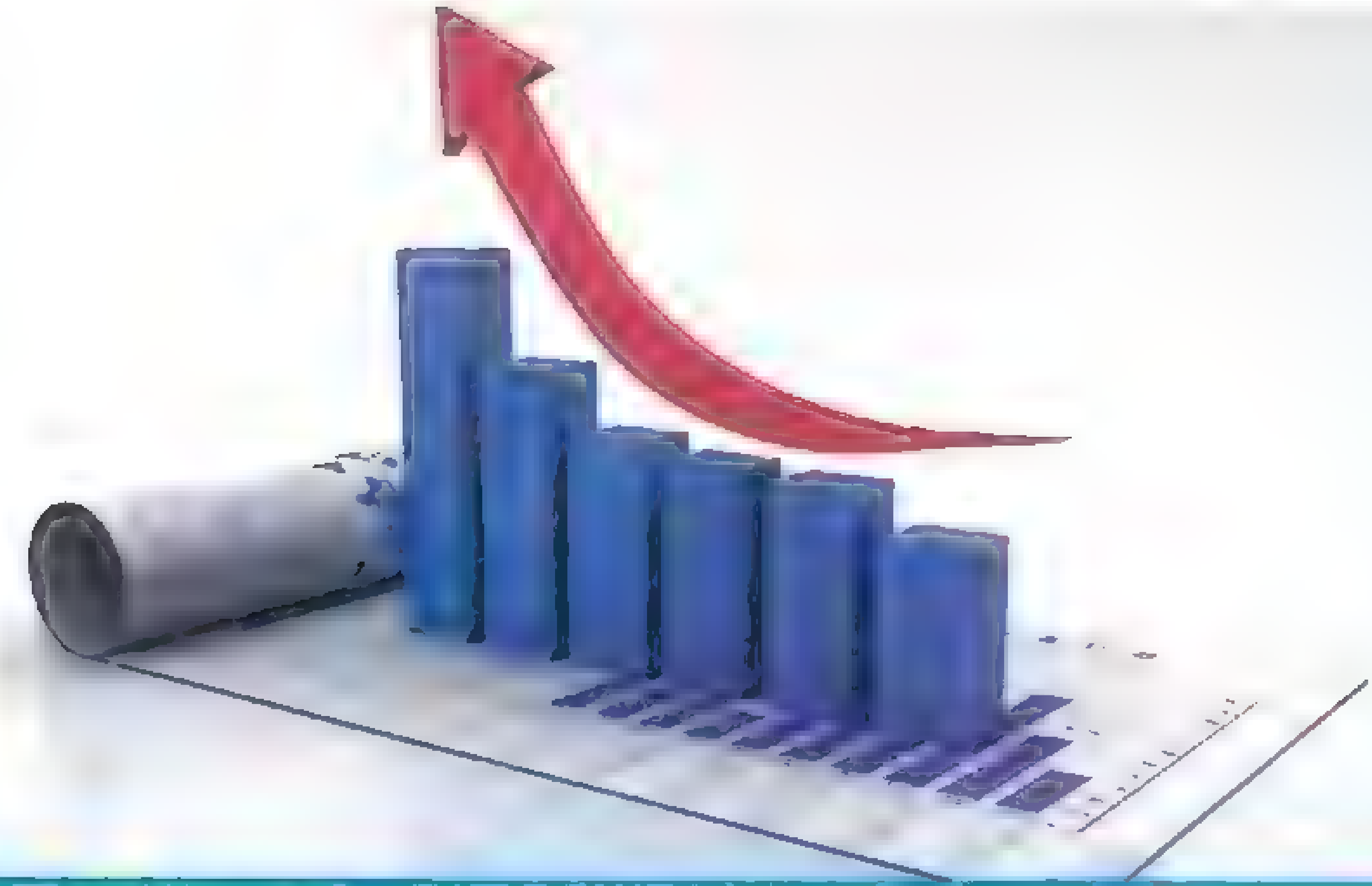
From the previous table , we deduce that :

- The set that has the greatest frequency is 44 –
- The set that has the least frequency is 20

The following is the weights of 50 persons :

52	35	40	57	43	40	36	49	43	58
47	48	51	30	59	36	45	41	44	37
42	54	38	55	42	47	46	34	53	44
47	32	41	62	50	39	58	46	43	49
40	41	64	44	54	45	38	40	48	41

Form the frequency table with sets.



Lesson 2 The ascending and descending cumulative frequency tables and their graphical representation

Prelude

- In the previous lesson , you learnt how to form a frequency table with sets and how to get some information from it as the following table which represents the distribution of weekly wages of 50 workers in one factory :

Sets of wages	54 -	58 -	62 -	66 -	70 -	Total
No. of workers (Frequency)	5	12	22	7	4	50

From this table , you can know the number of workers (the frequency) in each set.

For example:

– The number of workers whose wages lie between 58 and less than 62 pounds is 12 workers.

– The number of workers whose wages lie between 66 and less than 70 pounds is 7 workers.

- **But some other information cannot be obtained directly from this table such as :**

  – The number of workers who obtain wages less than 62 pounds.

  – The number of workers who obtain wages equal to 58 pounds or more.

- **In order to be able to know such information , you need to study how to form another type of tables called cumulative frequency tables (ascending and descending) and this what will be shown in the following examples :**

Example 1

The following frequency table shows the weekly wages in pounds of 50 workers in one factory :

Sets of wages	54	58	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the ascending cumulative frequency table and represent it graphically , then find :

1 The number of workers whose weekly wages are less than 60 pounds.

2 The percentage of the number of workers whose weekly wages are less than 60 pounds.

Solution

• Form the ascending cumulative frequency table as follows :

Sets of wages	54 -	58 -	62 -	66 -	70 -
Number of workers (Frequency)	5	12	22	7	4

Less than 54 = 0

Less than 58 = 5 + 0 = 5

Less than 62 = 5 + 12 = 17

Less than 66 = 5 + 12 + 22 = 39

Less than 70 = 5 + 12 + 22 + 7 = 46

Less than 74 = 5 + 12 + 22 + 7 + 4 = 50

The upper boundaries of sets	Frequency
Less than 54	zero
Less than 58	5
Less than 62	17
Less than 66	39
Less than 70	46
Less than 74	50

The ascending cumulative frequency table.

Notice that :

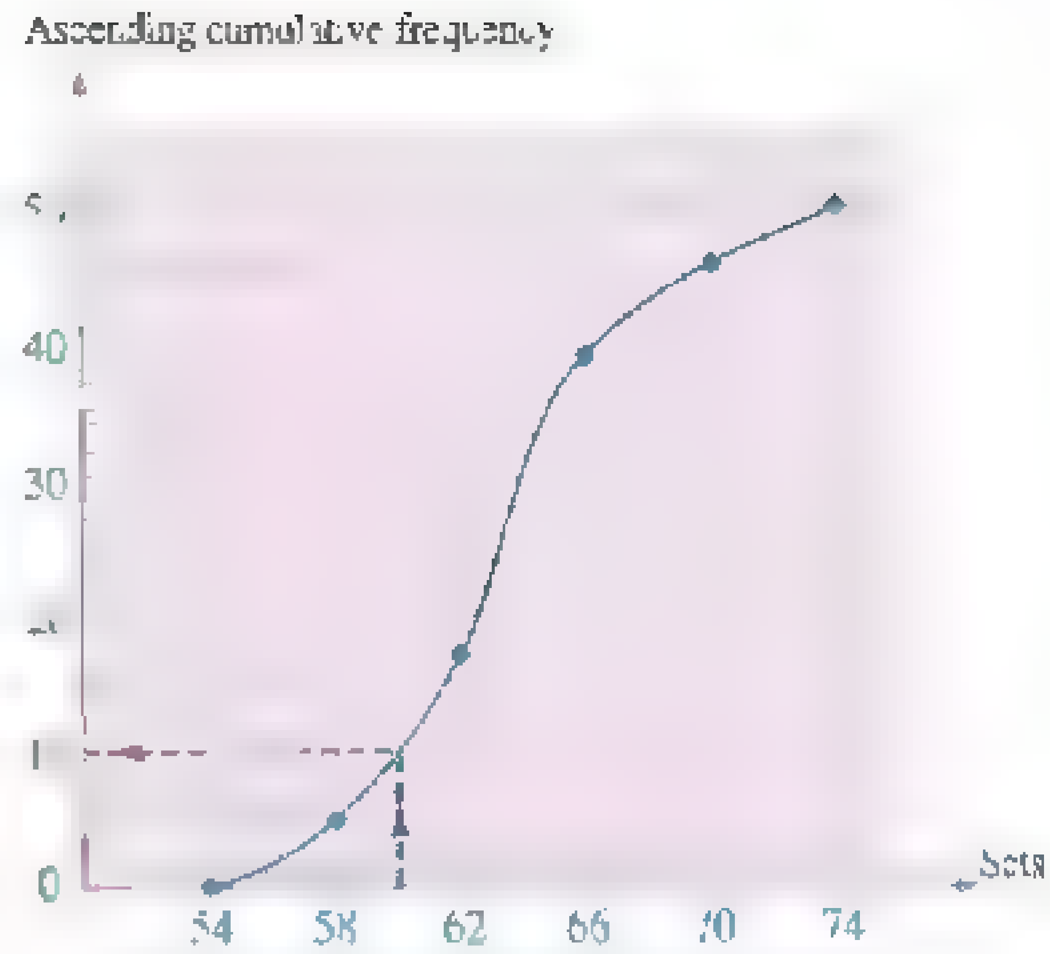
The ascending cumulative frequency begins with zero and ends at the total frequency.

To represent the ascending cumulative frequency table graphically , do as follows :

1 Specialize the horizontal axis for sets and the vertical axis for the ascending cumulative frequency.

2 Choose a suitable scale to represent data on the vertical axis so that it contains the ascending cumulative frequency easily.

Represent the ascending cumulative frequency of each set , then draw the graph (the curve) such that it passes through the points which we located as shown in the opposite figure.



The ascending cumulative frequency curve

• **From the graph , we find that :**

1 The number of workers whose weekly wages are less than 60 pounds = 10 workers.

2 The percentage of the number of workers whose weekly wages are less than 60 pounds $= \frac{10}{50} \times 100\%$

$= 20\%$

Example 2

The following frequency table shows the weekly wages of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the descending cumulative frequency table and represent it graphically , then find :

1 The number of workers whose weekly wages are 60 pounds or more.

2 The percentage of the number of workers whose weekly wages are 60 pounds or more.

Solution

• Form the descending cumulative frequency table as follows :

Sets of wages	54 -	58 -	62 -	66 -	70 -
Number of workers (Frequency)	5	12	22	7	4

54 and more = 5 + 12 + 22 + 7 + 4 = 50

58 and more = 12 + 22 + 7 + 4 = 45

62 and more = 22 + 7 + 4 = 33

66 and more = 7 + 4 = 11

70 and more = 4

74 and more = 0

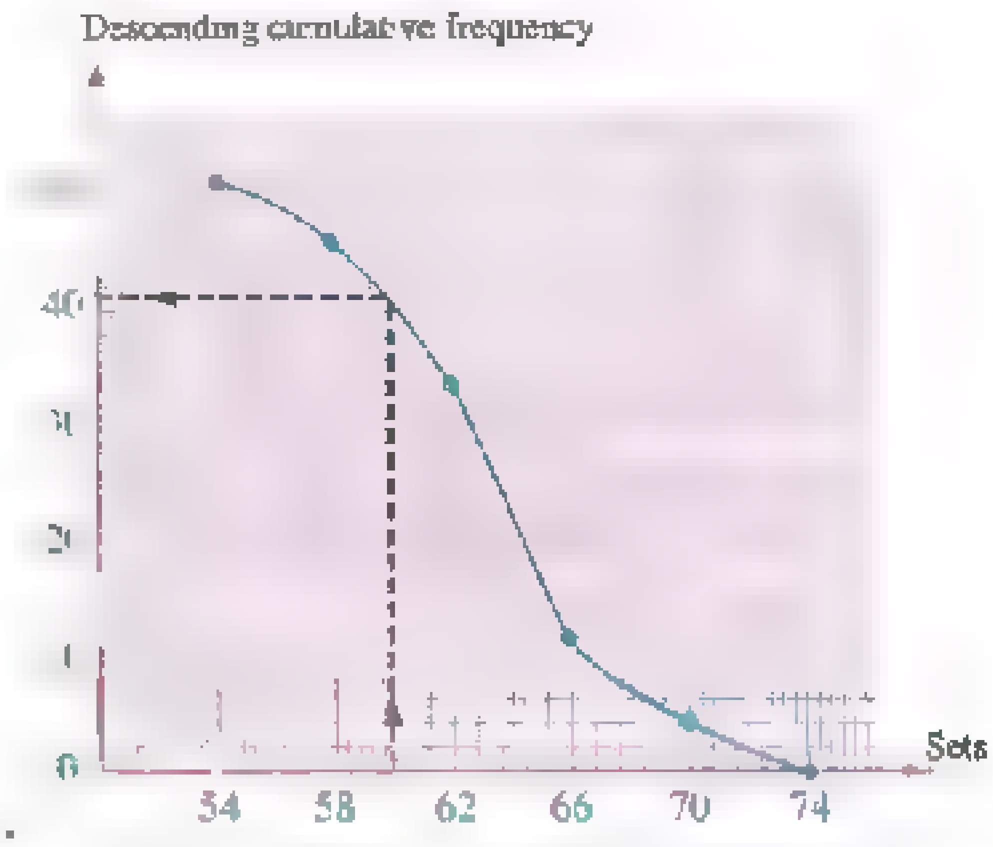
The lower boundaries of sets	Frequency
54 and more	50
58 and more	45
62 and more	33
66 and more	11
70 and more	4
74 and more	zero

The descending cumulative frequency table

Notice that

The descending cumulative frequency begins with the total frequency and ends with zero.

- To represent this table graphically , follow the same previous steps in the ascending cumulative frequency table to get the opposite graph.



The descending cumulative frequency curve

- **From the graph , we find that :**

1 The number of workers whose weekly wages are 60 pounds or more = 40 workers.

2 The percentage of those workers $= \frac{40}{50} \times 100\% = 80\%$

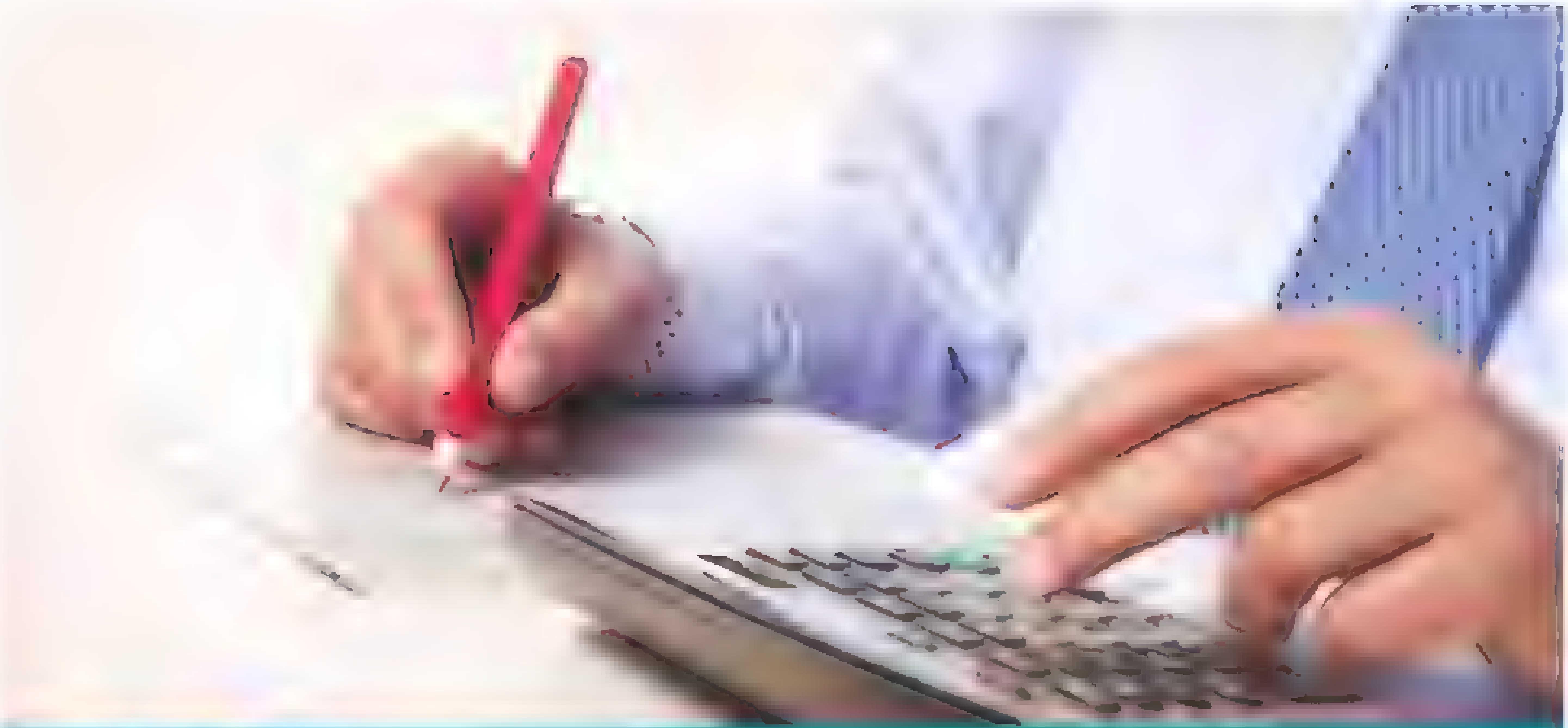
The following table shows the frequency distribution of marks of 40 students in math exam :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	4	8	12	10	6	**40**

Graph each of :

1 The ascending cumulative frequency curve.

2 The descending cumulative frequency curve.



Lesson 3 Mean

Remember that

To calculate the mean of a set of values , do as follows :

1. Find the sum of these values.
2. Divide this sum by the number of these values

i.e. The mean of a set of values = $\frac{\text{The sum of values}}{\text{Number of values}}$

For example:

If the marks of 5 students are 25 , 23 , 21 , 22 , 24

, then the mean of marks = $\frac{25 + 23 + 21 + 22 + 24}{5}$ = 23 marks.

Finding the mean of data from the frequency table with sets

Example **The following table shows the distribution of the marks of 50 students in mathematics :**

Sets	10 –	20 –	30	40	50 –	Total
Frequency	8	12	14	9	7	50

Find the mean of these marks.

1 **Determine the centres of sets according to the rule :**

$$\text{The centre of a set} = \frac{\text{the lower limit} + \text{the upper limit}}{2}$$

, then the centre of the first set $= \frac{10 + 20}{2} = 15$

, the centre of the second set $= \frac{20 + 30}{2} = 25$ ... and so on.

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

, then its centre $= \frac{50 + 60}{2} = 55$

2 **Form the vertical table :**

Set	Centre of the set « X »	Frequency « f »	$X \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

3 The mean $= \frac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \frac{1700}{50} = 34$ marks.

TRY

The following table shows the daily wages in pounds of 50 workers in a factory :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Find the mean of the wage of the worker in pounds.

Lesson 4 Median

Remember that

The median is the middle value in a set of values after arranging it ascendingly or descendingly such that the number of values which are less than it is equal to the number of values which are greater than it.

- To find the median of a set of values , do as follows :

Arrange the values ascendingly or descendingly

then

If the values number is odd

Then :

The median is the value lying in the middle exactly.

For example:

- **If the values are :**

42 , 23 , 17 , 30 , 20

- **We arrange them ascendingly as follows**

17 , 20 , 23 , 30 , 42

The median = 23

If the values number is [illegible]

Then :

$$\text{The median} = \frac{\text{The sum of the two values lying in the middle}}{2}$$

For example:

- **If the values are :**

27 , 13 , 23 , 24 , 13 , 21

- **We arrange them ascendingly as follows**

13 , 13 , 21 , 23 , 24 , 27

$$\text{The median} = \frac{21 + 23}{2} = 22$$

Unit 3

Finding the median of a frequency distribution with sets graphically

To find the median of a frequency distribution with sets graphically , do the following steps :

1. Form the ascending or the descending cumulative frequency table , then draw the cumulative frequency curve of it.
2. Find the order of the median = $\frac{\text{The total of frequency}}{2}$
3. Determine the point which represents the order of the median on the vertical axis, from this point , draw a horizontal straight line to intersect the curve at a point , then from this point , draw a perpendicular to the horizontal axis to intersect it at a point which represents the median.

The following example shows how to find the median using the two curves (the ascending or the descending cumulative frequency curve).

Example

The following table shows the frequency distribution of marks of 50 students in math exam :

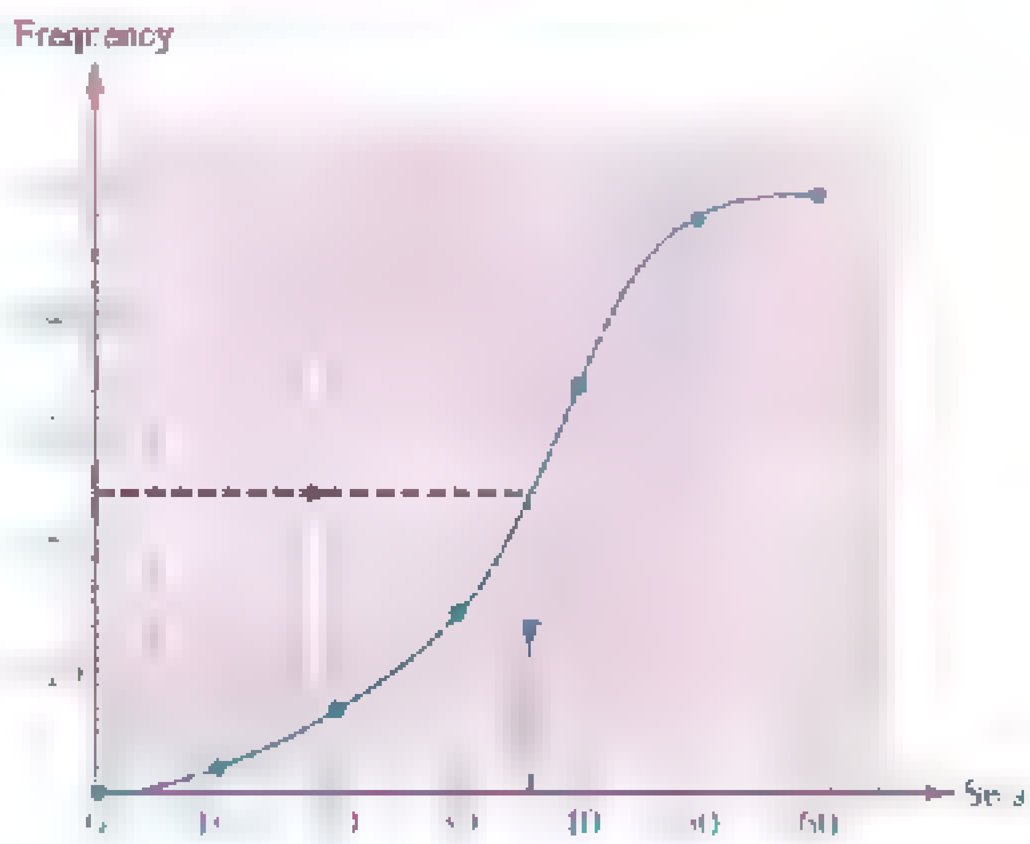
Sets of marks	0 –	10 –	20 –	30 –	40 –	50 –	Total
Number of students	2	5	8	19	14	2	50

Find the median mark of the students.

Solution

First : Using the ascending cumulative frequency curve :

The upper boundaries of sets	Frequency
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50

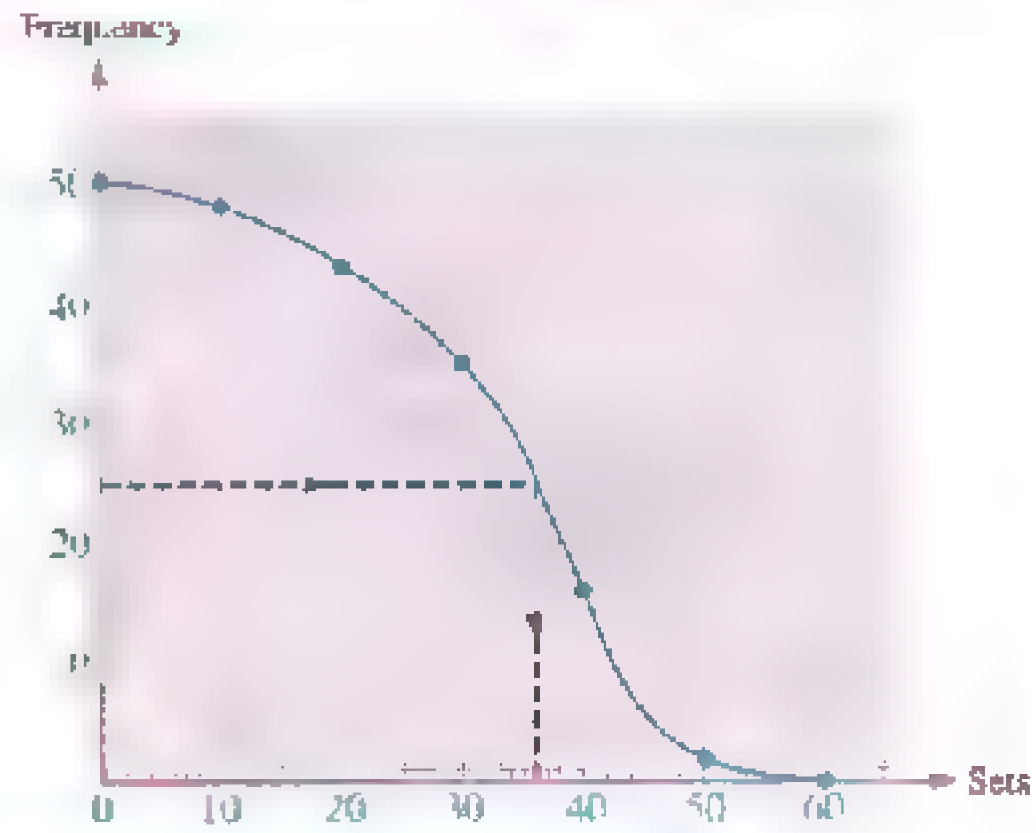


$\because$ The order of the median = $\frac{50}{2} = 25$

$\therefore$ From the graph , the median = 36 approximately

* Second : Using the descending cumulative frequency curve :

The lower boundaries of sets	Frequency
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0



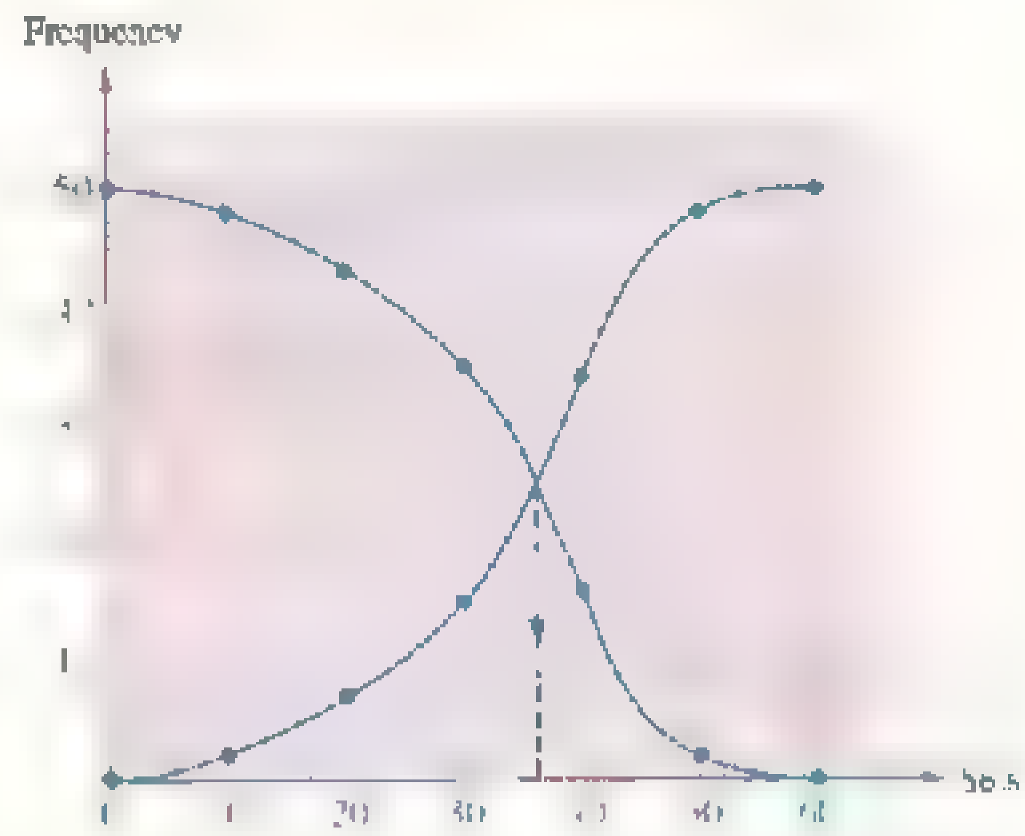
$\because$ The order of the median $= \dfrac{50}{2} = 25$

$\therefore$ From the graph , the median = 36 approximately

Remark

You can find the median by more accurate method , this by drawing the two curves (the ascending and descending cumulative frequency curves) together in one graph to intersect at one point.

From this point , draw a vertical straight line to meet the horizontal axis at a point which represents the median as shown in the opposite graph to get the median = 36 approximately.



TRY

Using the ascending or descending cumulative frequency curve , find the median of the following frequency distribution :

Sets	4 –	8 –	12 –	16 –	20 –	Total
Frequency	2	4	8	6	4	24

Lesson 5 Mode

Remember that

The mode of a set of values is the most common value in the set , or in other words , it is the value which is repeated more than any other values.

For example: The mode of the set of the values

7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is 7

Finding the mode for a frequency distribution with sets

The following example shows how to find the mode of a frequency distribution with sets :

Example **The following is the frequency distribution of marks of 100 students in an exam :**

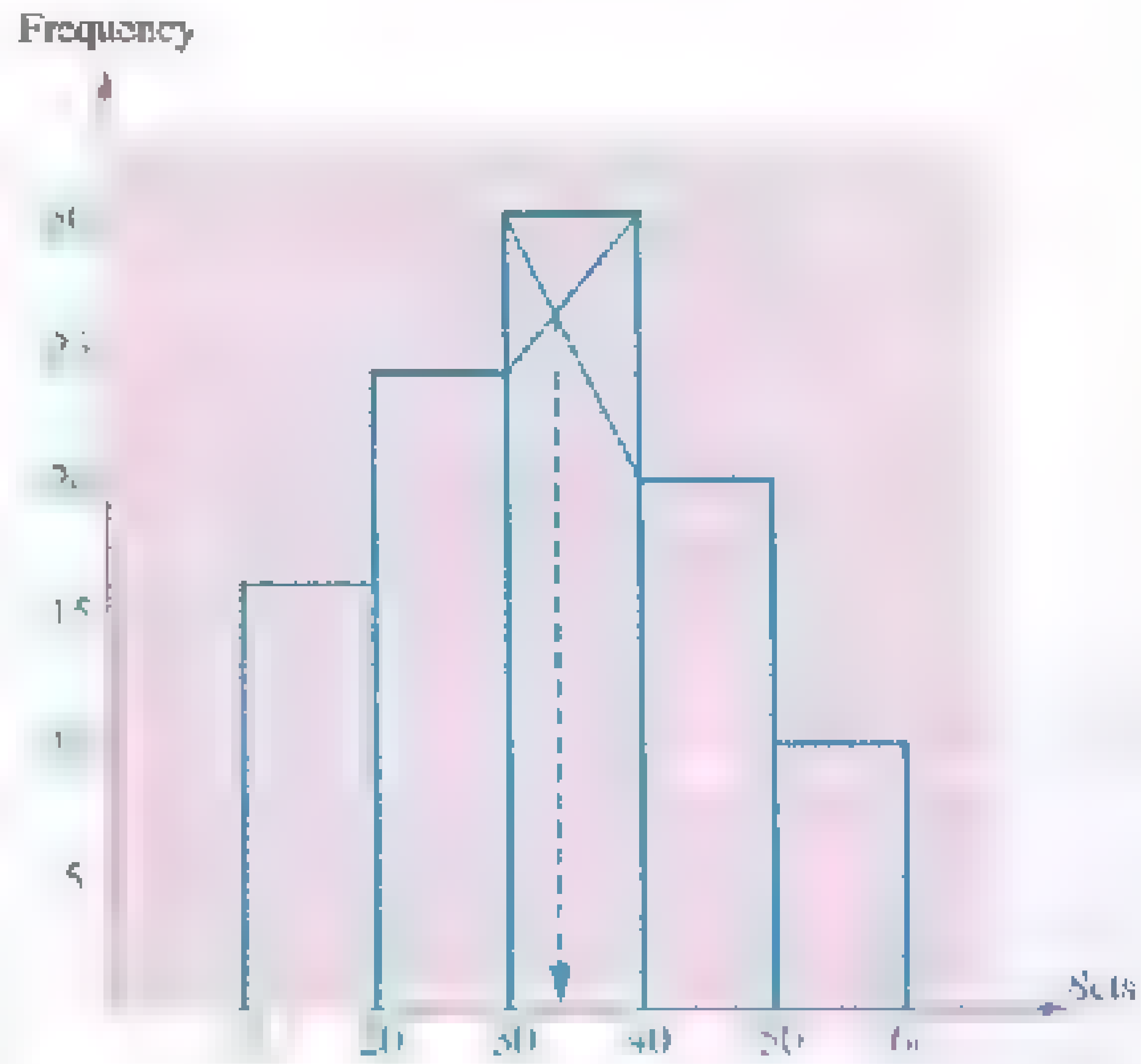
Sets of marks	10 –	20	30	40	50	Total
Number of students	16	24	30	20	10	100

Find the mode mark for these students.

Solution **You can find the mode of that distribution graphically using the histogram as follows :**

1. Draw two orthogonal axes , one of them is horizontal and the other is vertical to represent the frequency of each set.

2. Divide the horizontal axis into a number of equal parts with a suitable drawing scale to represent the sets.
3. Divide the vertical axis into a number of equal parts with a suitable drawing scale to represent the greatest frequency in the sets.
4. Draw a rectangle whose base is the set (10 –) and its height equals the frequency (16)
5. Draw a second rectangle adjacent to the first one whose base is the set (20 –) and its height equals the frequency (24)
6. Repeat drawing the remained adjacent rectangles till the last set (50 –)



7. Determine the set which has the greatest frequency then draw two lines as shown in the histogram to intersect at a point.

   From this point , draw a vertical line to intersect the horizontal axis at a point which represents the value of the mode.

   i.e. The mode mark is 34 approximately.

Find the mode for the following frequency distribution :

Sets	2 –	4 –	6 –	8 –	10	Total
Frequency	3	10	12	10	5	40

Second

Geometry

Notes

The notes found at the margin of some pages in **geometry** and referred to by (*) are theorems and corollaries have been studied before

UNIT FOUR

Medians of Triangle–Isosceles Triangle

Lessons of the unit :

1. Medians of triangle.
2. Medians of triangle "follow ".
3. The isosceles triangle.
4. The converse of the isosceles triangle theorem.
5. Corollaries of the isosceles triangle theorems.

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Unit Objectives : By the end of this unit, student should be able to :

- recognize the median of a triangle.
- recognize the intersection point of medians of a triangle and the ratio that the point divides each median
- deduce the relation between the length of the median from the vertex of the right angle in the right-angled triangle and the length of the hypotenuse.
- recognize thirty and sixty triangle.
- recognize the properties of isosceles triangle.
- recognize the properties of equilateral triangle.
- recognize the axis of symmetry of the line segment.
- recognize the axis of symmetry of the isosceles triangle
- solve miscellaneous problems on the equilateral triangle and the isosceles triangle
- appreciate the role of geometry in solving of real life problems

Lesson 1 Medians of triangle

Definition

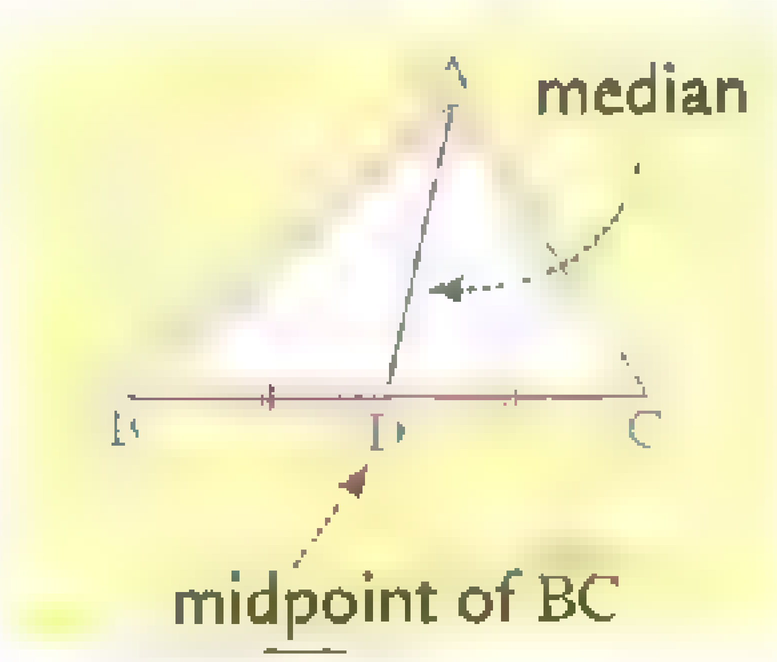
The median of a triangle is the line segment drawn from any vertex of this triangle to the midpoint of the opposite side of this vertex.

For example:

In the opposite figure :

If D is the midpoint of $\overline{BC}$

, then $\overline{AD}$ is a median of ΔABC



Notice that :

Any triangle has three medians.

Theorem 1

The medians of a triangle are concurrent.

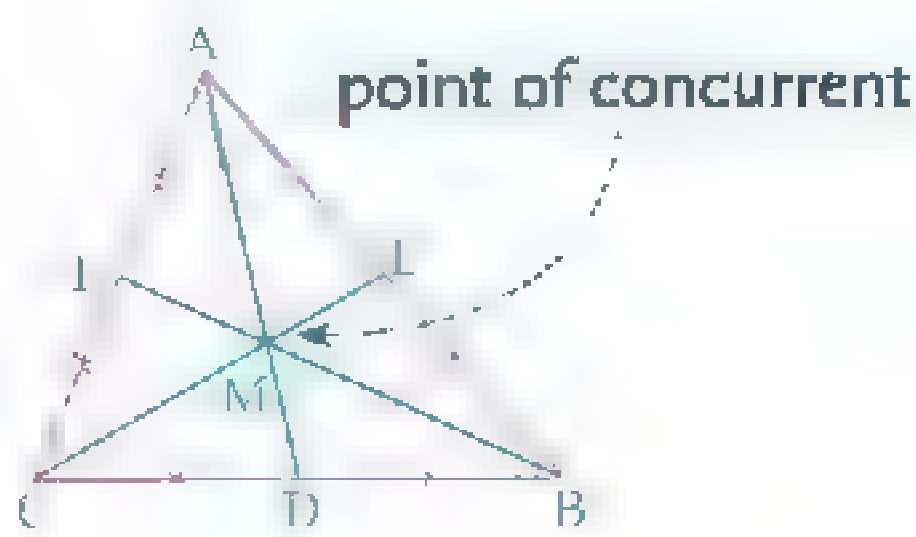
For example:

In the opposite figure :

$\overline{AD}$, $\overline{BF}$ and $\overline{CE}$ are the three medians of ΔABC ,

and they are concurrent at M

(i.e. $\overline{AD} \cap \overline{BF} \cap \overline{CE} = \{M\}$)



Example 11 **In the opposite figure :**

ABC is a right-angled triangle at B in which :

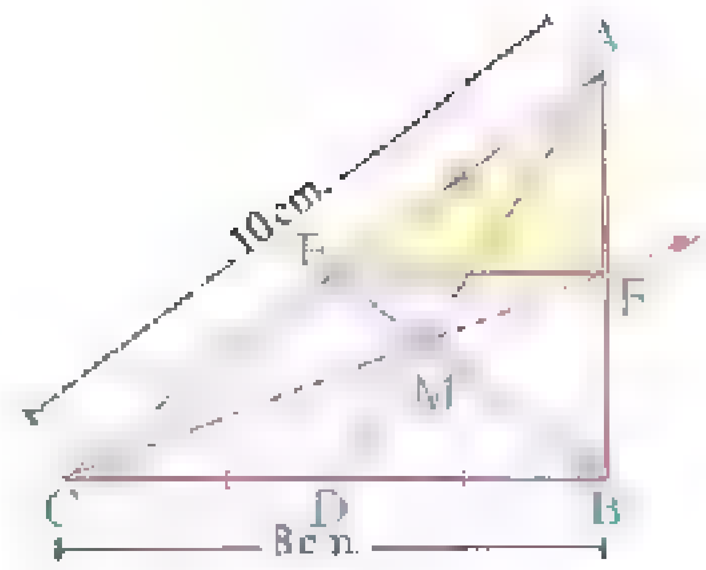
AC = 10 cm. , BC = 8 cm. ,

D and E are the midpoints of $\overline{BC}$ and $\overline{AC}$ respectively

where $\overline{AD} \cap \overline{BE} = \{M\}$

Draw $\overrightarrow{CM}$ to cut $\overline{AB}$ at F

Find the perimeter of Δ AFE



Solution

Given m ($\angle$ ABC) = 90° , AC = 10 cm. , BC = 8 cm. , D is the midpoint of $\overline{BC}$,
E is the midpoint of $\overline{AC}$

R.T.F. The perimeter of Δ AFE

Proof In Δ ABC :

$\because$ m ($\angle$ ABC) = 90°

$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 100 - 64 = 36$ (*) $\therefore$ AB = 6 cm.

, $\because$ D is the midpoint of $\overline{BC}$ $\therefore$ $\overline{AD}$ is a median in Δ ABC

, $\because$ E is the midpoint of $\overline{AC}$ $\therefore$ $\overline{BE}$ is a median in Δ ABC

, $\because \overline{AD} \cap \overline{BE} = \{M\}$

$\therefore$ M is the intersection point of the medians of Δ ABC

, $\because M \in \overline{CF}$ $\therefore$ $\overline{CF}$ is a median in Δ ABC

$\therefore$ F is the midpoint of $\overline{AB}$ $\therefore AF = \frac{1}{2} AB = 3$ cm.

, $\because$ E is the midpoint of $\overline{AC}$ $\therefore AE = \frac{1}{2} AC = 5$ cm.

, in Δ ABC :

$\because$ F and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

$\therefore FE = \frac{1}{2} BC = 4$ cm. (**)

$\therefore$ The perimeter of Δ AFE = AF + FE + AE

$= 3 + 4 + 5 = 12$ cm. (The req.)

(*) [illegible] In a right-angled triangle , the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides

(**) [illegible] The length of the line segment joining the midpoints of two sides in a triangle equals half the length of the third side

Theorem 3

The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base.

For example:

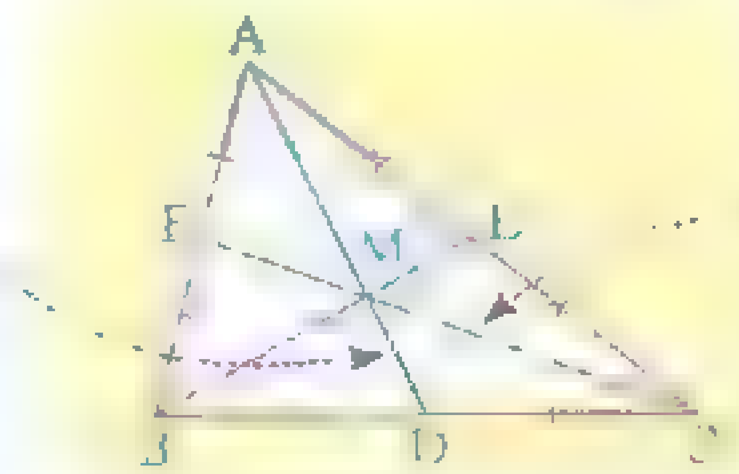
In the following figure :

M is the point of concurrence of the medians of Δ ABC , then :

(1) $MD = \frac{1}{2} AM$

If AM = 6 cm. ,

then MD = 3 cm.



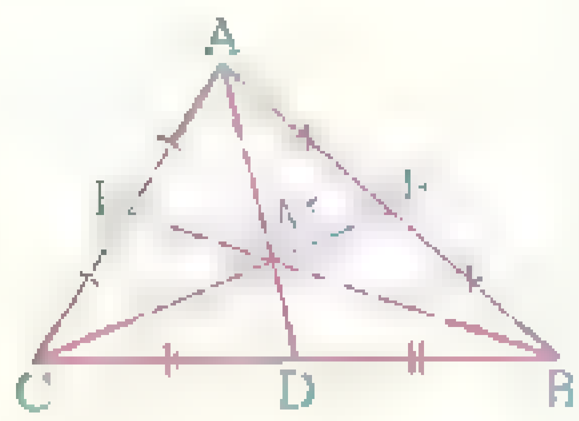
(2) CM = 2 FM

If FM = 4 cm. ,

then CM = 8 cm.

Remarks

- The point of concurrence of the medians of the triangle divides each of them in the ratio of 2 : 1 from the vertex.
- **In the opposite figure :**



If ABC is a triangle , M is the point of concurrence of its medians $\overline{AD}$, $\overline{BF}$ and $\overline{CE}$, then :

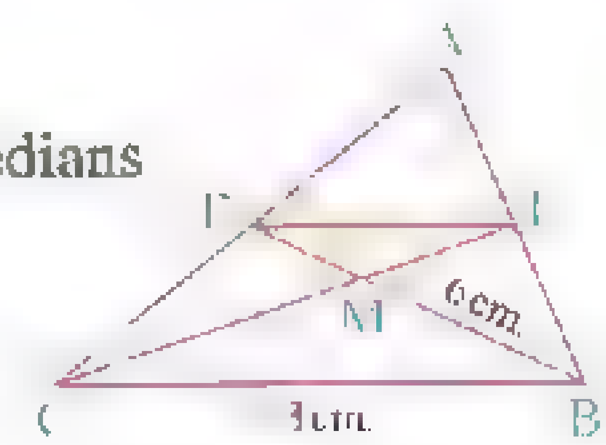
$MD = \frac{1}{3}$ AD and $AM = \frac{2}{3}$ AD

For example:

If AD = 9 cm. , then $MD = \frac{1}{3}$ AD = 3 cm. , $AM = \frac{2}{3}$ AD = 6 cm.

Similarly : $MF = \frac{1}{3}$ BF , $BM = \frac{2}{3}$ BF , $ME = \frac{1}{3}$ CE and $CM = \frac{2}{3}$ CE

Example 2 In the opposite figure :



ABC is a triangle in which : $\overline{CD}$ and $\overline{BE}$ are two medians intersecting at M , BM = 6 cm. , BC = 13 cm. and DC = 12 cm.

Find the perimeter of Δ DME

Solution

Given ABC is a triangle in which : $\overline{CD}$ and $\overline{BE}$ are two medians , M is the point of their intersection , BM = 6 cm. , BC = 13 cm. and DC = 12 cm.

R.T.F. The perimeter of Δ DME

Proof $\because$ $\overline{CD}$ and $\overline{BE}$ are medians intersecting at the point M

$\therefore$ M is the point of intersection of the medians of Δ ABC

$\therefore ME = \frac{1}{2} BM = \frac{1}{2} \times 6 = 3$ cm.

$, DM = \frac{1}{3} DC = \frac{1}{3} \times 12 = 4$ cm.

$\because$ CD and BE are two medians in ΔABC

$\therefore$ D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$

$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 13 = 6.5$ cm.(*)

$\therefore$ The perimeter of $\Delta DME = ME + DM + DE = 3 + 4 + 6.5 = 13.5$ cm.

(The req.)

Fact

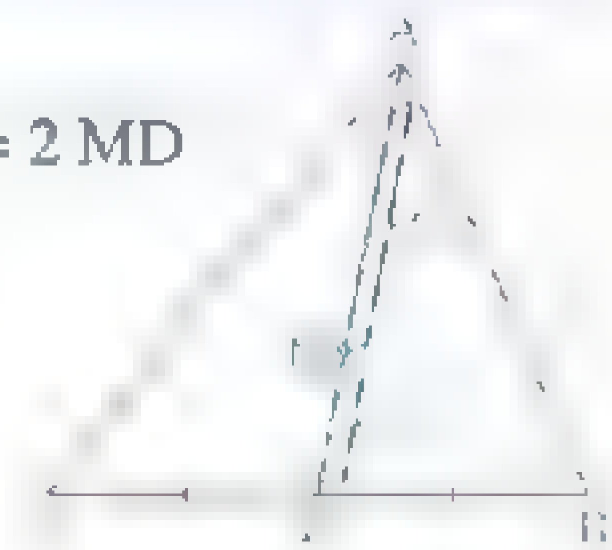
The point which divides the median in a triangle by the ratio of 1 : 2 from the base is the point of intersection of the medians of this triangle.

In the opposite figure :

If AD is a median in ΔABC and $M \in \overline{AD}$ such that $AM = 2 MD$,

then M is the point of intersection of the medians of ΔABC

$AM = 2 MD$



Example **In the opposite figure :**

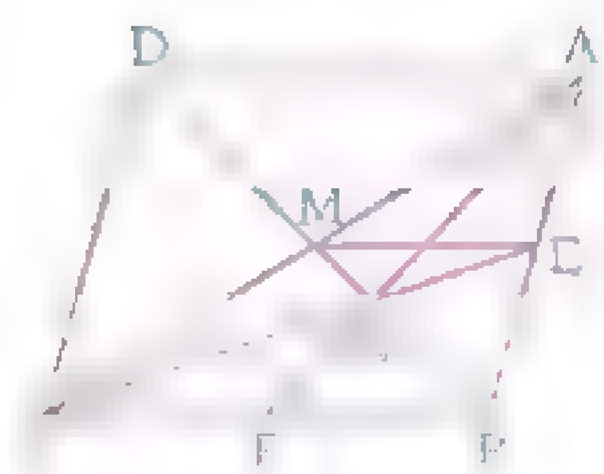
ABCD is a parallelogram ,

M is the point of intersection of its diagonals ,

$N \in \overline{BM}$ where $BN = 2 NM$

and $\overrightarrow{CN} \cap \overline{AB} = \{E\}$

Prove that : $EM = \frac{1}{2} BC$



Solution

Given ABCD is a parallelogram , M is the point of intersection of its diagonals , $BN = 2 NM$, $N \in \overline{BM}$ and $\overrightarrow{CN} \cap \overline{AB} = \{E\}$

R.T.P. $EM = \frac{1}{2} BC$

Proof $\because$ ABCD is a parallelogram.

$\therefore$ The two diagonals bisect each other.(**)

$\therefore$ M is the midpoint of $\overline{AC}$ $\therefore$ BM is a median in ΔABC

(*) Remember : The length of the line segment joining the midpoints of two sides in a triangle equals half the length of the third side.

(**) Remember : The two diagonals of a parallelogram bisect each other.

, ∵ $N \in \overline{BM}$ where $BN = 2\ NM$

∴ N is the point of intersection of the medians of $\Delta\ ABC$

, ∵ $\overline{CE}$ passes through the point N ∴ CE is a median in $\Delta\ ABC$

∴ E is the midpoint of $\overline{AB}$

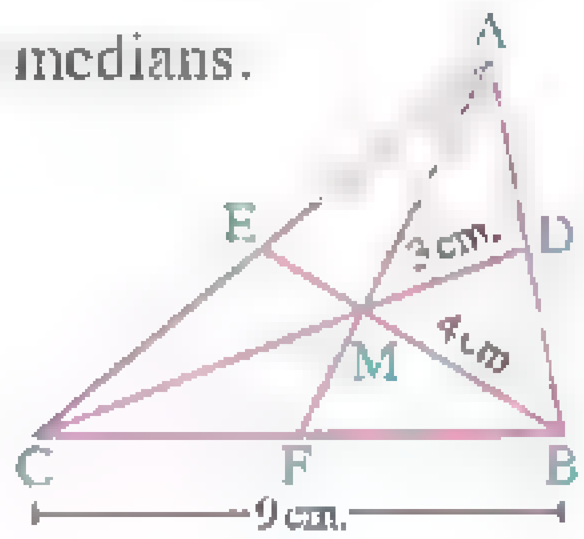
In $\Delta\ ABC$

∵ E is the midpoint of $\overline{AB}$ and M is the midpoint of $\overline{AC}$

∴ $EM = \frac{1}{2}\ BC$(*) (Q.E.D.)

TRY

In the opposite figure :



ABC is a triangle and M is the point of intersection of its medians.

If MD = 3 cm. , BM = 4 cm. and BC = 9 cm. ,

complete the following :

1 BF = ... cm.

2 MC = ... cm.

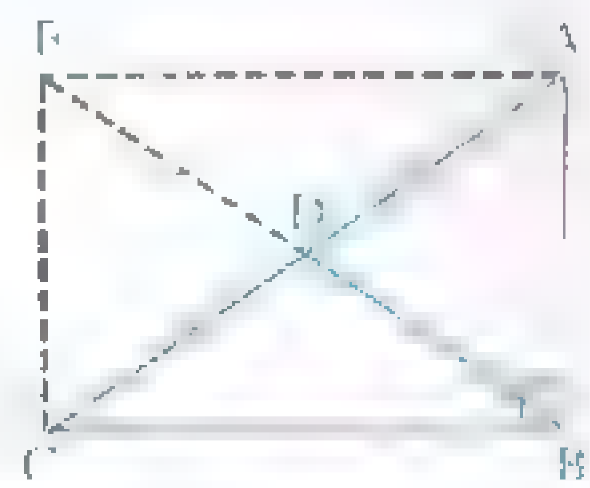
3 ME = ... cm.

(*) Remember The length of the line segment joining the midpoints of two sides in a triangle equals half the length of the third side.

Lesson 2 Medians of triangle "Follow"

Theorem

In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.



Given ABC is a triangle in which $m(\angle ABC) = 90°$,
$\overline{BD}$ is a median in the triangle ABC

R.T.P. $BD = \frac{1}{2} AC$

Construction Draw $\overrightarrow{BD}$ and take the point $E \in \overrightarrow{BD}$ such that $BD = DE$

Proof In the figure ABCE :

$\because$ $\overline{AC}$ and $\overline{BE}$ bisect each other.

$\therefore$ The figure ABCE is a parallelogram.(*)

, $\because m(\angle ABC) = 90°$ $\therefore$ The figure ABCE is a rectangle.(**)

$\therefore BE = AC$

, $\because BD = \frac{1}{2} BE$ $\therefore BD = \frac{1}{2} AC$ (Q.E.D.)

For example:

In the opposite figure :

ΔABC is a right-angled triangle at B ,

D is the midpoint of $\overline{AC}$ and AC = 10 cm. , then DB = 5 cm.

(*) Remember : A quadrilateral is a parallelogram if the two diagonals bisect each other.

(**) Remember : A parallelogram is a rectangle if one of its angles is right.

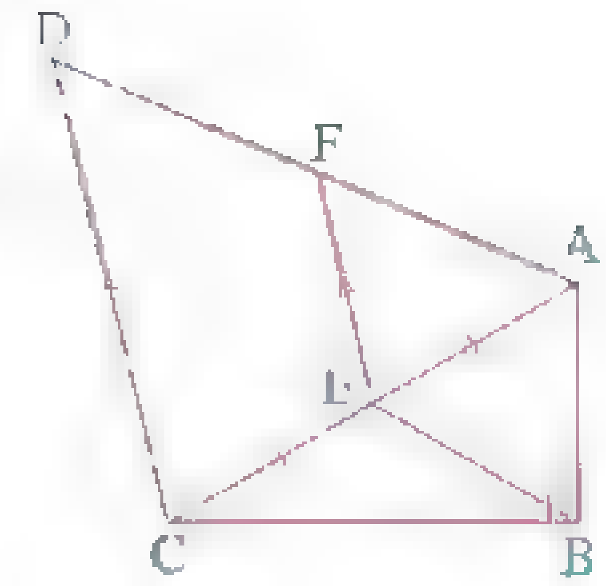
Example In the opposite figure :

ABCD is a quadrilateral in which m (∠ ABC) = 90°

, AC = CD , E is the midpoint of AC

and F ∈ AD such that EF // CD

Prove that : BE = EF



Solution

Given m (∠ ABC) = 90° , AC = CD , E is the midpoint of $\overline{AC}$ and $\overline{EF}$ // $\overline{CD}$

R.T.P. BE = EF

Proof In Δ ABC :

∵ m (∠ ABC) = 90° and $\overline{BE}$ is a median ∴ BE = $\frac{1}{2}$ AC

, ∵ AC = CD ∴ BE = $\frac{1}{2}$ CD (1)

In Δ ACD :

∵ E is the midpoint of $\overline{AC}$ and $\overline{EF}$ // $\overline{CD}$ ∴ F is the midpoint of $\overline{AD}$ (*)

∴ EF = $\frac{1}{2}$ CD (2)

From (1) and (2) : ∴ BE = EF (Q.E.D.)

TEST 1

Choose the correct answer from those given :

1 In the right-angled triangle , the ratio between the length of the hypotenuse and the length of the median drawn from the vertex of the right angle is

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 2 : 3

2 In Δ ABC which is right at B , if AC = 12 cm , D is the midpoint of $\overline{AC}$, then BD = ………… cm.

(a) 24 (b) 12 (c) 6 (d) 3

3 Δ ABC is right at A , the length of the median drawn from A is 4 cm. , then BC = ………… cm.

(a) 12 (b) 8 (c) 4 (d) 2

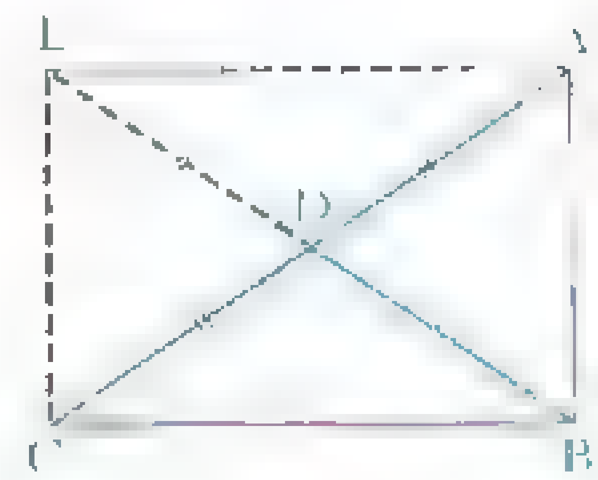
4 Δ XYZ is right at Y , if XY = 6 cm. , YZ = 8 cm , E is the midpoint of XZ , then YE = ………… cm.

(a) 4 (b) 5 (c) 10 (d) 20

(*) Remember : The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.

The converse of theorem 3

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.



Given | ABC is a triangle, $\overline{BD}$ is a median and DA = DB = DC

R.T.P. | m (∠ ABC) = 90°

Construction | Draw $\overrightarrow{BD}$, then take the point E $\in \overrightarrow{BD}$ such that BD = DE

Proof | $\because BD = \frac{1}{2} BE = \frac{1}{2} AC$

$\therefore$ BE = AC

$\therefore$ In the figure ABCE :

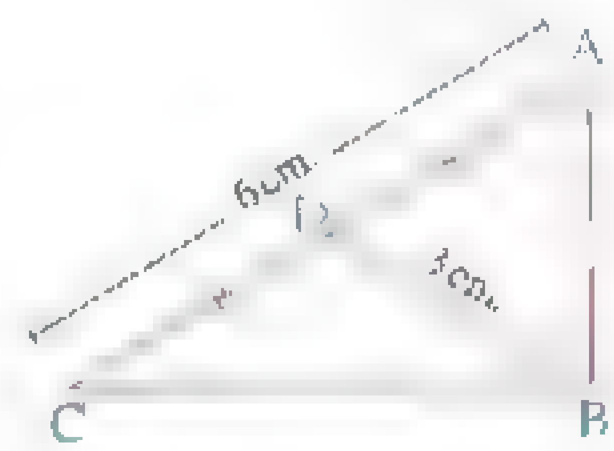
$\overline{AC}$ and $\overline{BE}$ are equal in length and bisect each other.

$\therefore$ The figure ABCE is a rectangle.(*)

$\therefore$ m (∠ ABC) = 90° (Q.E.D.)

For example:

In the opposite figure :



If $\overline{BD}$ is a median in Δ ABC,

BD = 3 cm. and AC = 6 cm.,

then m (∠ ABC) = 90° "because $BD = \frac{1}{2} AC$"

Example 3 **In the opposite figure :**

ABD is a right-angled triangle at D,

E is the midpoint of $\overline{AB}$ and CE = DE

Prove that : m (∠ ACB) = 90°

Solution

Given | E is the midpoint of $\overline{AB}$, m (∠ ADB) = 90°, CE = DE

R.T.P. m (∠ ACB) = 90°

(*) Remember: A quadrilateral is a parallelogram if the two diagonals bisect each other and if the two diagonals are equal in length, then it is a rectangle.

Proof In Δ ADB :

$\because$ m ($\angle$ ADB) = 90° , DE is a median $\therefore$ DE = $\frac{1}{2}$ AB

But CE = DE $\therefore$ CE = $\frac{1}{2}$ AB

$\therefore$ In Δ ACB :

$\overline{CE}$ is a median with length equals half the length of $\overline{AB}$

$\therefore$ m ($\angle$ ACB) = 90° (Q.E.D.)

TRY 2

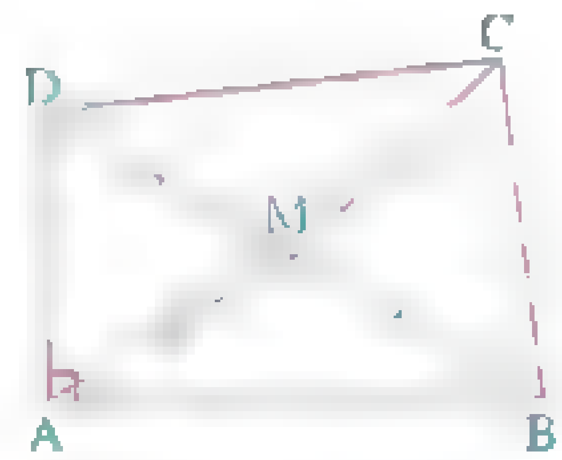
In the opposite figure :

ABCD is a quadrilateral in which m ($\angle$ BAD) = 90° ,

M is the midpoint of $\overline{BD}$ and CM = AM

Prove that :

m ($\angle$ BCD) = 90°



Corollary

The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

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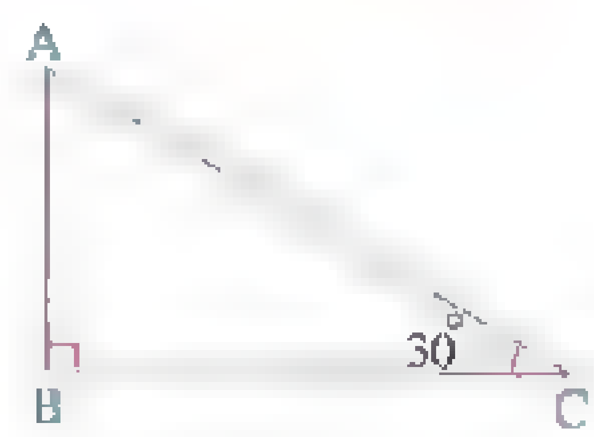
i.e.

In the opposite figure :

If Δ ABC is right-angled at B and

m ($\angle$ C) = 30° , then AB = $\frac{1}{2}$ AC



For example:

If AC = 20 cm. , then AB = 10 cm.

! Remark

The right angled triangle whose measure of one of its angles is 30° , then the measure of the third angle is 60° is called thirty and sixty triangle.

Example 3 **In the opposite figure :**

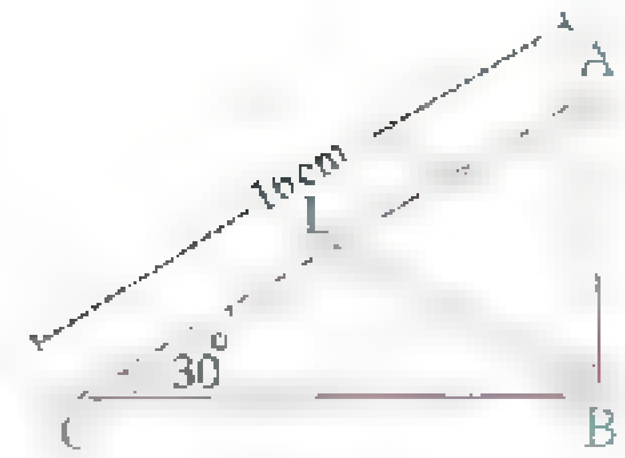
ABC is a triangle in which m ($\angle$ ABC) = 90° ,

m ($\angle$ C) = 30° , AC = 16 cm. and

L is the midpoint of $\overline{AC}$

Find : 1 The length of each of $\overline{AB}$ and $\overline{BL}$

2 The perimeter of Δ ABL



Solution

Given | m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30° ,
AC = 16 cm. and L is the midpoint of $\overline{AC}$

R.T.F. | 1 AB , BL   2 The perimeter of Δ ABL

Proof | $\because$ Δ ABC is right-angled at B , m ($\angle$ C) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC = 8 cm.

, $\because$ $\overline{BL}$ is a median in Δ ABC

$\therefore$ BL = $\frac{1}{2}$ AC = 8 cm. (First req.)

$\because$ AL = $\frac{1}{2}$ AC = 8 cm.

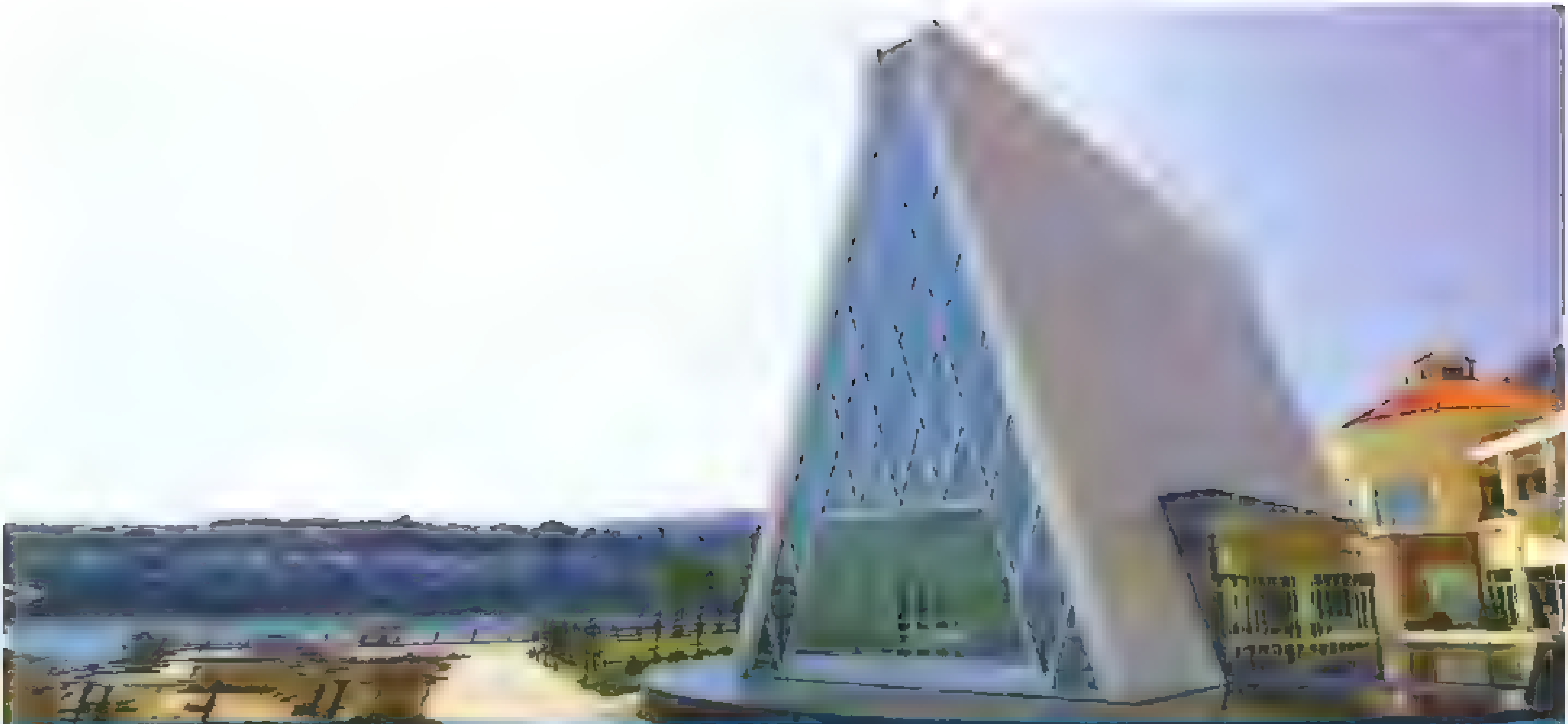
$\therefore$ The perimeter of Δ ABL = 8 + 8 + 8 = 24 cm. (Second req.)

TRY 3 ***In the opposite figure :***

m ($\angle$ ABC) = m ($\angle$ DBE) = 90° ,

D is the midpoint of $\overline{AC}$ and m ($\angle$ E) = 30°

Prove that : AC = DE



Lesson 3 The isosceles triangle

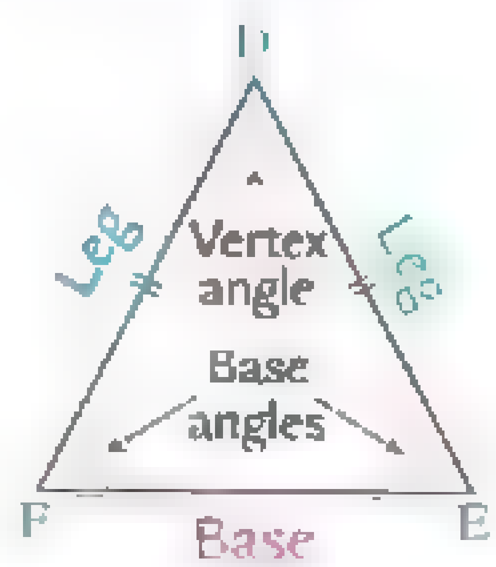
Prelude

Triangles are classified according to the lengths of their sides into three types which are :

1. Scalene triangle.

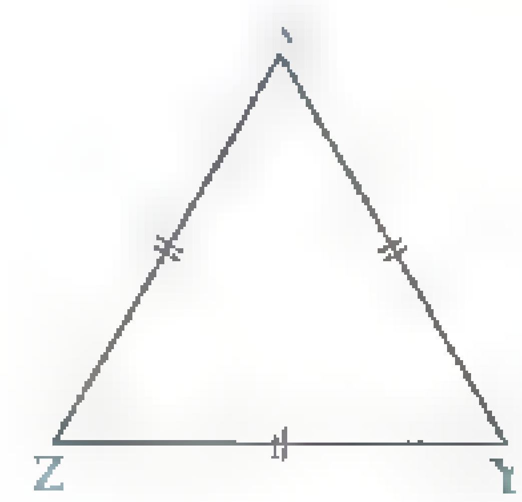
$AB \neq BC \neq CA$

2. Isosceles triangle. (two sides are congruent).



$DE = DF$

3. Equilateral triangle. (three sides are congruent).



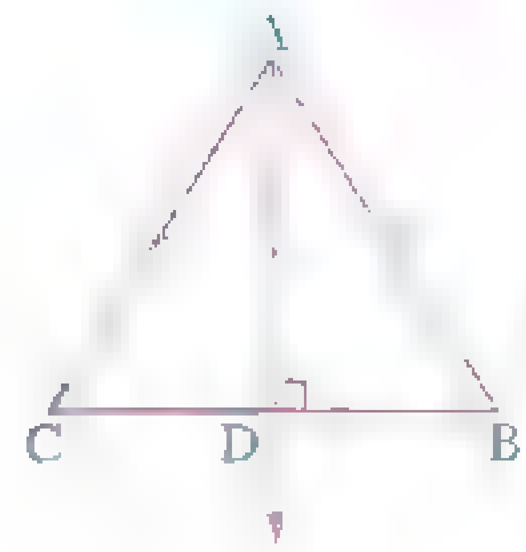
$XY = YZ = ZX$

- **In the following we will study the relations between the angles in the isosceles triangle and in the equilateral triangle.**

The isosceles triangle theorem

Theorem 1

The base angles of the isosceles triangle are congruent.



Given | ABC is a triangle in which $\overline{AB} = \overline{AC}$

R.T.P. | $\angle B \equiv \angle C$

Construction | Draw $\overline{AD} \perp \overline{BC}$ where $\overline{AD} \cap \overline{BC} = \{D\}$

Proof | $\because$ $\Delta\Delta$ ADB , ADC in which :

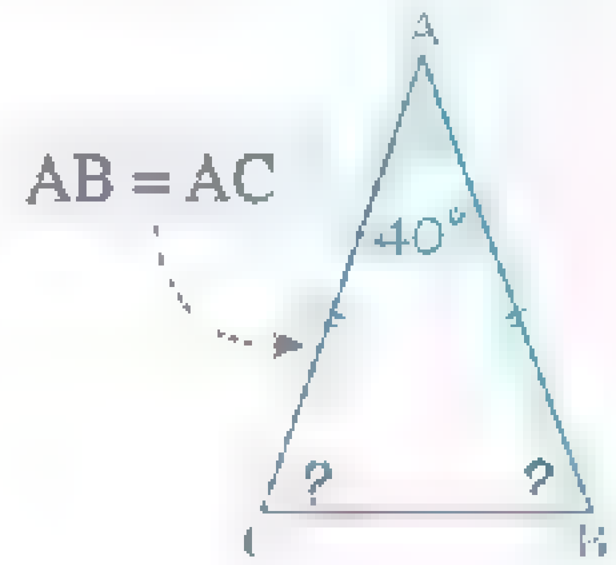
$\left\{\begin{array}{ll} m(\angle ADB) = m(\angle ADC) = 90^\circ & \text{(const.)} \\ \overline{AB} \equiv \overline{AC} & \text{(given)} \\ \overline{AD} \text{ is a common side} & \end{array}\right.$

$\therefore \Delta ADB \equiv \Delta ADC$ (*)

, then we deduce that $\angle B \equiv \angle C$ (Q.E.D.)

For example:

In the opposite figure :



If ABC is a triangle in which :

AB = AC , $m(\angle A) = 40^\circ$,

then $m(\angle B) = m(\angle C) = \dfrac{180^\circ - 40^\circ}{2} = 70^\circ$ (**)

! Remarks

1. Both of the base angles in the isosceles triangle are acute.
2. The vertex angle in the isosceles triangle may be acute , right or obtuse angle.

Example 1

Choose the correct answer from those given :

1 ABC is a triangle in which AB = AC , $m(\angle B) = 70^\circ$, then $m(\angle A) =$ ....

(a) 40° (b) 50° (c) 55° (d) 70°

2 In Δ XYZ , XY = XZ , $m(\angle X) = 100^\circ$, $m(\angle Z) =$ ....

(a) 20° (b) 40° (c) 80° (d) 100°

3 Δ XYZ is right at Y , if XY = YZ , then $m(\angle Z) =$ ....

(a) 30° (b) 45° (c) 60° (d) 90°

(*) Remember : Two right angled triangles are congruent , if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.

(**) Remember : The sum of measures of the interior angles of a triangle = 180°

4 LMN is a triangle in which LM = MN , then ∠ N is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

5 XYZ is an isosceles triangle , m (∠ X) = 110°

, then m (∠ Y) = ...

(a) 30° (b) 35° (c) 40° (d) 45°

Solution

1 **(a) The reason :** ∵ AB = AC

∴ m (∠ B) = m (∠ C) = 70°

∴ m (∠ A) = 180° – (70° + 70°) = 40°

2 **(b) The reason :** ∵ XY = XZ

∴ m (∠ Y) = m (∠ Z)

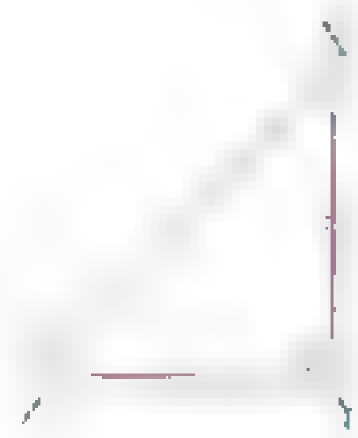
∴ m (∠ Z) = $\frac{180° - 100°}{2}$ = 40°

3 **(b) The reason :** ∵ XY = YZ

∴ m (∠ X) = m (∠ Z)

, ∵ m (∠ Y) = 90°

∴ m (∠ Z) = $\frac{180° - 90°}{2}$ = 45°



4 **(a) The reason :** ∵ Δ LMN is an isosceles triangle

∴ Each of the base angles is acute

∴ ∠ N is acute

5 **(b) The reason :** ∵ Δ XYZ is an isosceles triangle

∴ Each of the base angles is acute

∴ ∠ X is the vertex angle

∴ m (∠ Y) = m (∠ Z)

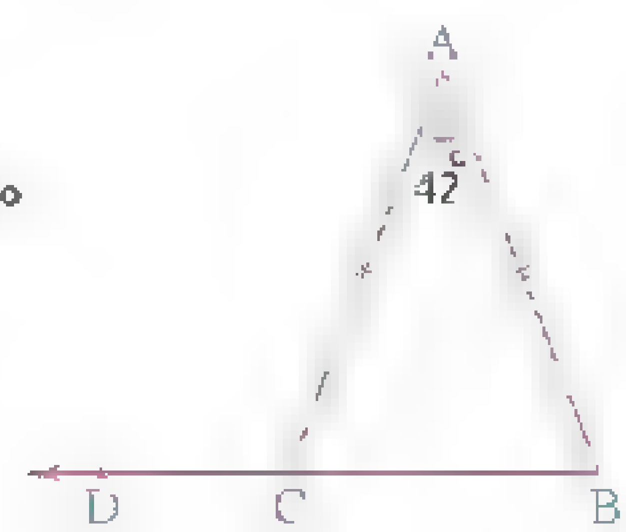
∴ m (∠ Y) = $\frac{180° - 110°}{2}$ = 35°

Example 2 **In the opposite figure :**

ABC is a triangle in which AB = AC , m (∠ A) = 42°

and D ∈ $\overrightarrow{BC}$

Find : m (∠ ACD)



Solution

Given $AB = AC$, $m(\angle A) = 42^\circ$ and $D \in \overrightarrow{BC}$

R.T.F. $m(\angle ACD)$

Proof $\because$ The sum of measures of the interior angles in $\Delta ABC = 180^\circ$

, $m(\angle A) = 42^\circ$

$\therefore m(\angle B) + m(\angle ACB) = 180^\circ - 42^\circ = 138^\circ$

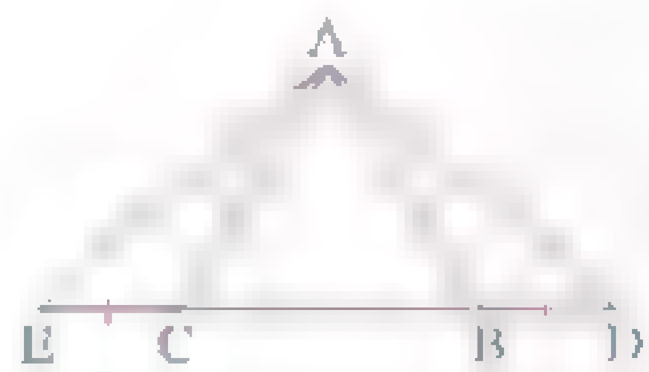
, $\because AB = AC$ (given)

$\therefore m(\angle B) = m(\angle ACB) = \frac{138^\circ}{2} = 69^\circ$

$\because \angle ACD$ is an exterior angle of ΔABC

$\therefore m(\angle ACD) = m(\angle A) + m(\angle B) = 42^\circ + 69^\circ = 111^\circ$ (*) (The req.)

Example 3 **In the opposite figure :**



$B \in \overline{DE}$, $C \in \overline{DE}$, $AB = AC$ and $BD = CE$

Prove that : $AD = AE$

Solution

Given $AB = AC$ and $BD = CE$

R.T.P. $AD = AE$

Proof $\because AB = AC$ (given)

$\therefore m(\angle ABC) = m(\angle ACB)$

$\because \angle ABD$ supplements $\angle ABC$

, $\angle ACE$ supplements $\angle ACB$

$\therefore m(\angle ABD) = m(\angle ACE)$ (**)

$\therefore$ In $\Delta\Delta$ ABD , ACE

$AB = AC$	(given)
$BD = CE$	(given)
$m(\angle ABD) = m(\angle ACE)$	(by proof)

$\therefore \Delta ABD \equiv \Delta ACE$ (***) , then we deduce that $AD = AE$ (Q.E.D.)

(*) Remember : The measure of the exterior angle of a triangle equals the sum of measures of its non adjacent interior angles

(**) Remember : The supplementaries of the equal angles in measures are equal in measures

(***) Remember : Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle.

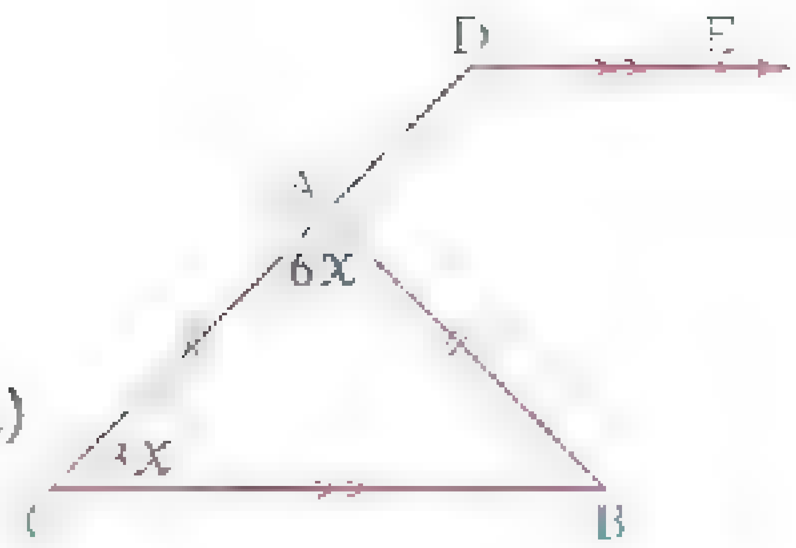
Example

In the opposite figure :

AB = AC , m (∠ BAC) = 6 X ,

m (∠ C) = 3 X and $\overleftrightarrow{BC}$ // $\overrightarrow{DE}$

Find : **1** The value of X **2** m (∠ EDA)



Solution

Given AB = AC , m (∠ BAC) = 6 X , m (∠ C) = 3 X and $\overleftrightarrow{BC}$ // $\overrightarrow{DE}$

R.T.F. 1 The value of X 2 m (∠ EDA)

Proof ∵ AB = AC ∴ m (∠ B) = m (∠ C) = 3 X

, ∵ the sum of measures of the interior angles of the triangle = 180°

∴ 6 X + 3 X + 3 X = 180° ∴ 12 X = 180°

∴ $X = \frac{180°}{12} = 15°$ (First req.)

∴ m (∠ C) = 45° ∵ $\overleftrightarrow{BC}$ // $\overrightarrow{DE}$ and $\overleftrightarrow{CD}$ is a transversal

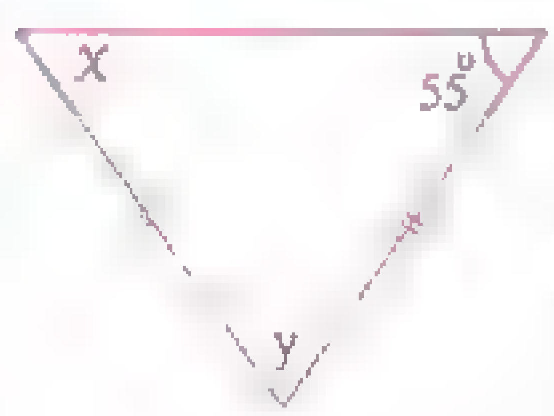
∴ m (∠ EDA) + m (∠ C) = 180° (two interior angles on the same side of the transversal)(*)

∴ m (∠ EDA) = 180° − 45° = 135° (Second req.)

Try 1

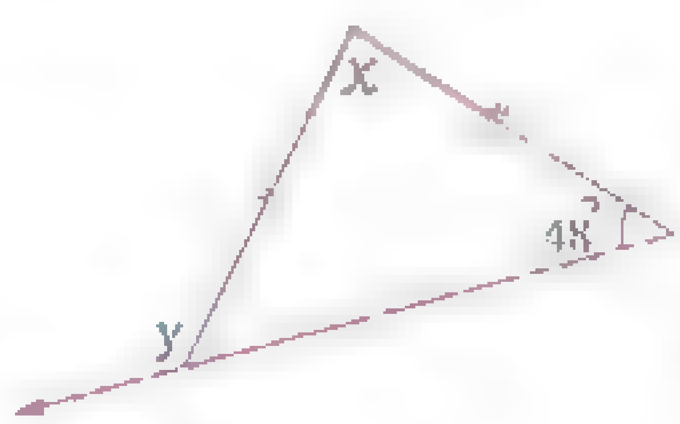
In each of the following figures, find the values of the symbols used as measures for the angles :

1



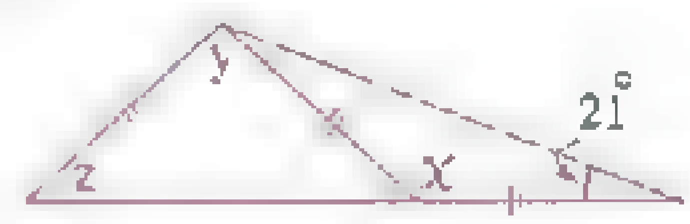
X = ...° , y = ...°

2



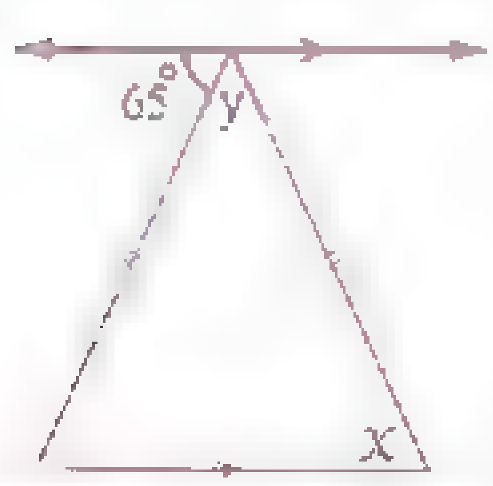
X = ...° , y = ...°

3



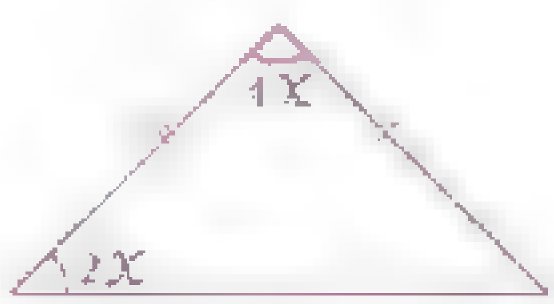
X = ...° , y = ...° , z = ...°

4



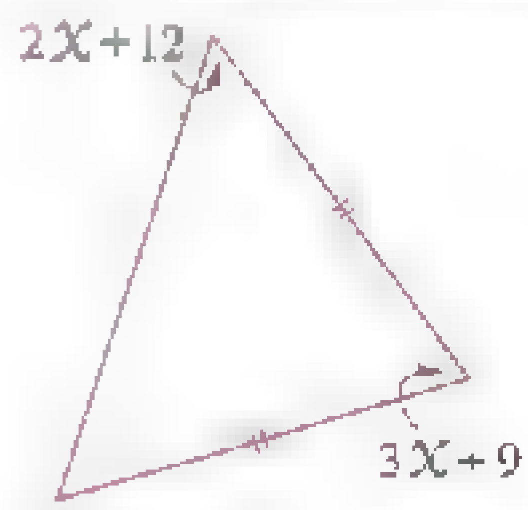
X = ...° , y = ...°

5



X = ...°

6



X = ...°

(*) Remember : If a straight line intersects two parallel straight lines , then each two interior angles in the same side of the transversal are supplementary

i.e. The sum of their measures is 180°

Corollary

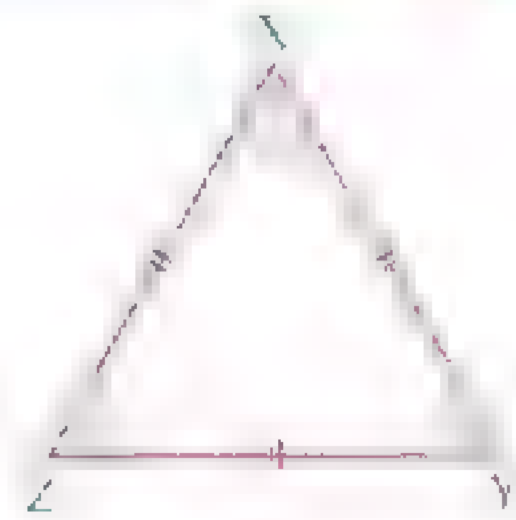
If the triangle is equilateral , then it is equiangular where each angle measure is 60°

For example:

In the opposite figure :

If XYZ is a triangle in which XY = YZ = ZX

, then m (∠ X) = m (∠ Y) = m (∠ Z) = 60°

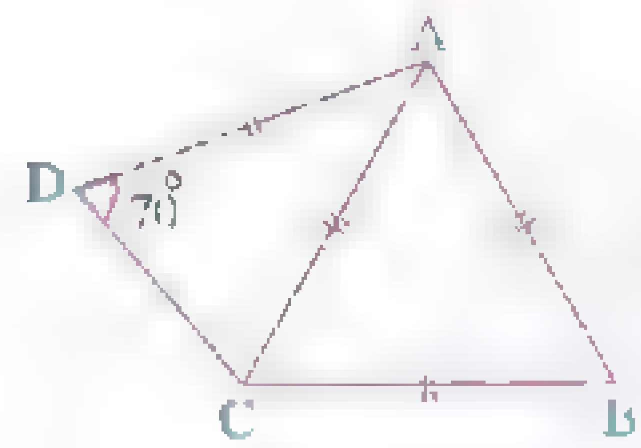


Example 5

In the opposite figure :

AB = BC = CA = AD and m (∠ D) = 70°

Find : 1 m (∠ BCD) 2 m (∠ BAD)



Solution

Given AB = BC = CA = AD and m (∠ D) = 70°

R.T.F. 1 m (∠ BCD) 2 m (∠ BAD)

Proof ∵ Δ ABC is an equilateral triangle. ∴ m (∠ BCA) = 60°

In Δ ACD : ∵ AC = AD ∴ m (∠ ACD) = m (∠ D) = 70°

∴ m (∠ BCD) = m (∠ BCA) + m (∠ ACD) = 60° + 70° = 130° (First req.)

∵ The sum of measures of the interior angles of the quadrilateral ABCD = 360° (*)

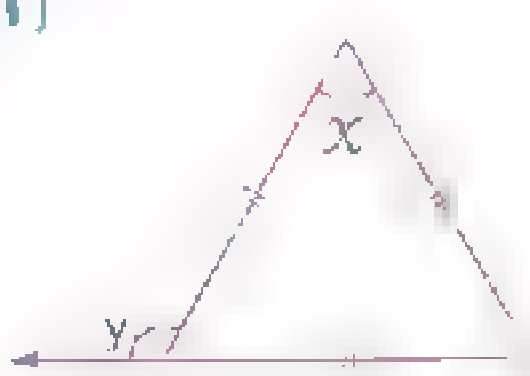
, m (∠ B) = 60°

∴ m (∠ BAD) = 360° – (60° + 130° + 70°) = 100° (Second req.)

TRY 2

In each of the following figures , find the values of the symbols used as measures for the angles :

1

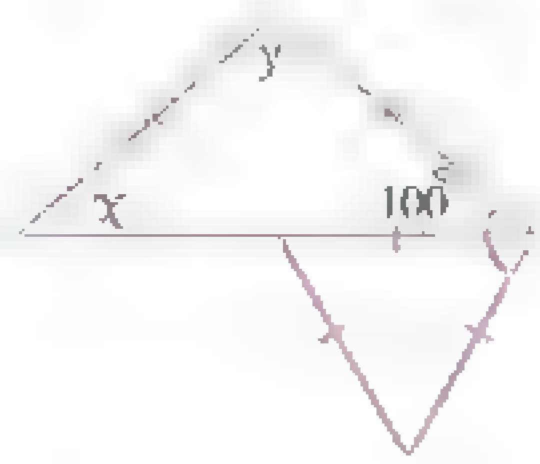


X = ……° , y = ……°

2

X = ……° , y = ……°

3



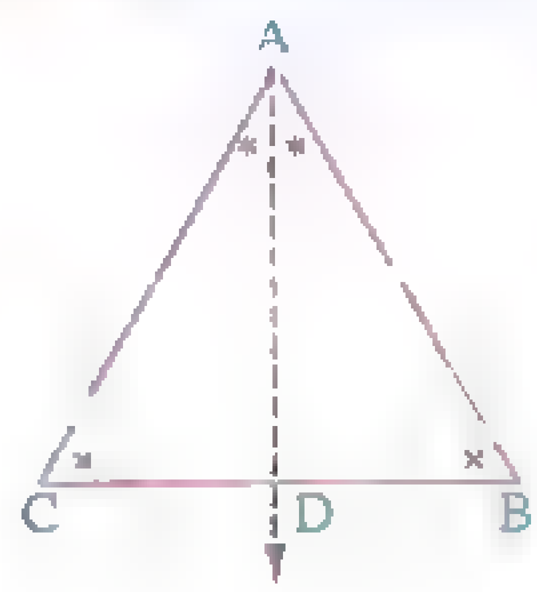
X = ……° , y = ……°

(*) Remember : The sum of measures of the interior angles of a polygon of n sides equals (n – 2) × 180°

Lesson 4 The converse of the isosceles triangle theorem

Theorem

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.



Given	ABC is a triangle in which $\angle B \equiv \angle C$
R.T.P.	$\overline{AB} \equiv \overline{AC}$
Construction	Bisect $\angle BAC$ by $\overrightarrow{AD}$ to intersect $\overline{BC}$ at D
Proof	$\because \angle B \equiv \angle C$

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\because \overrightarrow{AD}$ bisects $\angle$ BAC

$\therefore$ m ($\angle$ BAD) = m ($\angle$ CAD)

$\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore$ m ($\angle$ ADB) = m ($\angle$ ADC)

$\therefore$ In $\Delta\Delta$ ABD and ACD :

$\begin{cases} \text{AD is a common side} \\ m(\angle BAD) = m(\angle CAD) \text{ (const.)} \\ m(\angle ADB) = m(\angle ADC) \text{ (by proof)} \end{cases}$

$\therefore \Delta ABD \equiv \Delta ACD^{(*)}$, then we deduce that :

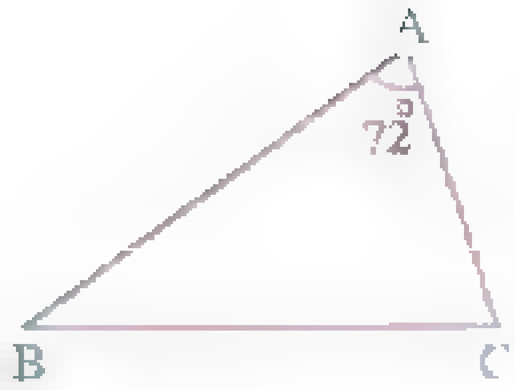
AB = AC , then Δ ABC is an isosceles triangle. (Q.E.D.)

(*) Two triangles are congruent, if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle.

Example 1 ABC is a triangle in which m (∠ A) = 2 m (∠ B) = 72°

Prove that : Δ ABC is an isosceles triangle.

Solution



Given m (∠ A) = 2 m (∠ B) = 72°

R.T.P. Δ ABC is an isosceles triangle.

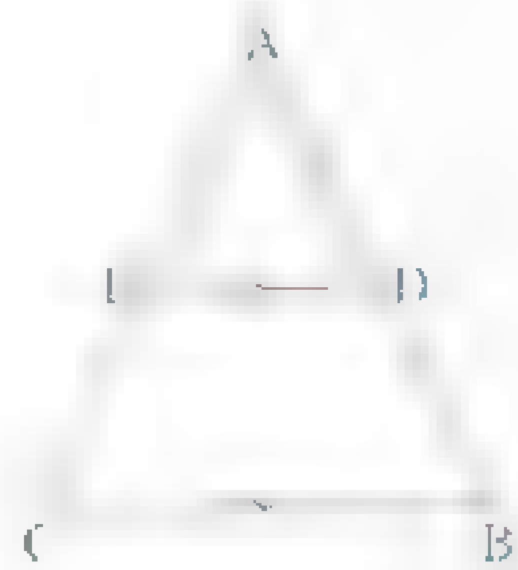
Proof In Δ ABC : ∵ 2 m (∠ B) = 72° ∴ m (∠ B) = $\frac{72^\circ}{2}$ = 36°

∵ m (∠ A) = 72° ∴ m (∠ C) = 180° − (36° + 72°) = 72°

∴ m (∠ A) = m (∠ C) ∴ BC = BA

∴ Δ ABC is an isosceles triangle. (Q.E.D.)

Example 2 **In the opposite figure :**



D ∈ $\overline{AB}$ and E ∈ $\overline{AC}$

where AD = AE and $\overline{DE}$ // $\overline{BC}$

Prove that : DB = EC

Solution

Given AD = AE and $\overline{DE}$ // $\overline{BC}$

R.T.P. DB = EC

Proof In Δ ADE : ∵ AD = AE ∴ m (∠ ADE) = m (∠ AED) (1)

∵ $\overline{DE}$ // $\overline{BC}$ and $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ B) = m (∠ ADE) (corresponding angles)(*) (2)

Similarly ∵ $\overline{DE}$ // $\overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal

∴ m (∠ C) = m (∠ AED) (corresponding angles)(*) (3)

From (1) , (2) and (3) : ∴ m (∠ B) = m (∠ C)

∴ AB = AC , ∵ AD = AE

Subtracting : ∴ AB − AD = AC − AE ∴ DB = EC (Q.E.D.)

(*) [illegible] If a straight line intersects two parallel straight lines, then each two corresponding angles are equal in measure.

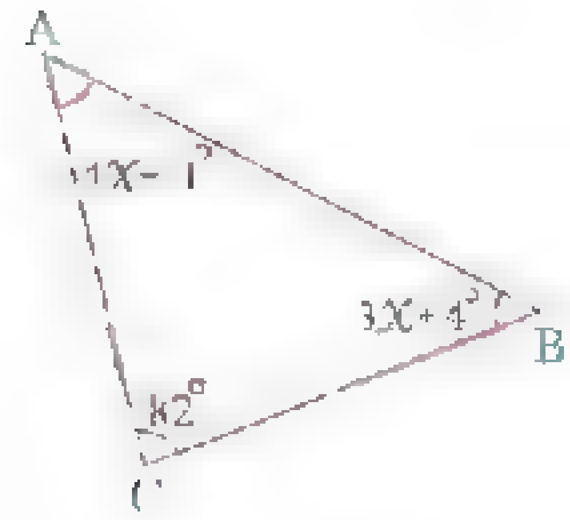
Example 3 **In the opposite figure :**

If m (∠ A) = $4X - 11°$

, m (∠ B) = $3X + 4°$

, m (∠ C) = 82°

, prove that : Δ ABC is an isosceles triangle.



Solution

Given m (∠ A) = $4X - 11°$, m (∠ B) = $3X + 4°$, m (∠ C) = 82°

R.T.P. Δ ABC is an isosceles triangle.

Proof ∵ The sum of measures of the interior angles of the triangle = 180°

∴ $4X - 11° + 3X + 4° + 82° = 180°$ ∴ $7X + 75° = 180°$

∴ $7X = 105°$

∴ $X = 15°$ ∴ m (∠ A) = 4 × 15° – 11° = 49°

, m (∠ B) = 3 × 15° + 4° = 49° ∴ m (∠ A) = m (∠ B)

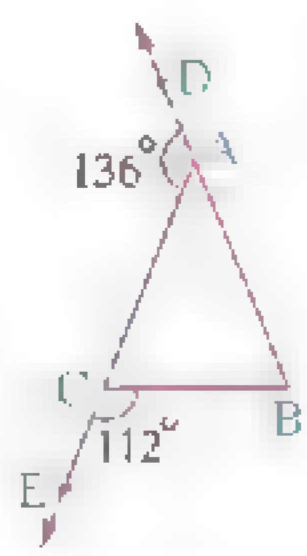
∴ BC = AC ∴ Δ ABC is an isosceles triangle. (Q.E.D.)

1

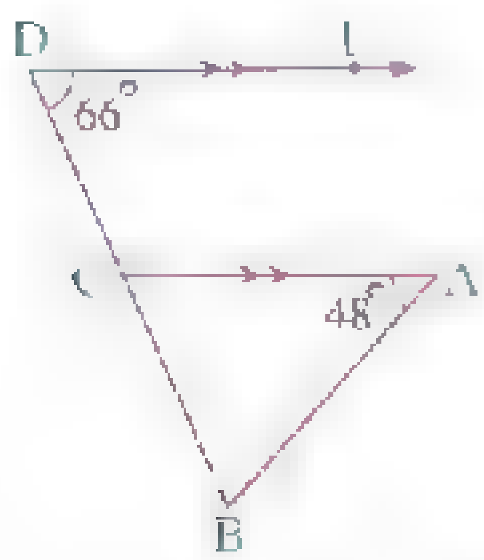
In each of the following figures , write the equal sides in length :

1

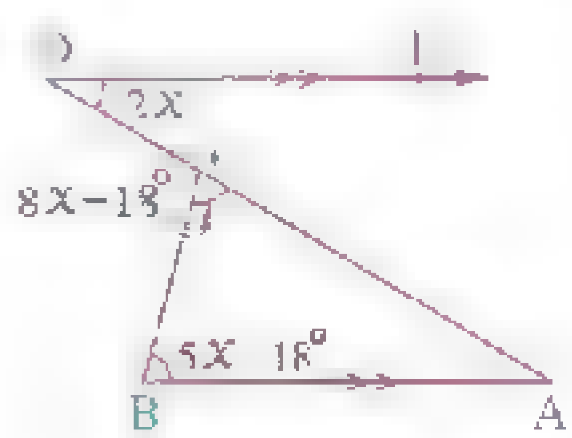
2



3



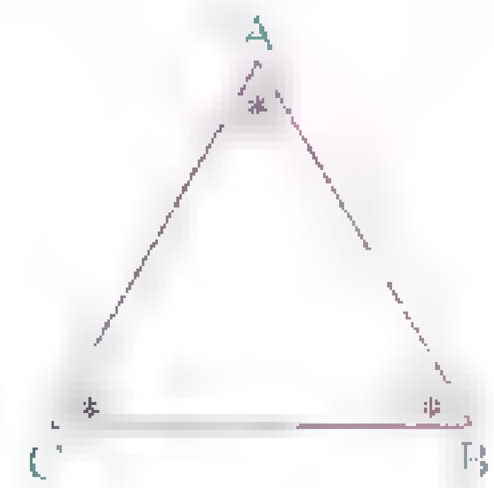
4



Corollary

If the angles of a triangle are congruent , then the triangle is equilateral.

For example:



If ABC is a triangle in which :

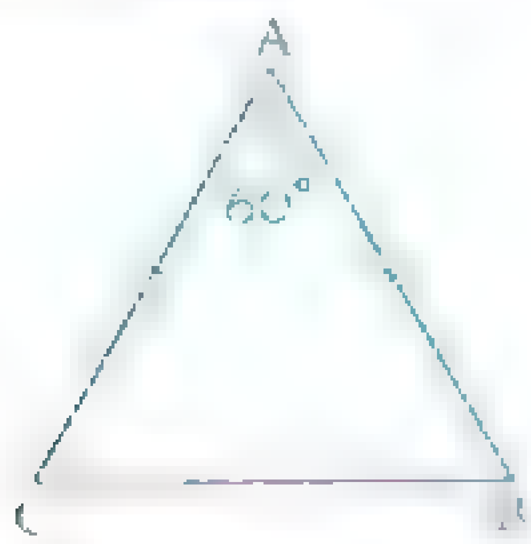
$\angle A \equiv \angle B \equiv \angle C$, then $AB = BC = CA$

i.e. Δ ABC is an equilateral triangle.

Remark

The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.

- **In the following figure :**



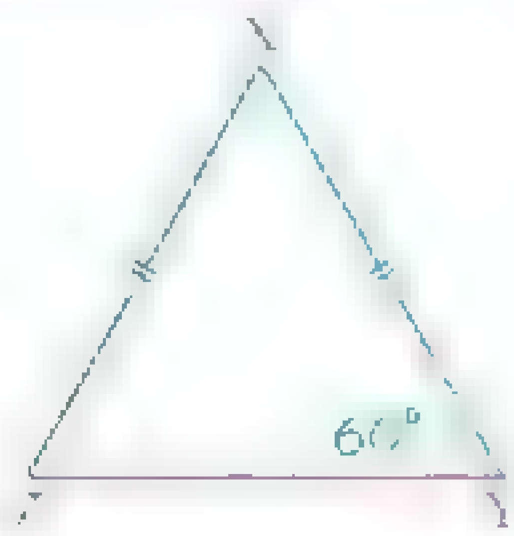
If $AB = AC$ and $m(\angle A) = 60°$

, then $m(\angle B) = m(\angle C) = \frac{180° - 60°}{2}$

$= 60°$

$\therefore \Delta$ ABC is an equilateral triangle

- **In the following figure :**



If $XY = XZ$ and $m(\angle Y) = 60°$

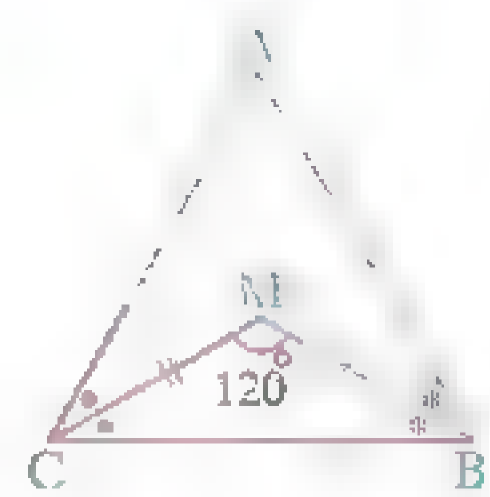
, then $m(\angle Z) = 60°$

, $m(\angle X) = 180° - (60° + 60°) = 60°$

$\therefore \Delta$ XYZ is an equilateral triangle.

Example 1

In the opposite figure :



$\overrightarrow{BM}$ bisects $\angle B$, $\overrightarrow{CM}$ bisects $\angle C$,

$MB = MC$ and $m(\angle BMC) = 120°$

Prove that : Δ ABC is an equilateral triangle.

Solution

Given $\overrightarrow{BM}$ bisects $\angle B$, $\overrightarrow{CM}$ bisects $\angle C$, $MB = MC$ and $m(\angle BMC) = 120°$

R.T.P. Δ ABC is an equilateral triangle.

Proof | In Δ MBC : $\because$ MB = MC and m ($\angle$ BMC) = 120°

$\therefore$ m ($\angle$ MBC) = m ($\angle$ MCB) = $\frac{180° - 120°}{2}$ = 30°

, $\because$ $\overrightarrow{BM}$ bisects $\angle$ B $\therefore$ m ($\angle$ ABC) = 2 m ($\angle$ MBC) = 60°

, $\because$ $\overrightarrow{CM}$ bisects $\angle$ C $\therefore$ m ($\angle$ ACB) = 2 m ($\angle$ MCB) = 60°

$\therefore$ In Δ ABC : m ($\angle$ BAC) = 180° – (60° + 60°) = 60°

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = m ($\angle$ BAC) = 60°

$\therefore$ Δ ABC is an equilateral triangle. (Q.E.D.)

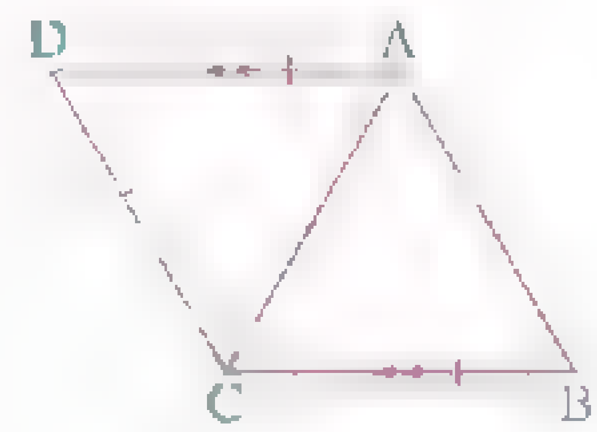
TRY 2

In the opposite figure :

ABCD is a quadrilateral in which :

AD = DC = CB = CA , $\overline{AD}$ // $\overline{BC}$

Prove that : Δ ABC is an equilateral triangle.

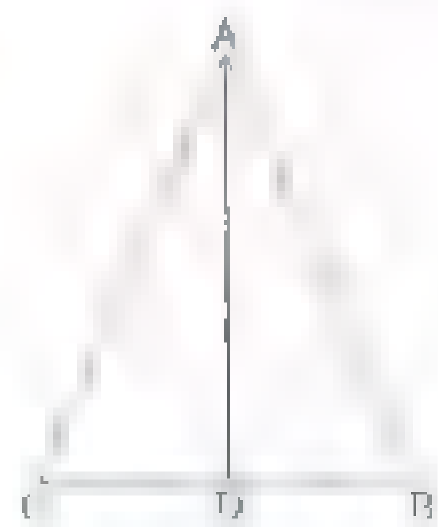


Lesson 5 Corollaries of the isosceles triangle theorems

Corollary 1

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base

In the opposite figure :



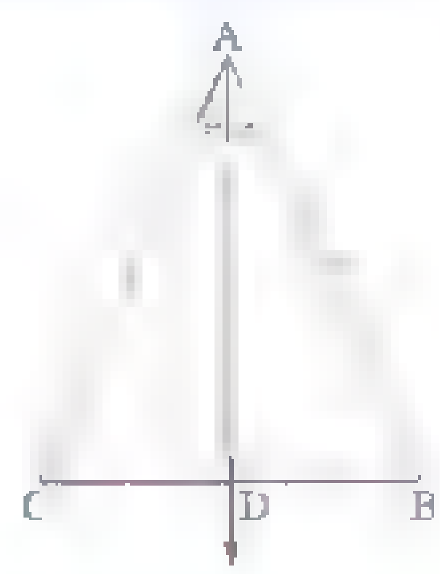
ABC is a triangle in which AB = AC and $\overline{AD}$ is a median , then :

1. $\overrightarrow{AD}$ bisects $\angle$ BAC i.e. m ($\angle$ BAD) = m ($\angle$ CAD)
2. $\overline{AD} \perp \overline{BC}$

Corollary 2

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

In the opposite figure :



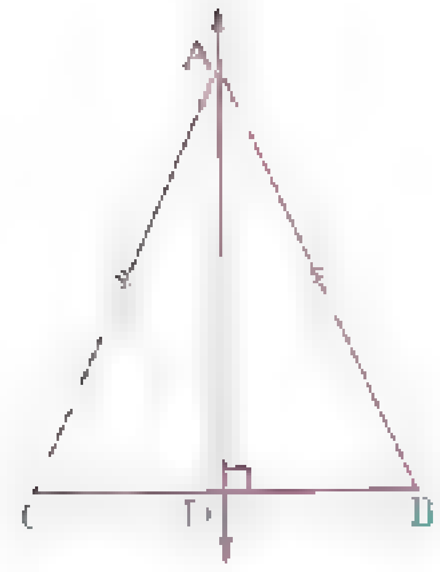
ABC is a triangle in which AB = AC and $\overrightarrow{AD}$ bisects $\angle$ BAC , then

1. D is the midpoint of BC i.e. BD = CD
2. $\overline{AD} \perp \overline{BC}$

Corollary 3

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle

In the opposite figure :



ABC is a triangle in which $AB = AC$ and $\overleftrightarrow{AD} \perp \overline{BC}$, then :

1 D is the midpoint of $\overline{BC}$ i.e. $BD = CD$

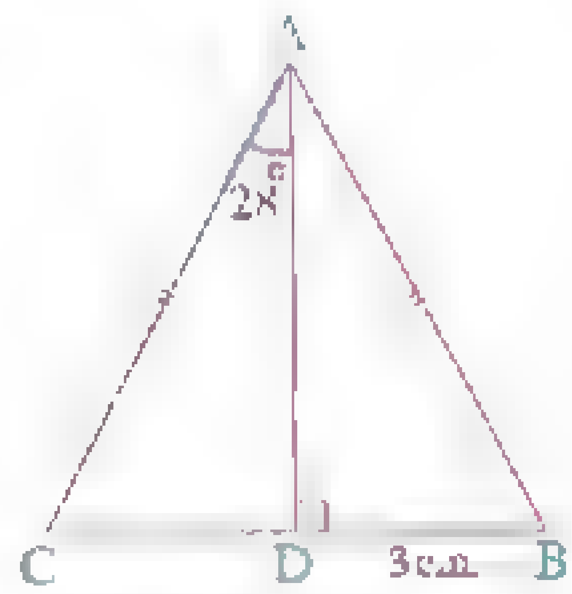
2 $m(\angle BAD) = m(\angle CAD)$

Notice that :

The previous three corollaries can be proved using the congruence of ΔABD and ΔACD

Example 1

In the opposite figure :



ABC is an isosceles triangle where $AB = AC$, $D \in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$, $m(\angle CAD) = 28°$ and $BD = 3$ cm. **Find :**

1 $m(\angle BAC)$

2 The length of $\overline{BC}$

Solution

Given $AB = AC$, $m(\angle CAD) = 28°$, $BD = 3$ cm. and $\overline{AD} \perp \overline{BC}$

R.T.F. 1 $m(\angle BAC)$ 2 The length of $\overline{BC}$

Proof In ΔABC : $\because AB = AC$ and $\overline{AD} \perp \overline{BC}$

$\therefore \overrightarrow{AD}$ bisects each of the vertex angle BAC and the base $\overline{BC}$

$\therefore m(\angle BAC) = 2\, m(\angle CAD) = 2 \times 28° = 56°$ (First req.)

, $BC = 2\,BD = 2 \times 3 = 6$ cm. (Second req.)

Example 2

Choose the correct answer from those given :

1 In ΔABC , if $AB = AC$, $\overline{AD}$ is a median , $m(\angle BAC) = 100°$, then $m(\angle BAD) = \ldots$

(a) 100° (b) 50° (c) 90° (d) 40°

2 In ΔXYZ , if $XY = XZ$, $\overrightarrow{XD}$ bisects $\angle YXZ$, then ΔXYD is

(a) acute-angled. (b) right angled.

(c) obtuse-angled . (d) isosceles.

3 In ΔLMN , $NM = NL$, $D \in \overline{LM}$ where $ND \perp \overline{LM}$, if $LM = 10$ cm. , then $LD = \ldots\ldots\ldots$ cm.

(a) 20 (b) 10 (c) 5 (d) 2.5

4 ABC is a triangle in which AB = AC , AX is a median , if BX = 5 cm. , m (∠ BAX) = 30° , then the perimeter of Δ ABC = ... cm.

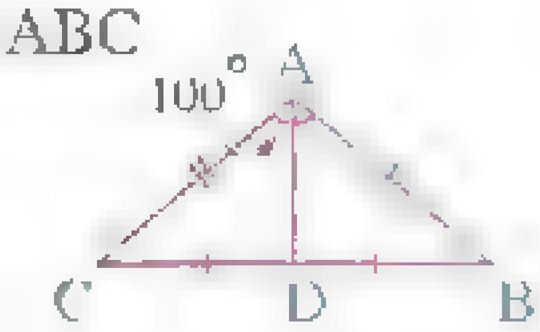
(a) 10 (b) 15 (c) 25 (d) 30

Solution

1 (b) The reason : ∵ AB = AC , $\overline{AD}$ is a median in Δ ABC

∴ m (∠ BAD) = m (∠ CAD)

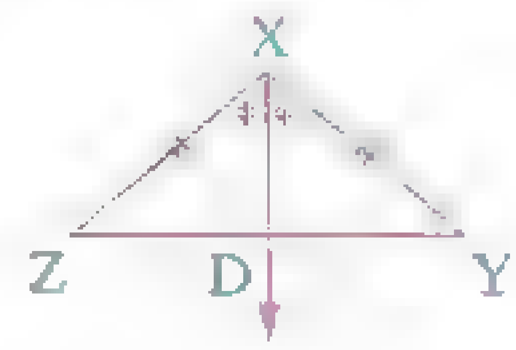
∴ m (∠ BAD) = $\frac{100°}{2}$ = 50°



2 (b) The reason : ∵ XY = XZ , $\overrightarrow{XD}$ bisects ∠ YXZ

∴ $\overrightarrow{XD}$ ⊥ YZ

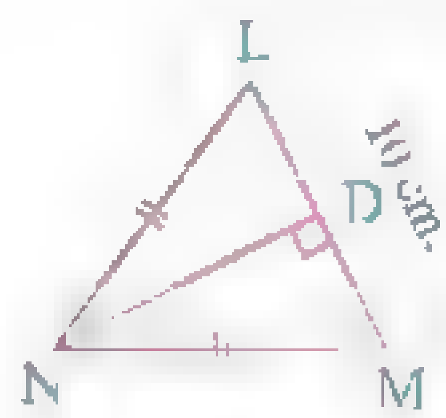
∴ Δ XYD is right-angled.



3 (c) The reason : ∵ NM = NL , $\overrightarrow{ND}$ ⊥ $\overline{LM}$

∴ D is the midpoint of $\overline{LM}$

∴ LD = $\frac{10}{2}$ = 5 cm.



4 (d) The reason : ∵ AB = AC , $\overline{AX}$ is a median

∴ $\overrightarrow{AX}$ bisects ∠ BAC

∴ m (∠ BAC) = 2 m (∠ BAX)

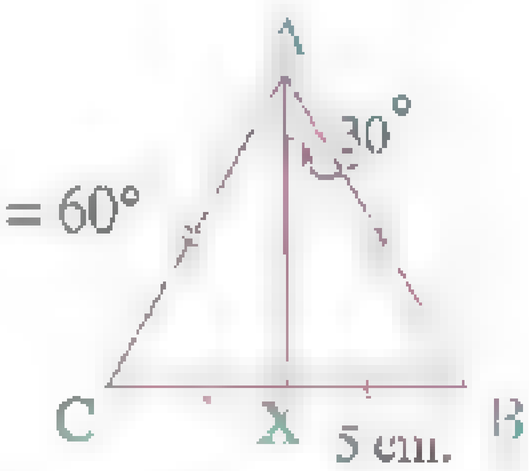
= 2 × 30° = 60°

, ∵ Δ ABC is isosceles , m (∠ BAC) = 60°

∴ Δ ABC is equilateral

, ∵ BC = 2 BX = 10 cm.

∴ The perimeter of Δ ABC = 3 × 10 = 30 cm.



TRY by yourself

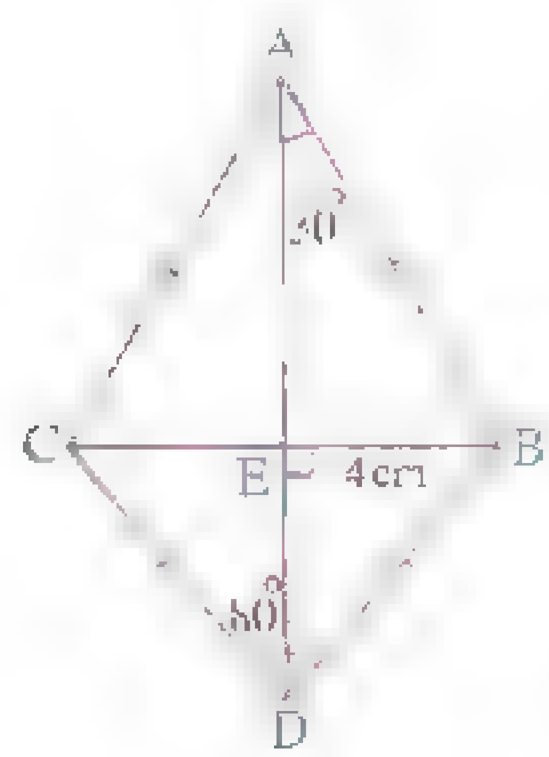
In the opposite figure :

ABDC is a quadrilateral in which :

AB = AC , BD = CD , $\overline{AD}$ ⊥ $\overline{BC}$,

$\overline{AD}$ ∩ $\overline{BC}$ = {E} , m (∠ BAD) = 30° ,

m (∠ BDC) = 80° and BE = 4 cm.



Complete the following :

[1] m (∠ DAC) = ... ° [2] m (∠ BDE) = ... °

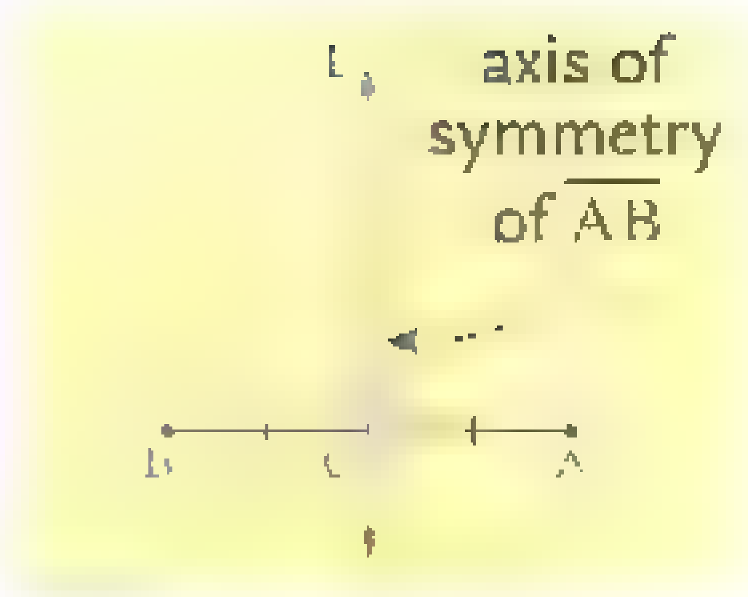
[3] m (∠ ACB) = ... ° [4] BC = ... cm.

[5] AC = ... cm. [6] AE = ... cm.

Axis of symmetry of a line segment

Definition

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment , in brief it is known as the axis of a line segment.

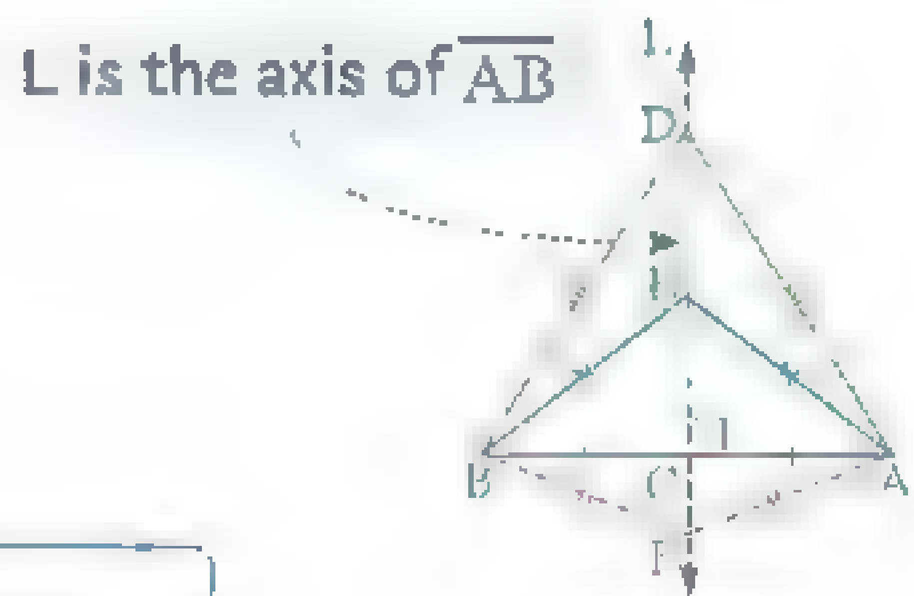


In the opposite figure :

If the straight line L $\perp$ $\overline{AB}$ and C $\in$ the straight line L where C is the midpoint of $\overline{AB}$, then the straight line L is called the axis of $\overline{AB}$

Property

Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).



In the opposite figure :

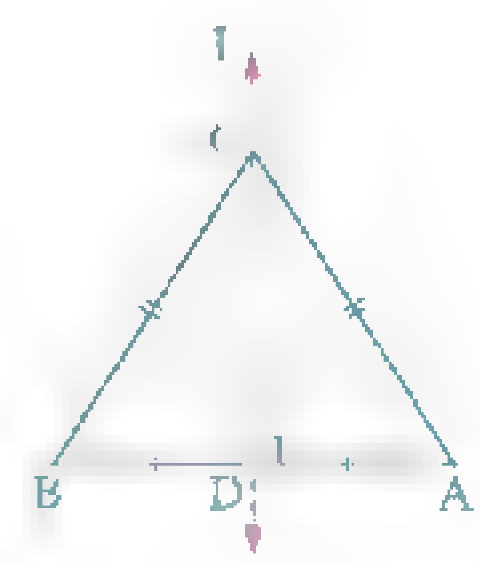
If the straight line L is the axis of $\overline{AB}$,

D $\in$ L , E $\in$ L and F $\in$ L , then

DA = DB , EA = EB and FA = FB

The converse of the previous property is true :

i.e. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.



In the opposite figure :

If C is a point such that CA = CB , then the point C lies on the axis of $\overline{AB}$

Example 3

In the opposite figure :

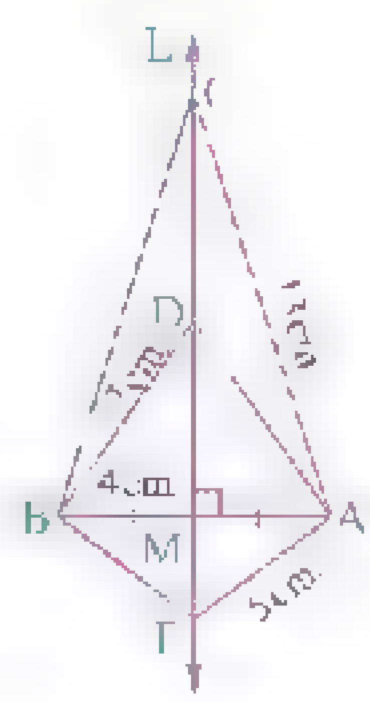
The straight line L is the axis of $\overline{AB}$

If the points C , D and E belong to the straight line L

, L $\cap$ $\overline{AB}$ = {M} where AC = 13 cm. ,

DB = 7 cm. , AE = 5 cm. and MB = 4 cm.

Find the length of each of : $\overline{CB}$, $\overline{DA}$, $\overline{EB}$ and $\overline{MA}$



Solution

Given The straight line L is the axis of AB , C , D and E belong to the straight line L , $L \cap \overline{AB} = \{M\}$

AC = 13 cm. , DB = 7 cm. , AE = 5 cm. and MB = 4 cm.

R.T.F. The lengths of : $\overline{CB}$, $\overline{DA}$, $\overline{EB}$ and $\overline{MA}$

Proof $\because$ C , D and E belong to L (the axis of $\overline{AB}$)

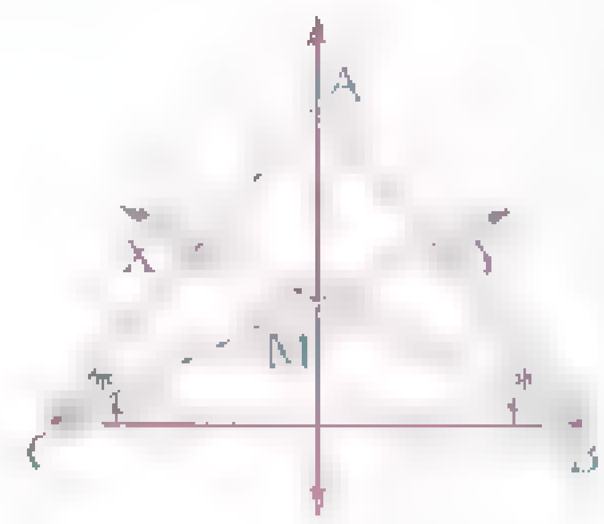
$\therefore$ CB = CA = 13 cm. , DA = DB = 7 cm. ,

EB = EA = 5 cm. , MA = MB = 4 cm. (The req.)

Example 8 Δ ABC is an isosceles triangle where AB = AC , $\overrightarrow{BX}$ bisects $\angle$ ABC and intersects $\overline{AC}$ at X , $\overrightarrow{CY}$ bisects $\angle$ ACB and intersects $\overline{AB}$ at Y

If $\overrightarrow{BX} \cap \overrightarrow{CY} = \{M\}$, **prove that :** $\overleftrightarrow{AM} \perp \overline{BC}$

Solution



Given AB = AC , $\overrightarrow{BX}$ bisects $\angle$ ABC and

$\overrightarrow{CY}$ bisects $\angle$ ACB

R.T.P. $\overleftrightarrow{AM} \perp \overline{BC}$

Proof $\because$ AB = AC $\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) (1)

, $\because$ $\overrightarrow{BX}$ bisects $\angle$ ABC $\therefore$ m ($\angle$ MBC) = $\frac{1}{2}$ m ($\angle$ ABC) (2)

Similarly

$\because$ $\overrightarrow{CY}$ bisects $\angle$ ACB $\therefore$ m ($\angle$ MCB) = $\frac{1}{2}$ m ($\angle$ ACB) (3)

From (1) , (2) and (3) , we deduce that :

m ($\angle$ MBC) = m ($\angle$ MCB) $\therefore$ MB = MC

i.e. M is at equal distances from B and C

$\therefore$ M $\in$ the axis of BC (4)

, $\because$ AB = AC **i.e.** A is at equal distances from B and C

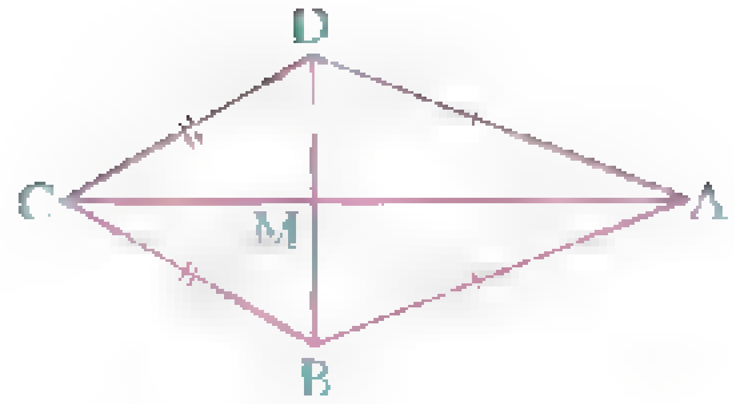
$\therefore$ A $\in$ the axis of $\overline{BC}$ (5)

From (4) and (5) : $\therefore$ $\overleftrightarrow{AM}$ is the axis of $\overline{BC}$ $\therefore$ $\overleftrightarrow{AM} \perp$ BC (Q.E.D.)

2 ***In the opposite figure :***

$\overline{BD} \cap \overline{AC} = \{M\}$, AB = AD and BC = DC

Prove that : M is the midpoint of $\overline{BD}$



Axis of symmetry of the isosceles triangle

The isosceles triangle has one axis of symmetry.

It is the straight line drawn from the vertex angle perpendicular to its base.

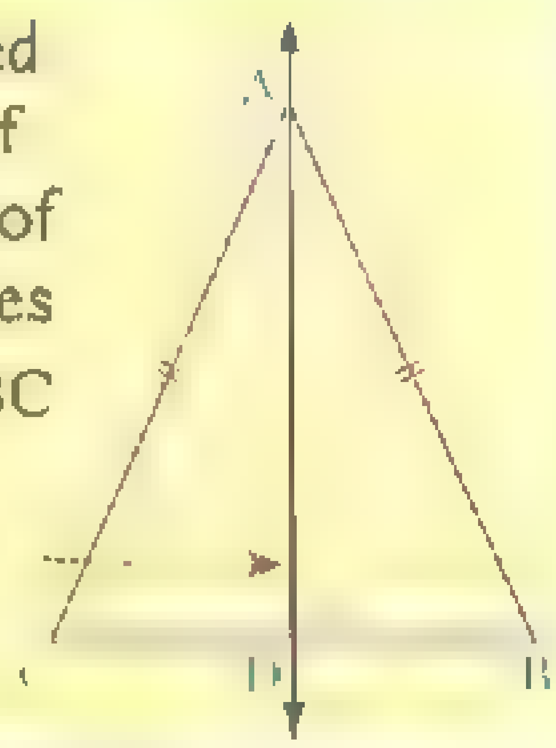
For example:

If ABC is an isosceles triangle where

AB = AC and $\overleftrightarrow{AD} \perp \overline{BC}$, then

$\overleftrightarrow{AD}$ is called the axis of symmetry

of the isosceles triangle ABC

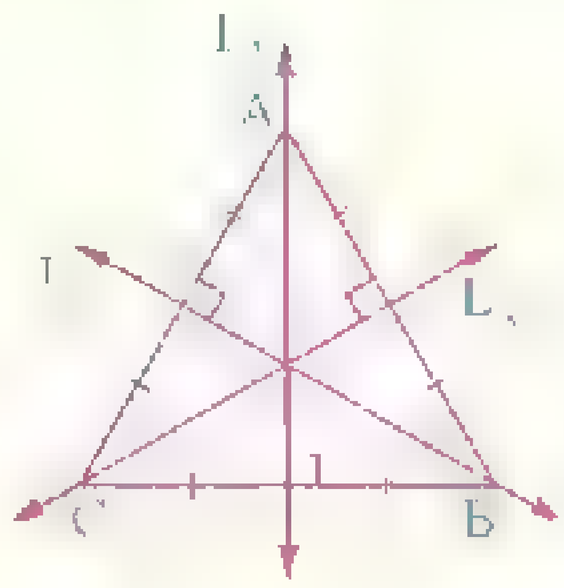


Remarks

1. The equilateral triangle has three axes of symmetry , they are the three perpendiculars drawn from its vertices to the opposite sides.

   In the opposite figure :

   The straight lines L_1 , L_2 and L_3 are the axes of symmetry of the equilateral triangle ABC



2. The scalene triangle has no axes of symmetry.

UNIT FIVE

Inequality

Lessons of the unit :

1. Inequality.
2. Comparing the measures of angles in a triangle.
3. Comparing the lengths of sides in a triangle.
4. Triangle inequality.

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Unit Objectives : By the end of this unit, student should be able to

- recognize the concept of inequality.
- recognize the axioms of the inequality relation
- compare between the measures of angles in the triangle
- deduce the relation between the measures of two angles in a triangle when the two opposite sides to these angles are not equal in length
- compare side lengths in the triangle
- deduce the relation between the lengths of two sides in a triangle when the two opposite angles to these sides are not equal in measure.
- recognize the triangle inequality
- use the axioms of the inequality relation and the triangle inequality in solving problems in geometry.

Lesson 1 Inequality

The concept of inequality

- Through our study of the sets of numbers , we had shown the relation of inequality that is used for comparing two different numbers , we expressed that by using one of the two signs $>$ that is read ⟶ « is greater than » (or) $<$ that is read ⟶ « is smaller than »
- Since the lengths of line segments and measures of angles are numbers , then we can use the relation of inequality to compare between the lengths of two line segments or between the measures of two angles

For example:

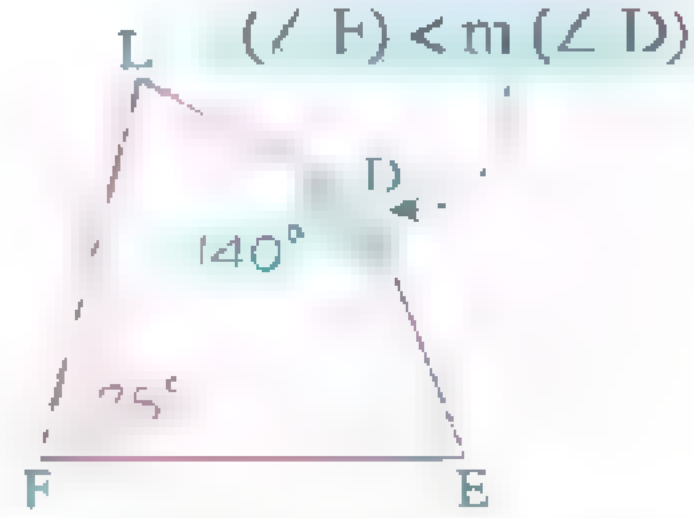
- **In Δ ABC :**

If AC = 5 cm. and AB = 3 cm. , then we deduce that :

The length of $\overline{AC}$ is greater than the length of $\overline{AB}$, then we write AC > AB

or the length of $\overline{AB}$ is smaller than the length of $\overline{AC}$, then we write AB < AC

- **Similarly in the figure DEFL :**



If m (∠ D) = 140° and m (∠ F) = 75° , then we deduce that :

m (∠ D) is greater than m (∠ F) ,

then we write : m (∠ D) > m (∠ F)

or m (∠ F) is smaller than m (∠ D)

, then we write : m (∠ F) < m (∠ D)

In the following , you will be given the axioms of inequality relation that you studied before.

Axioms of inequality relation

For any four numbers a , b , c and d :

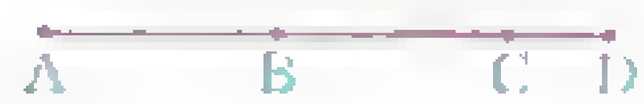
1. If $a > b$, then $a + c > b + c$
2. If $a > b$, then $a - c > b - c$
3. If $a > b$, $c > 0$, then $a\,c > b\,c$
4. If $a > b$, $b > c$, then $a > c$
5. If $a > b$, $c > d$, then $a + c > b + d$

Example

In the opposite figure :

If B and C belong to $\overline{AD}$ such that $AB > CD$

, **prove that :** $AC > BD$



Solution

Given B and C belong to $\overline{AD}$ and $AB > CD$

R.T.P. $AC > BD$

Proof $\because AB > CD$ (given) and adding BC to both sides

$\therefore AB + BC > CD + BC$

$\therefore AC > BD$ (Q.E.D.)

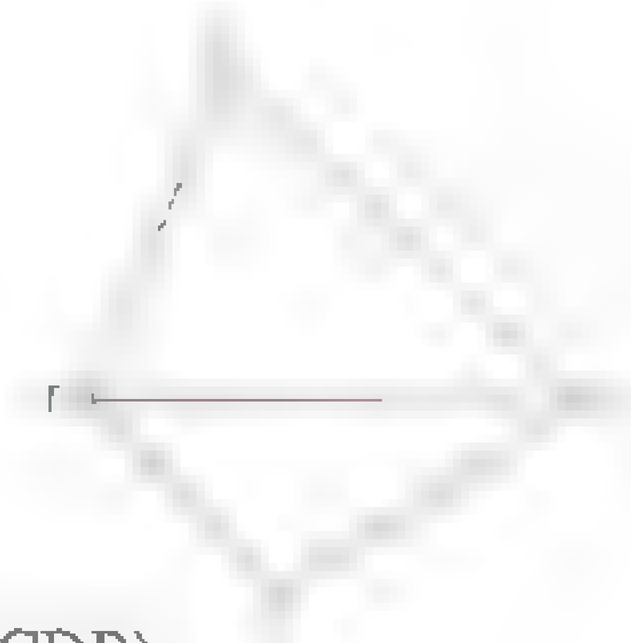
Example

In the opposite figure :

If $m(\angle ADB) > m(\angle ABD)$ and

$m(\angle CBD) < m(\angle CDB)$

, **prove that :** $m(\angle ADC) > m(\angle ABC)$



Solution

Given $m(\angle ADB) > m(\angle ABD)$ and $m(\angle CBD) < m(\angle CDB)$

R.T.P. $m(\angle ADC) > m(\angle ABC)$

Proof $\because m(\angle CBD) < m(\angle CDB)$ (given)

$\therefore m(\angle CDB) > m(\angle CBD)$ (1)

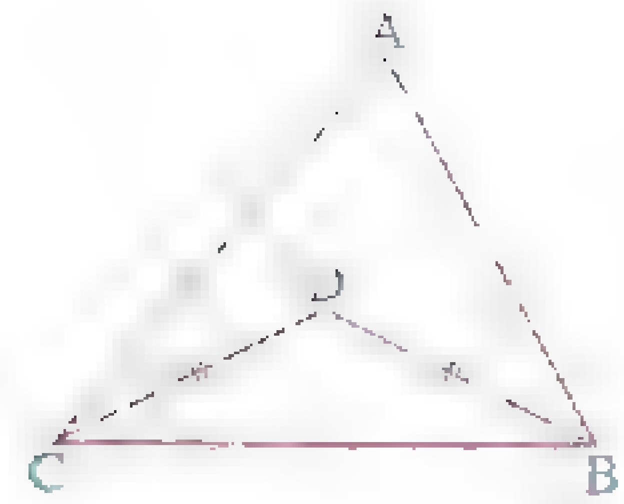
, $\because m(\angle ADB) > m(\angle ABD)$ (given) (2)

Adding (1) and (2) :

$\therefore m(\angle CDB) + m(\angle ADB) > m(\angle CBD) + m(\angle ABD)$

$\therefore m(\angle ADC) > m(\angle ABC)$ (Q.E.D.)

Example [illegible] **In the opposite figure :**



If m (∠ ABC) > m (∠ ACB)

and BD = DC

, prove that : m (∠ ABD) > m (∠ ACD)

Solution

Given | m (∠ ABC) > m (∠ ACB) and BD = DC

R.T.P. | m (∠ ABD) > m (∠ ACD)

Proof | ∵ DB = DC ∴ m (∠ DBC) = m (∠ DCB) (1)

, ∵ m (∠ ABC) > m (∠ ACB) (given) (2)

Subtracting (1) from (2) :

∴ m (∠ ABC) − m (∠ DBC) > m (∠ ACB) − m (∠ DCB)

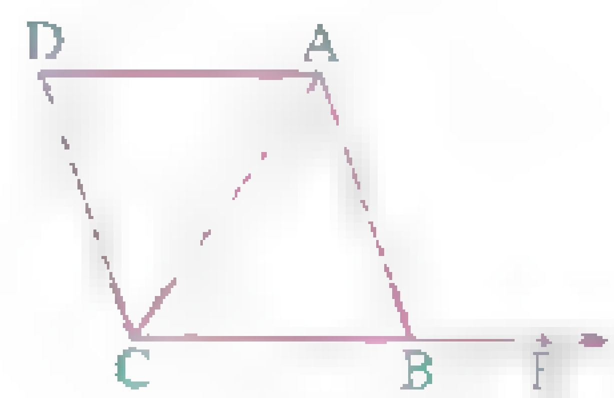
∴ m (∠ ABD) > m (∠ ACD) (Q.E.D.)

Remember that

The measure of any exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

[illegible]

In the opposite figure :



ABCD is a parallelogram and E ∈ $\overrightarrow{CB}$

Prove that :

m (∠ ABE) > m (∠ ACD)

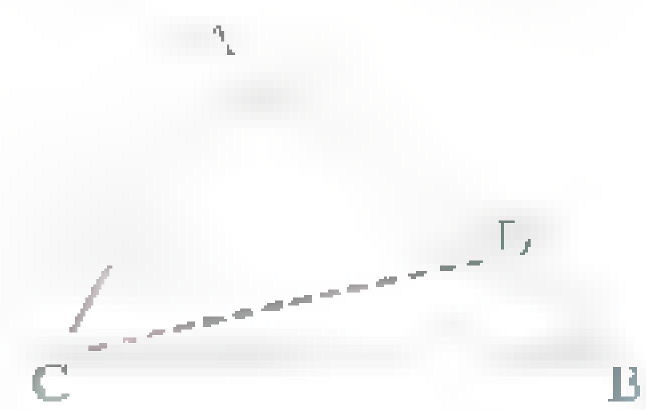
Lesson 2 Comparing the measures of angles in a triangle

From your study of the previous unit , you learnt that if two sides of a triangle are congruent , then the opposite angles to these sides are equal in measure

In the following , you shall study the relation between the measures of two angles of a triangle when the two opposite sides to these angles are not equal in length.

Theorem

In a triangle , if two sides have unequal lengths , then the longer is opposite to the angle of the greater measure.



Given | ABC is a triangle in which $AB > AC$

R.T.P. | $m(\angle ACB) > m(\angle ABC)$

Construction | Take $D \in \overline{AB}$ such that $AD = AC$

Proof | In ΔACD :

$\because AD = AC \quad \therefore m(\angle ADC) = m(\angle ACD)$

$\because \angle ADC$ is an exterior angle of ΔDBC

$\therefore m(\angle ADC) > m(\angle B)$

From (1) and (2) :

$\therefore m(\angle ACD) > m(\angle B)$

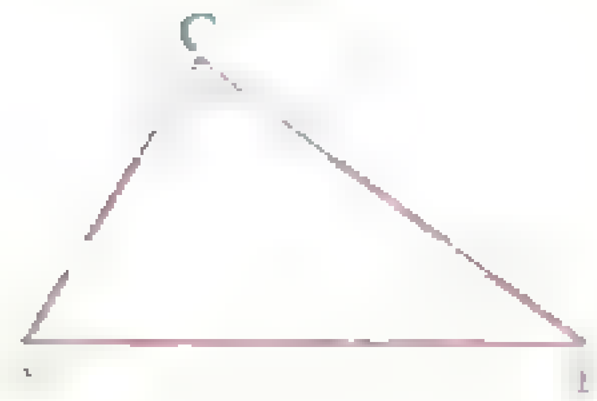
$, \because m(\angle ACB) > m(\angle ACD)$

$\therefore m(\angle ACB) > m(\angle ABC)$ (Q.E.D.

Remember The measure of the exterior angle is greater than the measure of any interior angle of the triangle except its adjacent angle

Remark

The greatest angle in measure of the triangle is opposite to the longest side of the triangle and its measure is greater than 60° and the smallest angle in measure of the triangle is opposite to the shortest side of the triangle and its measure is less than 60°



i.e. In ΔABC : If $AB > BC > AC$,

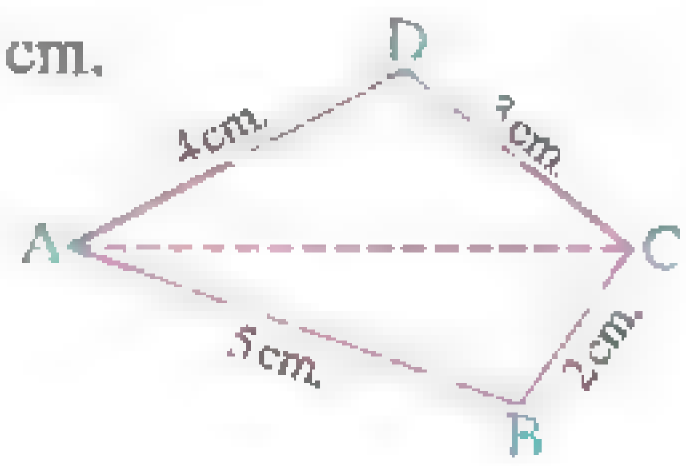
then $m(\angle C) > m(\angle A) > m(\angle B)$

, $m(\angle C) > 60°$ and $m(\angle B) < 60°$

Example 1 ABCD is a quadrilateral in which AB = 5 cm. , BC = 2 cm. , CD = 3 cm. and DA = 4 cm.

Prove that : $m(\angle DCB) > m(\angle DAB)$

Solution



Given | AB = 5 cm. , BC = 2 cm. , CD = 3 cm. and DA = 4 cm.

R.T.P. | $m(\angle DCB) > m(\angle DAB)$

Construction | Draw $\overline{AC}$

Proof | In ΔACD : $\because$ AD = 4 cm. and CD = 3 cm.

$\therefore AD > CD$ $\therefore m(\angle ACD) > m(\angle CAD)$ (1)

In Δ ABC : $\because$ AB = 5 cm. and CB = 2 cm.

$\therefore AB > CB$ $\therefore m(\angle ACB) > m(\angle CAB)$ (2)

Adding (1) and (2) : $\therefore m(\angle ACD) + m(\angle ACB) > m(\angle CAD) + m(\angle CAB)$

$\therefore m(\angle DCB) > m(\angle DAB)$ (Q.E.D.)

TRY 1

Choose the correct answer from those given :

1 In ΔXYZ , $XZ > XY$, then $m(\angle Z)$ .... $m(\angle Y)$

(a) > (b) < (c) = (d) ≥

2 In ΔABC , AB = 8 cm. , AC = 10 cm. , then ....

(a) $m(\angle A) > m(\angle B)$ (b) $m(\angle B) > m(\angle C)$

(c) $m(\angle B) < m(\angle C)$ (d) $m(\angle B) > m(\angle A)$

[3] In Δ XYZ , XY = 4 cm. , YZ = 8 cm. , XZ = 6 cm. , then ....

(a) m ($\angle$ Z) > m ($\angle$ Y) (b) m ($\angle$ Z) > m ($\angle$ X)

(c) m ($\angle$ X) < m ($\angle$ Y) (d) m ($\angle$ Z) < m ($\angle$ Y)

[4] In Δ ABC , AB = 3 cm. , BC = 5 cm. , AC = 4 cm. , then the ascending order of the measures of the angles of Δ ABC is ......

(a) $\angle$ C , $\angle$ B , $\angle$ A (b) $\angle$ C , $\angle$ A , $\angle$ B

(c) $\angle$ A , $\angle$ B , $\angle$ C (d) $\angle$ B , $\angle$ A , $\angle$ C

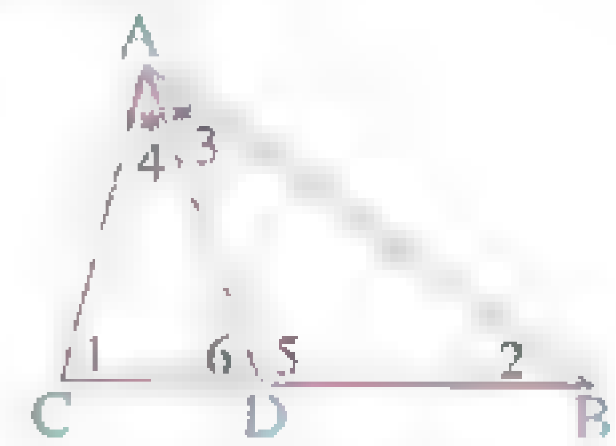
Example 11 ABC is a triangle in which AB > AC and $\angle$ BAC is bisected by $\overrightarrow{AD}$ which intersects $\overline{BC}$ at D

Prove that : ΔABD is an obtuse-angled triangle.

Solution

Given ABC is a triangle in which AB > AC and $\overrightarrow{AD}$ bisects $\angle$ BAC

R.T.P. ΔABD is an obtuse-angled triangle.



Proof In ΔABC :

$\because$ AB > AC $\therefore$ m ($\angle$ 1) > m ($\angle$ 2)

$\because$ $\overrightarrow{AD}$ bisects $\angle$ BAC

$\therefore$ m ($\angle$ 3) = m ($\angle$ 4)

$\therefore$ m ($\angle$ 1) + m ($\angle$ 4) > m ($\angle$ 2) + m ($\angle$ 3)

but m ($\angle$ 1) + m ($\angle$ 4) = m ($\angle$ 5)(*)

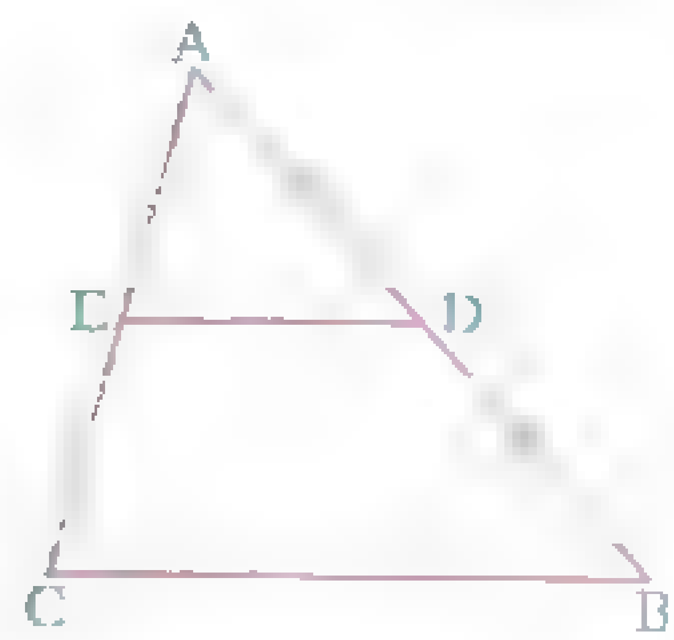
because $\angle$ 5 is an exterior angle of ΔADC

$\therefore$ m ($\angle$ 5) > m ($\angle$ 2) + m ($\angle$ 3)(**)

$\therefore$ ΔABD is an obtuse-angled triangle. (Q.E.D.)

TRY 2

In the opposite figure :



ABC is a triangle in which AB > AC

, D and E are the midpoints

of $\overline{AB}$ and $\overline{AC}$ respectively.

Prove that : m ($\angle$ AED) > m ($\angle$ ADE)

(*) Remember : The measure of an exterior angle of a triangle is equal to the sum of measures of its non adjacent interior angles.

(**) Remember : If the measure of an angle in a triangle is greater than the sum of measures of the two other angles , then this angle is an obtuse angle.

Lesson 3 Comparing the lengths of sides in a triangle

From your previous study , you learnt that , if two angles are equal in measure in a triangle , then the two opposite sides to these angles are equal in length

In the following , you shall study the relation between the lengths of two sides in a triangle when the two opposite angles are not equal in measure.

Theorem

In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

Given | ABC is a triangle in which m (∠ C) > m (∠ B)

R.T.P. AB > AC

Proof ∵ $\overline{AB}$ and $\overline{AC}$ are two line segments.

∴ One of the following cases should be verified :

1 AB > AC 2 AB = AC 3 AB < AC

Unless AB > AC , then either AB = AC or AB < AC

- If AB = AC , then m (∠ C) = m (∠ B) and this contradicts the given where m (∠ C) > m (∠ B)
- If AB < AC , then m (∠ C) < m (∠ B) according to the previous theorem. Again this contradicts the given where m (∠ C) > m (∠ B)

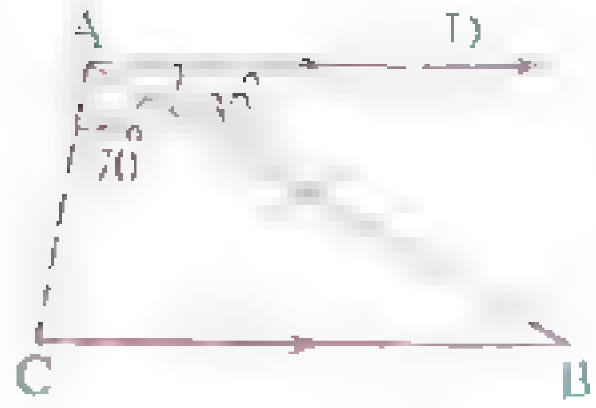
∴ It should be that AB > AC (Q.E.D.)

Example **In the opposite figure :**

ABC is a triangle in which m ($\angle$ BAC) = 70° ,

$\overrightarrow{AD}$ // $\overline{BC}$ and m ($\angle$ DAB) = 30°

Prove that : AB > AC



Solution

Given | $\overrightarrow{AD}$ // $\overline{BC}$, m ($\angle$ BAC) = 70° and m ($\angle$ DAB) = 30°

R.T.P. | AB > AC

Proof | $\because$ $\overrightarrow{AD}$ // $\overline{BC}$ and $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore$ m ($\angle$ B) = m ($\angle$ DAB) = 30° (alternate angles)(*)

$\therefore$ In Δ ABC : m ($\angle$ C) = 180° – (30° + 70°) = 80°

$\therefore$ m ($\angle$ C) > m ($\angle$ B) $\therefore$ AB > AC (Q.E.D.)

Corollaries

Corollary 1

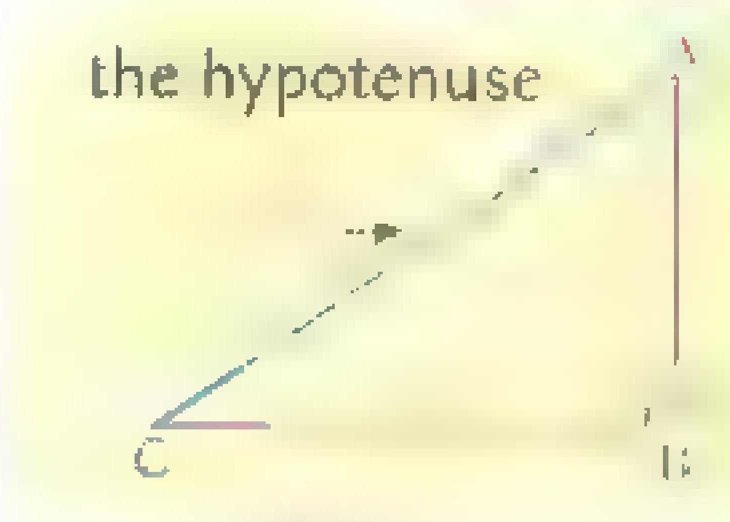
In the right-angled triangle , the hypotenuse is the longest side.

In the opposite figure :

If Δ ABC is right-angled at B , then m ($\angle$ B) > m ($\angle$ A) ,

m ($\angle$ B) > m ($\angle$ C) because $\angle$ B is a right angle and each of $\angle$ A and $\angle$ C is acute , so we find that :

AC > BC and AC > AB (according to the previous theorem).



Notice that :

In the obtuse-angled triangle , the side opposite to the obtuse angle is the longest side in the triangle.

Corollary 2

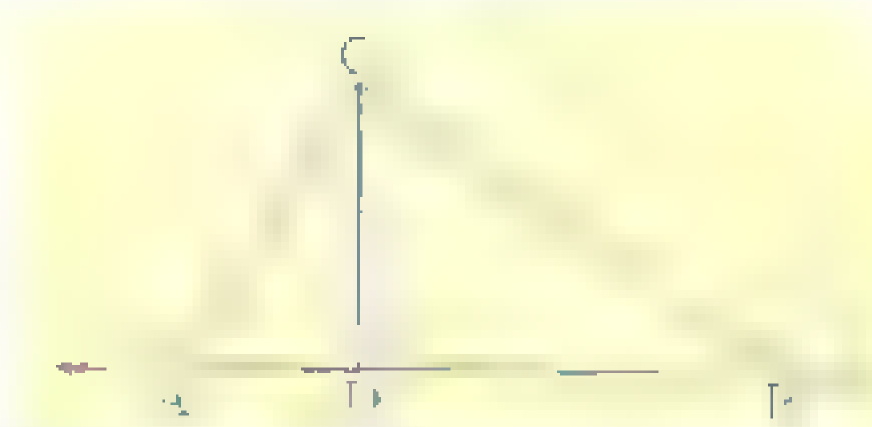
The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given straight line.

In the opposite figure :

If C $\notin$ $\overleftrightarrow{AB}$ and D $\in$ $\overleftrightarrow{AB}$ such that $\overline{CD}$ $\perp$ $\overleftrightarrow{AB}$,

then $\overline{CB}$ is the hypotenuse in Δ CBD which is right-angled at D ,

$\overline{CA}$ is the hypotenuse in Δ CDA which is right-angled at D and so on ..



According to corollary 1 , we find that CB > CD , CA > CD and so on ...

i.e. CD < CB and CD < CA

(*) [illegible] If a straight line intersects two parallel straight lines, then each two alternate angles are equal in measure.

Definition

The distance between any point and a given straight line is the length of the perpendicular line segment drawn from this point to the given line.

In the previous figure :

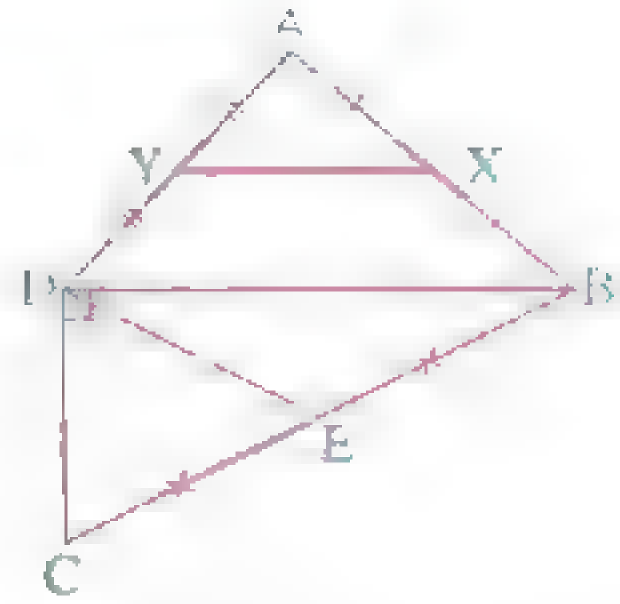
The distance between the point C and the straight line $\overline{AB}$ is the length of $\overline{CD}$

Example

In the opposite figure :

ABCD is a quadrilateral X , Y and E are the midpoints of $\overline{AB}$, $\overline{AD}$ and $\overline{BC}$ respectively and m (∠ BDC) = 90°

Prove that : DE > XY



Solution

Given X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AD}$, E is the midpoint of $\overline{BC}$ and m (∠ BDC) = 90°

R.T.P. DE > XY

Proof In Δ ABD : ∵ X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{AD}$

∴ XY = $\frac{1}{2}$ BD(*) (1)

In Δ DBC : ∵ m (∠ BDC) = 90° and E is the midpoint of $\overline{BC}$

∴ DE = $\frac{1}{2}$ BC (2)

, ∵ $\overline{BC}$ is the hypotenuse of Δ BDC ∴ BC > BD

∴ $\frac{1}{2}$ BC > $\frac{1}{2}$ BD (3)

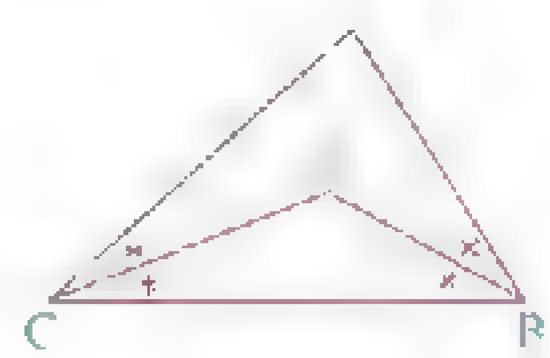
From (1) , (2) and (3) : ∴ DE > XY (Q.E.D.)

In the opposite figure :

ABC is a triangle in which AC > AB ,

$\overrightarrow{BM}$ bisects ∠ ABC and $\overrightarrow{CM}$ bisects ∠ ACB

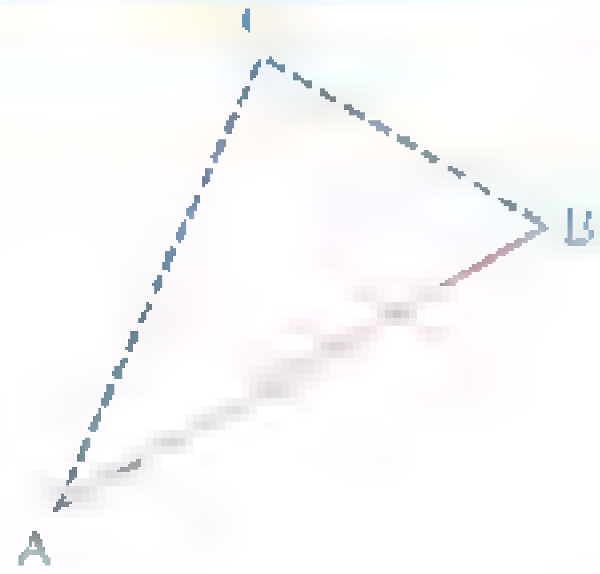
Prove that : MC > MB



(*) Remember The line segment joining the midpoints of two sides in a triangle is parallel to the third side and its length equals half the length of this side.

Lesson 4 Triangle inequality

We know that the shortest distance between two points is the length of the line segment joining them.



For example:

In the opposite figure :

The shortest distance between A and B is the length of $\overline{AB}$

So , for any point $C \notin \overleftrightarrow{AB}$, then $AB < AC + CB$

Generally

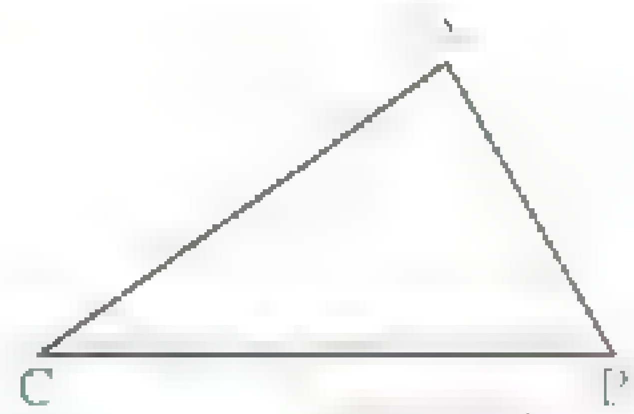
In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.

i.e. In any triangle such as ΔABC

, we get : $AB + BC > AC$

, $BC + CA > AB$

, $CA + AB > CB$



Corollary

The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum.

And you can prove that as follows :

In the opposite figure ABC is a triangle and from the triangle inequality :

$AC + AB > BC$

, $\because AB + BC > AC$ i.e. $BC > AC - AB$

From (1) and (2) , we deduce that : $\boxed{AC - AB < BC < AC + AB}$

! Remark

To check the possibility that three lengths can be side lengths of a triangle , do as follows :

Compare the greatest length with the sum of the other two lengths :

- If the greatest length is greater than or equal to the sum of the other two lengths , you deduce that the three given lengths **couldn't be** lengths of the three sides of a triangle (i.e. No triangle could be drawn with these side lengths).
- If the greatest length is less than the sum of the other two lengths , you deduce that the three given lengths **could be** lengths of the three sides of a triangle.
  (i.e. A triangle could be drawn with these side lengths).

Example 1

Is it possible to draw a triangle whose side lengths are as follows (giving reason) :

1. 5 cm. , 7 cm. , 12 cm.
2. 4 cm. , 6 cm. , 11 cm.
3. 14 cm. , 9 cm. , 7 cm.
4. 8 cm. , 18 cm. , 8 cm.

Solution

1. $\because 5 + 7 = 12$

   $\therefore$ It is not possible to draw a triangle of side lengths 5 cm. , 7 cm. and 12 cm.

2. $\because 4 + 6 < 11$

   $\therefore$ It is not possible to draw a triangle of side lengths 4 cm. , 6 cm. and 11 cm.

3. $\because 9 + 7 > 14$

   $\therefore$ It is possible to draw a triangle of side lengths 14 cm. , 9 cm. and 7 cm.

4. $\because 8 + 8 < 18$

   $\therefore$ It is not possible to draw a triangle of side lengths 8 cm. , 18 cm. and 8 cm.

Example 2

Find the interval to which the length of the third side of each of the following triangles belongs if the lengths of the other two sides are :

1. 4 cm. , 3 cm.
2. 4.5 cm. , 7.5 cm.
3. $2\sqrt{5}$ cm. , $2\sqrt{5}$ cm.

Solution ∵ The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum , and let the length of the third side be ℓ cm. , then

1 $4-3<\ell<4+3$ $\quad \therefore 1<\ell<7$ $\quad \therefore \ell \in]1 , 7[$

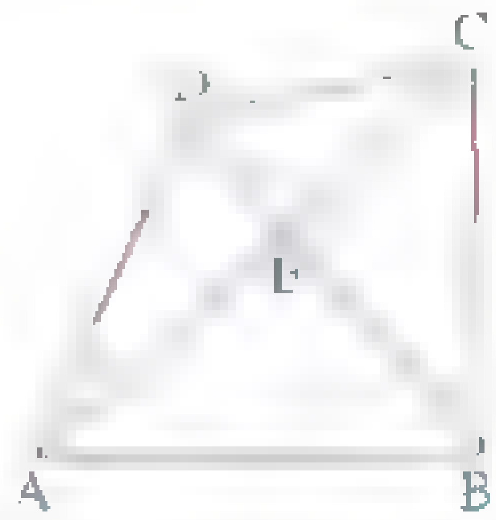
2 $7.5-4.5<\ell<7.5+4.5$ $\quad \therefore 3<\ell<12$ $\quad \therefore \ell \in]3 , 12[$

3 $2\sqrt{5}-2\sqrt{5}<\ell<2\sqrt{5}+2\sqrt{5}$ $\quad \therefore 0<\ell<4\sqrt{5}$ $\quad \therefore \ell \in]0 , 4\sqrt{5}[$

Example 3 **In the opposite figure :**

ABCD is a quadrilateral whose diagonals intersect at E

Prove that : $AC + BD > BC + AD$



Solution

Given | ABCD is a quadrilateral whose diagonals intersect at E

R.T.P. | $AC + BD > BC + AD$

Proof In Δ EBC : $EC + EB > BC$ (triangle inequality) (1)

In Δ EAD : $EA + ED > AD$ (triangle inequality) (2)

Adding (1) and (2) : $\therefore EC + EA + EB + ED > BC + AD$

, ∵ $EC + EA = AC$, $EB + ED = BD$ $\quad \therefore AC + BD > BC + AD$ (Q.E.D.)

TRY

1 *Put (✓) in the space in front of each group of the following lengths which can be side lengths of a triangle :*

1 2 cm. , 3 cm. , 4 cm. ( ) 2 3 cm. , 6 cm. , 2 cm. ( )

3 10 cm. , 3 cm. , 7 cm. ( ) 4 12 cm. , 5 cm. , 7.5 cm. ( )

2 *Find the interval to which the length of the third side of each of the following ngles belongs if the lengths of the other two sides are :*

1 6 cm. , 5 cm. 2 7.5 cm. , 7.5 cm.

EL-MOASSER

By a group of supervisors

EXERCISES

2nd PREP. 2025

FIRST TERM

Maths

Contents

First

Algebra and Statistics

UNIT ONE

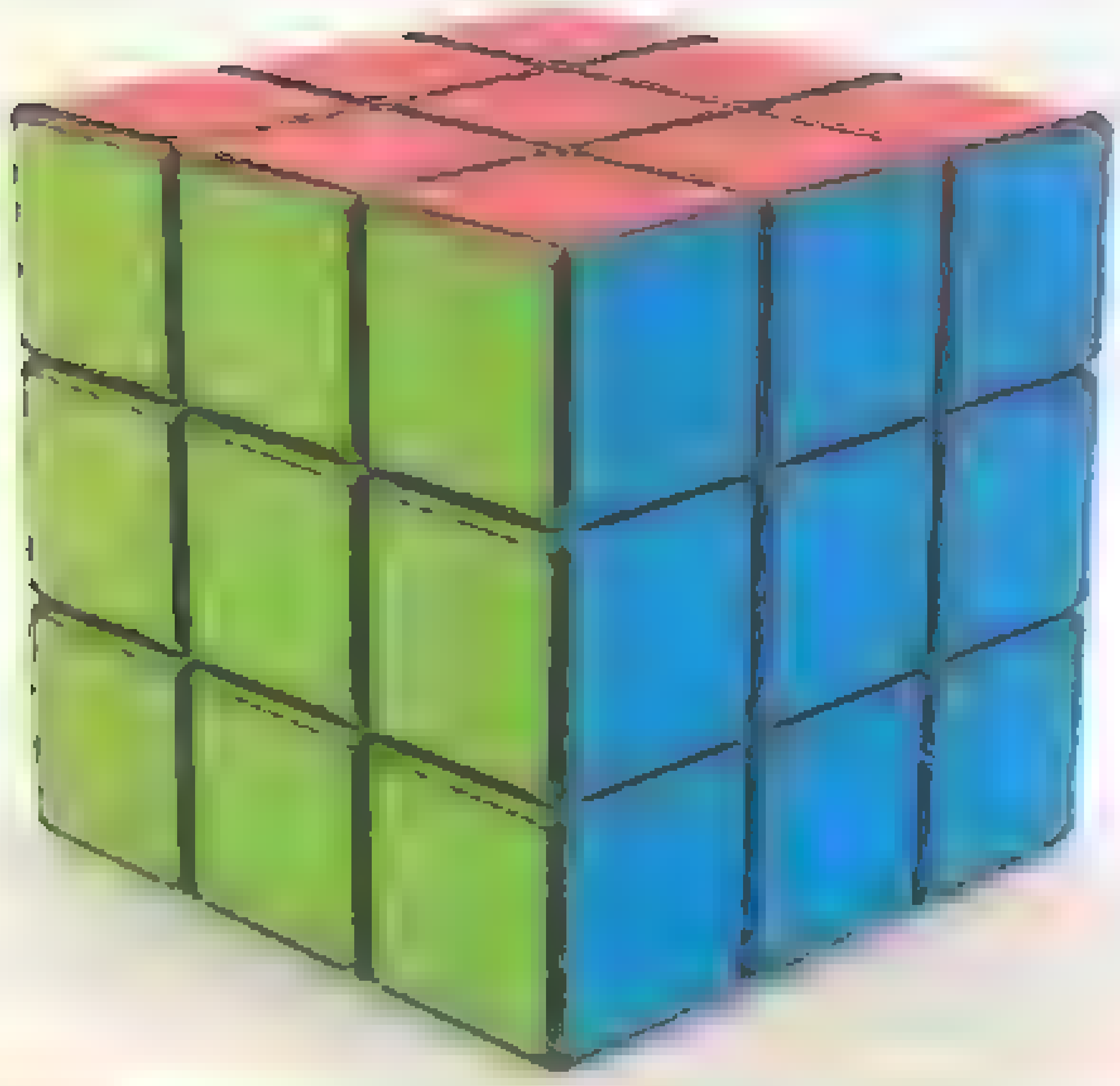
Real Numbers

Exercises of the unit :

1. The cube root of a rational number.
2. The set of irrational numbers $\acute{\mathbb{Q}}$
3. The set of real numbers $\mathbb{R}$
   - Ordering numbers in $\mathbb{R}$
4. Intervals.
5. Operations on the real numbers.
6. Operations on the square roots.
7. The two conjugate numbers.
8. Operations on the cube roots.
9. Applications on the real numbers.
10. Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$

Scan

the QR code to solve an interactive test on each lesson



From the school book

Exercise 1 The cube root of a rational number

Remember · Understand · Apply · Problem Solving

Interactive test

1 Complete :

$\sqrt[3]{8} = \ldots\ldots$

$\sqrt[3]{-27} = \ldots\ldots$

$\sqrt[3]{216} = \ldots\ldots$

$\sqrt[3]{-343} = \ldots\ldots$

$\sqrt[3]{0.001} = \ldots\ldots$

$\sqrt[3]{-\frac{8}{27}} = \ldots\ldots$

7 $\sqrt[3]{8} + \sqrt[3]{-8} = \ldots\ldots$

8 $\sqrt[3]{27} - \sqrt[3]{64} = \ldots\ldots$

9 $\sqrt[3]{27} - \sqrt[3]{-27} = \ldots\ldots$

10 $\sqrt{9} + \sqrt[3]{-8} = \ldots\ldots$

$-\sqrt[3]{-1} - \sqrt{1} = \ldots\ldots$

$\sqrt[3]{a^3} = \ldots\ldots$

$\sqrt[3]{-27a^6} = \ldots\ldots$

$\sqrt[3]{\ldots} = 4$

$\sqrt[3]{\ldots\ldots} = 10$

$\sqrt{16} = \sqrt[3]{\ldots\ldots}$

$\sqrt[3]{125} - \sqrt{\ldots}$

$\sqrt[3]{64 + \ldots} = 5$

2 Choose the correct answer from those given :

1 $\sqrt[3]{(-8)^2} = \ldots\ldots$

(a) 2 (b) -2 (c) 4 (d) -4

2 $\sqrt{25}-\sqrt[3]{-125} = \dots\dots\dots$

(a) 10 (b) zero (c) 5 (d) ± 5

3 $\sqrt{(-2)^2}+\sqrt[3]{(-2)^3} = \dots\dots\dots$

(a) -4 (b) 8 (c) 4 (d) zero

4 $\sqrt[3]{1000} \times \sqrt[3]{-0.008} = \dots\dots\dots$

(a) $\frac{1}{2}$ (b) 10 (c) 2 (d) -2

5 $\sqrt[3]{-27}+\sqrt{12\frac{1}{4}}+\sqrt[3]{0.125} = \dots\dots\dots$

(a) 1 (b) zero (c) -1 (d) $\frac{11}{2}$

6 If $-\sqrt{25} = \sqrt[3]{y}$, then $y = \dots\dots\dots$

(a) 5 (b) -5 (c) 125 (d) -125

7 If $x^3 = 64$, then $\sqrt{x} =$

(a) 4 (b) -4 (c) 2 (d) -2

8 If $x^3 = 27$, then $x^2 = \dots\dots\dots$

(a) 3 (b) 6 (c) 9 (d) 81

9 $\sqrt[3]{x^6} = \sqrt{\dots\dots\dots}$

(a) x^3 (b) x^2 (c) x (d) x^4

10 If $\frac{x}{3} = \frac{9}{x^2}$, then $x =$

(a) 1 (b) 3 (c) 9 (d) 27

Find the value of x in each of the following :

1 $\sqrt[3]{x} = 5$	2 $\sqrt[3]{x} = -\frac{1}{4}$	3 $\sqrt[3]{x} = -\sqrt{4}$
4 $\sqrt[3]{x} - 3 = -1$	5 $x^3 = -8$	6 $x^3 = 64$
7 $x^3 + 5 = 32$	8 $2x^3 = 54$	9 $\frac{1}{5}x^3 = -200$

Find the S.S. of each of the following equations in $\mathbb{Q}$:

1) $x^3 + 27 = 0$ | 2) $8x^3 + 7 = 8$ | 3) $2x^3 - 5 = x^3 + 3$

4) $(x+3)^3 = 343$ | 5) $(2x+1)^3 - 7 = 20$ | 6) $(5x-2)^3 + 10 = 18$

Find each of the following :

1) $\sqrt[3]{2^9 \times 3^6}$ | 2) $\sqrt{\sqrt[3]{729}}$ | 3) $\sqrt{27\sqrt[3]{27}}$

6 A cube of volume 27 cm^3 Find :

1) The area of one face. 2) The total area. « 9 cm^2 , 54 cm^2 »

Find the inner edge length of a cube vessel with capacity of one litre.

Find the diameter length of a sphere whose volume is $\frac{1372}{81}\pi$ cube unit.

Find the length of the diameter of a sphere whose volume is 113.04 cm^3 ($\pi = 3.14$)

For excellent pupils

Find the S.S. of each of the following equations in $\mathbb{Q}$:

1) $(x^2 + 6)^3 = 1000$ | 2) $(x^3 - 14)^2 = 169$

3) $\sqrt[3]{(x-1)^2} = \sqrt[3]{25}$

11 If $\sqrt[3]{\sqrt{x} + 19} = 3$, find the value of $\sqrt[3]{x}$ « 4 »

From the school book

Exercise 2 ? The set of irrational Numbers $\mathbb{Q}'$

● Remember ◆ Understand ○ Apply ⁂ Problem Solving

Interactive test

1 In each of the following , show which of them is a rational number and which of them is an irrational number :

1 -5	2 $2\frac{2}{3}$	3 2.06	4 2.3×10^5
5 $-\sqrt{36}$	6 $\sqrt[3]{36}$	7 $\sqrt{7}$	8 zero
9 $\|-5\|$	10 $\sqrt[3]{-\frac{64}{81}}$	11 $\sqrt{\frac{25}{16}}$	12 $\sqrt{\frac{1}{3}}$
13 $\sqrt[3]{3\frac{3}{8}}$	14 $\sqrt[3]{0.343}$	15 $\frac{\pi}{2}$	16 $(-5)^{\text{zero}}$
17 $\frac{\text{zero}}{3}$	18 $\frac{\sqrt{9}}{\sqrt{4}}$	19 $\sqrt{9}+\sqrt{16}$	20 $\sqrt{4}-\sqrt{11}$

2 Find an approximated value for each of the following numbers :

1 $\sqrt{11}$ "to the nearest hundredth".

2 $\sqrt[3]{7}$ "to the nearest tenth".

3 $\sqrt[3]{-9}$ "to the nearest tenth".

3 Find two consecutive integers for each of the following numbers to be included between them :

1 $\sqrt{5}$ 2 $\sqrt{12}$ 3 $\sqrt[3]{10}$ 4 $\sqrt[3]{-20}$

1 **If X is an integer , find the value of X in each of the following cases :**

1 $X < \sqrt{2} < X + 1$	« 1 »	2 $X < \sqrt{80} < X + 1$	« 8 »
3 $X < \sqrt[3]{5} < X + 1$	« 1 »	4 $X < \sqrt[3]{50} < X + 1$	« 3 »
5 $X < \sqrt[3]{-100} < X + 1$	« −5 »	6 $X < \lvert -\sqrt{35} \rvert < X + 1$	« 5 »

2 **Find an approximated value for each of the following numbers , then check your answer using the calculator :**

1 $\sqrt{20}$ 2 $\sqrt[3]{17}$ 3 $\sqrt{5} + 1$ 4 $\sqrt[3]{9} - 1$

3 **Choose the correct answer from the given ones :**

1 The irrational number in the following numbers is ......

(a) $\sqrt{\frac{1}{4}}$ (b) $\sqrt[3]{8}$ (c) $\sqrt{\frac{4}{9}}$ (d) $\sqrt{2}$

2 If $X = \sqrt{2}$, $y = 2$, then which of the following does not represent a rational number ?

(a) $X^2 + y$ (b) $X + y^2$ (c) $\sqrt{X^2 y}$ (d) $\sqrt{2}\, X y$

3 The irrational number located between 2 and 3 is ....

(a) $\sqrt{10}$ (b) $\sqrt{7}$ (c) 2.5 (d) $\sqrt{3}$

4 The irrational number located between −2 and −1 is .

(a) −3 (b) $-1\frac{1}{2}$ (c) $-\sqrt{3}$ (d) $\sqrt{2}$

5 $\sqrt{10} \simeq$

(a) 2.99 (b) 3.71 (c) 3 (d) −3.2

6 The nearest integer to $\sqrt[3]{25}$ is ....

(a) 5 (b) 3 (c) 2 (d) 12.5

7 If $n \in \mathbb{Z}_+$, $n < \sqrt{26} < n + 1$, then $n =$ ....

(a) 25 (b) 5 (c) −5 (d) 24

8 The side length of a square whose area is 6 cm^2 is ....

(a) a natural number. (b) an integer.

(c) a rational number. (d) an irrational number.

9 The area of a square whose side length is $\sqrt{3}$ cm. is .... cm².

(a) $4\sqrt{3}$ (b) 9 (c) 3 (d) 6

10 The square whose area is 10 cm², its side length is .... cm.

(a) 5 (b) -5 (c) $\sqrt{10}$ (d) $-\sqrt{10}$

The S.S. of the equation : $\left(x-\sqrt{5}\right)\left(x+\sqrt{3}\right)=0$ in $\mathbb{Q}'$ is

(a) $\{\sqrt{5}\}$ (b) $\{-\sqrt{3}\}$ (c) $\{\sqrt{5}, \sqrt{3}\}$ (d) $\{\sqrt{5}, -\sqrt{3}\}$

Find the value of x in each of the following cases and determine whether $x \in \mathbb{Q}$ or $x \in \mathbb{Q}'$:

1. $5x^2=10$ « $\pm\sqrt{2}$ »
2. $4x^2=9$ « $\pm\frac{3}{2}$ »
3. $x^3=125$ « 5 »
4. $3x^3=27$ « $\sqrt[3]{9}$ »
5. $0.1x^2=10$ « ± 10 »
6. $0.001x^3=-8$ « -20 »
7. $(x-1)^2=4$ « 3 or -1 »
8. $(x-5)^3=1$ « 6 »

Find in $\mathbb{Q}'$ the S.S. of each of the following equations :

1. $\frac{2}{5}x^2=\frac{25}{2}$
2. $\frac{5}{4}x^3=-2$
3. $125x^3-7=20$

$\frac{1}{4}x^2+2=66$

$(x^3+5)(x^2-3)=$ zero

$\left(x+\sqrt{7}\right)\left(x^3-6\right)=$ zero

Prove that :

1. $\sqrt{2}$ is included between 1.4 and 1.5
2. $\sqrt{11}$ is included between 3.31 and 3.32
3. $\sqrt[3]{2}$ is included between 1.2 and 1.3
4. $\sqrt[3]{15}$ is included between 2.4 and 2.5
5. $\sqrt[3]{-17}$ is included between -2.6 and -2.5
6. $\sqrt{3}+1$ is included between 2.7 and 2.8

1 **Determine the point that represents each of the following numbers on the number line :**

1. $\sqrt{3}$
2. $-\sqrt{11}$
3. $\sqrt{10}$
4. $\sqrt{5}+1$
5. $1 - \sqrt{7}$
6. $2\sqrt{5}$

2 Draw the number line and label point A which represents $\sqrt{2}$

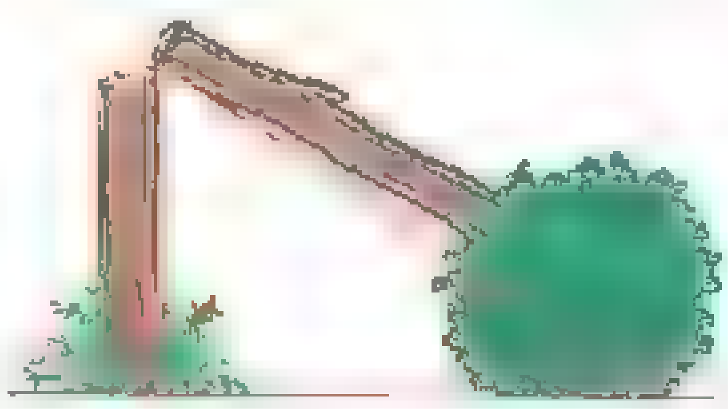
- Label point B which represents $1+\sqrt{2}$
- Label point C which represents $1-\sqrt{2}$

3 Calculate the side length and the diagonal length of a square whose area equals 10 cm^2

« $\sqrt{10}$ cm. , $\sqrt{20}$ cm. »

Life Application

4 A tree is 3 metres long. Its upper part was broken because of the wind and it made an angle with the surface of the ground. If the length of the left part of the tree is 1 metre , find the distance between the base of the tree and the point of touching of its top with the ground.



« $\sqrt{3}$ metres »

For excellent pupils

5 Without using the calculator , prove that $\sqrt{3}+\sqrt{2}$ is included between 3 and 4

From the school book

Exercise 3 The set of real numbers $\mathbb{R}$ and ordering numbers in $\mathbb{R}$

Remember · Apply · Problem Solving

Interactive test

1 Complete the following table by placing (✓) in the suitable place as shown in the first case :

The number	Natural	Integer	Rational	Irrational	Real
5	✗	✓	✓	✗	✓
$\sqrt{2}$					
$1\frac{1}{2}$					
$\sqrt[3]{9}$					
$\lvert -2 \rvert$					
$-\sqrt{4}$					
$\frac{5}{2}$					
0.3					
$\sqrt{\ }$ [illegible]					

2 Put the suitable sign (> , < or =) :

$\sqrt{5}$ 2

$\sqrt[3]{-24}$ .... -2

$\sqrt[3]{3}-1$ 0.2

2 $\sqrt{7}$ 2.6

5 $3-\sqrt{5}$ .... $\sqrt[3]{-1}$

8 $\sqrt{2}-1$ .... $1-\sqrt{2}$

$\sqrt[3]{24}$ 3

$\sqrt[3]{8}$ .... $\sqrt{4}$

1 Choose the correct answer from those given :

1 $\mathbb{R} = \dots\dots\dots$

(a) $\mathbb{Q} \cup \mathbb{Q}'$ (b) $\mathbb{Z}_+ \cup \mathbb{Z}_-$ (c) $\mathbb{R}_+ \cup \mathbb{R}$ (d) $\mathbb{N} \cup \mathbb{R}_-$

2 $\mathbb{Q} \cap \mathbb{Q}' = \dots\dots\dots$

(a) $\mathbb{Q}$ (b) $\mathbb{Q}'$ (c) $\mathbb{R}$ (d) $\varnothing$

3 $\mathbb{Q} \cup \mathbb{Q}' = \dots\dots\dots$

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $\mathbb{Q}$ (d) $\mathbb{Q}'$

4 $\mathbb{R}_+ \cap \mathbb{R}_- = \dots\dots\dots$

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $\mathbb{R}_+$ (d) $\mathbb{R}_-$

5 $\mathbb{R}_+ \cup \mathbb{R}_- = \dots\dots\dots$

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{R}_+$ (d) $\mathbb{R}^*$

6 $\mathbb{R} - \mathbb{Q}' = \dots\dots\dots$

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{Q}$ (d) $\{0\}$

7 $\mathbb{R} - \mathbb{Q} = \dots\dots\dots$

(a) $\mathbb{Q}'$ (b) $\mathbb{R}$ (c) $\varnothing$ (d) $\{0\}$

8 $\mathbb{R}_+ \cap \{-1 , 0 , 1\} = \dots\dots\dots$

(a) $\{0 , 1\}$ (b) $\{1\}$ (c) $\{0\}$ (d) $\varnothing$

9 $\{x : x \in \mathbb{R} , x < 0\} = \dots\dots\dots$

(a) $\mathbb{R}_+$ (b) $\mathbb{R}_-$ (c) $\mathbb{R}^*$ (d) $\mathbb{R}$

If x is a negative real number , then which of the following numbers is positive ?

(a) x^2 (b) x^3 (c) $2x$ (d) $\frac{x}{2}$

11 The S.S. of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is $\dots\dots\dots$

(a) $\{-1\}$ (b) $\{1 , -1\}$ (c) $\{1\}$ (d) $\varnothing$

$\sqrt{(2 - \pi)^2} \dots\dots\dots (2 - \pi)$ (where π is the ratio between the circumference of the circle and its diameter length)

(a) $=$ (b) $<$ (c) $>$ (d) $\leq$

If $\frac{1}{a}$ and $\frac{a}{\sqrt{5}}$ are two real numbers included between 0 and 1 , then a = ……

(a) -2 (b) 1 (c) $\sqrt{5}$ (d) 2

Arrange the following numbers ascendingly :

1. $\sqrt{8}$, $-\sqrt{3}$, $\sqrt{15}$, $\sqrt{5}$, $-\sqrt{7}$ and $-\sqrt{11}$
2. $\sqrt{27}$, $-\sqrt{45}$, $\sqrt{20}$, 0.6 and $\sqrt[3]{-1}$

Arrange the following numbers descendingly :

1. $\sqrt{62}$, 8 , $-\sqrt{50}$ and $\sqrt{70}$
2. $\sqrt{6}$, 9 , $-\sqrt{10}$, $-\sqrt{7}$, $-\sqrt{50}$ and $\sqrt{101}$

Write three positive irrational numbers less than 2

Write three negative irrational numbers greater than $-\sqrt{6}$

Write four irrational numbers included between 15 and 17

Prove that $\sqrt{3}$ is between 1.7 and 1.8 , then represent $\sqrt{3}$, 1.7 and 1.8 on the number line.

Solve the following equations to the nearest hundredth given $x \in \mathbb{R}$:

1. $x^2 - 6 = 0$
2. $\frac{3}{4} x^2 = 24$
3. $\frac{1}{2} x^2 - 5 = 0$
4. $\frac{2}{x^3} + 5 = 21 \quad (x \neq 0)$
5. $(x^2 - 9)(x^3 - 5) = 0$
6. $(2x^3 - 5)(x^2 + 1) = 0$

Geometric Applications

Find the side length of a square whose area is 5 cm^2. Is the side length a rational number ? «$\sqrt{5}$ cm.»

Find the edge length of a cube whose volume is 1.728 cm^3. Is the edge length a rational number ? «$\frac{6}{5}$ cm»

A cube whose total area is 13.5 cm^2. Find its edge length. Is the edge length a rational number ? «1.5 cm.»

A square is of side length 6 cm. Find its diagonal length. «$\sqrt{72}$ cm»

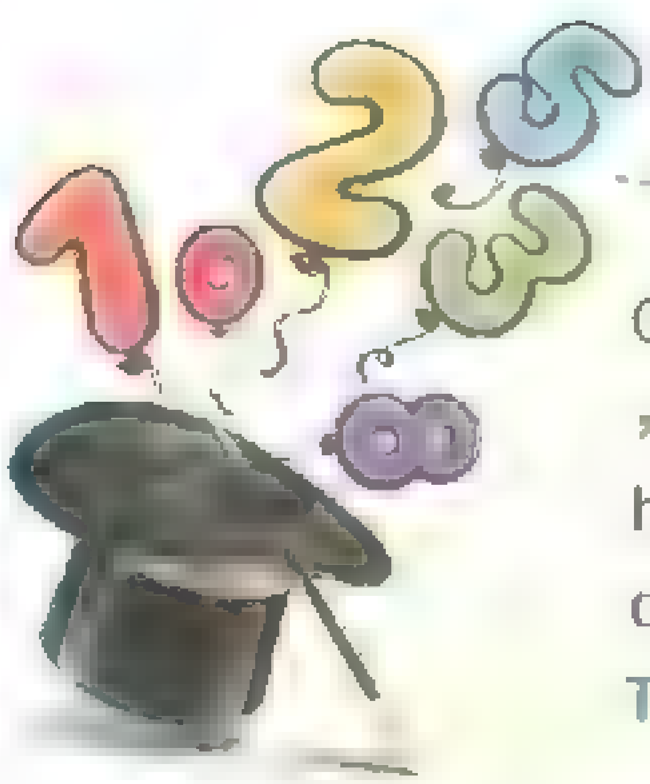
A rectangle with dimensions 5 cm. and 7 cm. Find the length of its diagonal. And if its area equals the area of a square , then find the side length of the square and its diagonal length. «$\sqrt{74}$ cm. , $\sqrt{35}$ cm. , $\sqrt{70}$ cm.»

For excellent pupils

Without using the calculator , prove that : $\sqrt[3]{3} > \sqrt{2}$

Two real numbers , the sum of their squares is 7 and the greater number is 2

Find the other number. «$\sqrt{3}$ or $-\sqrt{3}$»



Wonders of numbers

Choose a number from 1 to 9 , multiply it by 3 , add 3 to the product , and multiply the result by 3 once again "use calculator" Find the sum of the digits of the product.

The answer is always 9.

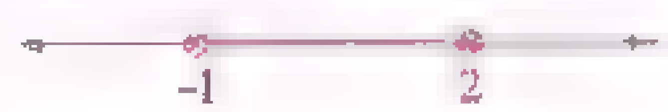
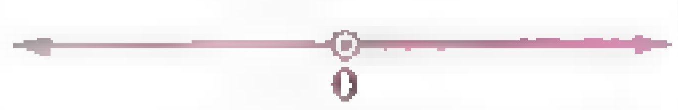
From the school book

Exercise 4 Intervals

● Remember ● Understand ○ Apply ● Problem Solving

Interactive test

Complete the following table :

The interval	Expression by description method	Its representation on the number line
1 $[-1 , 2]$	$\{x : -1 \le x \le 2 , x \in \mathbb{R}\}$	
2 $[1 , 3[$	....	....
3	$\{x : 0 < x \le 3 , x \in \mathbb{R}\}$	....
4	....	
5 $]-\infty , 1]$	....	....
6	....	
7	$\{x : x < 4 , x \in \mathbb{R}\}$	....
8 $[-2 , \infty[$	....	....

Choose the correct answer from the given ones :

● 1 $\mathbb{R} = \dots\dots\dots$

(a) $\mathbb{R}_+ \cap \mathbb{R}_-$ (b) $\mathbb{R}_+ \cup \mathbb{R}_-$ (c) $]-\infty , \infty[$ (d) $\mathbb{Q} \cap \acute{\mathbb{Q}}$

● 2 $\mathbb{R}_+ = \dots\dots\dots$

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

3 $\mathbb{R}_-$ = ...

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

4 The set of non-negative real numbers = ...

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

5 The set of non-positive real numbers = ...

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

3 Complete each of the following using one of the symbols $\in$ or $\notin$:

1 3 .... $[3 , 5]$

2 -2 .... $]-2 , 1]$

3 0 .... $[-1 , 4[$

4 $|-3|$ .... $[2 , \infty[$

5 $\sqrt{9}$ .... $]-3 , \infty[$

6 $\sqrt[3]{-1}$ .... $]-\infty , 1[$

7 1.3×10^{-5} .... $\mathbb{R}_+$

8 $\sqrt{2}$ .... $[2 , 5]$

9 5 .... $]\sqrt{5} , \sqrt{23}[$

10 $\sqrt[3]{-125}$ .... $]-\sqrt{25} , \sqrt{25}]$

4 If $X = [2 , 5[$ and $Y = [-1 , 3[$, find using the number line :

1 $X \cup Y$ | 2 $X \cap Y$ | 3 $X - Y$

4 $Y - X$ | 5 X' | 6 Y'

5 If $X =]-\infty , 3]$ and $Y = [-4 , \infty[$, find using the number line :

1 $X \cup Y$ | 2 $X \cap Y$ | 3 $X - Y$

4 $Y - X$ | 5 X' | 6 Y'

6 If $X = [-1 , 4]$, $Y = [3 , \infty[$ and $Z = \{3 , 4\}$, find the following using the number line :

1 $X \cup Y$ | 2 $X \cap Y$ | 3 $X - Y$ | 4 $X - Z$

5 $Y \cap Z$ | 6 $Y - X$ | 7 X' | 8 Y'

7 Find using the number line :

1 $[-1 , 4] \cap [2 , 5]$ | 2 $[-1 , 4] \cup [2 , 5]$ | 3 $]-2 , 3] \cap]0 , 1[$

4 $]-2 , 3] \cup]0 , 1[$ | 5 $[2 , 6] - [-1 , 3[$ | 6 $[-1 , 3[- [2 , 6]$

7 $[-3 , 0[\cup]0 , 2]$ | 8 $[-3 , 0] \cap]0 , 2]$ | 9 $[1 , 2] - [2 , 4]$

10 $[-2 , 4] - [1 , 2]$ | 11 $[-1 , 4] \cap [5 , 7[$ | 12 $[-1 , 5] -]-1 , 5[$

8 Find using the number line :

1. $[-1, \infty[\cup [-3, 4]$
2. $[2, \infty[\cap]-2, 3[$
3. $]-\infty, 3] \cap [-4, \infty[$
4. $[2, \infty[\cup]-\infty, 3]$
5. $]-\infty, 3] - [-1, \infty[$
6. $]-\infty, -3] - [-3, 1]$
7. $]-\infty, 2] -]-\infty, 0]$
8. $]-\infty, 3[\cup]4, \infty[$

9 Complete the following :

1. $[3, 5] \cup \{3, 5\} =$ ……….
2. $]3, 5[\cup \{3, 5\} =$ ……….
3. $[3, 5] \cap \{3, 5\} =$ ……….
4. $]3, 5[\cap \{3, 5\} =$ ……….
5. $[3, 5] - \{3, 5\} =$ ……….
6. $]3, 5[- \{3, 5\} =$ ……….
7. $\{3, 5\} \quad [3, 5] =$ ……….
8. $\{3, 5\} -]3, 5[=$ ……….
9. $]3, 5[\cup \{3\} =$ ……….
10. $[3, 5] - \{5\} =$ ……….
11. $]2, 5[\cap \{-2, 3, 4\} =$ ……….
12. $]-3, 5] \cup \{-2, 3, 4\} =$ ……….

Choose the correct answer from the given ones :

1. $[-3, 4] - \{-3, 5\} =$ ……….
   (a) $]-3, 4[$ (b) $]-3, 4]$ (c) $]-3, 5[$ (d) $[-3, 5[$
2. If $x \in [-3, \infty[$, then ……….
   (a) $x < -3$ (b) $x \leq -3$ (c) $x > -3$ (d) $x \geq -3$
3. If $X = \{x : x \in \mathbb{R}, 2 < x \leq 5\}$, then $[3, 4]$ ………. X
   (a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$
4. $\{3\} \cap [3, 6] =$ ……….
   (a) $\varnothing$ (b) $\{3\}$ (c) $]3, 6]$ (d) $\{6\}$
5. $\{8, 9, 10\} -]8, 10[=$ ……….
   (a) $\varnothing$ (b) $\{8, 10\}$ (c) $\{9\}$ (d) $\mathbb{N}$
6. The sum of all real numbers in $[\ \ 75, 75]$ is ……….
   (a) -75 (b) 75 (c) 150 (d) zero

11 **Complete the following :**

1. $\mathbb{R} \cap [-3, 3] = \dots\dots$
2. $\mathbb{R} \cup]-1, 4] = \dots\dots$
3. $\mathbb{R} - [-1, \infty[= \dots\dots$
4. $\mathbb{R}_- - [-3, 1] = \dots\dots$
5. $]-2, 5] - \mathbb{R}_+ = \dots\dots$
6. $[-2, 2] - \mathbb{R}_- = \dots\dots$
7. $]-3, 2] \cap \mathbb{Z}_+ = \dots\dots$
8. $\mathbb{N} \cap [-5, 2[= \dots\dots$
9. $\mathbb{Z} \cap [-1, 3[= \dots\dots$
10. $\mathbb{R}_+ \cap [0, 5] = \dots\dots$
11. $\mathbb{R}_- \cap [-3, 2] = \dots\dots$

For excellent pupils

12 **Choose the correct answer from the given ones :**

1. **In the opposite figure :**

If x is a real number , then $x \in \dots\dots$

(a) $\mathbb{R}_-$ (b) $\mathbb{R}_+$ (c) $]-\infty, -1]$ (d) $]-\infty, -1[$

2. If $x \in [-3, 4]$, then $x^2 \in \dots\dots$

(a) $[9, 16]$ (b) $[0, 9]$ (c) $[0, 16]$ (d) $[-9, 0]$

3. If $x \in [-5, 4]$, then $x^2 \in \dots\dots$

(a) $[0, 16]$ (b) $[16, 25]$ (c) $[0, 25]$ (d) $[-5, 0]$

4. If $x \in [1, 16]$, then $-\sqrt{x} \in \dots\dots$

(a) $[1, 4]$ (b) $[-1, 4]$ (c) $[-4, -1]$ (d) $[-4, 0]$

5. If $X \subset \mathbb{R}$, $[2, 5] - X =]2, 5[$, then $X = \dots$

(a) $[2, 5]$ (b) $\{2, 5\}$ (c) $[2, 5[$ (d) $]2, 5]$

6. If $X \subset \mathbb{R}$, $]4, 7] \cup X = [1, 7]$, then $X = \dots\dots$

(a) $[1, 3[$ (b) $[1, 3]$ (c) $[1, 4[$ (d) $[1, 5]$

7. If $M \subset \mathbb{R}$, $M \cap [3, 8[= [3, 8[$, then $M = \dots\dots$

(a) $]3, 8[$ (b) $]3, 8]$ (c) $[3, 9]$ (d) $[3, 7]$

8. If $]-\infty, k[\cap [-2, 5] = [-2, 3[$, then $k = \dots\dots$

(a) -2 (b) 5 (c) 3 (d) zero

9. If $[-1, x] \cap [y, 5] = [2, 3]$, then $x^y = \dots\dots$

(a) 8 (b) $\frac{1}{5}$ (c) 9 (d) 1

13 If $X \cap Y = [4, 7]$, $X \cup Y = [3, 7]$ and $X \subset Y$, find : X , Y and $Y - X$

From the school book

Exercise 5 Operations on the real numbers

Remember Understand Apply Problem Solving

Interactive test

1 Find each of the following in the simplest form :

1) $\sqrt{3}+2\sqrt{3}$
2) $2\sqrt{5}-3\sqrt{5}+\sqrt{5}$
3) $5\sqrt[3]{7}-8\sqrt[3]{7}+2\sqrt[3]{7}$
4) $4\sqrt{5}\quad 2\sqrt{5}+5\sqrt{5}-\sqrt{5}$

2 Find each of the following in the simplest form :

1) $\sqrt{5}-\sqrt{3}+2\sqrt{5}+\sqrt{3}$
2) 📖 $2\sqrt{3}+5+\sqrt{3}\quad 6$
3) 📖 $2\sqrt{7}-3\sqrt{2}+\sqrt{7}+5\sqrt{7}$
4) $2\sqrt{2}-3\sqrt[3]{2}+5\sqrt{2}+\sqrt[3]{2}$
5) $\frac{1}{4}\sqrt{2}+\frac{2}{7}\sqrt{5}+\frac{3}{4}\sqrt{2}-\frac{2}{7}\sqrt{5}$
6) $8\sqrt{\frac{1}{4}}+2\sqrt[3]{3}-\sqrt[3]{64}-5\sqrt[3]{3}$

3 Find the result of each of the following :

1) $\sqrt{3}\times\sqrt{3}$
2) $-2\sqrt{5}\times 3\sqrt{5}$
3) $2\times 3\sqrt{2}$
4) $\frac{1}{3}\sqrt{3}\times\sqrt{3}$
5) $\left(\sqrt[3]{5}\right)^3\times 3\sqrt{3}$
6) $2\sqrt{3}\times\frac{2\sqrt{7}}{7}\div\frac{20\sqrt{3}}{5\sqrt{7}}$

4 Find the result of each of the following in the simplest form :

1) $2\left(\sqrt{2}+\sqrt{5}\right)$
2) 📖 $\sqrt{2}\left(5+\sqrt{2}\right)$
3) 📖 $\sqrt{7}\left(\sqrt{7}+2\right)$
4) 📖 $-\sqrt{3}\left(-5\quad\sqrt{3}\right)$
5) $-2\sqrt{5}\left(3-\sqrt{5}\right)$
6) $\sqrt{7}\left(\frac{2}{\sqrt{7}}-\sqrt{7}+3\right)$
7) $-3\left(8+2\sqrt{3}\right)+6\sqrt{3}$
8) 📖 $\sqrt{5}\left(3-\sqrt{5}\right)-2\left(1+\sqrt{5}\right)$

Find the result of each of the following operations :

$(\sqrt{2}+1)(\sqrt{2}-1)$	$(4-3\sqrt{2})(4+3\sqrt{2})$	$(\sqrt{5}-1)^2$
$(2\sqrt{3}+4)^2$	$(\sqrt{3}+2)(\sqrt{3}-1)$	$(5-\sqrt{3})^2 - 28$

6 Make the denominator in each of the following an integer :

1 $\frac{10}{\sqrt{5}}$	2 $-\frac{6}{\sqrt{3}}$	3 $\frac{2}{3\sqrt{2}}$
4 $\frac{6}{2\sqrt{3}}$	5 $\frac{\sqrt{2}+3}{\sqrt{2}}$	6 $\frac{\sqrt{5}-15}{2\sqrt{5}}$

Choose the correct answer from those given :

1. $2\sqrt{3}+3\sqrt{3}=$ ....
   (a) $5\sqrt{6}$ (b) $5\sqrt{3}$ (c) $6\sqrt{3}$ (d) $5\sqrt[3]{3}$

2. $5+7\sqrt{2}-4+\sqrt{2}=$ ....
   (a) 15 (b) $1+7\sqrt{2}$ (c) $1+8\sqrt{2}$ (d) $1+6\sqrt{2}$

3. $-2\sqrt{3}\times\sqrt{3}=$ ....
   (a) -6 (b) $-2\sqrt{3}$ (c) $2\sqrt{3}$ (d) 6

4. $(2\sqrt[3]{5})^3=$ ....
   (a) 10 (b) 20 (c) $4\sqrt[3]{5}$ (d) 40

5. The additive inverse of the number $\frac{6}{\sqrt{2}}$ is ....
   (a) $-2\sqrt{3}$ (b) $2\sqrt{3}$ (c) $-3\sqrt{2}$ (d) $3\sqrt{2}$

6. The additive inverse of the number $(\sqrt{2}-\sqrt{5})$ is ....
   (a) $\sqrt{2}+\sqrt{5}$ (b) $\sqrt{5}-\sqrt{2}$ (c) $\sqrt{2}-\sqrt{5}$ (d) $-\sqrt{2}-\sqrt{5}$

7. The multiplicative inverse of the number $\sqrt{5}$ is ....
   (a) -5 (b) $\frac{-1}{5}$ (c) $\frac{5}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{5}$

8. The multiplicative inverse of the number $\frac{\sqrt{2}}{6}$ is ....
   (a) $\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\sqrt{6}$ (d) $\frac{\sqrt{2}}{2}$

19 If $x = \sqrt{2} + 10$, $y = \sqrt{2} - 10$, then $(x + y)^2 = \cdots\cdots$

(a) 4 (b) 6 (c) 8 (d) $4\sqrt{2}$

Complete the following :

1 The multiplicative neutral in $\mathbb{R}$ is ...... and the additive neutral in $\mathbb{R}$ is ......

2 The additive inverse of the number $1 - \sqrt{2}$ is ....

3 The multiplicative inverse of the number $\frac{2\sqrt{3}}{5}$ is $\frac{\cdots\cdots}{6}$

4 The multiplicative inverse of the number $\frac{3}{\sqrt{3}}$ is $\frac{\cdots}{\sqrt{3}}$

5 $7 + \sqrt{3} = 5 + (\cdots\cdots + \cdots\cdots)$

6 If $a = \frac{\sqrt{2}}{\sqrt{3}}$, $b = \frac{\sqrt{3}}{\sqrt{2}}$, then $\frac{a}{b} = \cdots\cdots$

7 $(\sqrt{3} - 2)^2 = 7 - \cdots\cdots$

8 If $\sqrt{x} = \sqrt{2} + 1$, then $x = \cdots\cdots$

9 If $x^2 = (2\sqrt{3} - \sqrt{7})(2\sqrt{3} + \sqrt{7})$, then $x = \cdots\cdots$

10 If $x^2 - y^2 = 16$, $x - y = \sqrt{2}$, then $x + y = \cdots\cdots$

11 If the side length of a square is ℓ cm. and its area is 15 cm^2 , then the area of the square of side length 2ℓ cm. is ......

12 If $a \in \mathbb{R}$ and $b \in \mathbb{R}$, then $a - b$ means the sum of the number a and ...... of the number b

13 If $a \in \mathbb{N}$, $b \in \mathbb{Q}$ and $c \in \mathbb{R}$, then $a + b + c \in \cdots\cdots$

If $x = \sqrt{5} - 2$ and $y = \sqrt{5} + 2$, find the value of each of the following :

1 $x + y$

2 $x - y$

3 $x y$

4 $x^2 - y^2$

5 $x^2 + 2xy + y^2$

6 $x^2 - 2xy + y^2$

If $x = \sqrt{15} + 2$ and $y = 4 - \sqrt[3]{25}$, estimate the value of each of the following :

1 x , y

2 $x \times y$

3 $x + y$

Check the reasonability of each value using your calculator.

Geometric Application

A rectangle is of dimensions $\left(6+\sqrt{5}\right)$ cm. and $\left(6-\sqrt{5}\right)$ cm.

Calculate its perimeter and its area. « 24 cm. , 31 cm^2 »

For excellent pupils

If the multiplicative inverse of the number $\sqrt{a}-1$ is $\frac{\sqrt{a}+1}{4}$, find the numerical value of a

If $x = 2y = 4z = \sqrt{2}$, **find the value of :** $x^2 + 2y^2 + 4z^2$

If the number y is the additive inverse of x and $\frac{1}{2}(y-x) = 1-\sqrt{2}$

Prove that : $xy - 2\sqrt{2} = -3$

Wonders of numbers

- $1 \times 1 = 1$
- $11 \times 11 = 121$
- $111 \times 111 = 12321$
- $1111 \times 1111 = 1234321$

What happens when you multiply 11111×11111 ?

From the school book

Exercise 6 Operations on the square roots

Remember — Understand — Apply — Problem Solving

Interactive test

1 **Put each of the following in the form $a\sqrt{b}$ where a and b are two integers , b is the least possible value :**

1 $\sqrt{12}$	2 $\sqrt{28}$	3 $2\sqrt{72}$
4 $\frac{2}{5}\sqrt{1000}$	5 $2\sqrt{\frac{1}{2}}$	6 $6\sqrt{\frac{2}{3}}$

2 **Simplify each of the following to the simplest form :**

1. $\sqrt{50}+\sqrt{8}$ « $7\sqrt{2}$ »
2. $\sqrt{20}-\sqrt{45}$ « $-\sqrt{5}$ »
3. $3\sqrt{2}+\sqrt{8}-\sqrt{18}$ « $2\sqrt{2}$ »
4. $\sqrt{98}-\sqrt{128}-\sqrt{18}+4\sqrt{2}$ « zero »
5. $2\sqrt{18}+\sqrt{50}+\frac{1}{3}\sqrt{162}$ « $14\sqrt{2}$ »
6. $\sqrt{98}+\sqrt{50}-\frac{1}{2}\sqrt{200}-\sqrt{2}$ « $6\sqrt{2}$ »
7. $\sqrt{27}+5\sqrt{18}-\sqrt{300}$ « $15\sqrt{2}-7\sqrt{3}$ »

3 **Put each of the following in the simplest form :**

1. $2\sqrt{5}+4\sqrt{20}-5\sqrt{\frac{1}{5}}$ « $9\sqrt{5}$ »
2. $\sqrt{32}-\sqrt{72}+6\sqrt{\frac{1}{2}}$ « $\sqrt{2}$ »
3. $2\sqrt{5}+6\sqrt{\frac{1}{3}}-\sqrt{12}-5\sqrt{\frac{1}{5}}$ « $\sqrt{5}$ »
4. $\sqrt{3}+\frac{3}{\sqrt{3}}-\sqrt{2}\times\sqrt{6}$ « zero »
5. $\sqrt{18}-\frac{\sqrt{12}}{\sqrt{6}}$ « $2\sqrt{2}$ »
6. $\sqrt{(-5)^2}+\sqrt{18}-\frac{6}{\sqrt{2}}$ « 5 »

4 Simplify each of the following to the simplest form :

1 $2\sqrt{3} \times 5\sqrt{2}$ « $10\sqrt{6}$ »

2 $2\sqrt{18} \times 3\sqrt{2}$ « 36 »

3 $\sqrt{5} \times 2\sqrt{10}$ « $10\sqrt{2}$ »

4 $\sqrt{\frac{2}{7}} \times \sqrt{\frac{7}{2}}$ « 1 »

5 $\frac{3\sqrt{15}}{\sqrt{5}}$ « $3\sqrt{3}$ »

6 $12\sqrt{\frac{2}{3}} \times \sqrt{54}$ « 72 »

5 Simplify each of the following to the simplest form :

1 $\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)$

2 $5\sqrt{2}\left(2\sqrt{2}+\sqrt{12}\right)$

3 $\left(3\sqrt{5}-\sqrt{7}\right)\left(3\sqrt{5}+\sqrt{7}\right)$

4 $\left(\sqrt{3}-\sqrt{2}\right)^2$

5 $\left(\sqrt{3}+\sqrt{5}\right)^2-\sqrt{60}$

6 $\sqrt{18}-\frac{12}{\sqrt{6}}+\sqrt{2}\left(2\sqrt{3}-3\right)$

6 Write each of the following such that the denominator is an integer :

1 $\frac{\sqrt{3}}{\sqrt{2}}$

2 $\sqrt{\frac{5}{3}}$

3 $\frac{5\sqrt{3}}{\sqrt{5}}$

4 $\frac{4\sqrt{3}-\sqrt{2}}{2\sqrt{3}}$

7 Choose the correct answer from those given :

1 $\frac{\sqrt{63}}{\sqrt{7}} = \dots\dots\dots$

(a) 3 (b) $\sqrt{3}$ (c) 9 (d) ± 3

2 $\sqrt{8}-\sqrt{2} = \dots\dots\dots$

(a) $\sqrt{6}$ (b) $\sqrt{2}$ (c) 2 (d) 1

3 $\left(\sqrt{8}+\sqrt{2}\right)^2 = \dots\dots\dots$

(a) $\sqrt{10}$ (b) 10 (c) 18 (d) $\sqrt{18}$

4 $\left(\sqrt{7}-\sqrt{5}\right)\left(\sqrt{7}+\sqrt{5}\right) = \dots\dots\dots$

(a) 2 (b) 12 (c) $2\sqrt{7}$ (d) $-2\sqrt{5}$

5 $\sqrt{5}+\sqrt{5} = \dots\dots\dots$

(a) $\sqrt{10}$ (b) $\sqrt{20}$ (c) 5 (d) 10

6. $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}} = \dots\dots$

(a) 1 (b) $\sqrt{\frac{1}{4}}$ (c) $\sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$

7. $\frac{\sqrt{27}}{\sqrt{3}} \div \frac{\sqrt{72}}{\sqrt{2}} =$

(a) $\frac{1}{2}$ (b) 2 (c) -2 (d) 4

8. The multiplicative inverse of the number $\sqrt{50}$ is $\dots\dots\dots$

(a) $\frac{\sqrt{2}}{10}$ (b) $\frac{-\sqrt{2}}{10}$ (c) $-5\sqrt{2}$ (d) $5\sqrt{2}$

9. If $x = \frac{\sqrt{6}}{\sqrt{2}}$, then $x^{-1} = \dots\dots\dots$

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{3}$ (d) $2\sqrt{3}$

10. If $x = \sqrt{7} + \sqrt{3}$ and $y = \sqrt{28} + \sqrt{12}$, then $x = \dots\dots\dots$

(a) y (b) $\frac{1}{2}$ y (c) 2 y (d) y^2

Complete the following :

1. $\frac{3\sqrt{2}}{2\sqrt{18}} = \dots\dots\dots$

2. $\sqrt{3} \times \sqrt{6} = 3 \times \dots\dots\dots$

3. $\frac{1}{2}\sqrt{48} = 2 \times \dots\dots\dots$

4. $\sqrt{3\frac{3}{8}} = \frac{3}{2}\sqrt{\dots\dots\dots}$

5. If $2\sqrt{27} - 2\sqrt{48} = x\sqrt{3}$, then $x = \dots\dots$

6. $\sqrt{5}$, $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, $\dots\dots\dots$ (in the same pattern).

7. If $x^2 = \frac{8}{9}$, then x in the simplest form $= \dots\dots\dots$

8. If $x^2 = 5$, then $(x + \sqrt{5})^2 = \dots\dots\dots$ or $\dots\dots\dots$

Find the value of each of $x + y$, $x \times y$ in each of the following cases :

1. $x = 3 + \sqrt{5}$, $y = 1 - \sqrt{5}$ « 4 , $2 - 2\sqrt{5}$ »

2. $x = \sqrt{3} - \sqrt{2}$, $y = \sqrt{3} + \sqrt{2}$ « $2\sqrt{3}$, 1 »

3. $x = 5 - 3\sqrt{2}$, $y = 5 - 3\sqrt{2}$ « $10 - 6\sqrt{2}$, $43 - 30\sqrt{2}$ »

10 If $x = \frac{\sqrt{2}}{\sqrt{3}}$ and $y = \frac{\sqrt{3}}{\sqrt{2}}$, find the value of : $6(x + y)$ « $5\sqrt{6}$ »

11 If $x = 2\sqrt{5} + \sqrt{2}$, $y = 2\sqrt{5} - \sqrt{2}$

, find the value of the expression : $x^2 + 2xy + y^2$ « 80 »

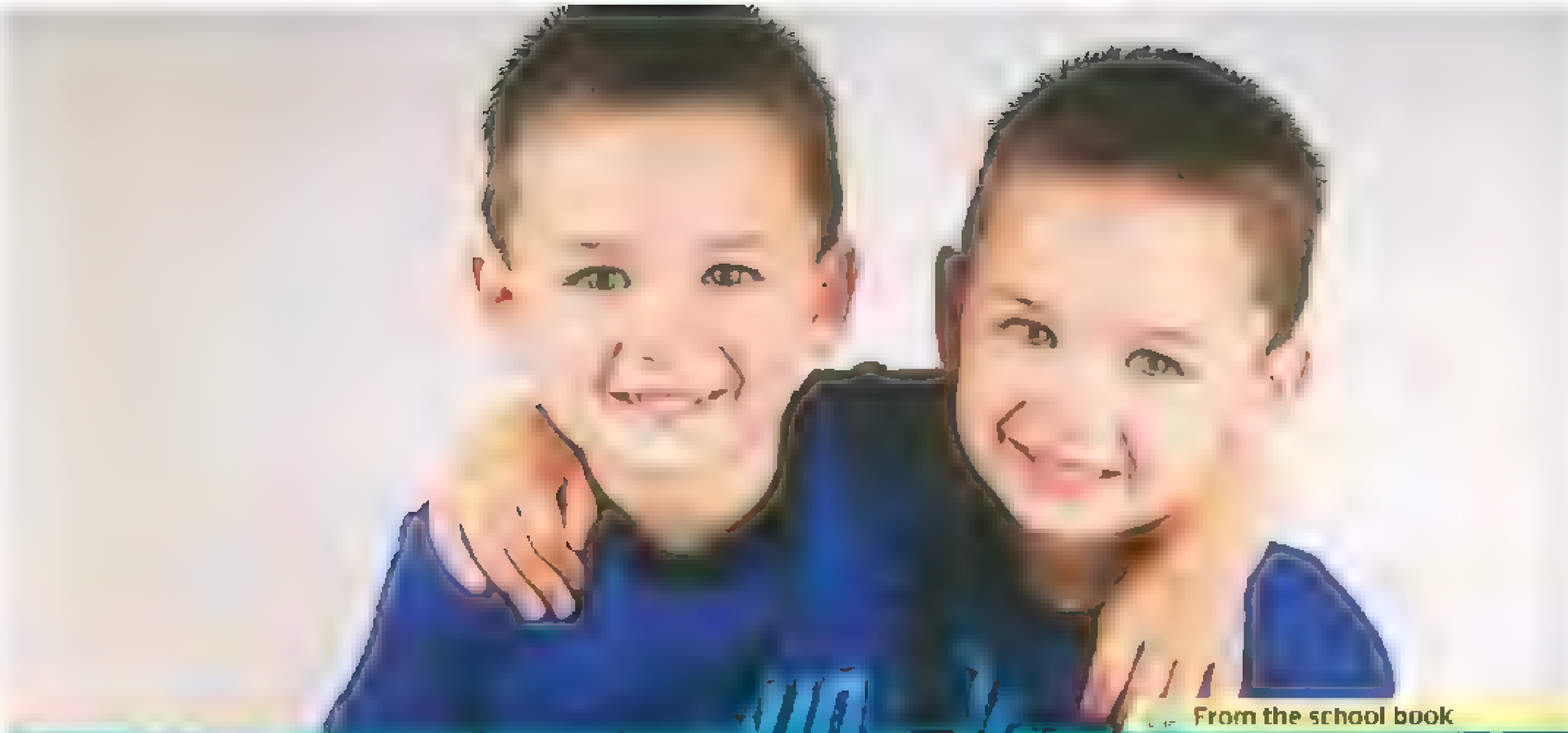
For excellent pupils

12 If $a^x = 6$ and $a^{-y} = \sqrt{3}$, find the value of : a^{x+y} « $2\sqrt{3}$ »

Simplify each of the following to the simplest form :

$\frac{(\sqrt{5})^3 \times (\sqrt{5})^5}{(\sqrt{10})^6}$ « $\frac{5}{8}$ »

$\frac{2\sqrt{2} \times (\sqrt{6})^3}{(\sqrt{3})^{-3}}$



Exercise 7 ? The two conjugate numbers

● Remember ● Understand ○ Apply ● Problem Solving

Interactive test

1 **Write the conjugate number of each of the following numbers :**

1) $\sqrt{5}+\sqrt{3}$ 2) $5-2\sqrt{7}$ 3) $\sqrt{5}+\dfrac{2}{\sqrt{2}}$

2 **Make the denominator of each of the following a rational number :**

1) $\dfrac{5}{\sqrt{7}-\sqrt{2}}$ 2) $\dfrac{\sqrt{3}}{2-\sqrt{3}}$ 3) $\dfrac{\sqrt{7}+3}{\sqrt{7}-3}$

3 If $x=\dfrac{2}{\sqrt{7}-\sqrt{5}}$ and $y=\sqrt{7}-\sqrt{5}$, find the value of : $(x+y)^2$ « 28 »

4 📖 If $x=\dfrac{4}{\sqrt{7}-\sqrt{3}}$ and $y=\dfrac{4}{\sqrt{7}+\sqrt{3}}$, find the value of : $x^2 y^2$ « 16 »

5 If $x=\sqrt{5}+\sqrt{3}$, prove that : $\dfrac{4}{x}+2x=4\sqrt{5}$

6 📖 If $a=\sqrt{3}+\sqrt{2}$ and $b=\dfrac{1}{\sqrt{3}+\sqrt{2}}$, find the value of : a^2-b^2 in its simplest form. « $4\sqrt{6}$ »

7 📖 If $x=\sqrt{5}-\sqrt{3}$ and $y=\dfrac{2}{\sqrt{5}-\sqrt{3}}$, find the value of : $x^2+2xy+y^2$ « 20 »

8 📖 If $x=\sqrt{5}-\sqrt{2}$ and $y=\dfrac{3}{\sqrt{5}-\sqrt{2}}$, prove that x and y are conjugate numbers , then find the value of : $x^2-2xy+y^2$ « 8 »

9 **If $X = 3 + \sqrt{5}$ and $y = \dfrac{4}{3 + \sqrt{5}}$, prove that X and y are conjugate numbers , then find :**

[1] Their product. [2] $X^2 + y^2$ « 4 , 28 »

10 If $X = \dfrac{2}{\sqrt{5} - \sqrt{3}}$ and $y = \dfrac{2}{\sqrt{5} + \sqrt{3}}$, find the value of : $X^2 - X y + y^2$ « 14 »

11 If $X = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$, find the value of : $\dfrac{X + y}{X y - 1}$ in its simplest form. « $\sqrt{5}$ »

12 If $a = \dfrac{4}{\sqrt{7} - \sqrt{3}}$ and $b = \dfrac{4}{\sqrt{7} + \sqrt{3}}$, find the value of : $\dfrac{a - b}{a b}$ « $\dfrac{\sqrt{3}}{2}$ »

13 If $X = 2\sqrt{2} - \sqrt{3}$ and $y = \dfrac{5}{\sqrt{8} - \sqrt{3}}$

, prove that X and y are conjugate numbers and calculate : $\dfrac{X + y}{X y}$ « $\dfrac{4\sqrt{2}}{5}$ »

14 If $X = \dfrac{5\sqrt{2} + 3\sqrt{5}}{\sqrt{5}}$ and $y = \dfrac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$

, find the value of each of : [1] $X^2 + y^2$ [2] $X y$ « 38 , 1 »

, then prove that : $X^2 + y^2 = 38\, X y$

15 If $X = \dfrac{1}{2 + \sqrt{3}}$ and $y = \dfrac{12}{\sqrt{3}}$, find the value of : $X^2 + y$ « 7 »

16 If $X = \dfrac{1}{\sqrt{3} - \sqrt{2}}$ and y is the multiplicative inverse of X

, find y , then prove that : $(X + y)^2 = 12$ « $\sqrt{3} - \sqrt{2}$ »

17 If $X = \sqrt{13} + \sqrt{6}$, $X y = 1$, find the value of : $X^2 - 49\, y^2$ « $4\sqrt{78}$ »

18 If $X = \dfrac{4}{\sqrt{7} - \sqrt{3}}$ and $y^{-1} = \dfrac{1}{\sqrt{7} - \sqrt{3}}$ (Remember that $y^{-1} = \dfrac{1}{y}$)

, prove that X and y are conjugate numbers , then find the value of : $X^2 y^2$ « 16 »

19 If $x=\sqrt{7}+\sqrt{5}$ and $y=\frac{2}{x}$, find the value of : $\frac{x+y}{xy}$ in its simplest form.

20 If $x=\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$, prove that : $x+\frac{1}{x}=22$

21 **Complete the following :**

1. $\left(\sqrt{7}+\sqrt{3}\right)\left(\sqrt{7}-\sqrt{3}\right)=$ ....

2. If $x=3+\sqrt{2}$, then its conjugate is .... and the product of multiplying x by its conjugate is ....

3. The conjugate number of the number $\frac{2}{\sqrt{5}-\sqrt{3}}$ is ....

4. The conjugate number of the number $1+\frac{7}{\sqrt{7}}$ in the simplest form is ....

5. The multiplicative inverse for $\left(\sqrt{3}+\sqrt{2}\right)$ in its simplest form is ....

6. If $x=2+\sqrt{5}$ and y is the conjugate number of x , then $(x-y)^2=$ ....

7. If $\frac{x}{5-\sqrt{5}}=5+\sqrt{5}$, then the value of x in its simplest form is ....

8. If $\frac{1}{x}=\sqrt{5}-2$, then the value of x in its simplest form is ....

9. If $x=\sqrt{3}+2$, $y=\sqrt{3}-2$, then $(xy , x+y)=$ ....

10. $\left(\sqrt{2}+\sqrt{3}\right)^{9}\left(\sqrt{2}-\sqrt{3}\right)^{-9}=$ ....

22 **In each of the following , if a and b are two integers , find the value of each of them :**

1. $\frac{11}{2\sqrt{5}+3}=a\sqrt{5}+b$ «2 , −3»

2. $\frac{3}{2\sqrt{2}-\sqrt{5}}=a\sqrt{2}+b\sqrt{5}$ «2 , 1»

3. $\frac{7}{\sqrt{8}+1}=a+b\sqrt{2}$ «−1 , 2»

For excellent pupils

If $x = \sqrt{4+\sqrt{7}}$, $y = \sqrt{4-\sqrt{7}}$

, find in the simplest form : $(x+y)^2$ « 14 »

If $x = \sqrt{5}+1$ and $y = \sqrt{5} - 1$, find the value of : $x\,y^{-1} + y\,x^{-1}$

If $\frac{x}{y} = \sqrt{3}-\sqrt{2}$, find the value of $\frac{3x^2+3y^2}{xy}$ « $6\sqrt{3}$ »

26 If $x = \sqrt{7}+\sqrt{6}$ and $y = \sqrt{7}-\sqrt{6}$, find the value of : $\frac{x^8 y^9 - y}{(x+y)^5}$

From the school book

Exercise 8 Operations on the cube roots

Remember | Problem Solving

Interactive test

1 **Put each of the following in the form $a\sqrt[3]{b}$ where a and b are two integers , b is the least possible positive value :**

1 $\sqrt[3]{16}$	2 $\sqrt[3]{-54}$	3 $2\sqrt[3]{250}$
4 $\frac{2}{3}\sqrt[3]{-135}$	5 $3\sqrt[3]{\frac{1}{3}}$	6 $-10\sqrt[3]{\frac{2}{5}}$

2 **Find the result of each of the following in its simplest form :**

1 $\sqrt[3]{2}\times\sqrt[3]{32}$	2 $\frac{\sqrt[3]{72}}{\sqrt[3]{9}}$	3 $\frac{4\sqrt[3]{-54}}{2\sqrt[3]{-2}}$
4 $\frac{1}{2}\sqrt[3]{10}\times 6\sqrt[3]{100}$	5 $\sqrt[3]{\frac{2}{5}}\times\sqrt[3]{\frac{4}{25}}$	6 $\sqrt[3]{\frac{3}{4}}\div\sqrt[3]{\frac{2}{9}}$

3 **Find the result of each of the following in its simplest form :**

1 $\sqrt[3]{16}-\sqrt[3]{2}$	« $\sqrt[3]{2}$ »	2 $\sqrt[3]{125}-\sqrt[3]{24}$	[illegible]
3 $\sqrt[3]{81}+\sqrt[3]{-24}$	« $\sqrt[3]{3}$ »	4 $\sqrt[3]{54}+\sqrt[3]{16}-\sqrt[3]{250}$	« zero »
5 $2\sqrt[3]{54}-5\sqrt[3]{2}+\sqrt[3]{16}$	[illegible]	6 $\sqrt[3]{16}-\frac{1}{3}\sqrt[3]{54}+\sqrt[3]{-2}$	« zero »
7 $\sqrt[3]{16}+\sqrt[3]{10}\times\sqrt[3]{25}$	[illegible]	8 $\sqrt[3]{24}-6\sqrt[3]{13\frac{8}{9}}$	[illegible]

4 Prove that :

1. $\sqrt[3]{128}+\sqrt[3]{16}-2\sqrt[3]{54}=\text{zero}$
2. $\sqrt[3]{54}\times\sqrt[3]{16}\div\left(\sqrt[3]{4}\times 6\right)=1$

Simplify each of the following to the simplest form :

$\sqrt[3]{81}+\sqrt[3]{-24}-3\sqrt[3]{\frac{1}{9}}$ « zero »	$\sqrt[3]{54}+8\sqrt[3]{-\frac{1}{4}}+5\sqrt[3]{16}$ « $9\sqrt[3]{2}$ »
$\sqrt[3]{108}-2\sqrt[3]{4}-\sqrt[3]{\frac{1}{2}}$ « $\frac{1}{2}\sqrt[3]{4}$ »	$\sqrt[3]{3}-\sqrt[3]{4}\times\sqrt[3]{6}+3\sqrt[3]{\frac{1}{9}}$ « zero »

Simplify each of the following to its simplest form :

1. $\frac{7}{3}\sqrt{18}+\sqrt[3]{54}-7\sqrt{2}+\sqrt[3]{16}$ « $5\sqrt[3]{2}$ »
2. $\sqrt{27}+\frac{1}{3}\sqrt[3]{27}-9\sqrt{\frac{1}{3}}-1$ « zero »
3. $\sqrt[3]{-16}+\frac{14}{\sqrt{7}}-\sqrt{28}+\sqrt[3]{54}$ « $\sqrt[3]{2}$ »
4. $\sqrt{18}+\sqrt[3]{54}-\frac{\sqrt{216}}{\sqrt{12}}-\sqrt[3]{16}$ « $\sqrt[3]{2}$ »
5. $5\sqrt{2}-\frac{1}{2}\sqrt{200}+\left(\sqrt[3]{5}\times\sqrt[3]{25}\right)$ « 5 »

Choose the correct answer from those given :

1. $\sqrt[3]{54}+\sqrt[3]{-2}=$ .....

   (a) $\sqrt[3]{52}$ (b) $\sqrt[3]{2}$ (c) $2\sqrt[3]{2}$ (d) $4\sqrt[3]{2}$

2. $\sqrt[3]{-64}+\sqrt{16}=$ .....

   (a) zero (b) 8 (c) -8 (d) ± 8

3. $\frac{\sqrt[3]{16}}{\sqrt[3]{2}}=$ ....

   (a) 8 (b) -2 (c) 2 (d) $2\sqrt[3]{2}$

4. $\sqrt[3]{2}+\sqrt[3]{2}=$ ....

   (a) $\sqrt[3]{2}$ (b) $\sqrt[3]{4}$ (c) $\sqrt[3]{8}$ (d) $\sqrt[3]{16}$

$\sqrt[3]{\frac{2}{9}} = \ldots\ldots$

(a) $\frac{\sqrt[3]{6}}{3}$ (b) $\sqrt[3]{\frac{1}{6}}$ (c) $\sqrt[3]{6}$ (d) $\sqrt[3]{2}$

Complete the following :

$\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{-12} = \ldots\ldots$

$\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt{\ldots\ldots}$

3 $\sqrt[3]{54} - \sqrt[3]{-16} = \sqrt[3]{\ldots\ldots\ldots}$

4 $\frac{1}{2}\sqrt[3]{56} - \sqrt[3]{\frac{7}{27}} = \ldots\ldots\ldots$

If $x = 2$, $y = \sqrt[3]{-16}$, then $\left(\frac{x}{y}\right)^3 = \ldots\ldots$

$\frac{\sqrt[3]{250} - \sqrt[3]{16}}{\sqrt[3]{54}} = \ldots\ldots$

If $a = \sqrt[3]{5} + 1$, $b = \sqrt[3]{5} - 1$, find the value of each of the following :

1 $(a - b)^5$ 2 $(a + b)^3$ « 32 , 40 »

If $x = 3 + \sqrt[3]{6}$, $y = 3 - \sqrt[3]{6}$, find the value of : $\left(\frac{x - y}{x + y}\right)^3$

For excellent pupils

11 If $x = \sqrt[3]{2} + 1$, $y = \sqrt[3]{2} - 1$, prove that : $x^2 + y^2 = 2\sqrt[3]{4} + 2$

Make the denominator of $\frac{2}{\sqrt[3]{2}}$ a rational number.

From the school book

9 Applications on the real numbers

Remember · Understand · Apply · Problem Solving

Interactive test

1 Complete the following :

1. If the edge length of a cube is 5 cm. , then its volume = ... cm^3.
2. The edge length of a cube is 4 cm. , then its total area = ... cm^2.
3. The lateral area of a cube whose edge length is ℓ cm. is .... cm^2.
4. The cube whose volume is ℓ^3 cm^3 , its total area = ... cm^2.
5. The cube whose edge length is 2 ℓ cm. , then its volume = ... cm^3.

2 A cube whose lateral area is 36 cm^2. Find :

1. Its total area.
2. Its volume.

« 54 cm^2 , 27 cm^3 »

3 The perimeter of one face of a cube is 12 cm. Find :

1. Its volume.
2. Its lateral area.

« 27 cm^3 , 36 cm^2 »

4 The sum of lengths of all edges of a cube is 60 cm. Find :

1. Its volume.
2. Its total area.

« 125 cm^3 , 150 cm^2 »

5 Choose the correct answer from those given :

1. The volume of a cube is 1 cm^3 , then the sum of its edge lengths = ... cm.

(a) 1 (b) 6 (c) 8 (d) 12

2 The volume of a cube is 64 cm^3, then its lateral area = ... cm^2

(a) 4 (b) 8 (c) 64 (d) 96

3 A cube of volume 27 cm^3, then its total area = ... cm^2

(a) 9 (b) 27 (c) 36 (d) 54

4 If the total area of a cube is 96 cm^2, then the area of one face = ... cm^2

(a) 16 (b) 64 (c) 24 (d) 48

5 A cube of total area 150 cm^2, then its lateral area = ... cm^2

(a) 25 (b) 100 (c) 125 (d) 150

6 If the area of the six faces of a cube = 54 cm^2, then its volume = ... cm^3

(a) 54 (b) 44 (c) 72 (d) 27

7 If the volume of a cube = 64 cm^3, then the length of a diagonal of one face = ... cm.

(a) 16 (b) $4\sqrt{2}$ (c) 32 (d) 64

8 The edge length of a cube whose volume is $2\sqrt{2}$ cm^3 is .... cm.

(a) $\sqrt{2}$ (b) 2 (c) 8 (d) 1.5

The cuboid

The dimensions of the base of a cuboid are 9 cm. and 10 cm. and its height is 5 cm. Find :

1 Its volume. 2 Its lateral area. 3 Its total area.

« 450 cm^3, 190 cm^2, 370 cm^2 »

The dimensions of a cuboid are $\sqrt{2}$ cm., $\sqrt{3}$ cm. and $\sqrt{6}$ cm. Find its volume. « 6 cm^3 »

The lateral area of a cuboid is 480 cm^2 and its base is in the shape of a square whose side length is 10 cm. Calculate its height. « 12 cm. »

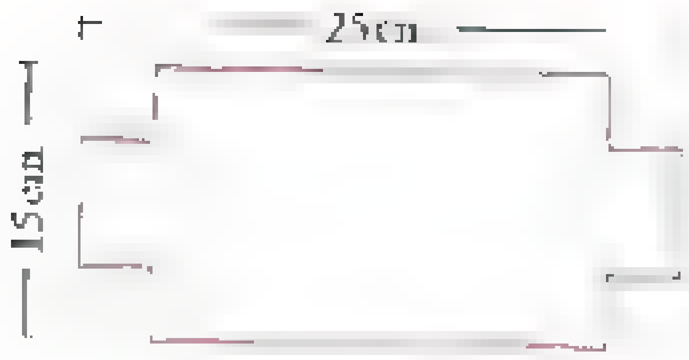
Find the total area of a cuboid whose volume is 720 cm^3 and its height is 5 cm. with a squared-shape base. « 528 cm^2 »

Which is more in size :

A cube whose total area is 294 cm^2 or a cuboid with dimensions $7\sqrt{2}$ cm., $5\sqrt{2}$ cm. and 5 cm. ?

11 **In the opposite figure :**

A rectangular piece of cardboard has a length of 25 cm. and a width of 15 cm. A square whose side length = 4 cm. was cut from each of its four corners , then the projected parts were folded to form a basin in the shape of a cuboid. Find the volume and the total area of that cuboid.



« 476 cm^3 , 311 cm^2 »

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

A circle is of radius length 10.5 cm. Find each of its circumference and its area.

« 66 cm. , 346.5 cm^2 »

The area of a circle is 154 cm^2. Find its circumference and its diameter length.

« 44 cm. , 14 cm. »

A circle whose area is 64 π cm^2. Find the length of its radius , then find its circumference approximating it to the nearest integer. ($\pi = 3.14$)

« 8 cm. , 50 cm. »

In the opposite figure :

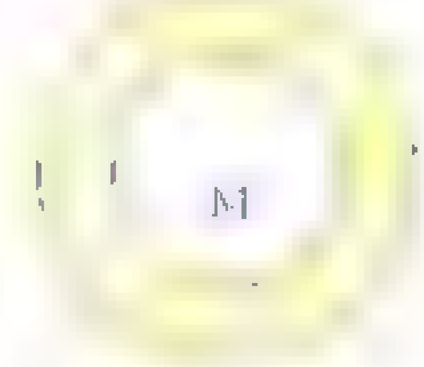
$\overline{AB}$ is a diameter of the semicircle. If the area of this region is 12.32 cm^2 , find the perimeter of the figure.

« 14.4 cm. »

In the opposite figure :

These are two concentric circles at M and their radii lengths are 3 cm. and 5 cm.

Find the area of the shaded part in terms of π



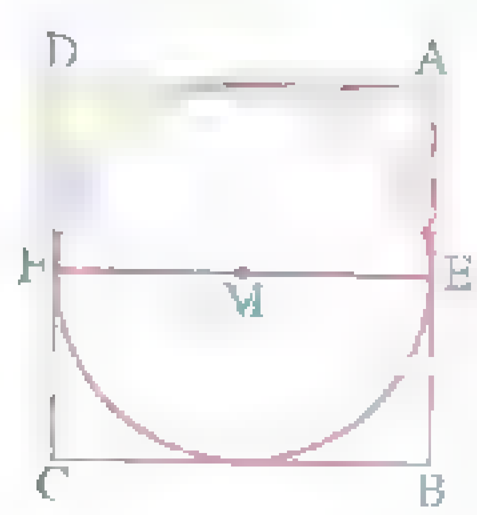
« 16 π cm^2 »

17 **In the opposite figure :**

The circle M is inside the square ABCD

If the area of the shaded part = $10\frac{5}{7}$ cm^2 ,

find the perimeter of this part.



« $35\frac{5}{7}$ cm. »

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

A right circular cylinder , the radius length of its base is 14 cm. and its height is 20 cm. Find the volume and the total area of the cylinder. « 12320 cm^3 , 2992 cm^2 »

Find the lateral area for a right circular cylinder of volume 924 cm^3 and of a height 6 cm. « 264 cm^2 »

Find the total area of a right circular cylinder of volume 7536 cm^3. and its height is 24 cm. ($\pi = 3.14$) « 2135.2 cm^2 »

Which is more in volume :

A right circular cylinder with base radius length 7 cm. and its height = 10 cm. or a cube whose edge length is equal to 11 cm. ?

22 **Complete the following :**

1. A right circular cylinder whose base radius length is r cm. and its height = h cm. , then its lateral area = ... cm^2 and its volume = ... cm^3
2. A right circular cylinder with volume 40 π cm^3 and its height = 10 cm. , then its base radius length =
3. A right circular cylinder with volume 500 π cm^3 and its base radius length = 5 cm. , then its height = ...
4. A right circular cylinder with volume πr^3 cm^3 , then its height = ...
5. If the lateral area of a right circular cylinder is $2 \pi r^2$ cm^2 , then its height = .

The circumference of the base of a right circular cylinder is 44 cm. and its height = 25 cm. Find its volume. « 3850 cm^3 »

The lateral area of a right circular cylinder is 52 cm^2 and the length of the diameter of its base is 8 cm. Find its volume. « 104 cm^3 »

A right circular cylinder of volume 36 π cm^3 and height 4 cm. , the radius length of its base equals the edge length of a cube.

Find : The total area of the cube. « 54 cm^2 »

26 Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is $72\ \pi\ cm^3$. « $2\sqrt[3]{9}$ cm. »

27 A piece of paper has a shape of a rectangle ABCD in which AB = 10 cm. and BC = 44 cm. It was folded to form a right circular cylinder such that $\overline{AB}$ is coincident to $\overline{DC}$
Find the volume of the resulted cylinder. « $1540\ cm^3$ »

The sphere **Consider $\pi = \frac{22}{7}$ if there are not any other values given.**

28 Find the volume and the surface area of a sphere if the length of its diameter is 4.2 cm.
« $38.808\ cm^3$, $55.44\ cm^2$ »

29 The volume of a sphere is $4188\ cm^3$. Find its radius length. ($\pi = 3.141$)

30 The volume of a sphere is $562.5\ \pi\ cm^3$.
Find its surface area in terms of π « $225\ \pi\ cm^2$ »

31 **Choose the correct answer from those given :**

1. The volume of the sphere = ……

(a) $4\ \pi\ r^2$ (b) $\frac{4}{3}\ \pi\ r^3$ (c) $\frac{3}{4}\ \pi\ r^3$ (d) $\frac{4}{3}\ \pi\ r^2$

2. The sphere whose radius length is $\sqrt[3]{3}$ cm. , its volume = …… cm^3.

(a) $4\ \pi$ (b) $4\sqrt{3}\ \pi$ (c) $\frac{4}{3}\ \pi$ (d) $\frac{9}{4}\ \pi$

3. The volume of the sphere whose diameter length is 6 cm. equals …… cm^3.

(a) 288 (b) $12\ \pi$ (c) $36\ \pi$ (d) $288\ \pi$

4. If the volume of a sphere = $\frac{9}{16}\ \pi\ cm^3$, then its radius length = …… cm.

(a) 3 (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

5. If the surface area of a sphere is $9\ \pi\ cm^2$, then its diameter length = …… cm.

(a) 9 (b) 3 (c) 1.5 (d) 6

6. If three quarters of the volume of a sphere equals $8\ \pi\ cm^3$, then the length of its radius equals ………. cm.

(a) 64 (b) 8 (c) 4 (d) 2

7. If the radius length of a sphere is r cm. , then which of the following represents the ratio between the area of the sphere and its volume?

(a) $\frac{4}{r}$ (b) $\frac{3}{r}$ (c) $\frac{r}{4}$ (d) $\frac{r}{\pi}$

● Remember ● Understand ○ Apply ♣ Problem Solving

Find the radius length of a sphere if its volume equals the volume of a right circular cylinder whose height is 18 cm. and its base radius length is 4 cm. « 6 cm. »

Find the volume of a sphere if its radius length equals the radius length of a right circular cylinder with volume 7536 cm^3 and height 24 cm ($\pi = 3.14$) « $4186\frac{2}{3}$ cm^3 »

A lead cuboid is of dimensions 77 cm. , 24 cm. and 21 cm. It was melted to make a sphere. Find the radius length of that sphere. « 21 cm. »

A metallic sphere , with diameter length 6 cm. has got melt and changed into a right circular cylinder with base radius length 3 cm. Find its height. « 4 cm. »

A sphere with volume $36\,\pi$ cm^3 is placed inside a cube. If the sphere touches the six faces of the cube , **find :**

1) The radius length of the sphere.

2) The volume of the cube. « 3 cm. , 216 cm^3. »

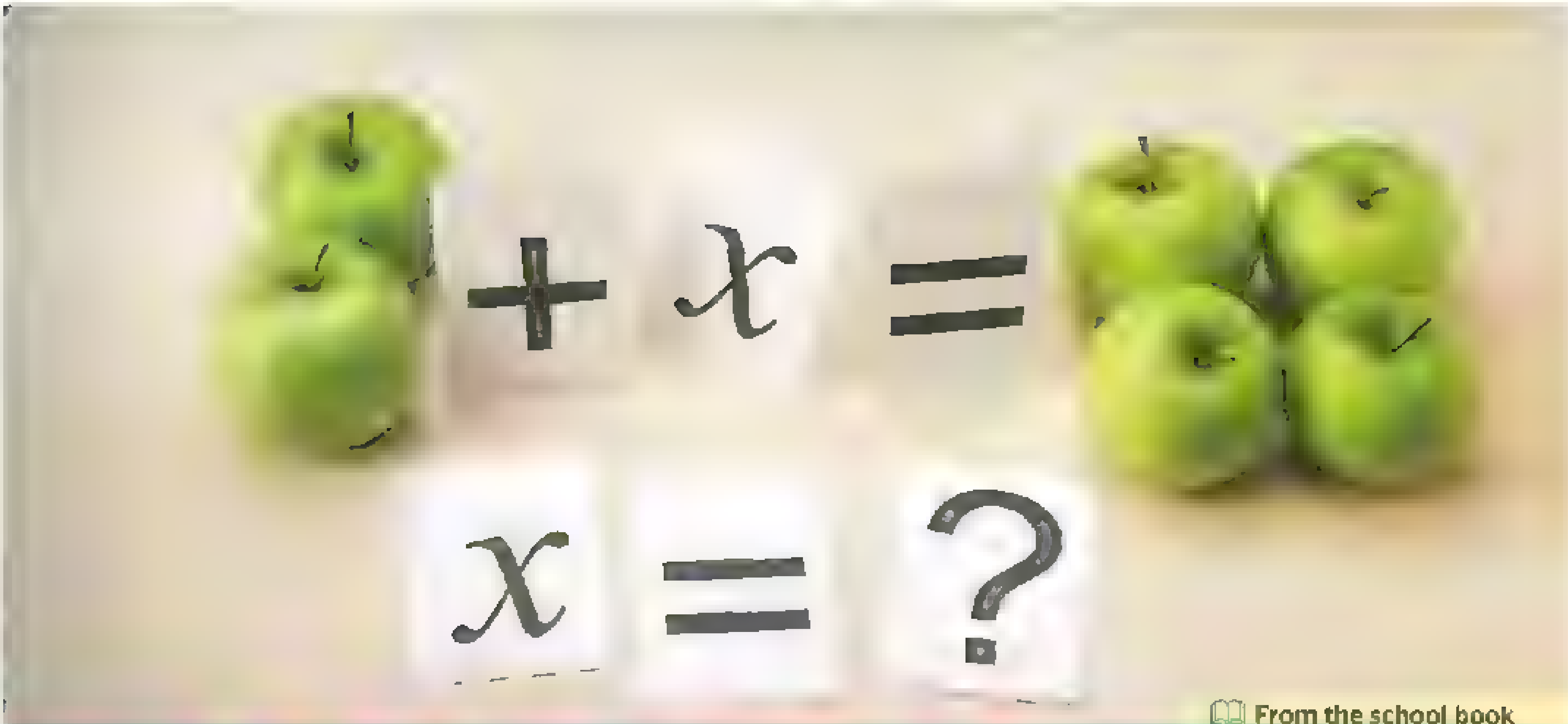
A metallic sphere is of radius length 16.8 cm. It is melted and it is converted to 8 small spheres which are equal in volume. Find the radius length of each small sphere. « 8.4 cm. »

A right circular cylinder has a height of 20 cm. Find its base radius length if its volume equals $\frac{4}{9}$ of the volume of a sphere with a diameter length of 30 cm. « 10 cm. »

For excellent pupils

A cuboid has a square-shaped base whose height = 3 cm. If the sum of lengths of its edges is 52 cm. , find its volume. « 75 cm^3 »

A hollow metal sphere is with internal radius length 2.1 cm. and external radius length 3.5 cm. Find its mass approximated to the nearest gram taking into consideration that the mass of a cubic centimetre of such a metal is 20 gm. « 2817 gm. »



From the school book

Exercise 10 Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$

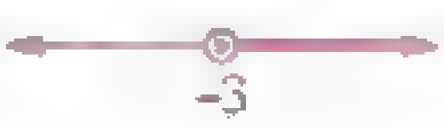
Remember · Understand · Apply · Problem Solving

Interactive test

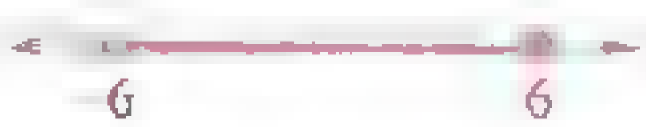
1 Find the solution set for each of the following equations in $\mathbb{R}$, then represent the solution on the number line :

1 $x + 5 = 0$	2 $5x + 6 = 1$	3 $2x + 4 = 3$
4 $2x - 3 = 4$	5 $4x - 1 = \vert -2 \vert$	6 $\sqrt{5}\,x - 1 = 4$
7 $x - 1 = \sqrt{3}$	8 $2\sqrt{6}\,x = \vert -8 \vert$	9 $x + 2\sqrt{3} = 3$

2 Choose the correct answer from those given :

1 The figure  represents the solution set of the inequality .... in $\mathbb{R}$

(a) $x > -3$ (b) $x \geq -3$ (c) $x < -3$ (d) $x \leq -3$

2 The figure  represents the solution set of the inequality .... in $\mathbb{R}$

(a) $-6 < x < 6$ (b) $-6 \leq x < 6$ (c) $-6 < x \leq 6$ (d) $-6 \leq x \leq 6$

3 If $x \in]3 , \infty[$, then ....

(a) $x < 3$ (b) $x \leq 3$ (c) $x > 3$ (d) $x \geq 3$

4 The S.S. of the inequality : $x > 7$ in $\mathbb{R}$ is ....

(a) $]-7 , \infty[$ (b) $[7 , \infty[$ (c) $]-\infty , 7[$ (d) $]7 , \infty[$

5 The S.S. of the inequality : $-1 < x \le 5$ in $\mathbb{R}$ is ………

(a) $]-1 , 5]$ (b) $[-1 , 5]$ (c) $\{-1 , 5\}$ (d) $[-1 , 5[$

6 The S.S. of the inequality : $x > 3$ in $\mathbb{R}$ is ……

(a) $\{-3\}$ (b) $]3 , \infty[$ (c) $]-\infty , 3[$ (d) $]-\infty , -3[$

Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :

1 $2x > 6$
2 $-7x \ge -14$
3 $x + 3 \le 5$
4 $5 - x > 3$
5 $2x + 5 \ge 3$
6 $1 - 5x < 6$
7 $\frac{1}{2}x + 1 \le 2$
8 $3 - 2x \le 7$

Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :

1 $3 < x + 2 \le 6$
2 $-5 < x + 3 < 9$
3 $-3 \le -x < 3$
4 $1 < 5 - x \le 3$
5 $\sqrt[3]{-8} \le x + 1 \le \sqrt{9}$
6 $5 < 3 - x \le 3^2$
7 $-8 \le 3x + 1 \le 4$
8 $|-3| < 2x - 1 < 5$
9 $-3 < \frac{1}{2}x - 2 \le$ zero
10 $0 \le \frac{-2x + 6}{3} < 4$

Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :

1 $3x < 2x + 4$
2 $7x - 9 \ge 4x$
3 $5x - 3 < 2x + 9$
4 $7x - 12 \ge 5x - 8$
5 $x - 1 \le 3 - x$
6 $1 - x \ge -2x - 3$

Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :

1 $x + 3 \ge 2x \ge x - 2$
2 $-x < x < 4 - x$
3 $4x \le 5x + 2 < 4x + 3$
4 $x - 1 < 3x - 1 \le x + 1$
5 $2 + 2x \le 3x + 3 < 5 + 2x$
6 $\frac{3x - 4}{6} < x + 1 < \frac{x + 3}{2}$

Complete the following :

1 If $x - 3 \ge 0$, then x ………
2 If $5x < 15$, then x ………
3 If $1 - x > 4$, then x ………
4 If $-2x \le 3$, then x ………

5 If $\sqrt{2}\,X \le 4$, then X ………

6 The S.S. of the inequality : $4 < 2\,X < 8$ in $\mathbb{R}$ is ………

7 The S.S. of the inequality : $-5 \le \quad X < 2$ in $\mathbb{R}$ is ………

8 The S.S. of the inequality : $2 \quad X < 0$ in $\mathbb{R}$ is ………

9 If $-3 < X < 3$ where $X \in \mathbb{R}$, then $2\,X \in]-6$, ……… [

Choose the correct answer from those given :

1 The S.S. of the inequality : $X + 3 < 3$ in $\mathbb{R}$ is ………

(a) $]-\infty , 0[$ (b) $]-\infty , 0]$ (c) $[0 , \infty[$ (d) $]0 , \infty[$

2 The S.S. of the inequality : $1 > X - 5 > -1$ in $\mathbb{R}$ is ………

(a) $[4 , 6]$ (b) $]4 , 6[$ (c) $]4 , 6]$ (d) $[4 , 6[$

3 If $X > 5$, then $-X$ ………

(a) < -9 (b) ≥ -5 (c) < -5 (d) > -5

4 If $-2 < X < 2$, then $2\,X + 3$ belongs to ……

(a) $[-1 , 7]$ (b) $]-1 , 5[$ (c) $]-1 , 7[$ (d) $]-4 , 6[$

5 The number 5 belongs to the S.S. of the inequality ………

(a) $X > 5$ (b) $X < 5$ (c) $-X \ge -5$ (d) $X \ge 5$

Life Application

A lift for carrying goods can carry 2200 kg. as a maximum weight. If we have 60 boxes of cans and the weight of one box is 45 kg , what is the maximum number of boxes can the lift carry in one time without carrying any person ? « 48 boxes »

For excellent pupils

10 Prove that $\sqrt{3}$ belongs to the S.S. of the inequality : $0 < 4 - 2\,X < 6$ in $\mathbb{R}$

11 If $[4 , 7]$ is the S.S. of the inequality : $a < X - 3 \le b$, find the value of each of a and b

If $[m , m + n]$ is the S.S. of the inequality : $\frac{1}{5} \le \frac{2\,X + 1}{5} < 1$, find the value of n

If $5 \le \frac{2\,X}{3} + 1 \le 7$, find the smallest value of the expression : $X - 2$

Find in $\mathbb{R}$ the S.S. of the inequality : $\frac{X}{\sqrt{3} - \sqrt{5}} \ge \sqrt{3} + \sqrt{5}$

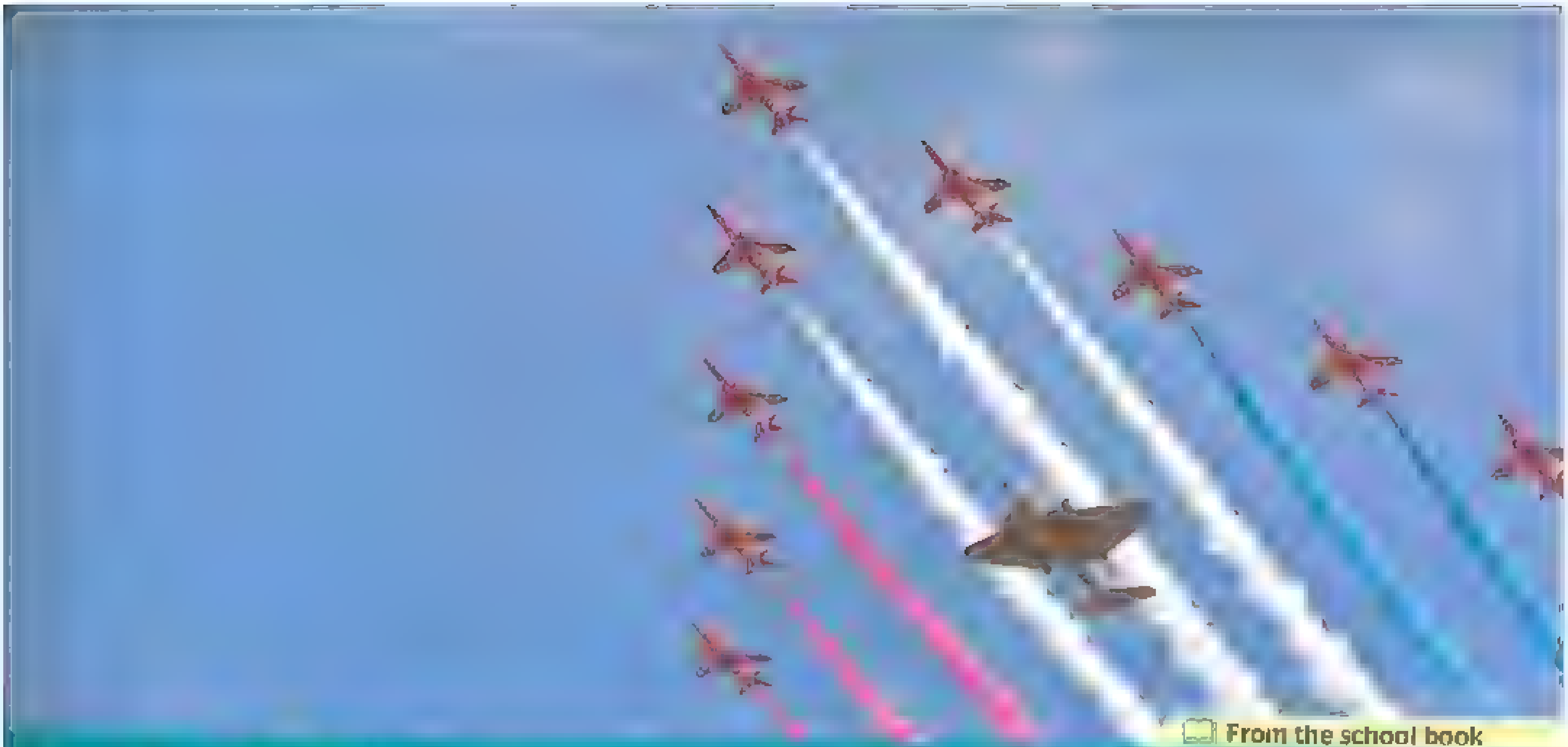
UNIT TWO

Relation between Two Variables

Exercises of the unit :

11. Relation between two variables.
12. Slope of straight line.
13. Real life applications on the slope.

Scan the QR code to solve an interactive test on each lesson



11 ? Relation between two variables

● Remember ● Understand ● Apply ● Problem Solving

Interactive test

1 **Complete the following ordered pairs which satisfy the relation :** $y = 3X - 1$

(5 , ...) , (2 , ...) , (0 , ...) , (−3 , ...)

2 **Show which of the following ordered pairs satisfies the relation :** $y - 4X = 7$

1 (1 , 2) 2 (3 , −5) 3 (−1 , 3)

3 **Find four ordered pairs satisfying each of the following relations :**

1 $2X - y = 5$

2 $y = \frac{1}{2}X + 5$

3 $y = 2$

4 $2X - 5$

4 **Using the linear relations , complete the following tables :**

1 $4X - y = -1$

X	0	1	2	3
y	....	....	....	....

2 $y = 5X + 15$

X	−4	−3	−2
y	....	....	....

3 $a - b = 4$

a	1	....	....
b	....	0	−1

4 $a - 3b = 5$

a	2	....	−1
b	....	0	

If $y - 2X = 1$, find :

1. y at $X = 3$
2. y at $X = -5$
3. X at $y = 1$
4. X at $y = 1$

If $(3, 6)$ satisfies the relation : $y = kX$, find the value of k

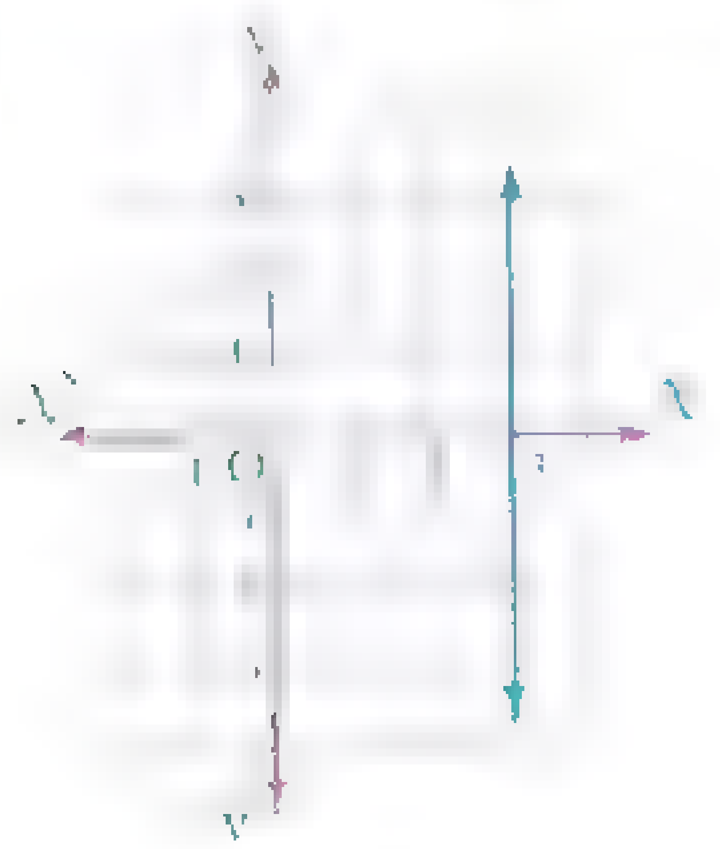
If $(3, 1)$ satisfies the relation : $y - 3X = a$, find the value of a « – 8 »

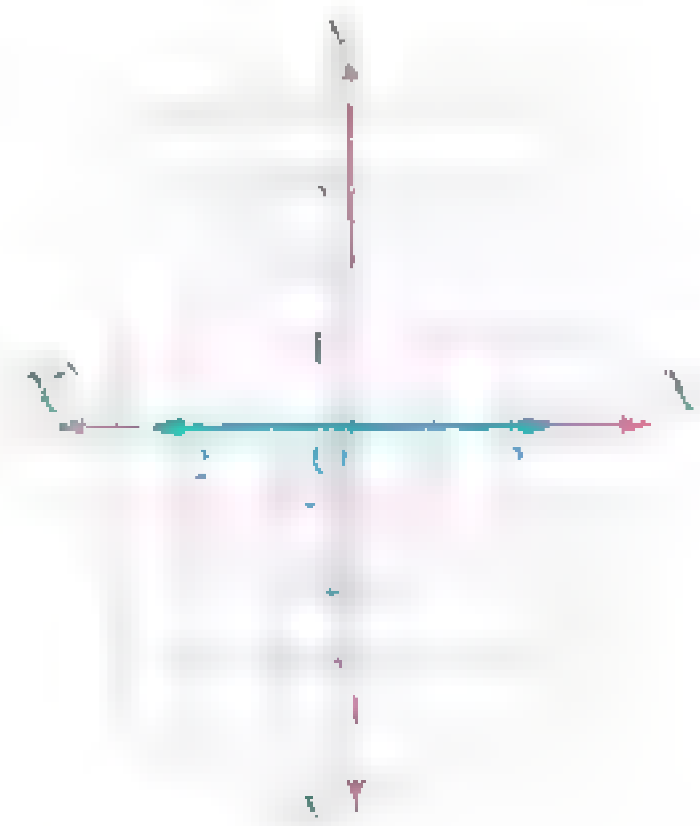
Find the value of b, where $(3, 2)$ satisfies the relation : $3X + by = 1$

If $(3, a)$ satisfies the relation : $y - 2X = 4$, find the value of a

Find the value of k, where $(k, 2k)$ satisfies the relation : $X + y = 15$

Find the relation that is represented by the line in each figure below :





Represent graphically each of the following relations :

1. $X + y = 2$
2. $X - y = 3$
3. $X + 2y = 3$
4. $y - 3X = 1$
5. $y = -2X$
6. $y - 2X = -1$
7. $2X = 5$
8. $y + 1 = 0$

Graph the relation : $2X + 3y = 6$, if the straight line representing this relation intersects the X-axis at the point A and the y-axis at the point B, find the area of the triangle OAB where O is the origin point. « 3 square units »

11 If the straight line which represents the relation : $2X - y = a$ intersects the X axis at the point $(3, b)$, find a and b «6, 0»

12 **Choose the correct answer from those given :**

1. Which of the following ordered pairs satisfies the relation : $2X + y = 5$?
   (a) $(-1, 3)$ (b) $(1, 3)$ (c) $(3, 1)$ (d) $(2, 2)$
2. $(3, 2)$ does not satisfy the relation ....
   (a) $y + X = 5$ (b) $3y - X = 3$ (c) $y + X = 7$ (d) $X - y = 1$
3. The relation : $5X = 7y$ is represented by a straight line passes through the point
   (a) $(5, 7)$ (b) $(0, 0)$ (c) $(5, 0)$ (d) $(0, 7)$
4. The point $(3, 5)$ lies on the straight line which represents the relation ..
   (a) $y = 3X - 5$ (b) $2X - y = 1$ (c) $3X + y = 1$ (d) $y = 3X - 1$
5. If $(2, -5)$ satisfies the relation : $3X - y + c = 0$, then $c =$ ....
   (a) 1 (b) -1 (c) 11 (d) -11
6. If $(-1, 5)$ satisfies the relation : $3X + ky = 7$, then $k =$ ....
   (a) 2 (b) -2 (c) 1 (d) 10
7. Which of the following relations is represented by a straight line parallel to the y-axis ?
   (a) $y = -5$ (b) $X = -5$ (c) $X = y$ (d) $X + y = 0$
8. Which of the following relations is represented by a straight line parallel to the X-axis ?
   (a) $2y = 6$ (b) $2X = 6$ (c) $X = -y$ (d) $X - y = 0$
9. Which of the following relations is represented by a straight line passes through the origin point ?
   (a) $y = 5$ (b) $X = 3$ (c) $y = X + 2$ (d) $y = 3X$
10. The relation : $3X + 8y = 24$ is represented by a straight line intersecting the y-axis at the point
    (a) $(0, 8)$ (b) $(8, 0)$ (c) $(0, 3)$ (d) $(3, 0)$
11. The relation : $2X + 7y = 14$ is represented by a straight line intersecting the X-axis at the point ....
    (a) $(2, 0)$ (b) $(0, 2)$ (c) $(7, 0)$ (d) $(0, 7)$

The opposite table represents the relation between X and y , which of the following expresses this relation ?

X	1	2	3	4
y	-2	5	-8	-11

(a) $X + y = -1$ (b) $X - y = 3$ (c) $3X + y = 1$ (d) $y = -X - 3$

13 The opposite table shows the relation between X and y , which is ....

X	1	2	3	4	5
y	1	3	5	7	9

(a) $y = X + 4$ (b) $y = X + 1$

(c) $y = 2X - 1$ (d) $y = 3X - 2$

The relation which expresses the two ordered pairs (2 , 1) and (4 , 3) together is ....

(a) $y = \frac{1}{2}X$ (b) $y = 2X - 5$ (c) $y = X - 1$ (d) $y = 3X + 3$

Two even natural numbers , twice the first plus the second equals 12

Find the different possibilities of the two numbers.

The perimeter of a rectangle is 14 cm. What are the different possibilities of the length and the width given that each of them belongs to $\mathbb{Z}_+$?

Essam has 10 bills of L.E. 5 and other bills of L.E. 20 He bought some goods from a shopping centre for L.E. 65 Determine the different possibilities to pay this amount of money.

Find the relation and graph it.

The selling price of a computer table is L.E. 100 and its chair is L.E. 50 If the store sells in one week with L.E. 500 , what are the represented expectations to the number of sold computer tables and chairs ? Represent the relation graphically.

For excellent pupil

The perimeter of an isosceles triangle is 19 cm. What are the different possible lengths of its sides given that its sides lengths $\in \mathbb{Z}_+$?

Notice that : The sum of the lengths of any two sides of the triangle is greater than the length of the third side.

From the school book

12 ? Slope of straight line

● Remember ● Understand ○ Apply ● Problem Solving

Interactive test

1 **Classify the slope of the straight line in each of the following figures showing whether it is (positive – negative – zero – undefined) :**

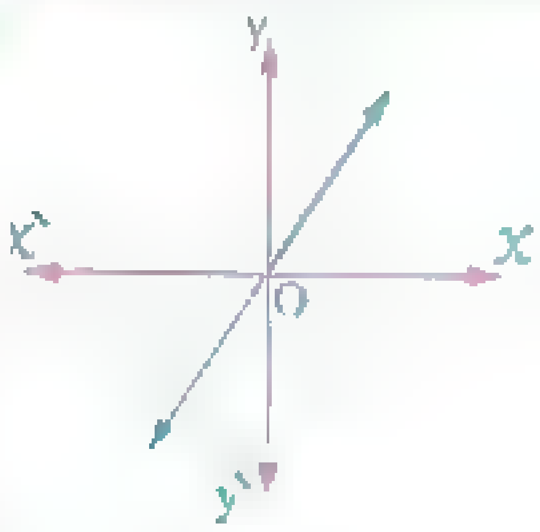


Fig. (1)

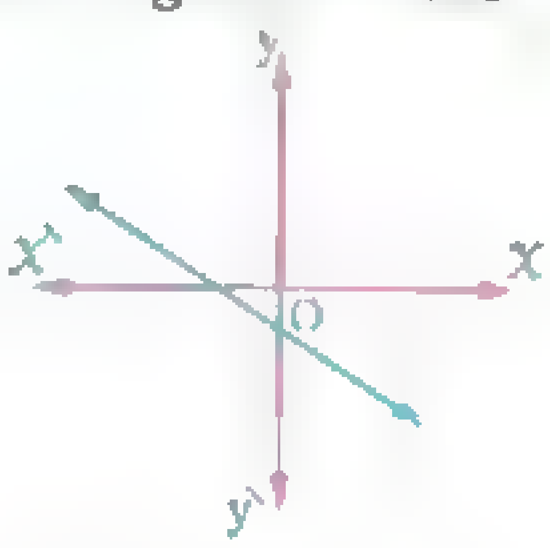


Fig. (2)

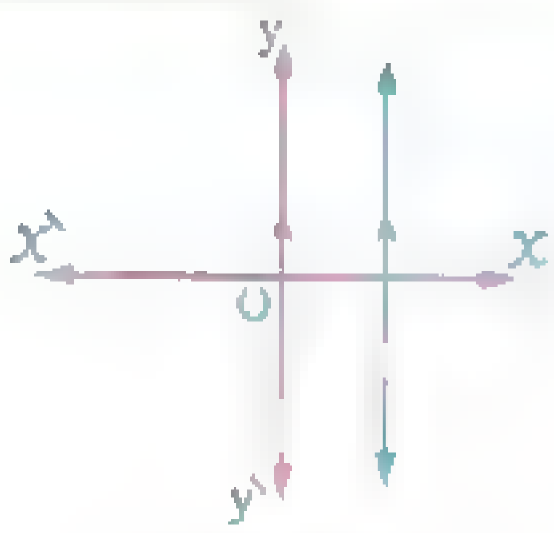


Fig. (3)

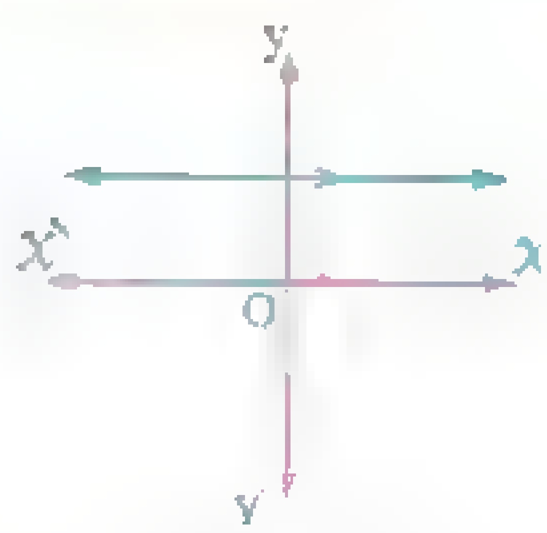
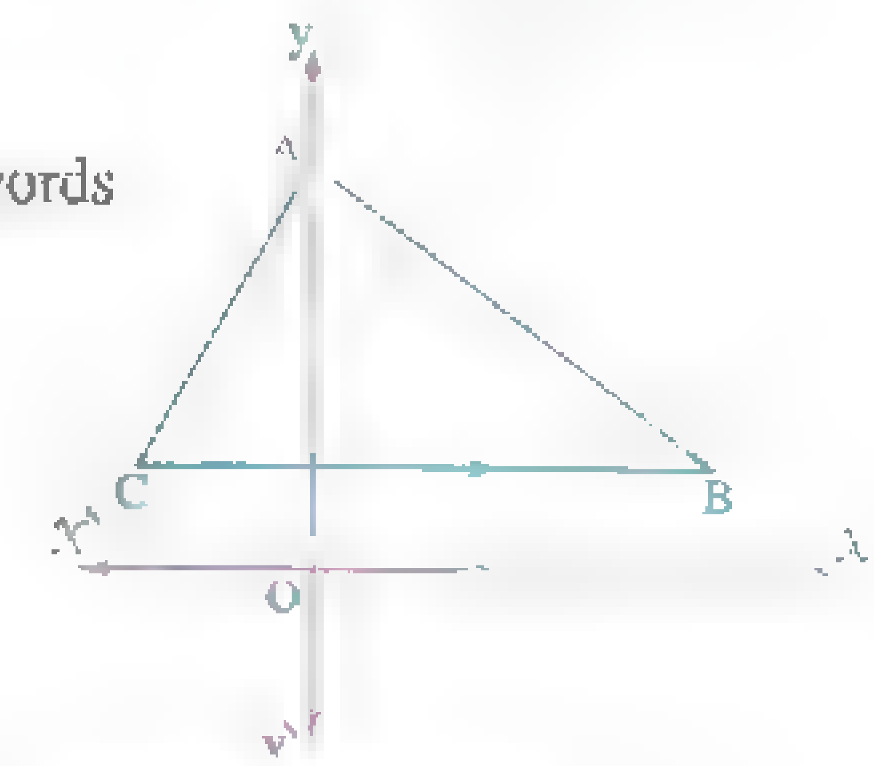


Fig. (4)

2 **In the opposite figure :**



ABC is a triangle. Complete by using one of the following words (positive , negative , zero , undefined)

1. The slope of $\overleftrightarrow{AB}$ is ....
2. The slope of $\overleftrightarrow{BC}$ is ....
3. The slope of $\overleftrightarrow{AO}$ is ....
4. The slope of $\overleftrightarrow{AC}$ is ....

3 **Complete the following :**

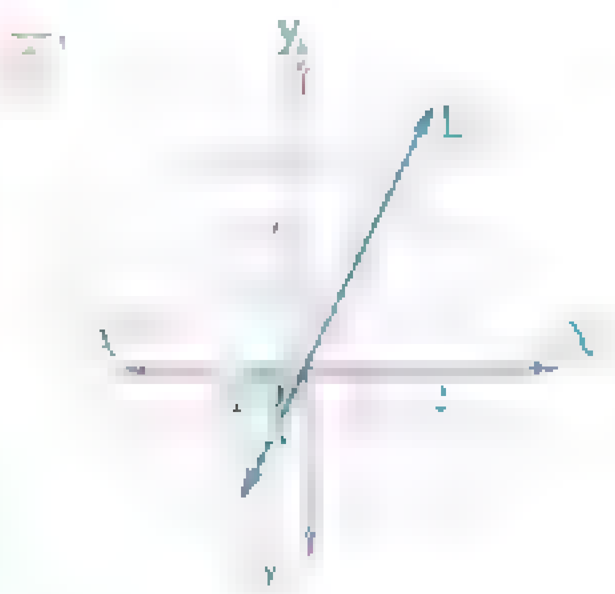
1. The slope of any horizontal straight line equals ....
2. The slope of any straight line parallel to y-axis is ....
3. The straight line whose slope = zero is parallel to ....
4. If A , B and C are collinear , then the slope of $\overleftrightarrow{AB}$ = the slope of ....

Find the slope of the straight line passing through the two points in each of the following :

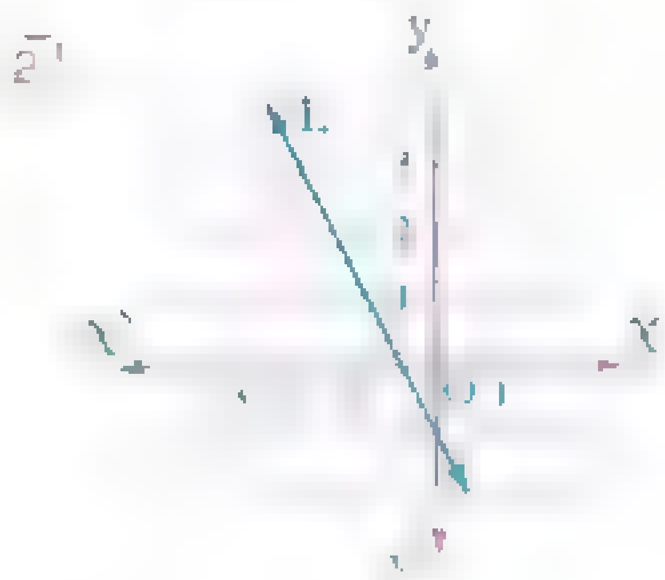
1. A (1 , 3) , B (3 , 4)
2. A (1 , 2) , B (5 , 0)
3. A (3 , 2) , B (6 , 5)
4. A (2 , – 1) , B (4 , – 1)
5. A (1 , 3) , B (2 , 3)
6. A (5 , 2) , B (5 , 4)
7. A (3 , – 1) , B (3 , 2)
8. A (3 , – 2) , B (4 , 1)
9. A (– 1 , 3) , B (2 , 1)
10. N (4 , – 2) , K (1 , – 7)
11. E (– 3 , – 1) , O (0 , 0)
12. A (– 6 , – 9) , B (– 1 , – 1)

Find the slope of the straight line L in each of the following graphs :

1



2



In the opposite figure :

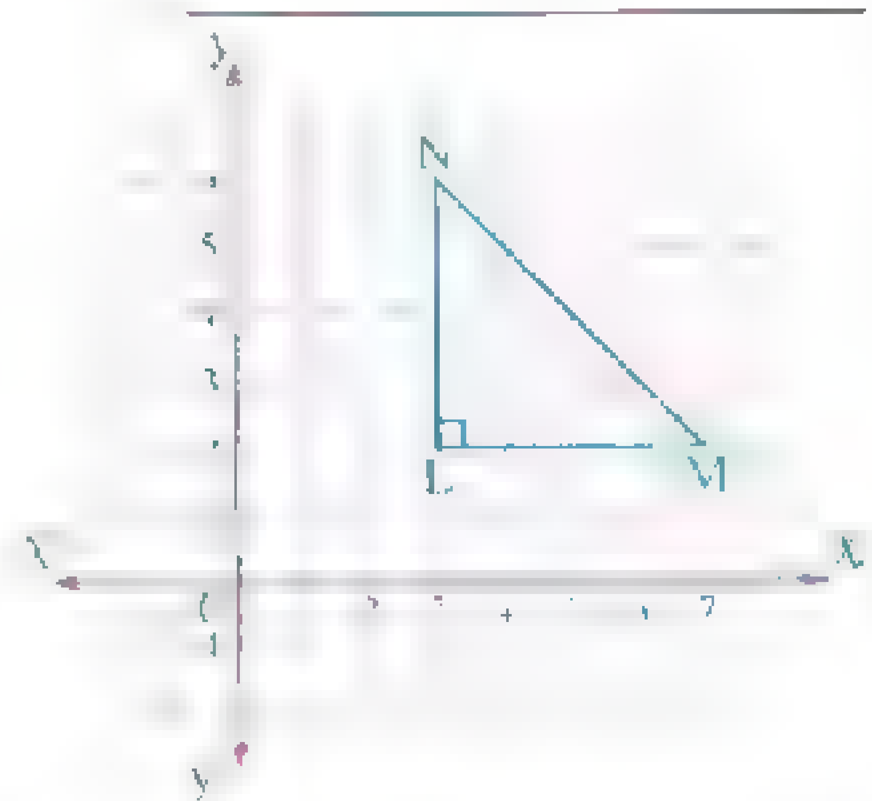
LMN is a right-angled triangle at L

, where m (∠ M) = 45°

Given that L (3 , 2) and M (7 , 2)

, find the coordinates of N

and calculate the slope of $\overleftrightarrow{MN}$



If A (2 , – 1) , B (10 , 3) and C (2 , 3) , find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CA}$

Draw the triangle ABC on a square grid , then mention the type of the triangle according to the measures of its angles.

If the slope of the straight line which passes through the two points (1 , 3) and (3 , k) equals 3 , find the value of k « 9 »

If the slope of the straight line which passes through the two points (3 , c) and (5 , – 2) equals 3 , find the value of c « 4 »

If A (– 1 , 4) , B (X , 2) and the slope of $\overleftrightarrow{AB}$ equals – 2

, find the value of X « zero »

11 If the straight line which passes through the two points $(-2, y)$ and $(3, 1)$ has a slope -0.6, find the value of y « 2 »

12 Find the value of k such that the straight line passing through the two points $(3, 4)$ and $(2, k)$ is parallel to X-axis. « 4 »

13 Find the value of X such that the straight line which passes through the two points $(2X, 3)$ and $(6, 7)$ is parallel to y-axis. « 3 »

14 Find the value of y such that the straight line passing through the two points $(3, 6)$ and $(-2, 3y)$ is perpendicular to y-axis. « 2 »

15 Are the points $(-5, 11)$, $(0, 8)$ and $(5, 5)$ collinear ?

16 Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{AC}$, where $A(2, 1)$, $B(3, 2)$ and $C(4, 5)$ and represent each line graphically. What do you observe ?

17 **In each of the following, prove that the points A, B and C are collinear :**

1. $A(1, 1)$, $B(2, 2)$, $C(-3, -3)$
2. $A(4, -3)$, $B(-6, 7)$, $C(5, -4)$
3. $A(-2, 12)$, $B(2, 4)$, $C(6, -4)$

18 **In each of the following, prove that the points A, B and C are not collinear :**

1. $A(2, 1)$, $B(3, 0)$, $C(5, -1)$
2. $A(-1, 2)$, $B(3, 1)$, $C(7, 2)$
3. $A(0, -3)$, $B(2, 2)$, $C(-3, 3)$

19 Find the slope of the line $\overleftrightarrow{AB}$, where $A(-1, 3)$ and $B(2, 5)$

Is the point $C(8, 1) \in \overleftrightarrow{AB}$? « $\frac{2}{3}$ »

20 Find the value of y such that the points $(4, 1)$, $(-2, 7)$ and $(3, y)$ are collinear.

For excellent pupils

21 If the straight line which passes through the points $(3, -1)$, $(X, 1)$ and $(9, y)$ has a slope $= \frac{2}{3}$, find the value of each of X and y « 6, 3 »

From the school book

Exercise 13 ? Real life applications on the slope

Remember · Understand · Apply · Problem Solving

Interactive test

1 A car moves with uniform velocity such that it covers 180 km. per 3 hours. If the car moves for 5 hours , what is the covered distance ? « 300 km. »

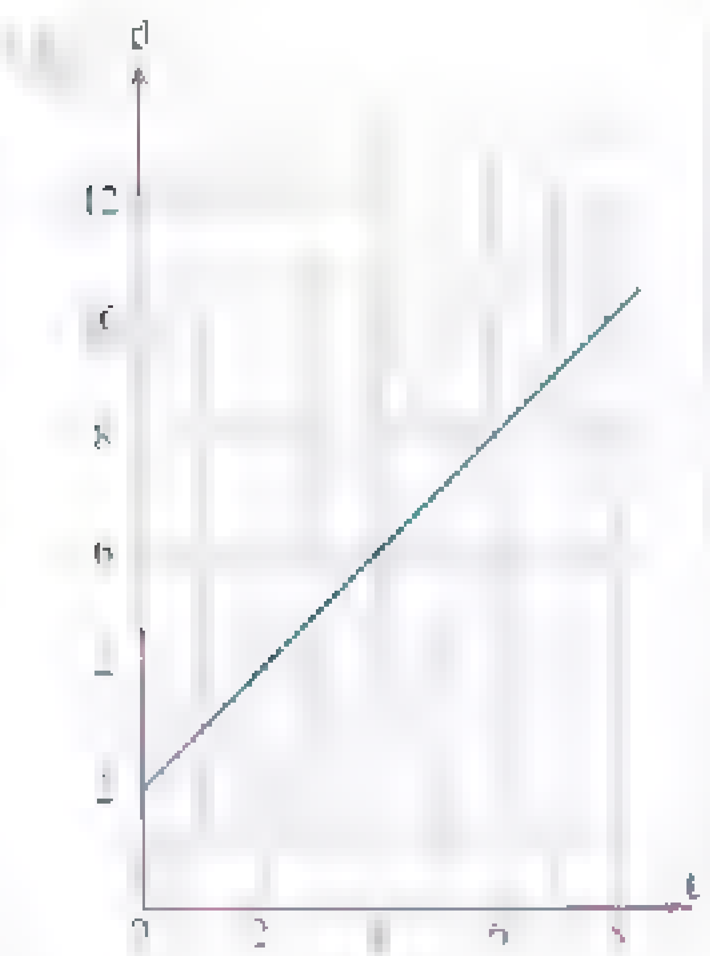
---

2 An irrigation machine consumes 2.47 litres of diesel to work for 3 hours. If the machine works for 10 hours , how many litres of diesel will the machine consume ? « $8\frac{7}{30}$ litres »

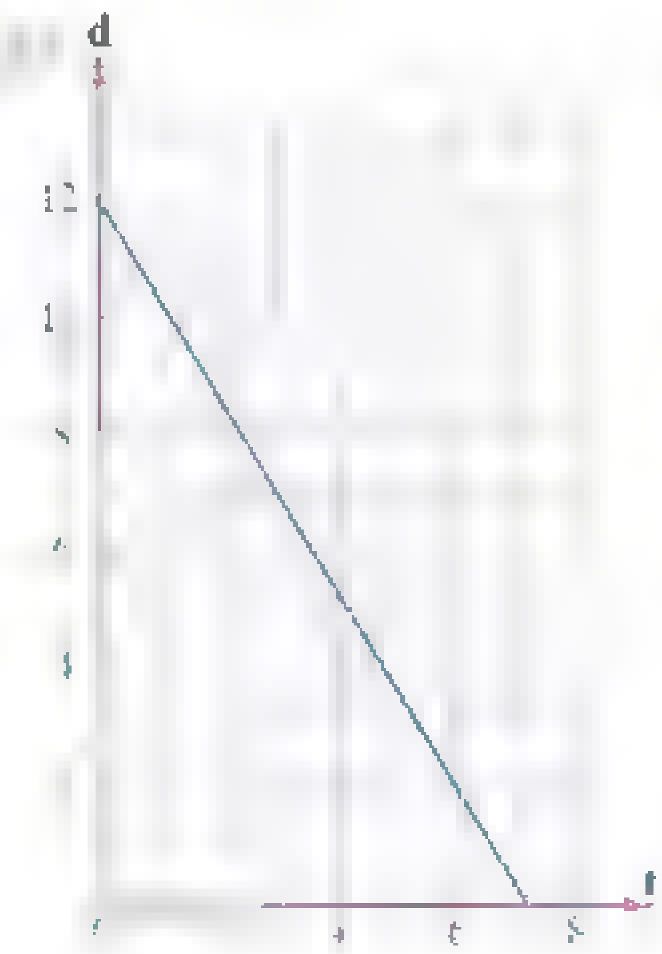
---

3 The following diagrams show the relation between the covered distance (in m.) and the elapsed time (in sec.) of an object. Determine the position of the object at the starting of motion and its position after 6 seconds (when t = 6 sec.) Find the slope of the line in each case and state what it represents.

(1)

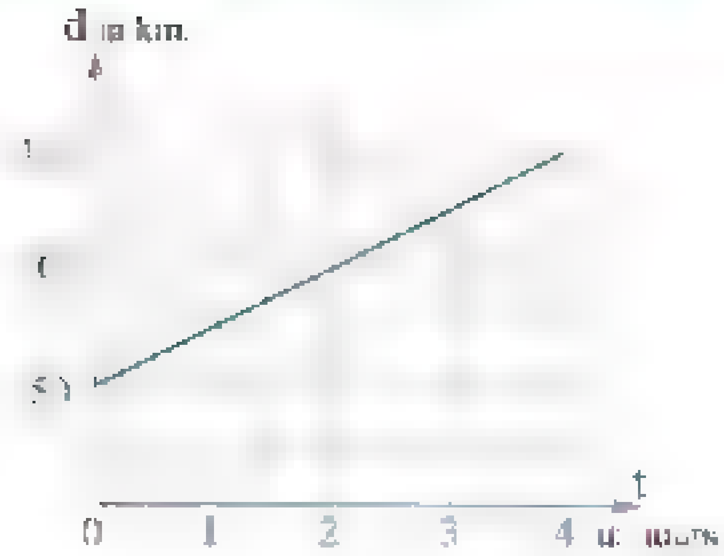


(2)



(3)

4 The opposite graph represents the motion of a car moving with uniform velocity.
Determine the velocity of the car.

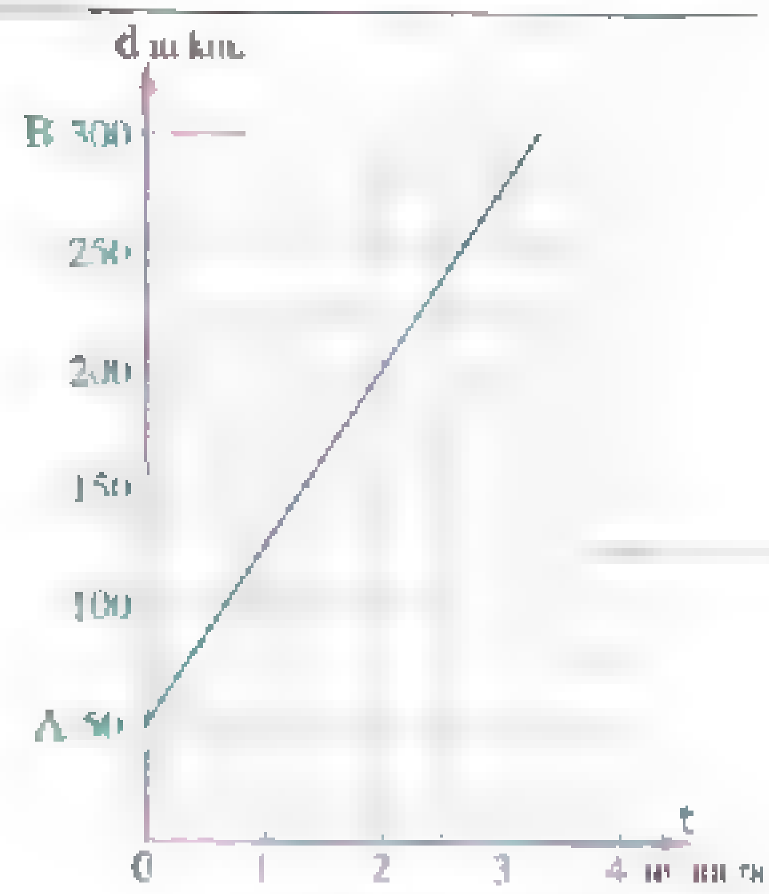


« 25 km./hr. »

Bassem drove his car from the city A to the city B
The opposite graph shows the relation between the distance d in km. and the time t in hours.

Answer the following :

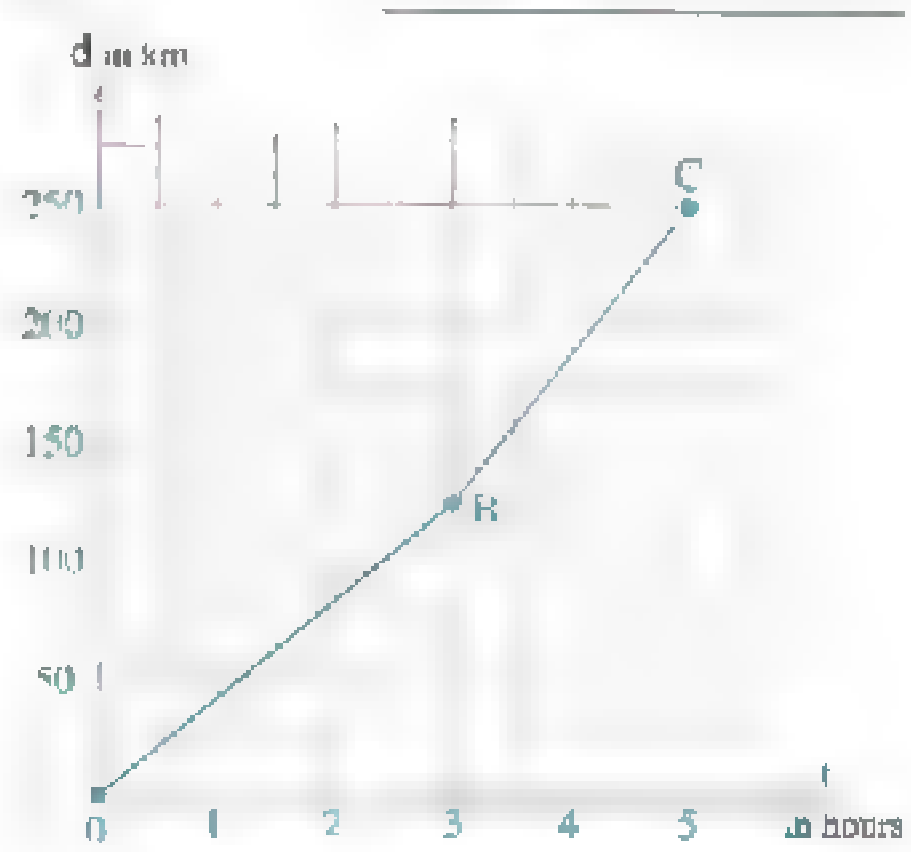
1. What is the uniform velocity of the car of Bassem ?
2. Find the distance between the car and the point 0 after three hours from the moment of beginning.



« 75 km./hr. , 275 km. »

The opposite graph represents the motion of a car :

1. Find the velocity of the car within the first three hours from the beginning , then find the velocity within the next two hours.
2. Find the average velocity of the car within the total time.

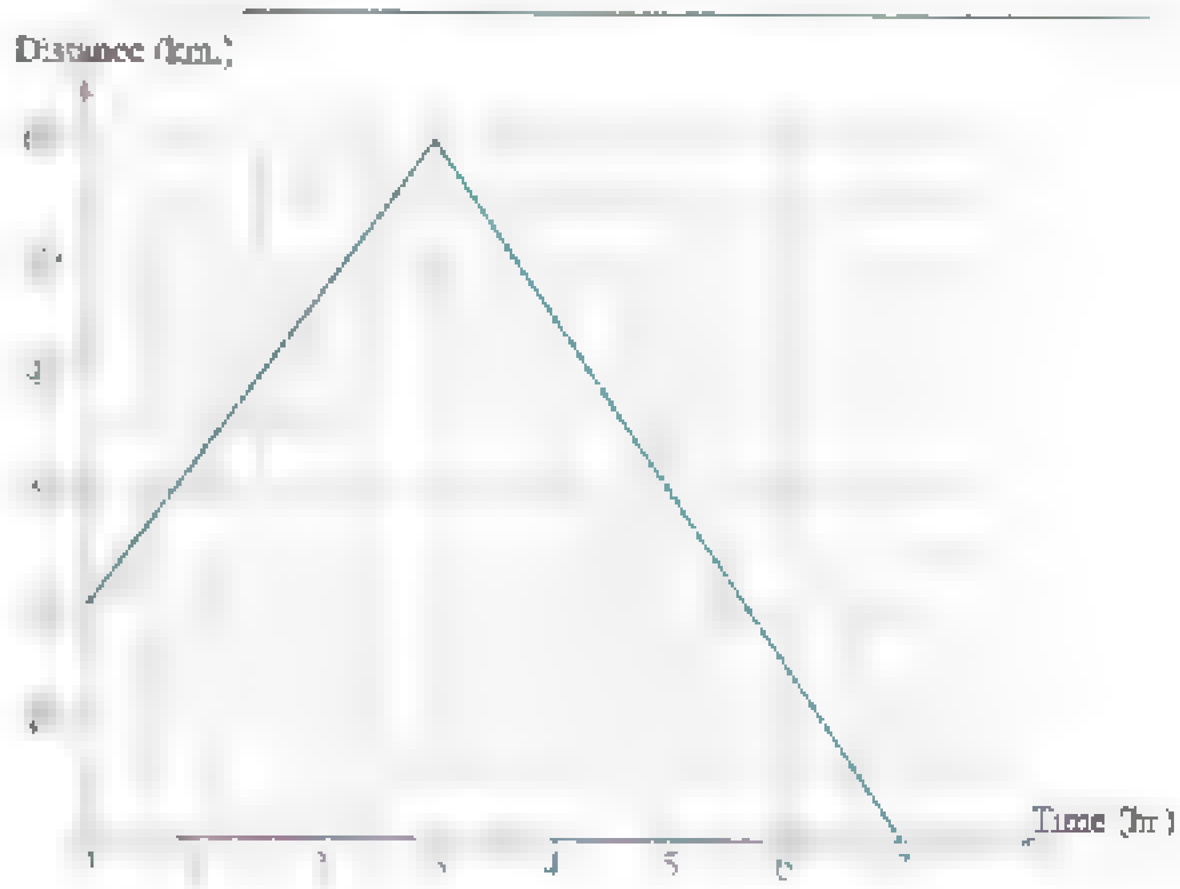


« $41\frac{2}{3}$ km./hr. , $62\frac{1}{2}$ km./hr. , 50 km./hr. »

The opposite figure represents the motion of a bicycle measured from a constant point.
Find the regular velocity of the bicycle during :

1. The first three hours.
2. The next four hours.

Find the total distance covered by the bicycle.



« $13\frac{1}{3}$ km./hr. , 15 km./hr. , 100 km. »

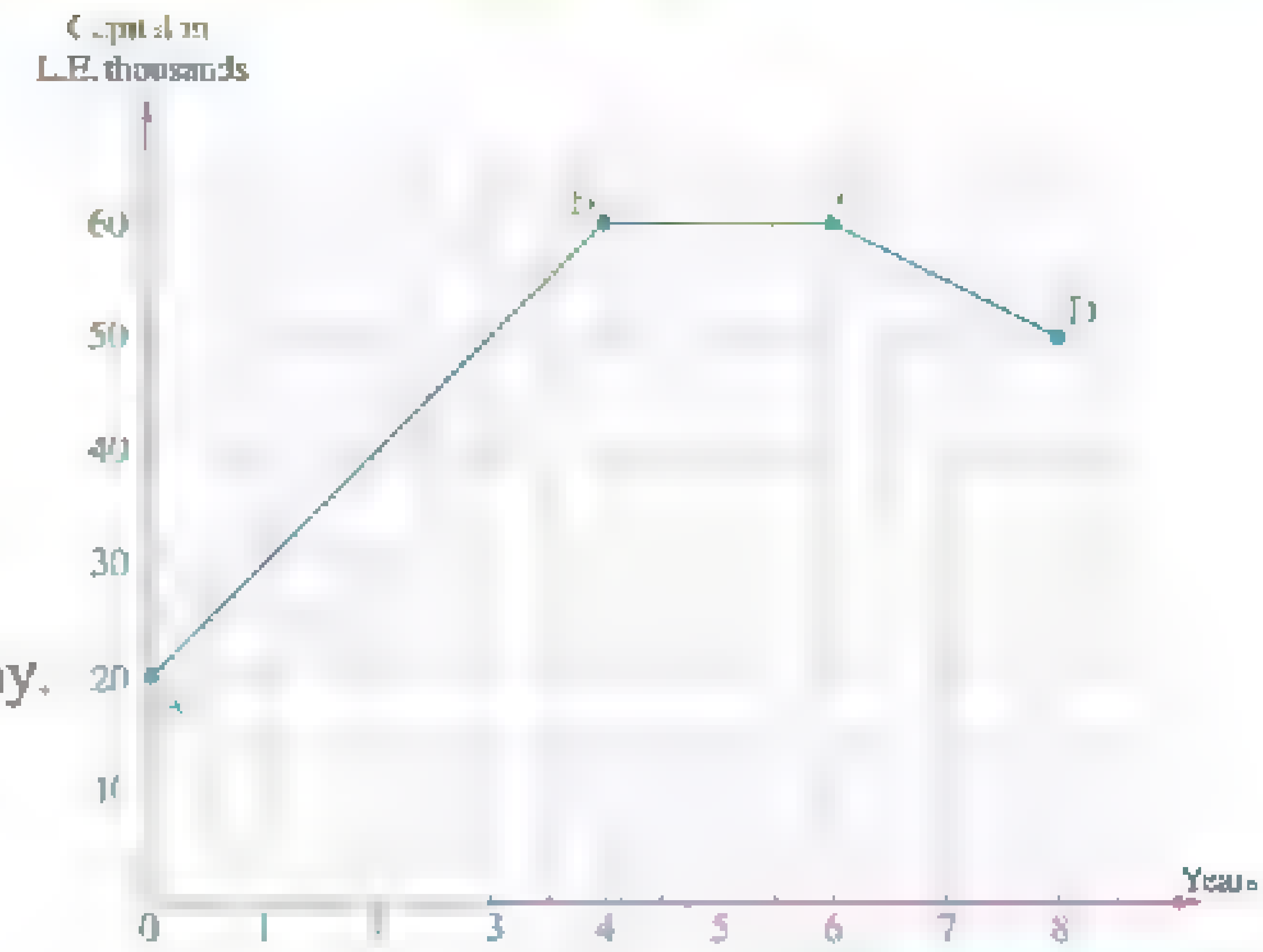
● Remember ● Understand ○ Apply ● Problem Solving

The opposite figure shows the capital change of a company during 8 years :

1. Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$

   What is the meaning of each ?

2. Find the starting capital of the company.

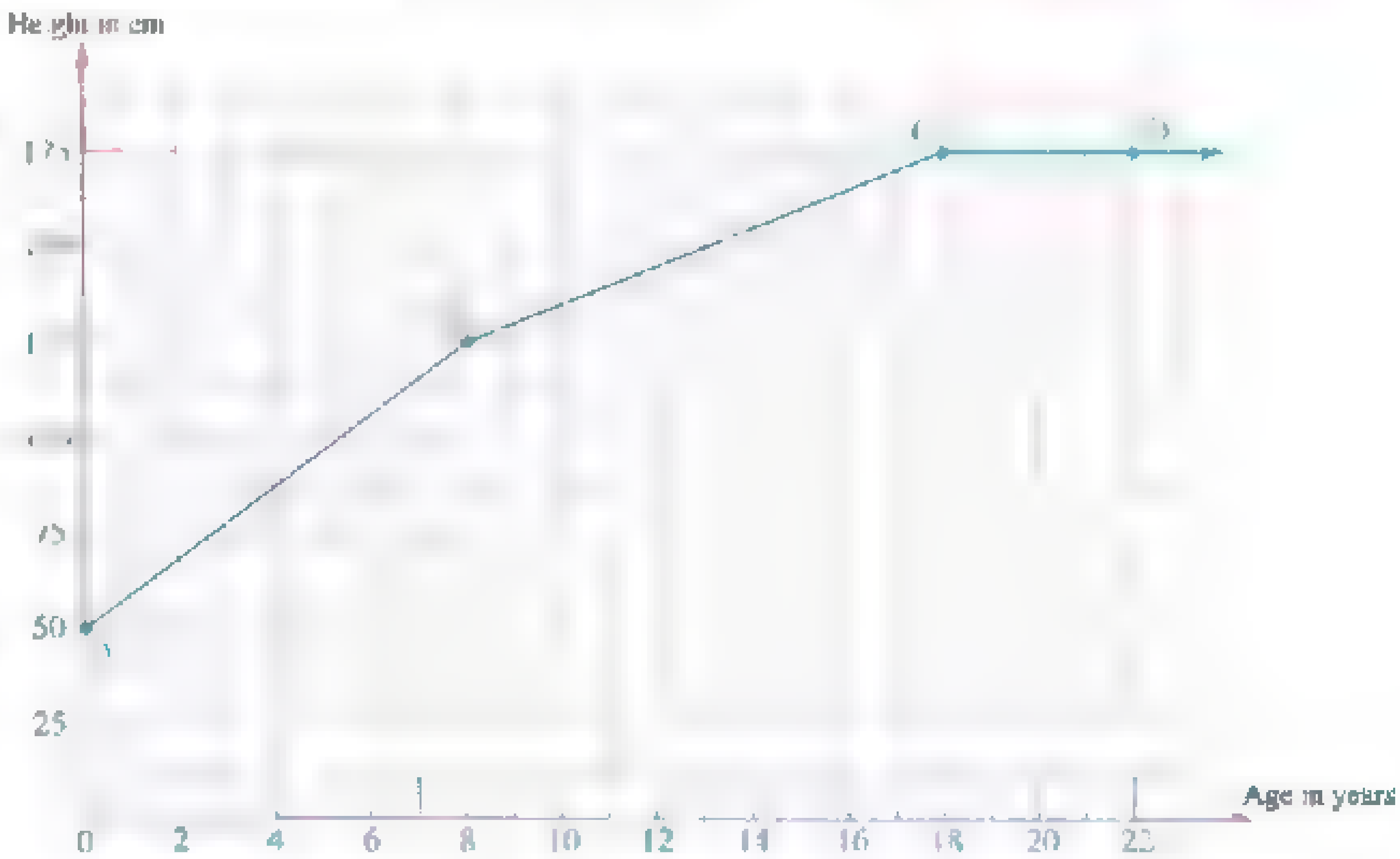


« 10 , 0 , – 5 , 20 thousand pounds »

The opposite figure shows the relation between the height of a person (in cm.) and his age (in years) :

1. Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$

   What is the meaning of each ?

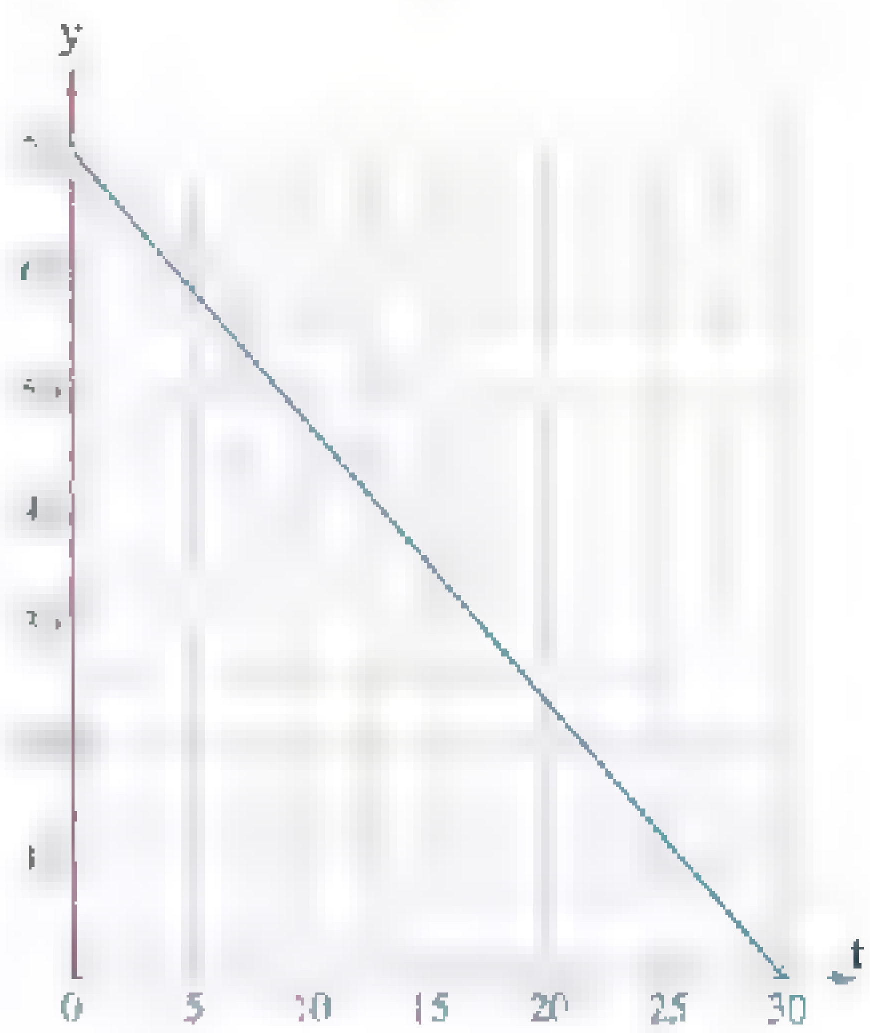


2. Calculate the difference between the height of this person when he was 8 years old and his height when he was 30 years old.

« $9\frac{3}{8}$, 5 , 0 , 50 cm. »

Magdi filled the tank of his car by fuel. The opposite figure represents the relation between the time (t) in hours and the amount of remained fuel in the tank (y) in litres :

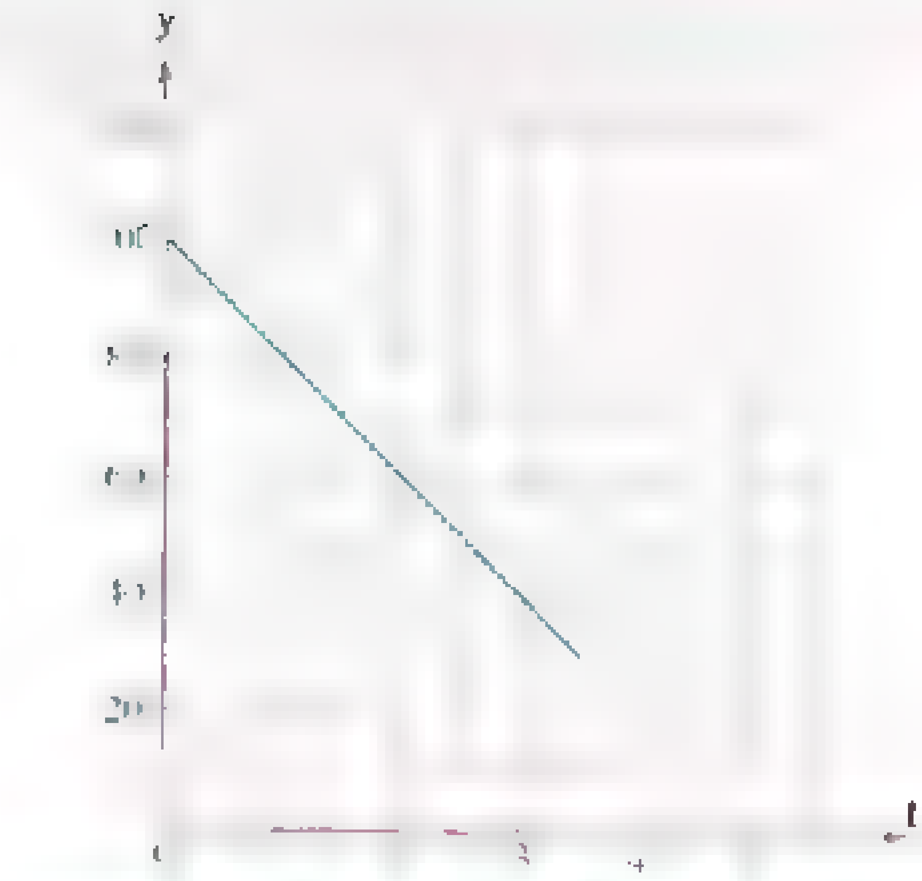
1. What is the greatest capacity of the tank ?
2. When will the tank become empty ?
3. What is the amount of remained fuel after 15 hours ?
4. What is the range of consumption of fuel in each hour ?



« 70 L , 30 hr. , 35 L , $2\frac{1}{3}$ L/hr. »

11 **A person read a book.**

The opposite graph shows the relation between the time (t) in hours and the number of remained pages (y) :



1. How many pages are remained in the beginning ?
2. Find the rate of reading pages per hour.
3. When does this person finish reading this book ?

« 60 pages , 12 pages/hr. , after 5 hours »

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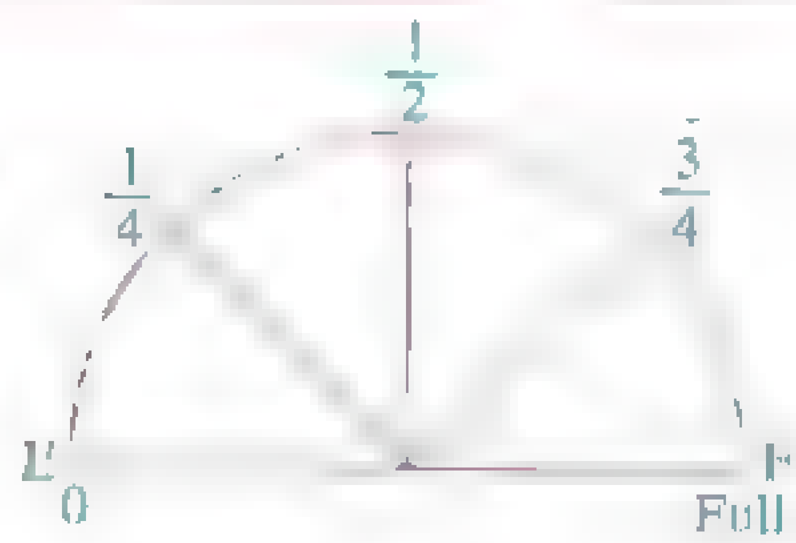
12 Hazem filled up the 40 L tank of his car.

After covering a distance of 120 km. , the fuel gauge shows that the rest of fuel is $\frac{3}{4}$ of the tank.



Draw a diagram to show the relation between the amount of fuel in the tank and the covered distance

(This relation is linear).

Calculate the covered distance until the tank totally gets empty

« 480 km. »

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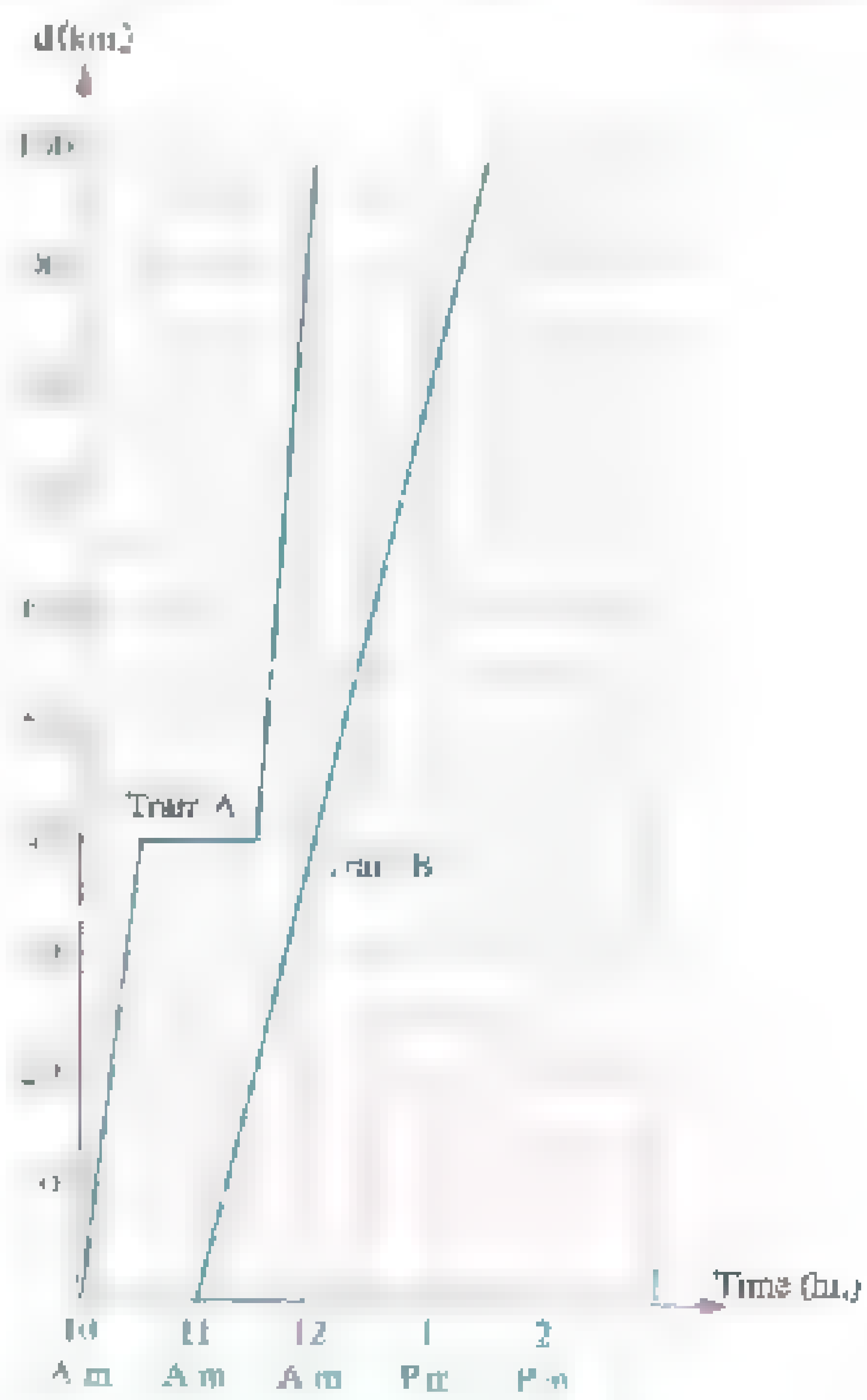
13 The opposite diagram shows the relation between the covered distance (in km.) and the elapsed time (in hr.) for two trains A and B between two railway stations.



Use the diagram to find :

1. The distance between the two railway stations.
2. The elapsed time of each train.
3. The average speed of each train.
4. The meaning of the horizontal segment in the diagram of train A

« 100 km. , 2 hr. , 2.5 hr. , 50 km./hr. , 40 km./hr. »

For excellent pupils

During the motion of a bicycle with a uniform velocity in a straight line , the distances between the bicycle and a fixed point have been registered after periods measured in hours from the moment of beginning the motion in the following table :

The distance between the bicycle and the fixed point	125	150	175	200
The passed time in hours	2	4	6	8

Graph the relation between the distance between the bicycle and the fixed point and the passed time. **From the graph , find :**

1. The velocity of the bicycle in km./hr.
2. The distance between the bicycle and the fixed point after 300 minutes.
3. The time at which the bicycle is at a distance = 187.5 km. from the fixed point.
4. The distance between the starting point of the bicycle and the fixed point.

« 12.5 km./hr. , 162.5 km. , 7 hr. , 100 km. »

UNIT THREE

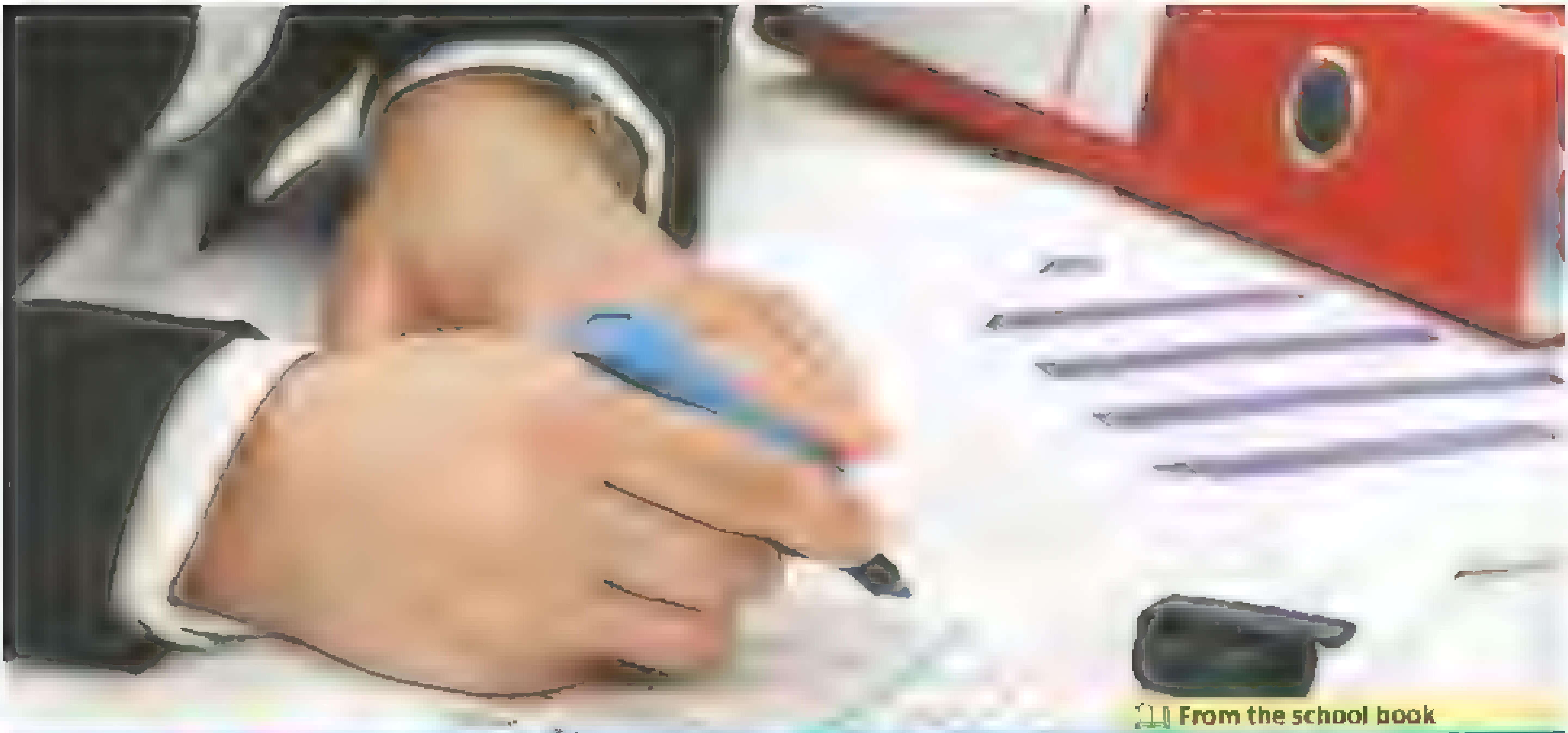
Statistics

Exercises of the unit

14. Collecting and organizing data.
15. The ascending and descending cumulative frequency tables and their graphical representation.
16. Mean.
17. Median.
18. Mode.

Scan

the QR code to solve an interactive test on each lesson



From the school book

Exercise

14 Collecting and organizing data

Remember … Problem Solving

1 The following are the weekly wages of 40 workers in a factory in L.E. :

47	71	36	94	54	64	87	89	62	57
51	61	44	52	70	66	56	32	69	36
79	48	77	90	65	99	96	67	60	55
95	75	81	84	78	38	49	94	48	59

Required : Form a frequency table with sets (use the subsets : 30– , 40– , 50– , ... , 90–)

What is the set with the highest frequency? What is the set with the lowest frequency ?

2 The following are the scores of 30 students in a monthly math exam :

25	35	40	20	30	37	40	33	22	38
35	36	28	37	39	28	32	26	29	37
23	34	35	36	29	38	40	35	37	31

1. From a frequency table with sets for these scores.
2. Find the total number of excellent students. The excellence rate is 36 marks or more.

« 12 students »

In a military camp , the heights of 55 soldiers were measured in centimetres , their measures were as follows :

169	194	200	185	165	188	166	186	181	176	173
177	179	188	170	193	180	173	173	184	192	167
182	168	186	189	171	179	172	175	175	181	166
185	177	175	165	190	172	177	178	184	166	174
178	177	172	174	175	179	195	176	189	187	189

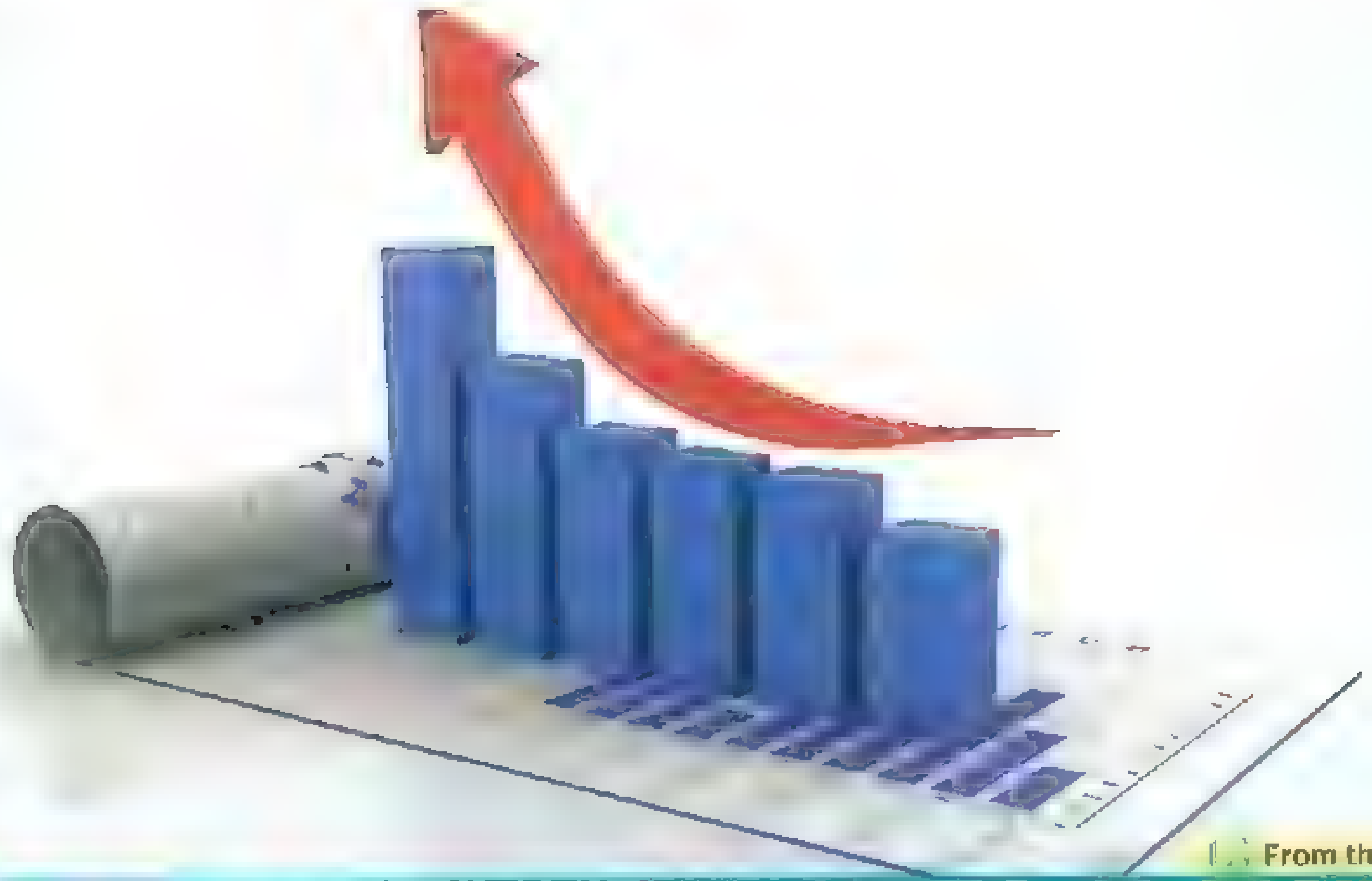
Find the least height , greatest height and the range.

Form a frequency table using the sets (165– , 170 , 175– , ...)

From the table , find :

The number of soldiers whose heights are less than 185 cm. « 39 soldiers »

The percentage of the of soldiers whose heights are 180 cm. at least. « 40 % »



From the school book

Exercise 15 The ascending and descending cumulative frequency tables and their graphical representation

● Remember ● Understand ○ Apply ● Problem Solving

Problems on the ascending cumulative frequency curve

1 **The following table shows the frequency distribution of the scores of 50 students in an experimental math exam :**

Sets	2–	6–	10–	14	18–	22–	26–	Total
Frequency	3	5	9	10	12	7	4	50

Graph the ascending cumulative frequency curve.

2 **The following frequency table represents the marks of 60 pupils in math :**

Sets	10–	20–	30–	40–	50–	Total
Frequency	9	11	13	17	10	60

Graph the ascending cumulative frequency curve and if the success mark is 30 marks , find the number of failed pupils. « 20 pupils »

Problems on the descending cumulative frequency curve

3 **The following table shows the frequency distribution of the daily wages of some workers :**

Sets	5 -	10	15 -	20–	25–	30–	Total
Frequency	10	14	24	30	12	10	100

Graph the descending cumulative frequency curve.

A class has 50 pupils , the following table shows the distribution of studying hours among them every day :

Sets	1–	2–	3–	4–	5	6–	7–	Total
Freq.	2	3	5	12	15	7	6	50

1. Graph the descending cumulative frequency curve of this distribution.
2. From the graph , find the number of pupils who study 6 hours or more daily. « 13 pupils »
3. Find the percentage of the number of pupils who study 6 hours or more daily. « 26 % »

Third Problems on the two curves together

Graph the ascending and descending curves for the following frequency distribution :

Sets	8	12–	16–	20–	24–	28 -	32–	36–	40–	Total
Freq.	4	7	12	18	20	19	11	6	3	100

The following table shows the frequency distribution of the scores of 1000 students in a final year exam :

Percentage	20–	30–	40–	50–	60–	70–	80–	90–	Total
Number of students	30	70	160	260	150	130	110	90	1000

1. Graph the ascending and descending cumulative frequency curves
2. Find the number of students whose scores are less than 75% « 740 students »
3. Find the number of students whose scores are 85% or more. « 140 students »

Exercise 16 Mean

● Remember ● Understand ○ Apply ● Problem Solving

Interactive test

1 Complete the following :

1. The mean of a set of values = $\dfrac{\dots\dots\dots\dots \quad \dots\dots\dots}{\dots\dots\dots\dots}$
2. The centre of the set = $\dfrac{\dots\dots + \dots\dots}{2}$
3. The arithmetic mean of the values : 5 , 12 , 17 , 6 is ....
4. If the lower limit of a set is 8 and the upper limit of the same set is 14 , then its centre is ....
5. If the lower limit of a set is 4 and its centre is 9 , then its upper limit is ....
6. If the mean of a frequency distribution is 39.4 and the total of frequencies is 100 , then the total of the products of frequencies of the sets by their centres is ....

2 Choose the correct answer from those given :

1. The mean of the values : 2 – a , 4 , 1 , 5 , 3 + a is ....
   (a) 1 (b) 2 (c) 3 (d) 15
2. If the mean of marks of 5 pupils is 20 , then the sum of their marks is .... marks.
   (a) 4 (b) 15 (c) 25 (d) 100
3. The centre of the first set of the sets : 7 – , 13 – , 19 – , 25 – is ....
   (a) 6 (b) 7 (c) 10 (d) 13
4. If the upper limit of a set is 14 and its centre is 10 , then its lower limit is ....
   (a) 5 (b) 6 (c) 20 (d) 24
5. If the beginning of a set is 5 and its centre is 7.5 , then the length of the set is ....
   (a) 5 (b) 7.5 (c) 10 (d) 12.5

3 **Find the mean of the following frequency distribution :**

Sets	5 –	15 –	25	35 –	Total
Frequency	6	8	4	2	20

« 21 »

4 **The following table shows the frequency distribution of marks of 10 students in mathematics :**

Sets	10 –	20 –	30	40 –	50 –	Total
Frequency	1	2	4	2	1	10

1 Calculate the mean of marks of students.

2 If the mark of success is 30 , calculate the number of failed students.

« 35 marks , 3 students »

5 **The following table shows the frequency distribution of extra wages of 30 workers :**

Sets	15 –	25 –	35 -	45 –	55 –	65 –	75	Total
Freq.	2	3	5	8	6	4	2	30

Find the arithmetic mean.

« 51 »

6 **The following table shows the frequency distribution of the heights of 120 students in centimetres :**

Height (in cm.)	140 –	144	148 –	152 –	156	160 –	Total
Frequency	12	20	38	22	17	11	120

Find the mean.

« 151.5 cm. »

7 **The following table shows the distribution of marks of 40 students in one exam :**

Sets	5	15 –	......	35	45	Total
Number of students	3	......	12	10	5	40

1 Complete the table.

2 Calculate the mean.

3 Find the number of students whose marks are not less than 35 marks.

« 31 marks , 15 students »

The following table shows the frequency distribution of the weights of 30 children in kg. :

Weight (kg.)	6 –	10	14	18	22	26	30 –	Total
Frequency	2	3	.....	8	6	4	2	30

Complete the table , then find the mean of this distribution. « [illegible] »

The following table shows the frequency distribution of weights of 50 pupils in kg. in one school :

Weight in kg.	30 –	35 –	40 –	45 –	50	55	Total
Number of pupils	7	3 k	4 k	10	8	4	50

1 Calculate the value of k

2 Find the mean of this distribution. « 3 , 44 kg. »

The following table shows the frequency distribution of 50 workers days-off :

Sets	2 –	6 –	10 –	14 –	18 –	22	26 –	Total
Frequency	4	5	8	k – 2	7	5	1	50

Find :

1 The value of k

2 The mean. « 22 , 15.2 days »

If the mean of the scores of a student during the first 5 months is 23.8 , what is the score of the 6^{th} month if the mean of his scores is 24 marks ? « 2[illegible] marks »

If the mean of marks of Magdi in 4 exams is 16 marks , what is the mark which he should obtain in the fifth exam so that his mean in the five exams will be 18 marks ?

« 26 marks »

From the school book

Exercise 17 ? Median

Remember · Understand · Apply · Problem Solving

Interactive test

1 Choose the correct answer from those given :

1. The median of the values : 9 , 4 , 8 , 1 and 3 is ....
   (a) 3 (b) 4 (c) 5 (d) 8
2. The median of the values : 3 , 7 , 2 , 9 , 5 and 11 is ....
   (a) 12 (b) 7 (c) 6 (d) 5
3. The order of the median of the values : 7 , 6 , 5 , 8 and 4 is ....
   (a) third. (b) fourth. (c) fifth. (d) sixth.
4. If the order of the median of a set of values is the fourth , then the number of these values equals ....
   (a) 4 (b) 5 (c) 6 (d) 7
5. If the median of the values : k + 1 , k + 2 , k + 5 , k + 4 and k + 3 where k is a positive integer is 13 , then k = ...
   (a) – 10 (b) 10 (c) 13 (d) 16
6. The point of intersection of the ascending and descending cumulative frequency curves determines .... on the set-axis.
   (a) the mean (b) length of the set
   (c) centre of the set (d) the median
7. If the the point of intersection of the ascending and descending frequency curves is (30 , 50) , then the sum of frequencies is ....
   (a) 30 (b) 50 (c) 60 (d) 100

Using the ascending cumulative frequency curve , find the median of the following frequency distribution :

Sets	0 –	2 –	4	6	Total
Frequency	1	2	2	5	10

« 6 »

The following table shows the frequency distribution of 40 persons according to the percentage of intelligence of each of them :

Sets of intelligence percentage	40 –	50 –	60 –	70 –	80 –	90 –	Total
Number of persons	1	3	8	14	10	4	40

Using the ascending cumulative frequency curve , find the madian of percentage of intelligence. « Approximately 75 % »

The following table shows the frequency distribution of 100 factories according to the number of weekly working hours :

Sets of hours	50 –	60	70	80	90 –	100 –	Total
Number of factories	5	8	12	28	33	14	100

Find using the descending cumulative frequency curve the median number of hours of work of these factories. « 89.5 hours »

The following table shows the frequency distribution of 50 workers' wages in pounds :

Sets of wages	300 –	400 –	500 –	600 –	700 –	Total
Number of workers	8	12	18	7	5	50

Graph the descending cumulative frequency curve , then find the median. « 520 pounds »

The following table shows the frequency distribution of marks of 60 students in mathematics exam :

Sets of marks	5 –	10 –	15	20	25	30	35 –	Total
Number of students	2	5	14	20	13	5	1	60

Find the median mark. « 22 marks »

The following table shows the frequency distribution of weights of 20 children in kg. :

Sets	5	15–	25–	35	45	Total
Frequency	3	4	7	4	2	**20**

Find the median weight in kg. using the ascending and descending cumulative frequency curves of this distribution. « 29 kg »

The following table shows the frequency distribution for the scores of 50 students in an examination :

Sets	2–	6–	10	14–	18–	22–	26	Total
Frequency	3	5	9	10	12	7	4	50

Find : 1 The mean of the student's score.

2 The median. « 16.8 , 17.6 »

From the following frequency table with equal sets in range :

Sets	10–	20–	X–	40–	50–	60	Total
Frequency	10	17	20	32	k + 2	4	100

1 Find the value of each of X and k « $X = 30$, $k = 15$ »

Graph the ascending and descending cumulative curves on one figure, then calculate the median. « 41 »

From the school book

Exercise 18 Mode

Remember Problem Solving

Interactive test

Choose the correct answer from those given :

1 The mode of a set of values is ....

(a) $\frac{\text{sum of values}}{\text{number of these values}}$ (b) the most common value.

(c) the middle value after rearranging the values ascendingly or descendingly.

(d) the point of intersection of the ascending and descending cumulative frequency curves.

2 The mode of the values : 5 , 3 , 8 , 5 , 9 is .....

(a) 3 (b) 5 (c) 8 (d) 9

3 The mode of the values : 8 , 7 , 8 , 7 , 6 , 5 , 8 is ....

(a) 8 (b) 7 (c) 6 (d) 5

4 If the mode of the values : 4 , a , 5 , 3 is 3 , then a = ...

(a) 5 (b) 4 (c) 3 (d) 6

5 If the mode of the values : 12 , 7 , $X + 1$, 7 , 12 is 7 , then $X =$ .....

(a) 12 (b) 11 (c) 7 (d) 6

6 If the mode of the values : 4 , 11 , 8 , $2X$ is 4 , then $X =$ ....

(a) 1 (b) 2 (c) 4 (d) 8

7 If the mode of the values : 5 , 3 , $\sqrt{X-1}$, 4 is 3 , then $X =$ ..

(a) 3 (b) 4 (c) 8 (d) 10

A factory has 600 workers. A sample of 120 workers is taken such that it represents the all groups very well. It is found that the distribution of their ages in years is as the following table :

Age	25–	30–	35–	40–	45–	50	Total
Number of workers	12	17	18	40	25	8	120

Draw the histogram , then deduce the mode age. « 43 years »

The following table shows the frequency distribution of marks of 100 pupils in an exam :

Sets of marks	10–	14–	18–	22	26–	30–	34–	Total
Number of pupils	2	10	15	40	25	6	2	100

Find the mode mark using the histogram of this distribution.

Find the mode of the following frequency distribution for the scores of 40 students in an examination :

Sets of marks	30–	40–	50	60–	70–	80–	Total
Frequency	3	4	12	8	7	6	40

« 57 »

The following table shows the frequency distribution of the weights of 50 students in kg. :

Weight in kg.	30–	35–	40–	45–	50–	55–	Total
Number of students	$k+4$	$3k$	$4k$	$3k+1$	$3k-1$	$k+1$	50

1 Find the value of k « 3 »

Graph the frequency histogram , then find the mode.

The following table shows the frequency distribution with equal range sets for the weekly wages of 100 workers in a factory :

Sets of wages in L.E.	70	80	90–	100–	X–	120	130–
Number of workers	10	13	k 4	20	16	14	11

Find : 1 The value of each of X and k « $X = 110$, $k = 20$ »

2 The mode of wages in L.E. «L.E. 105»

The following table shows the frequency distribution for the weights of 50 students in kg. at a school :

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
Number of students	7	3 k	4 k	10	8	4	50

1 Find the value of k « 3 »

2 Calculate the mean. « 44 kg. »

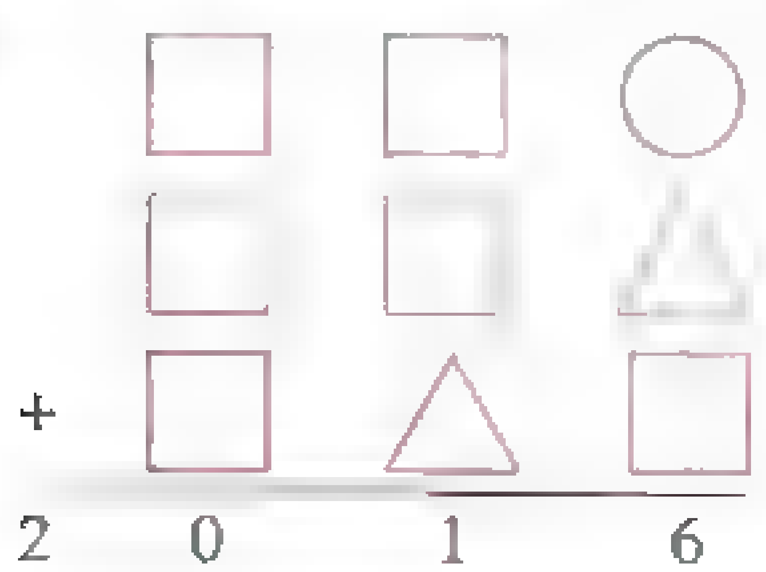
3 Draw the ascending cumulative frequency curve.

4 Draw the histogram and find the mode of weights. « 43 kg. »

5 Find the median. « 43.5 kg. »

1 Complete the following :

1. A turtle covers 80 metres per hour , then it covers 8 metres in ……… minutes.
2. The sum of the real numbers in the interval [−12 , 12] equals ………
3. If ◯ + □ = 20 , ◯ + ◯ + □ = 35 , then ◯ = ………
4. In three games of bowling , Sara gained 139 , 143 , 144 points , then the number of points she needs in the 4^{th} game so that the mean of points is 145 , is ………
5. Two boxes of apples , the sum of their weights is 54 kg. The first has 12 kg. more than the second , then the number of kilograms in the second box is ……… kg.
6. $300 \div 200 = 1 \div$ ………
7. $(301 + 302 + 303 + \cdots + 325) - (1 + 2 + 3 + \cdots + 25) =$ ………
8. If four times a number is 48 , then $\frac{1}{3}$ this number is ………
9. Gamal has 3 sisters and 5 brothers , his sister Sara has X sisters and y brothers , then X y = ………
10. If a + b + c = 26 , a + b = 15 , b + c = 20 , then b = ………
11. Three girls can perform a work in 36 hours , then the needed hours for four girls to perform the same work is ……… hours.
12. If



, then □ = ……… , △ = ……… , ◯ = ………

Remember   Understand   Apply   Problem Solving

Choose the correct answer from the given ones :

1 The number 3.015 lies on the number line between ·········

(a) $\frac{5}{2}$, 3   (b) $\frac{7}{2}$, $\frac{11}{3}$ (c) 3 , $\frac{16}{5}$ (d) 3.12 , 3.15

2 Which of the following numbers lies between 0.07 , 0.08 ?

(a) 0.00075   (b) 0.0075   (c) 0.075   (d) -0.75

3 Which of the following is different in value ?

(a) $1 \div 9 + 9 - 1$ (b) $1 + 9 \div 9 - 1$ (c) $1 - 9 + 9 \times 1$ (d) $1 \times 9 - 9 + 1$

If X is a negative number , which of the following is a positive number ?

(a) X^2 (b) X^3 (c) $2X$ (d) $\frac{X}{2}$

5 The greatest number of the following is ··········

(a) -1.25 (b) 0.125   (c) -0.0125 (d) -0.00125

The best estimation to the number opposite to X is ·

(a) 1.1   (b) 1.2   (c) 1.5   (d) 1.7

7 If 10% of X equals y , then $X =$ ··········

(a) 0.1 y   (b) y   (c) 9 y   (d) 10 y

8 If $X = (-2)^4$, $y = -2^4$, then ··········

(a) $X = y$ (b) $X > y$ (c) $X < y$ (d) $X \leq y$

9 $\sqrt{\sqrt{81 \times 81 \times 81 \times 81}} =$ ··········

(a) 3   (b) 9   (c) 27   (d) 81

10 For any number k , then $k + k + (k \times k \times k)$ can be written as ··········

(a) $2k^2 + 3k$ (b) $5k$ (c) k^5 (d) $2k + k^3$

11 A machine produces two kinds of rods , one is red and of length (10 ± 0.5) cm. and the other is white and of length (6 ± 0.5) cm.

If we put two rods as shown in the opposite figure , then the smallest difference between their lengths may be ······

(a) 4 cm.   (b) 5 cm.   (c) 3 cm.   (d) 8.5 cm.

12 All numbers divisible by 4 and 15 are divisible by ··

(a) 6   (b) 8   (c) 24   (d) 45

Second

Geometry

UNIT FOUR

Medians of Triangle– Isosceles Triangle

Exercises of the unit

1. Medians of triangle.
2. Medians of triangle "Follow".
3. The isosceles triangle.
4. The converse of the isosceles triangle theorem.
5. Corollaries of the isosceles triangle theorem.

Scan the QR code to solve an interactive test on each lesson

From the school book

Exercise **1**

Medians of triangle

Problem Solving

Interactive test

Complete the following :

In ΔABC , if D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is called ....

The number of medians of the triangle is ....

The medians of the triangle intersect at ....

The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... from its base.

The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... from the vertex.

The point of intersection of the medians of the triangle divides each of them in the ratio 2 : .... from the base.

The point of intersection of medians of the triangle divides each of them in the ratio .... : 8 from the vertex.

Choose the correct answer from those given :

The number of medians of the obtuse-angled triangle is ....

(a) zero (b) 1 (c) 2 (d) 3

If $\overline{YD}$ is a median in ΔXYZ , M is the point of intersection of medians , then MD = ... YM

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

If M is the point of intersection of medians of ΔABC , $\overline{BD}$ is a median , then BD : MD = ...

(a) 2 : 3 (b) 1 : 3 (c) 3 : 2 (d) 3 : 1

If $\overline{AD}$ is a median in ΔABC , M is the point of intersection of medians , then AD = ... AM

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

If $\overline{AD}$ is a median in ΔABC of length 9 cm. , M is the point of intersection of medians , then DM = ... cm.

(a) 3 (b) 4.5 (c) 6 (d) 9

If M is the point of intersection of the medians of ΔABC , $\overline{AD}$ is a median of length 6 cm. , then AM = ... cm.

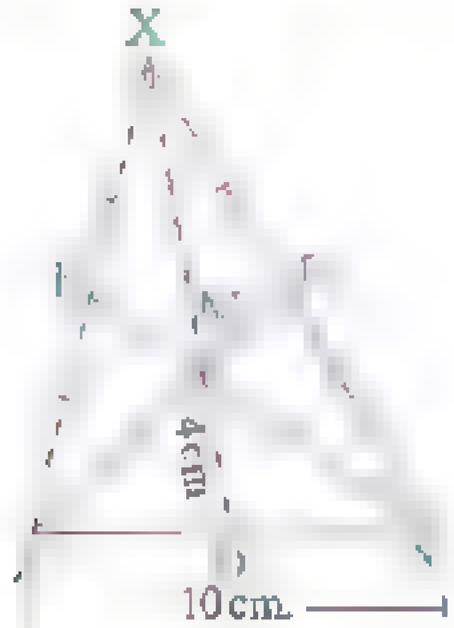
(a) 1 (b) 2 (c) 3 (d) 4

If M is the point of intersection of the medians of ΔABC , D is the midpoint of $\overline{BC}$, then AD = ...

(a) 2 AM (b) $\frac{2}{3}$ MD (c) $\frac{3}{2}$ AM (d) 4 MD

Using data given for each of the following figures , find the required below each figure :

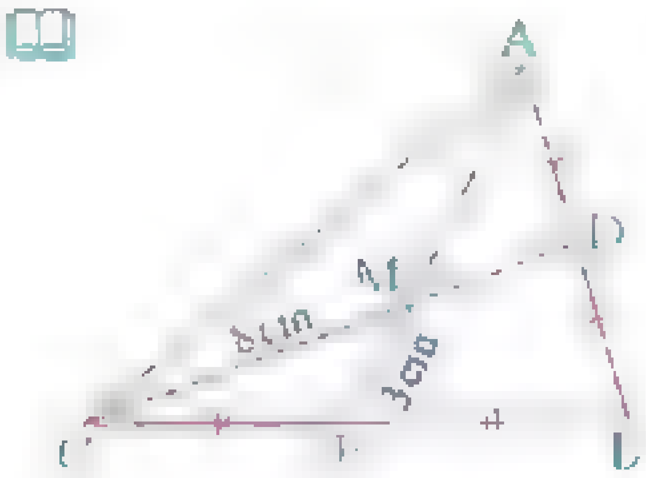
1



XM = ... cm. and

YD = ... cm.

2

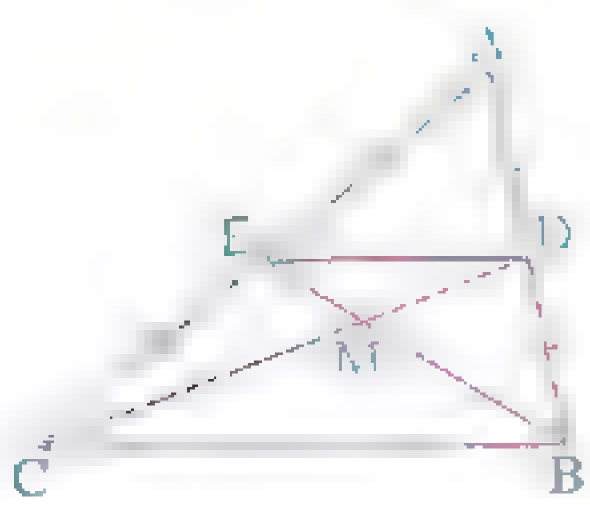


MA = ... cm. ,

MD = ... cm. ,

ME = ... AE

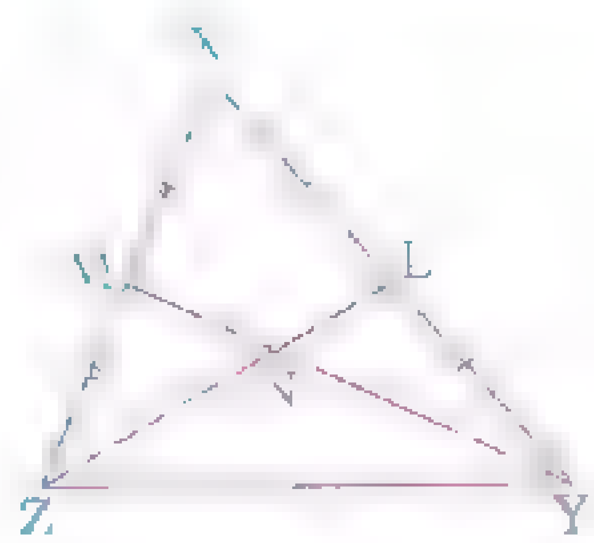
and MC = ... CD



If BC = 12 cm. , BE = 9 cm. and MC = 8 cm. , then DE = ... cm. ,

ME = ... cm. and

MD = ... cm.



If LZ = 15 cm. , YM = 18 cm. and XY = 20 cm. , then NL = ... cm. ,

NY = ... cm. and the perimeter of Δ NLY = ... cm.

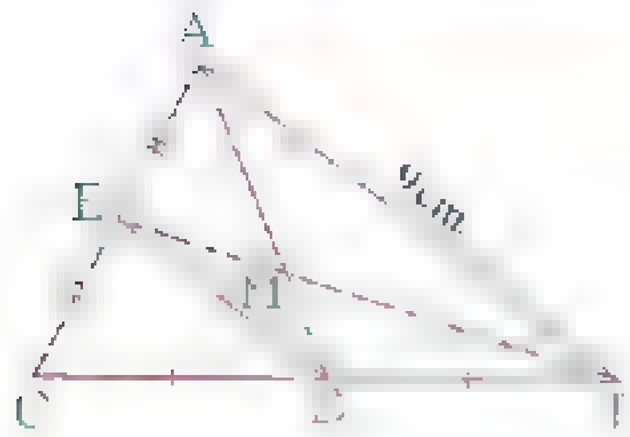
4 **In the opposite figure :**

ABC is a triangle in which D is the midpoint of $\overline{BC}$

, E is the midpoint of $\overline{AC}$ and $\overline{AD} \cap \overline{BE} = \{M\}$

If AD = 6 cm. and AB = BE = 9 cm.

Calculate : The perimeter of Δ MDE



« 9.5 cm »

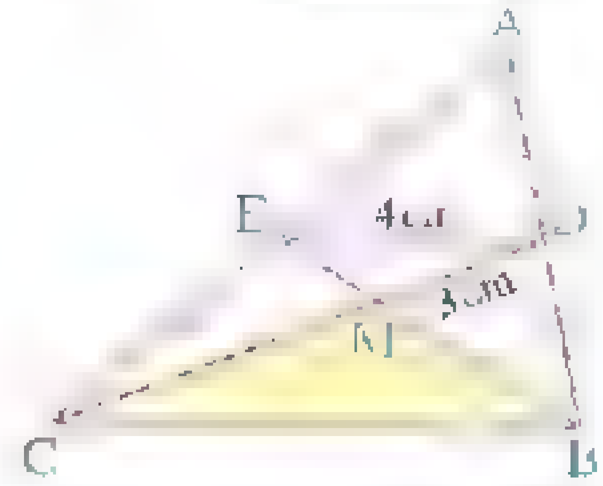
5 **In the opposite figure :**

If D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

and $\overline{BE} \cap \overline{DC} = \{M\}$, DE = 4 cm. ,

DM = 3 cm. and BE = 6 cm.

Find : The perimeter of Δ BMC



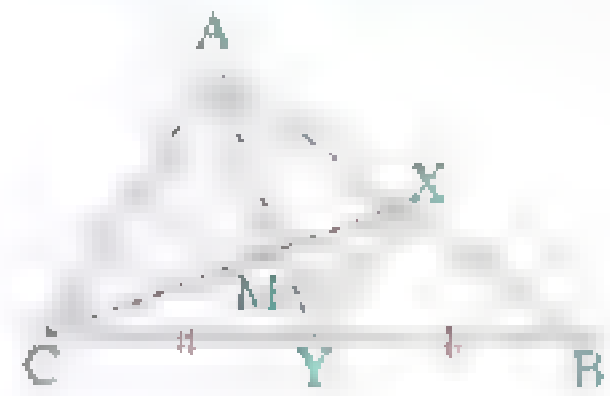
« 18 cm »

6 **In the opposite figure :**

ABC is a triangle , X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{BC}$, XY = 5 cm. and $\overline{XC} \cap \overline{AY} = \{M\}$

where CM = 8 cm. , YM = 3 cm. **Find :**



1 The perimeter of Δ MXY 2 The perimeter of Δ MAC « 12 cm. , 24 cm. »

7 In Δ ABC , BC = 8 cm. , F and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively and $\overline{BE} \cap \overline{CF} = \{M\}$ If BM = 4 cm. and CM = 6 cm. **Find :** The perimeter of Δ MFE « 9 cm »

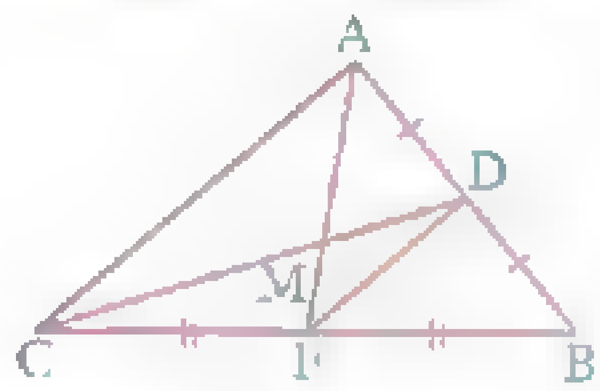
8 **In the opposite figure :**

$\overline{AF}$ and $\overline{CD}$ are two medians in Δ ABC ,

$\overline{AF} \cap \overline{CD} = \{M\}$

If the perimeter of Δ AMC = 36 cm.

Find : The perimeter of Δ MFD



« 18 cm »

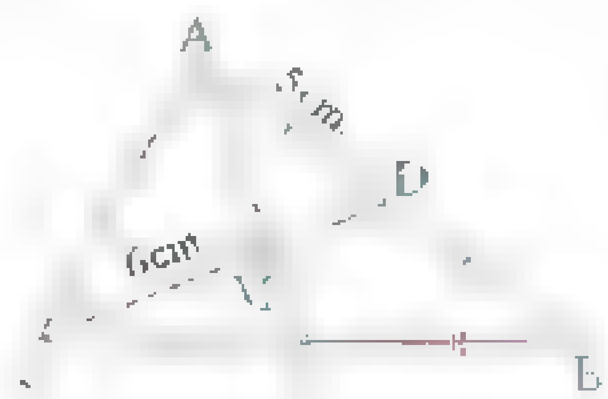
9 **In the opposite figure :**

M is the point of concurrence of the medians

of Δ ABC , $\overline{AM} \perp \overline{CD}$

, MC = 6 cm. , AD = 5 cm.

Find : The length of $\overline{ME}$



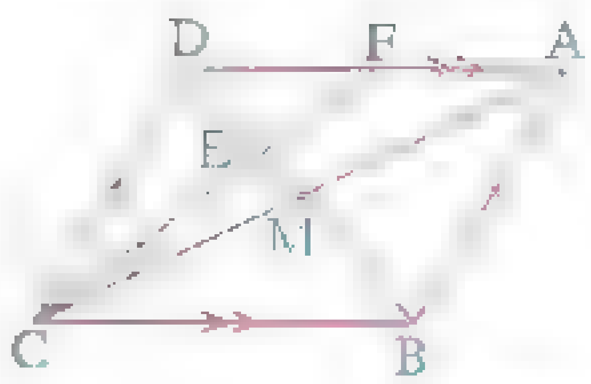
« 2 cm »

10 **In the opposite figure :**

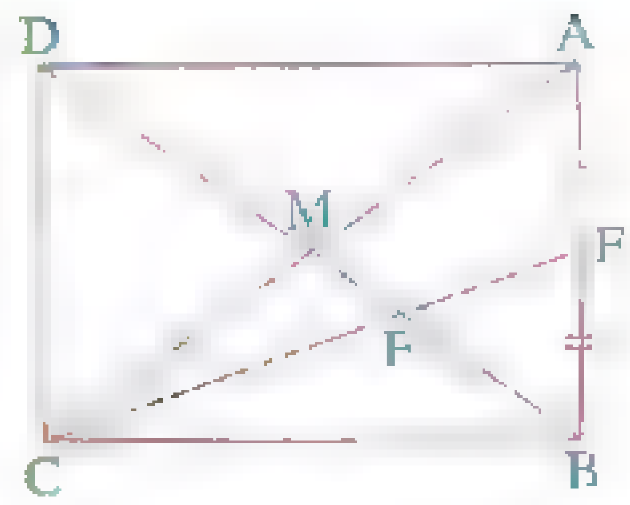
ABCD is a parallelogram , its diagonals intersect at M ,

E $\in$ $\overline{DM}$ where DE = 2 EM , draw $\overrightarrow{CE}$ to cut $\overline{AD}$ at F

Prove that : AF = FD



11 **In the opposite figure :**



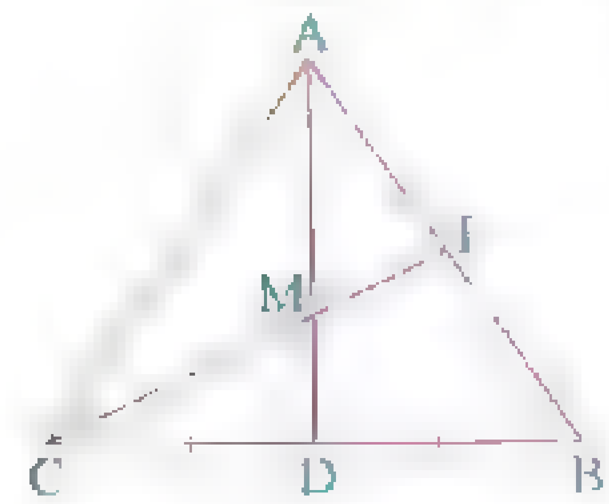
ABCD is a rectangle , its diagonals intersect at M ,

E is the midpoint of $\overline{AB}$, $\overline{CE} \cap \overline{BD} = \{F\}$

1 **Prove that :** F is the intersection point of the medians of the triangle ABC

2 If BF = 4 cm. , **find :** the length of $\overline{AM}$ « 6 cm. »

12 **In the opposite figure :**



ABC is a triangle in which D is the midpoint of $\overline{BC}$,

AB = AC , M ∈ $\overline{AD}$ where AM = $\frac{2}{3}$ AD and

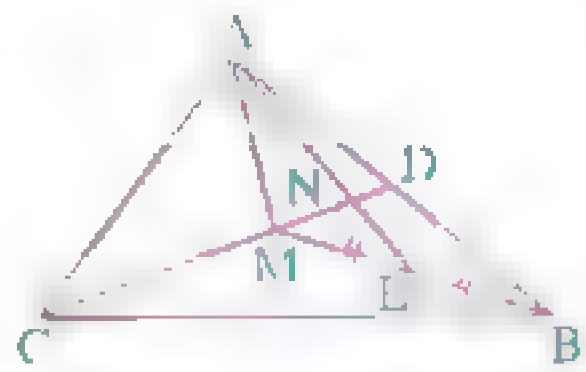
$\overrightarrow{CM} \cap \overline{AB} = \{F\}$

Prove that : BF = $\frac{1}{2}$ AC

13 ABC is a triangle where point D is the midpoint of $\overline{BC}$ and point M ∈ $\overline{AD}$, AM = 2 MD

Draw $\overrightarrow{CM}$ to intersect $\overline{AB}$ at point E If EC = 12 cm. , **then find :** the length of $\overline{EM}$ « 4 cm. »

14 **In the opposite figure :**



M ∈ $\overline{CD}$, M is the point of concurrence of the medians

of Δ ABC , N ∈ $\overline{DM}$ where ND = $(X - 1)$ cm.

, MN = $(X + 3)$ cm. , $\overrightarrow{AN}$ is drawn to intersect $\overline{BM}$ at E

which is the midpoint of $\overline{BM}$

Find : The length of $\overline{MC}$ « 24 cm. »

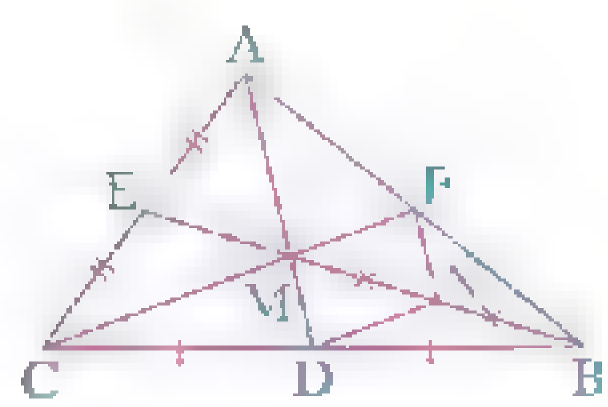
15 ABCD is a parallelogram whose diagonals intersect at M , E is the midpoint of $\overline{BC}$,

$\overline{DE}$ intersects $\overline{AC}$ at F

Prove that : 1 $\overrightarrow{BF}$ bisects $\overline{CD}$ 2 CF = $\frac{1}{3}$ AC

For excellent pupils

16 **In the opposite figure :**



AD and BE are medians in the triangle ABC intersecting at M ,

$\overrightarrow{CM} \cap \overline{AB} = \{F\}$, if N is the midpoint of $\overline{MB}$

Prove that : The figure FNDM is a parallelogram.

17 **In the opposite figure :**

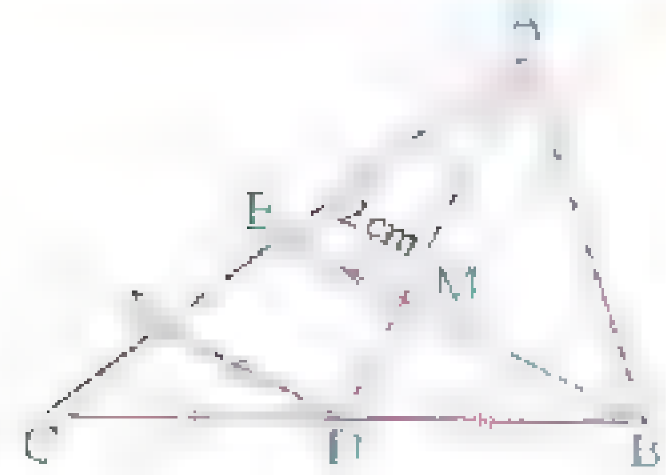
ABC is a triangle in which D is the midpoint of $\overline{BC}$

, $M \in \overline{AD}$ where $AM = 2\,MD$

, $\overrightarrow{BM} \cap \overline{AC} = \{E\}$

, $ME = 2$ cm. , draw $\overrightarrow{DF} \parallel \overline{BE}$ and cut $\overline{AC}$ at F

Find : The length of $\overline{DF}$



« 3 cm »

18 ABC is a triangle , D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$

If $\overline{CD} \cap \overline{BE} = \{M\}$ Draw $\overrightarrow{AM}$ to intersect $\overline{BC}$ at F

Prove that : The figure DBFE is a parallelogram.

From the school book

Exercise 2 Medians of triangle "Follow"

● Remember ● Understand ○ Apply ● Problem Solving

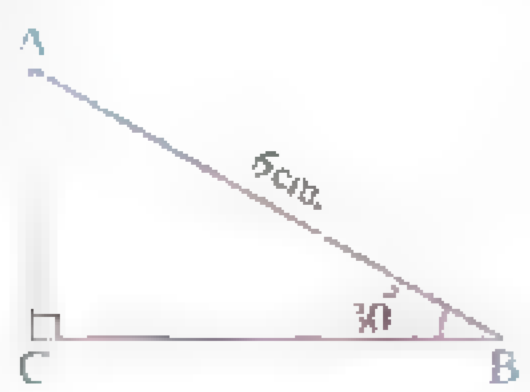
Interactive test

1 Complete the following :

1. The number of medians in the right-angled triangle is ....
2. The length of the median from the vertex of the right angle in the right-angled triangle equals ....
3. If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is ....
4. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ....
5. The length of the hypotenuse in thirty and sixty triangle equals .... the length of the side opposite to the angle whose measure is 30°
6. The length of the hypotenuse in the right-angled triangle equals .... the length of the median drawn from the vertex of the right angle.

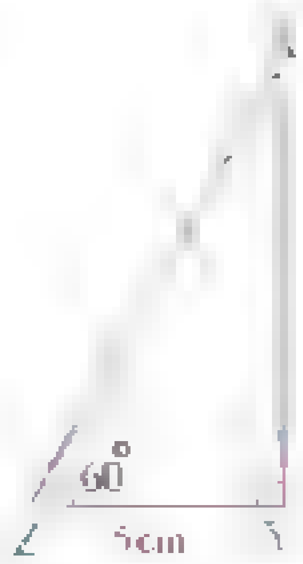
2 Using data given for each of the following figures , find the required below each figure :

1



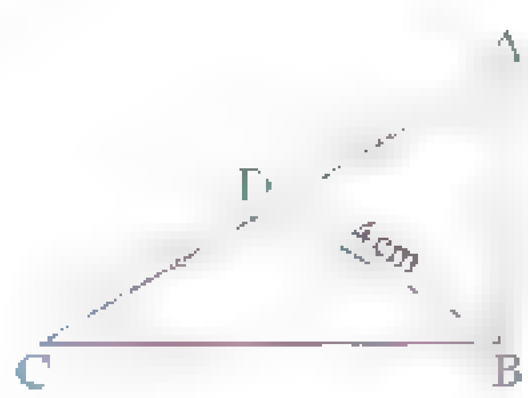
AC = ... cm.

2

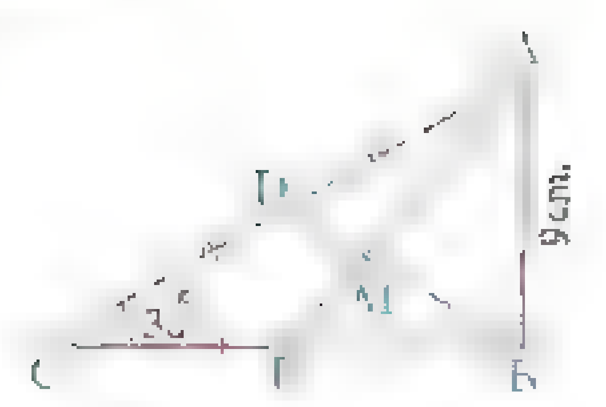


XZ = ... cm.

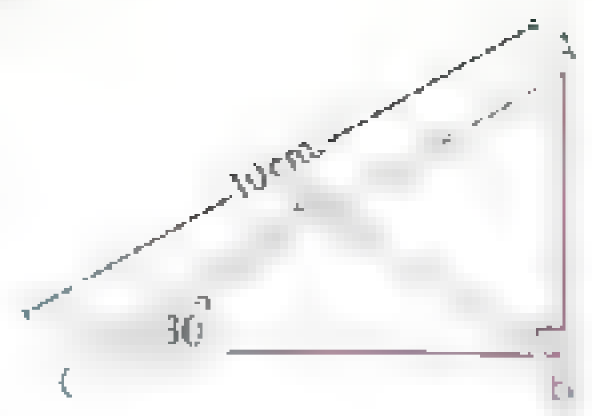
3



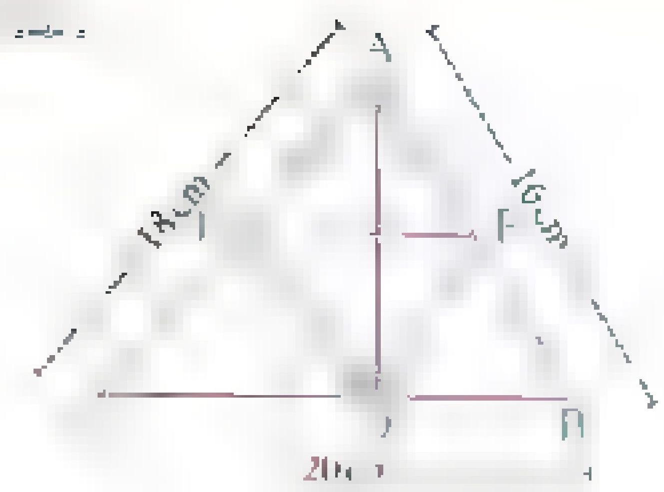
AC = ... cm.



AC = ... cm. ,
BD = ... cm. ,
MD = ... BD
and MD = ... cm.



BD = ... cm. ,
AB = ... cm.
and the perimeter of
Δ ABD = ... cm.



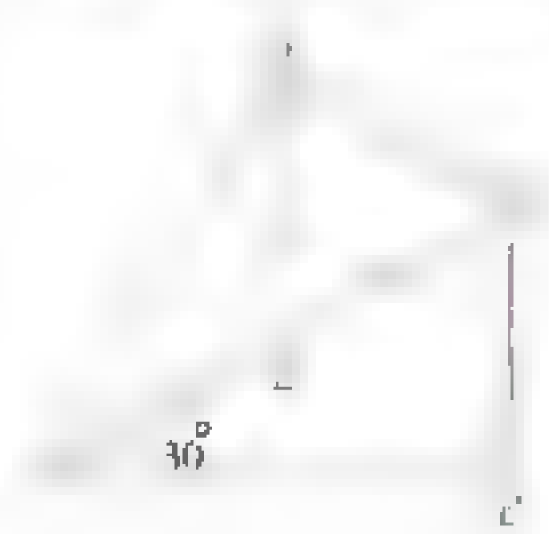
DF = ... cm. ,
DE = ... cm. ,
FE = ... cm.
and the perimeter of
Δ DEF = ... cm.

Choose the correct answer from those given :

1. In the right-angled triangle , the ratio between the length of the median drawn from the vertex of the right angle and the length of the hypotenuse is
   (a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 3 : 2
2. In the thirty-sixty triangle , the ratio between the length of the hypotenuse and the length of the side opposite to the angle of measure 30° is ....
   (a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3
3. In the thirty-sixty triangle , the ratio between the length of the median drawn from the vertex of the right angle and the length of the side opposite to the angle of measure 30° is ....
   (a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 2 : 3
4. ABC is a right-angled triangle at B , D is the midpoint of $\overline{AC}$, then BD ....
   (a) $\frac{1}{2}$ AC (b) AC (c) $\frac{1}{2}$ BC (d) AB
5. ABC is a triangle in which m ($\angle$ A) = 90° , AC = $\frac{1}{2}$ BC , then m ($\angle$ C) = ...
   (a) 30° (b) 60° (c) 90° (d) 120°
6. In Δ ABC , m ($\angle$ B) = 90° , if 2 AB − AC = 0 , then m ($\angle$ C) = ...
   (a) 30° (b) 60° (c) 90° (d) 120°

4 In the opposite figure :

m ($\angle$ ABC) = m ($\angle$ ADC) = 90° ,
m ($\angle$ ACB) = 30° and
E is the midpoint of AC
Prove that : AB = DE



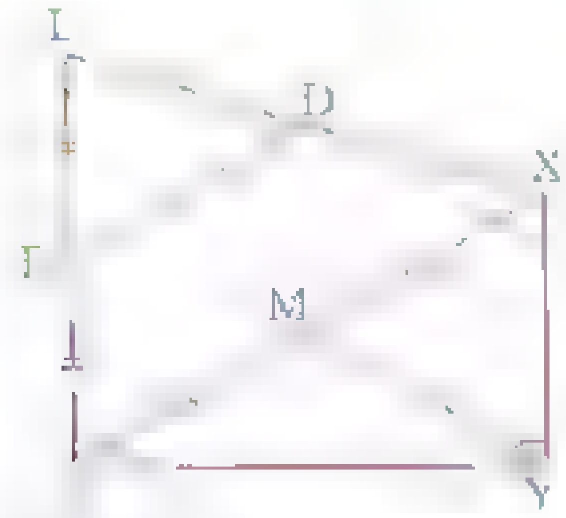
5 **In the opposite figure :**

m (∠ XYZ) = 90° , D is the midpoint of $\overline{XL}$,

E is the midpoint of $\overline{ZL}$ and

M is the midpoint of $\overline{XZ}$

Prove that : DE = YM



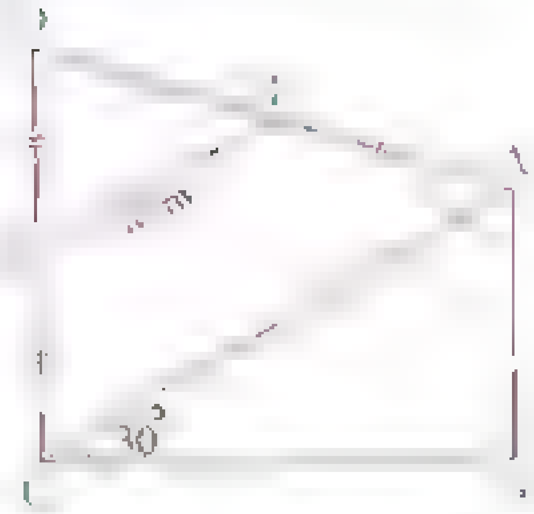
6 **In the opposite figure :**

ABCD is a quadrilateral in which m (∠ B) = 90° ,

E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{CD}$,

m (∠ ACB) = 30° and EF = 4 cm.

Find by proof : The length of $\overline{AB}$ « 4 cm. »



7 **In the opposite figure :**

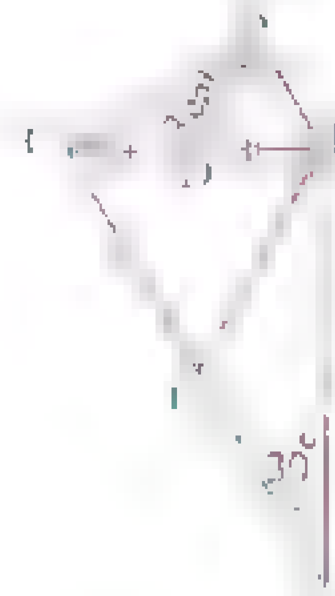
m (∠ BAC) = m (∠ CBE) = 90°

, m (∠ BEC) = 30°

, D and F are the midpoints

of $\overline{BC}$ and $\overline{CE}$ respectively and AD = 3 cm.

Find : The length of $\overline{BF}$ « 6 cm »



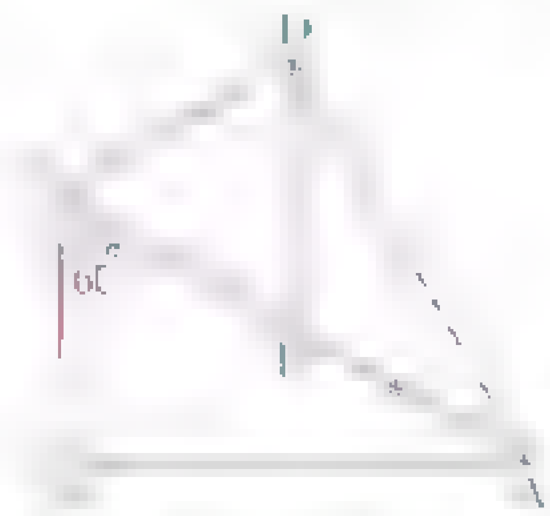
8 **In the opposite figure :**

ABC is a right-angled triangle at B , m (∠ ACB) = 60° ,

E is the midpoint of $\overline{AC}$ and

DE = BC

Prove that : m (∠ ADC) = 90°



9 **In the opposite figure :**

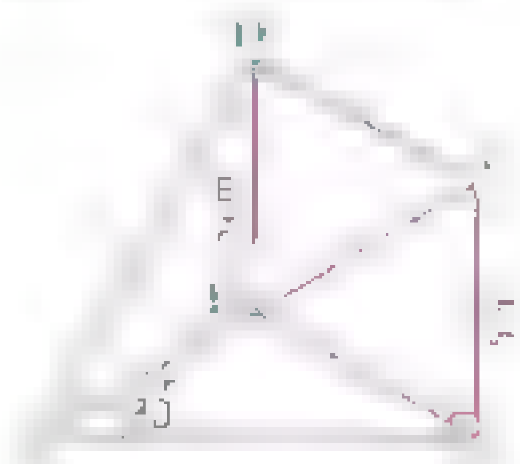
ABC is a right-angled triangle at B ,

m (∠ ACB) = 30° , AB = 5 cm. and

E is the midpoint of $\overline{AC}$

If DE = 5 cm.

Prove that : m (∠ ADC) = 90°



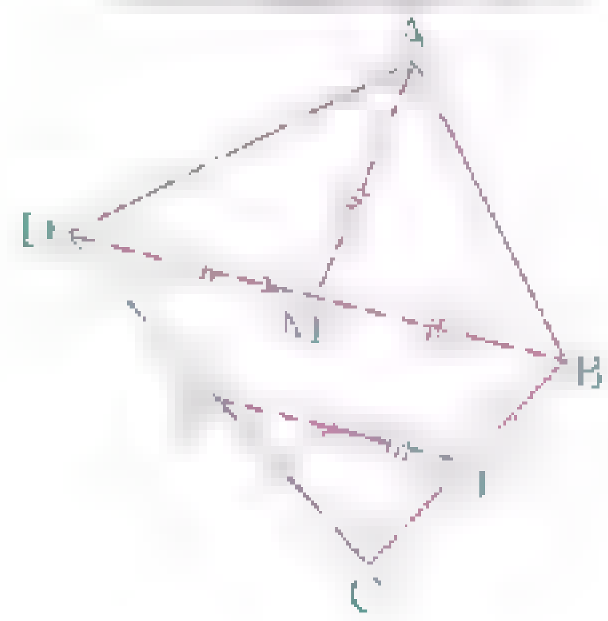
10 **In the opposite figure :**

ABD is a triangle , M is the midpoint of $\overline{BD}$,

E is the midpoint of $\overline{BC}$,

F ∈ $\overline{CD}$, $\overline{EF}$ // $\overline{BD}$ and AM = EF

Prove that : m (∠ BAD) = 90°



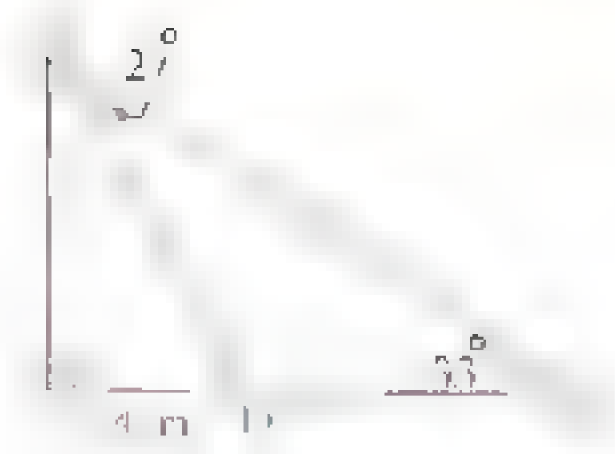
In the opposite figure :

ABC is a triangle in which m (∠ B) = 33°

, m (∠ C) = 90° , D ∈ $\overline{BC}$ where CD = 4 cm.

, m (∠ BAD) = 27°

Find : The length of $\overline{AD}$ « 8 cm. »



In the opposite figure :

ADB is a right-angled triangle at D ,

ACB is a right-angled triangle at C and E is the midpoint of $\overline{AB}$

Prove that : Δ CED is an isosceles triangle.

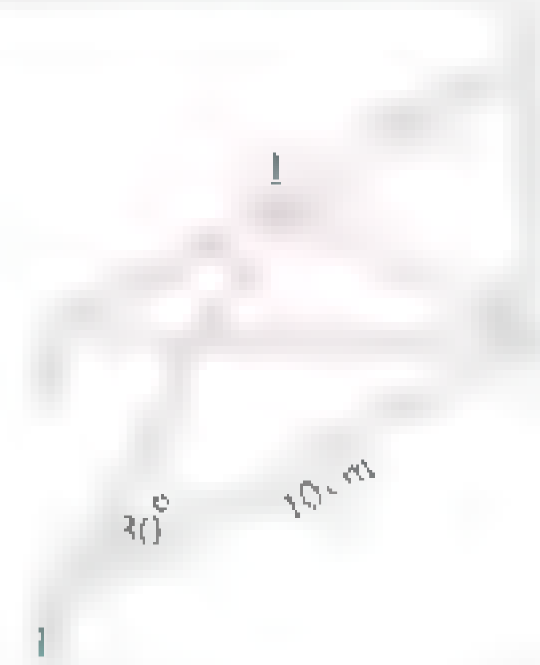
In the opposite figure :

m (∠ YLE) = 90° , m (∠ E) = 30° , YE = 10 cm. ,

m (∠ XYZ) = 90° and

L is the midpoint of $\overline{XZ}$

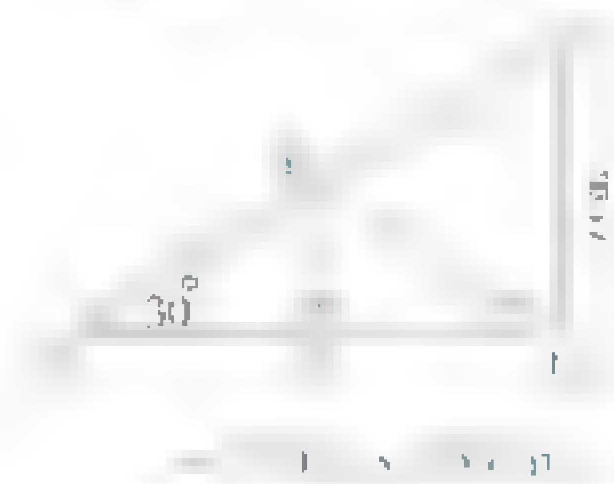
Find by proof : The length of $\overline{XZ}$ « 10 cm »



In the opposite figure :

ABC is a right-angled triangle at B , D is the midpoint of $\overline{AC}$, $\overline{DE} \perp \overline{BC}$, AB = 7 cm. and m (∠ C) = 30°

Find the length of each of : $\overline{BD}$ and $\overline{DE}$



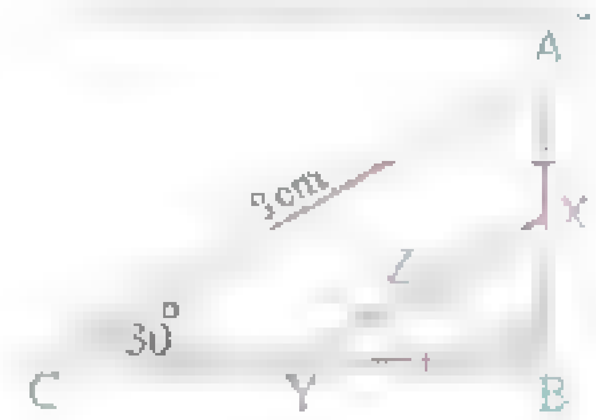
In the opposite figure :

ABC is a triangle in which m (∠ ABC) = 90° , m (∠ C) = 30° ,

X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{XY}$

respectively and AC = 8 cm.

Find the length of each of : $\overline{AB}$, $\overline{XY}$ and $\overline{BZ}$



« 4 cm. , 4 cm , 2 cm »

In the opposite figure :

ABC is a right-angled triangle at A

, M is the point of concurrence of its medians

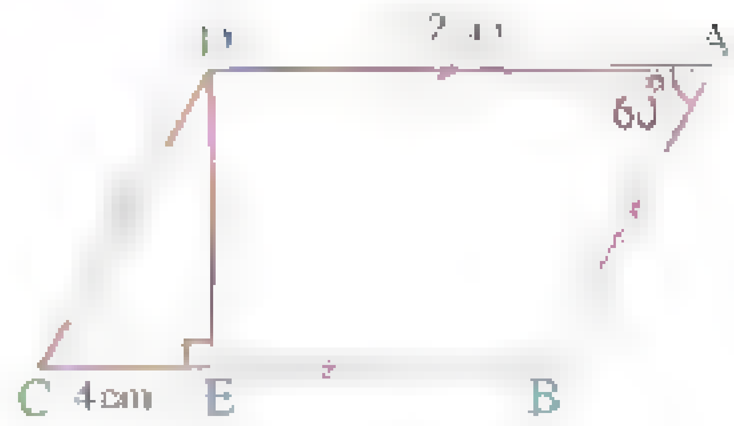
, E ∈ $\overline{DC}$ where $\overline{ME} \perp \overline{DC}$, DE = 3 cm.

and ME = 4 cm.

Find : The length of $\overline{BC}$

« 30 cm. »

17 **In the opposite figure :**



ABCD is a parallelogram in which

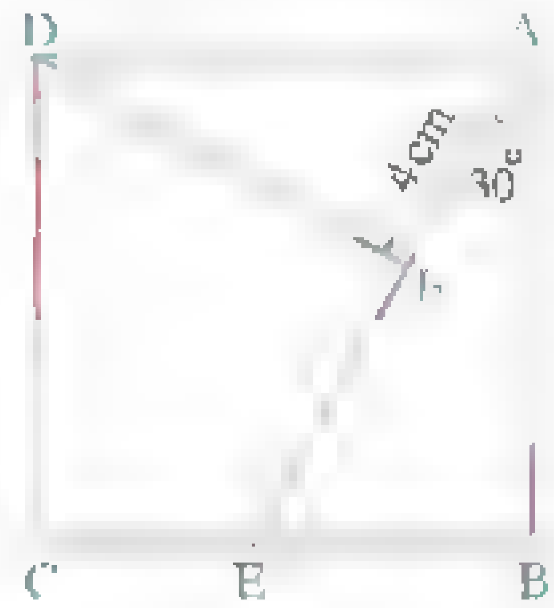
m (∠ A) = 60° , $\overline{DE} \perp \overline{BC}$

, AD = 12 cm. and EC = 4 cm.

Find : The perimeter of the parallelogram ABCD

« 40 cm. »

18 **In the opposite figure :**



ABCD is a square , E ∈ $\overline{BC}$ where m (∠ BAE) = 30° and

$\overline{DF} \perp \overline{AE}$ If AF = 4 cm.

Calculate : The area of the square ABCD

« 64 cm.² »

19 **In the opposite figure :**

ABCD is a rectangle , E ∈ $\overline{DC}$

where m (∠ CBE) = 30°

and m (∠ AEB) = 90°

Prove that : CE = $\frac{1}{4}$ AB

20 **In the opposite figure :**

ABC is a right-angled triangle at B ,

m (∠ A) = 30° ,

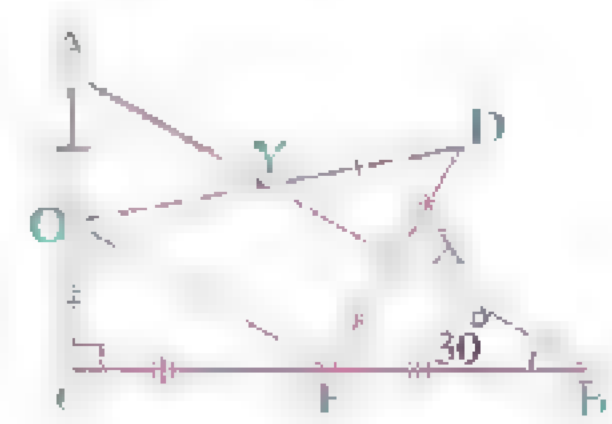
D ∈ $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$

If BC = 8 cm.

Find : The length of $\overline{AD}$

« 12 cm. »

21 **In the opposite figure :**



ABC is a right-angled triangle at C in which m (∠ B) = 30°

, E , O , X , Y are the midpoints of $\overline{BC}$, $\overline{AC}$

, $\overline{DE}$, $\overline{DO}$ respectively

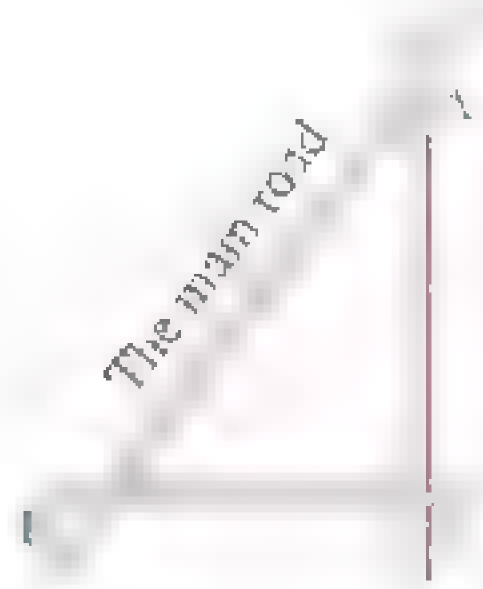
Prove that : XY = $\frac{1}{2}$ AC

22 ABC is a triangle in which AB = AC and $\overrightarrow{AD}$ is drawn to be perpendicular to BC where $\overline{AD} \cap \overline{BC} = \{D\}$ If E and F are the two midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

Prove that : DE + DF = AB

Life Application

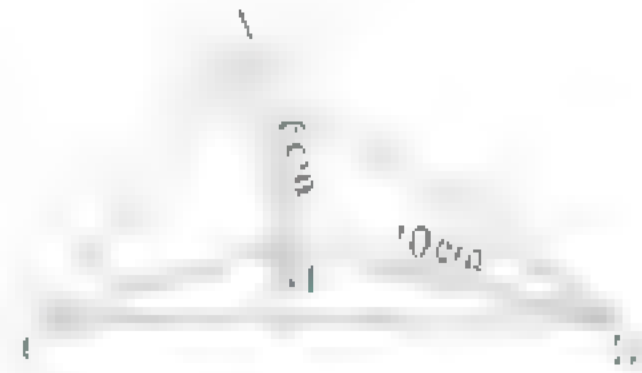
23 The opposite figure is a sketch for three towns A , B and C such that the distance between the towns A and C is 40 km. and the distance between the towns B and C is 30 km.



If we want to build a service station lying on the main road at the half-way between the towns A and B , also we want to build a road linking this station to the town C , then how long will this road be ?

For excellent pupils

24 **In the opposite figure :**



M is the point of concurrence of the medians of Δ ABC

, AM = 6 cm. , BM = 10 cm. , m ($\angle$ AMC) = 90°

Find by proof : The length of $\overline{AC}$ The length of $\overline{MC}$

25 ABCD is a parallelogram , X is an interior point in it such that $\overrightarrow{DX}$ bisects $\angle$ ADC , $\overrightarrow{CX}$ bisects $\angle$ DCB , if the point Y is the midpoint of $\overline{DC}$

Prove that : XY = YC

Wonders of numbers

- Pick any positive 2-digit number, add the two digits, and subtract the sum from the original number.
- Is the difference divisible by 9 ?

Try other numbers.

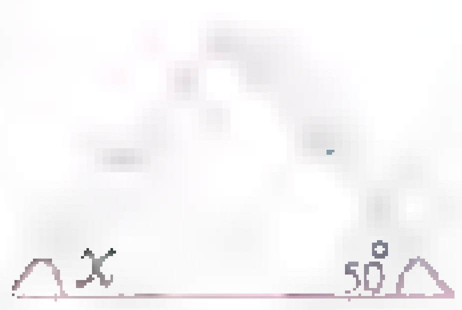
From the school book

Exercise 3 ? The isosceles triangle

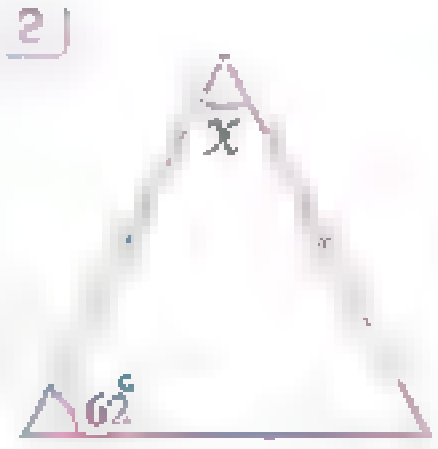
● Remember ● Understand ○ Apply ● Problem Solving

Interactive test

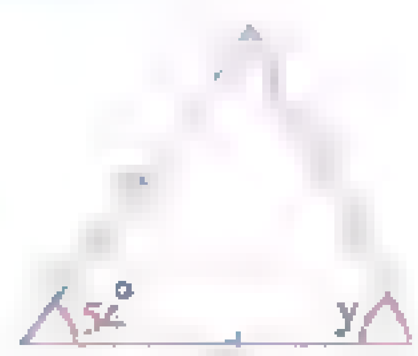
1 In each of the following , find the value of the symbol used for the measure of the angle :

1 

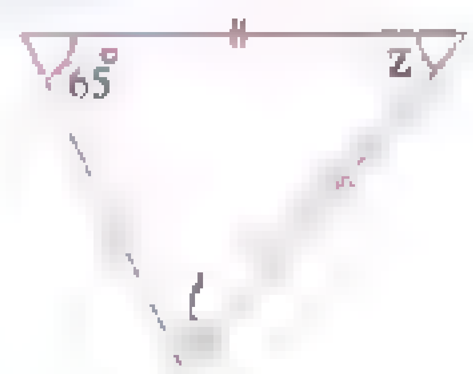
x = ...°

2 

x = ...°

3 

y = ...°

4 

ℓ = ...° , z = ...°

5 

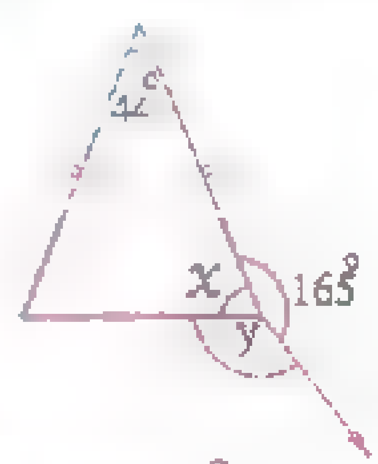
x = ...° , y = ...°

6 

x = ...° , y = ...°

7 

x = ...°

8 

x = ...° , y = ...°

2 Complete the following :

● 1 The base angles of the isosceles triangle are ......

● 2 The measure of each angle in the equilateral triangle equals ....°

● 3 In Δ DEF , if DE = DF , then m ($\angle$ E) = m ($\angle$ ....)

4 In the isosceles triangle , if the measure of one of the two base angles is 65° , then the measure of its vertex angle equals ....°

In the isosceles triangle , if the measure of the vertex angle equals 40° , then the measure of one of the two base angles equals ....°

An isosceles triangle , the measure of its vertex angle is 80° , if the measure of one of its base angles is $(X + 30°)$, then $X =$ ..

Choose the correct answer from those given :

1 In Δ XYZ , if XY = YZ = XZ , then m ($\angle$ X) = ...

(a) 30° (b) 60° (c) 90° (d) 180°

The measure of the exterior angle of the equilateral triangle equals ....

(a) 60° (b) 90° (c) 120° (d) 180°

LMN is a triangle in which LM = MN , m ($\angle$ M) = 70° , m ($\angle$ N) = ...

(a) 20° (b) 35° (c) 55° (d) 70°

4 In Δ ABC , AB = AC , m ($\angle$ C) = 65° , then m ($\angle$ A) = ...

(a) 30° (b) 50° (c) 55° (d) 130°

5 In Δ XYZ , ZY = ZX , m ($\angle$ Z) = 120° , then m ($\angle$ X) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

If Δ ABC is right-angled at A and AB = AC , then m ($\angle$ B) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

XYZ is an isosceles triangle in which , m ($\angle$ Y) = 100° , then m ($\angle$ Z) = ...

(a) 100° (b) 80° (c) 50° (d) 40°

If the measure of one of the two base angles in the isosceles triangle is 30° , then the triangle is ....

(a) obtuse-angled. (b) acute-angled.

(c) right-angled. (d) equilateral.

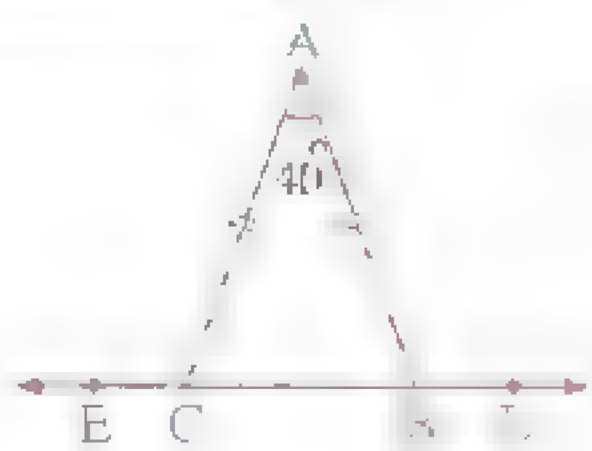
9 In Δ ABC , AB = AC , m ($\angle$ B) = $6X°$, m ($\angle$ A) = $3X°$, then $X =$ ....

(a) 30° (b) 12° (c) 60° (d) 90°

10 In Δ XYZ , if XY = XZ , then the exterior angle at the vertex Z is ....

(a) acute. (b) obtuse. (c) right. (d) reflex.

In the opposite figure :



ABC is an isosceles triangle in which AB = AC ,

m ($\angle$ A) = 40° and D $\in \overrightarrow{CB}$, E $\in \overrightarrow{BC}$

1 **Find : m ($\angle$ ABC)**

« 70° »

2 **Prove that : $\angle$ ABD $\equiv$ $\angle$ ACE**

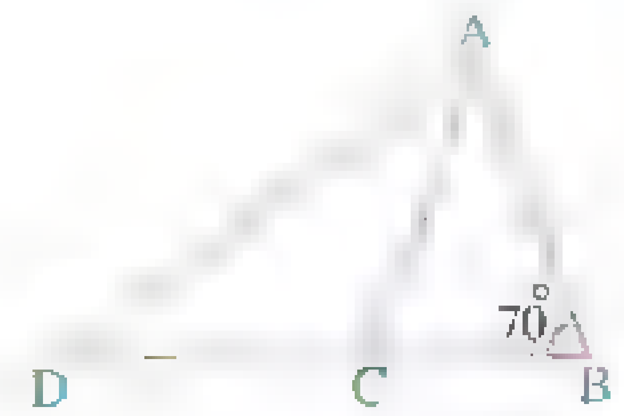
In the opposite figure :

AB = AC = CD

and m (∠ B) = 70°.

Find by proof :

m (∠ BAD)



« 75° »

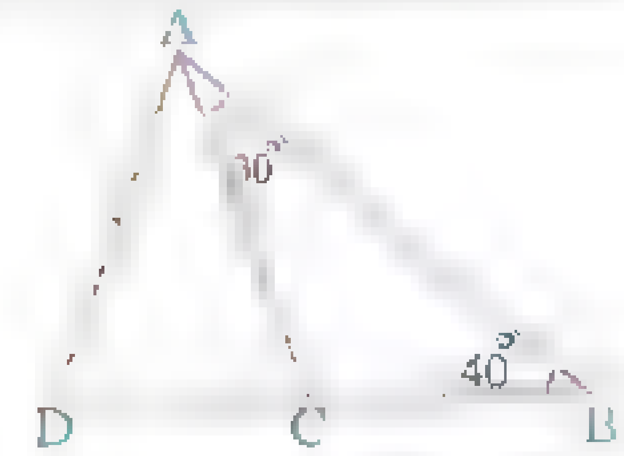
In the opposite figure :

m (∠ B) = 40° , m (∠ BAC) = 30°

and AC = AD

Find by proof :

1 m (∠ D)   2 m (∠ CAD)



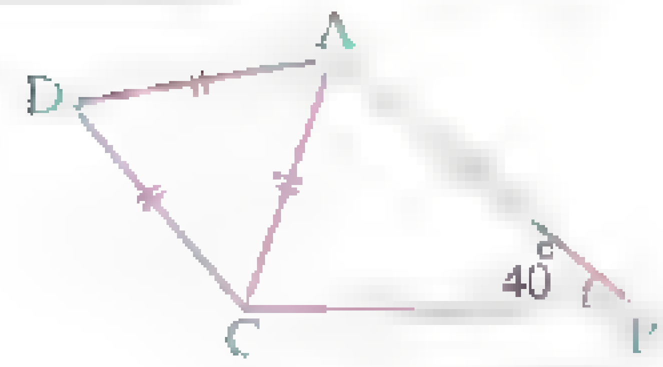
« 70° , 40° »

In the opposite figure :

AD = DC = AC , AB = BC

and m (∠ ABC) = 40°

Find : m (∠ BAD)



« 130° »

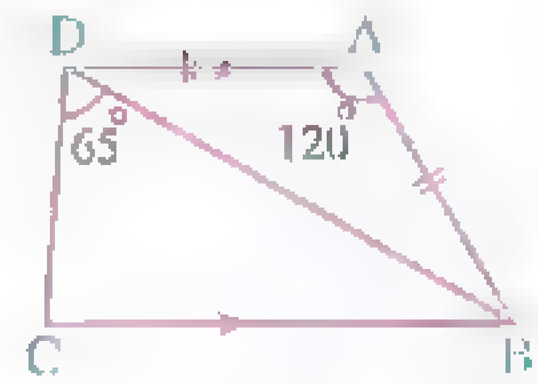
In the opposite figure :

AB = AD , $\overline{AD}$ // $\overline{BC}$,

m (∠ BAD) = 120° and m (∠ BDC) = 65°

Find :

1 m (∠ ADB)   2 m (∠ C)



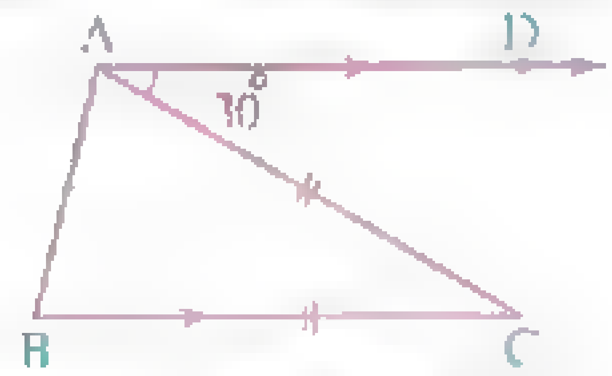
« 30° , 85° »

In the opposite figure :

ABC is a triangle in which AC = BC ,

$\overline{AD}$ // $\overline{BC}$ and m (∠ DAC) = 30°

Find : The measures of the angles of Δ ABC



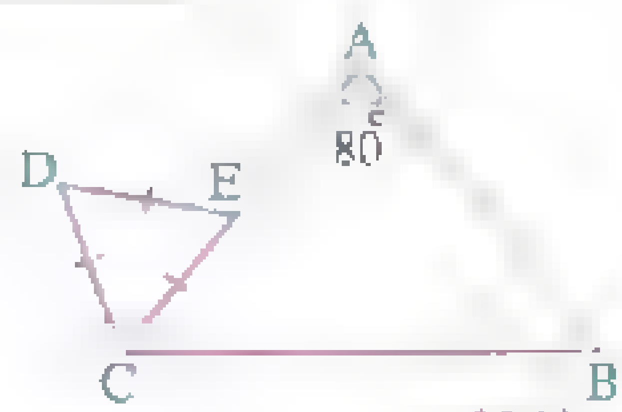
« 30° , 75° , 75° »

In the opposite figure :

AB = AC , m (∠ BAC) = 80°

and CE = ED = CD

Find by proof : m (∠ BCD)



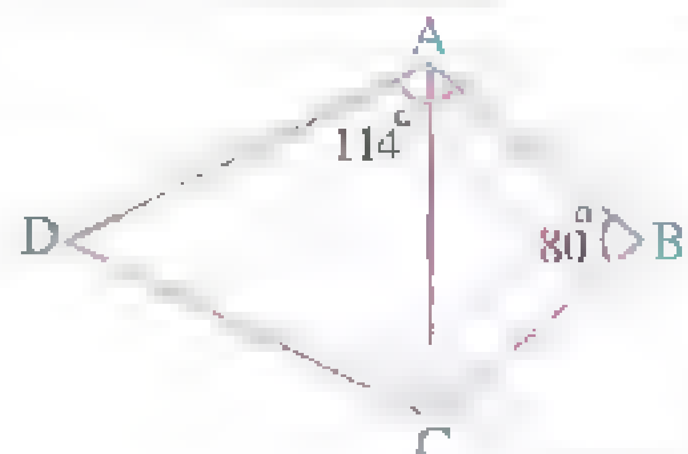
« 110° »

In the opposite figure :

AB = BC , AD = CD , m (∠ BAD) = 114°

and m (∠ B) = 80°

Find : m (∠ ADC)



« 52° »

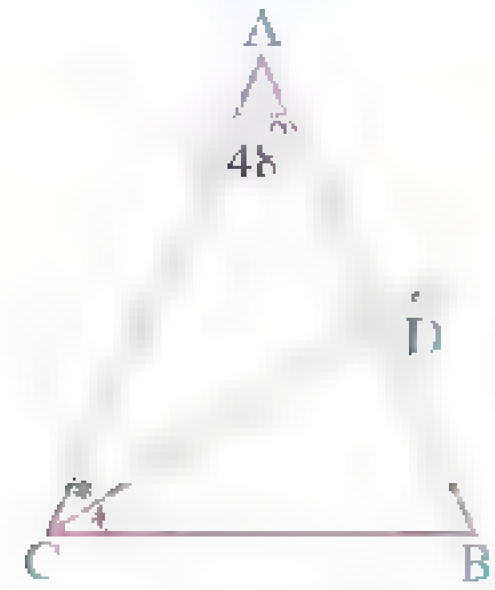
12 **In the opposite figure :**

$AB = AC$, $m(\angle BAC) = 48°$, $\overline{CD}$ bisects $\angle BCA$

and intersects $\overline{AB}$ at D

Find :

1 $m(\angle B)$ 2 $m(\angle BCD)$



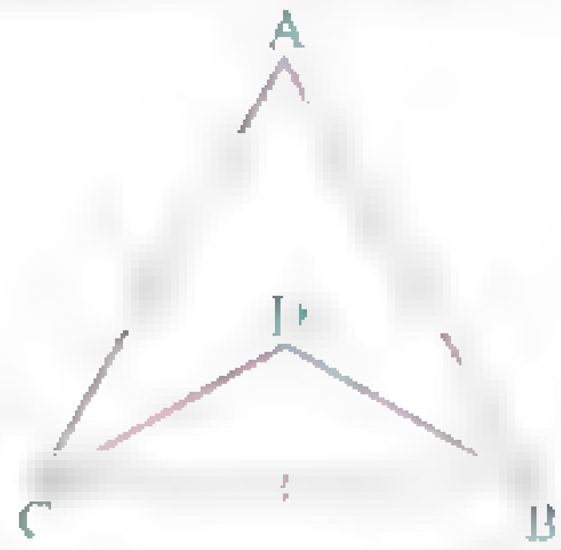
« 66° , 33° »

13 **In the opposite figure :**

ABC is an equilateral triangle and the two bisectors of

$\angle B$ and $\angle C$ intersect together at D

Find : $m(\angle BDC)$



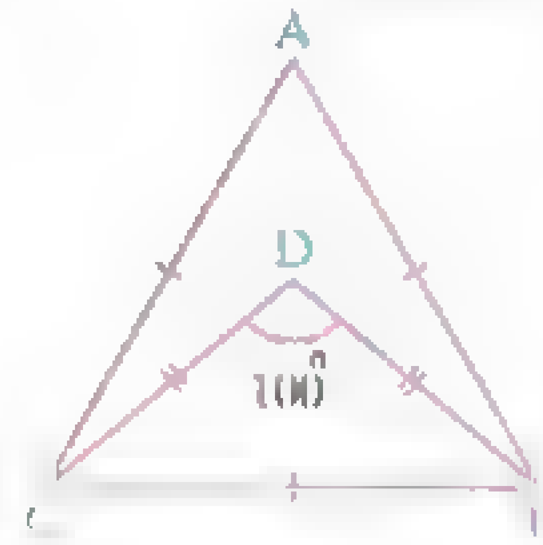
« 20° »

14 **In the opposite figure :**

ABC is an equilateral triangle , $DB = DC$

and $m(\angle BDC) = 100°$

Find by proof : $m(\angle ABD)$



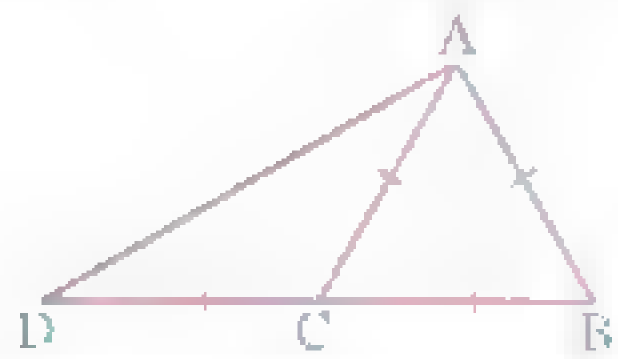
« 20° »

15 **In the opposite figure :**

ABC is an equilateral triangle.

$D \in \overrightarrow{BC}$ such that $BC = CD$

Prove that : $\overline{BA} \perp \overline{AD}$



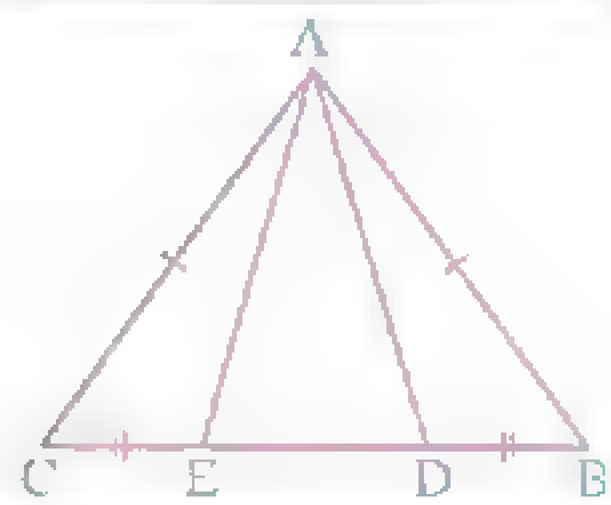
16 **In the opposite figure :**

ABC is an isosceles triangle in which $AB = AC$, $D \in \overline{BC}$

and $E \in \overline{BC}$, such that $BD = EC$

Prove that : 1 ΔADE is an isosceles triangle.

2 $\angle AED \equiv \angle ADE$

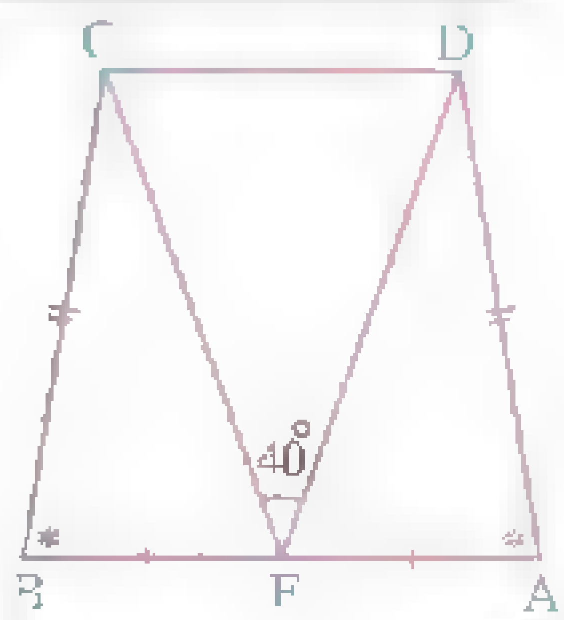


17 **In the opposite figure :**

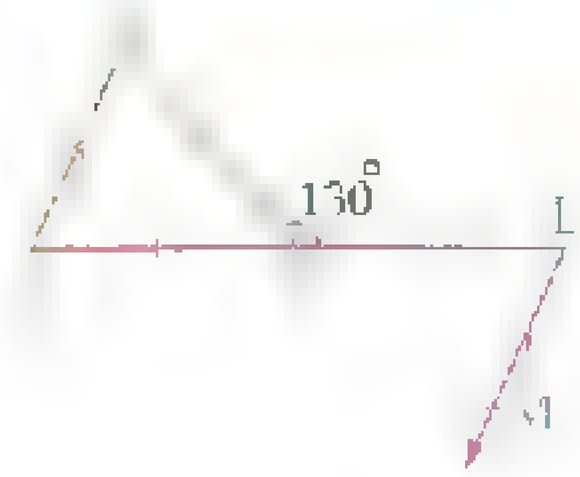
E is the midpoint of $\overline{AB}$, $AD = BC$, $m(\angle A) = m(\angle B)$

and $m(\angle DEC) = 40°$

Find : $m(\angle EDC)$



In the opposite figure :



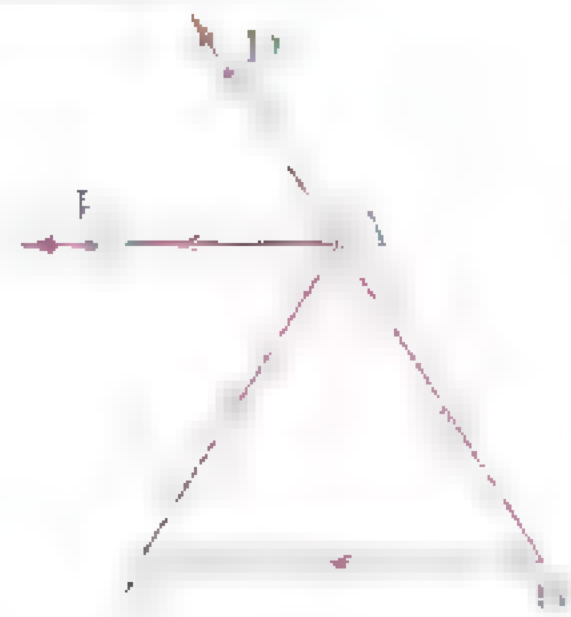
$Z \in \overleftrightarrow{LY}$, $XZ = YZ$, $m(\angle LZX) = 130°$

and $\overrightarrow{LM} \parallel \overline{XY}$

Find : $m(\angle MLY)$

« 65° »

In the opposite figure :

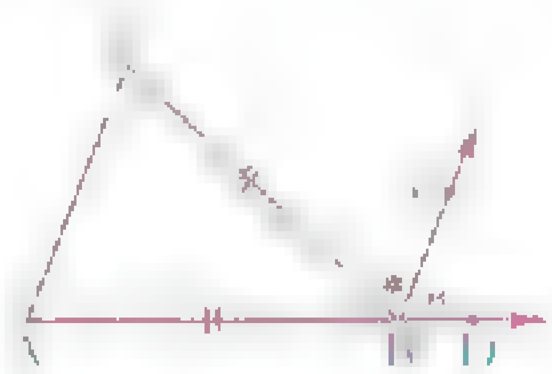


$A \in \overrightarrow{BD}$, $AB = AC$ and $\overrightarrow{AE} \parallel \overline{BC}$

Prove that :

$\overrightarrow{AE}$ bisects $\angle DAC$

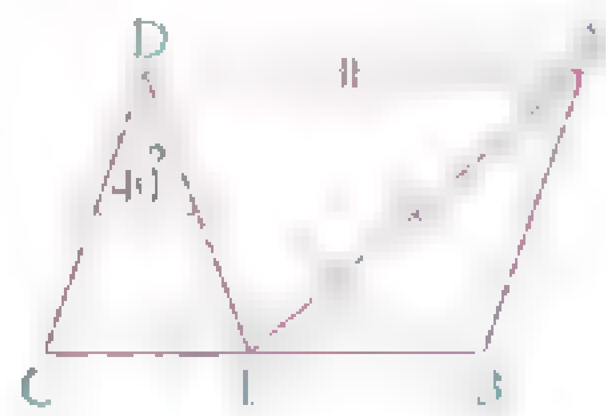
20 **In the opposite figure :**



$AB = BC$ and $\overrightarrow{BE}$ bisects $\angle CBD$

Prove that : $\overrightarrow{BE} \parallel \overline{AC}$

21 **In the opposite figure :**



ABCD is a parallelogram , $E \in \overline{BC}$,

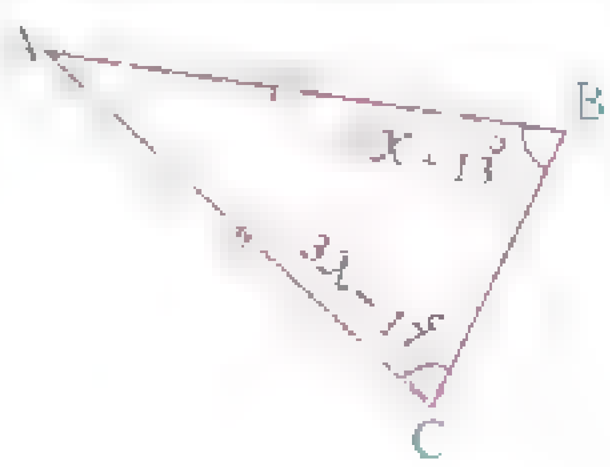
where $AE = AD$, $DE = DC$ and $m(\angle EDC) = 40°$

Find : (1) $m(\angle AED)$

(2) $m(\angle BAE)$

« 70° , 30° »

22 **In the opposite figure :**



$AB = AC$, $m(\angle B) = 2X + 13°$

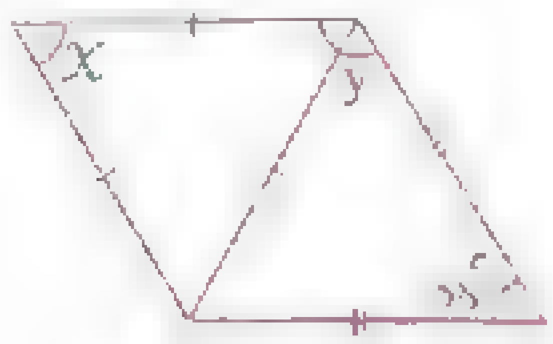
and $m(\angle C) = 3X - 17°$

Find : The measures of the angles of ΔABC

« 73° , 73° , 34° »

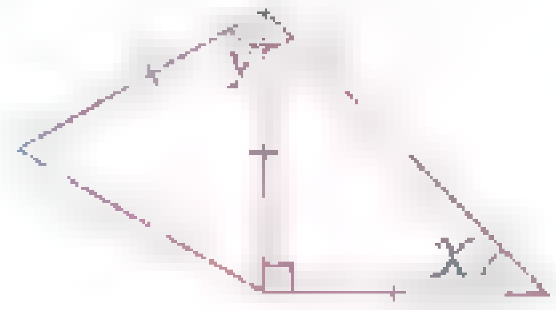
In each of the following figures , find the value of the symbol used for the measure of the angle :

1



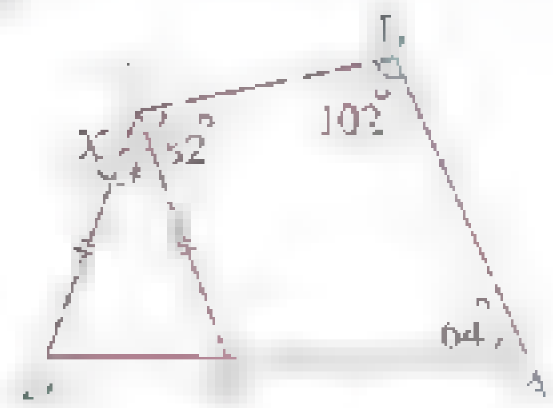
$x = \dots\dots^\circ$, $y = \dots\dots^\circ$

2



$x = \dots\dots^\circ$, $y = \dots\dots^\circ$

3



$x = \dots\dots^\circ$

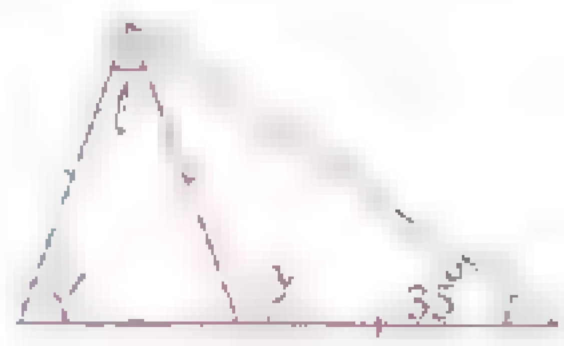
4

$x = \dots\dots^\circ$, $y = \dots\dots^\circ$

5

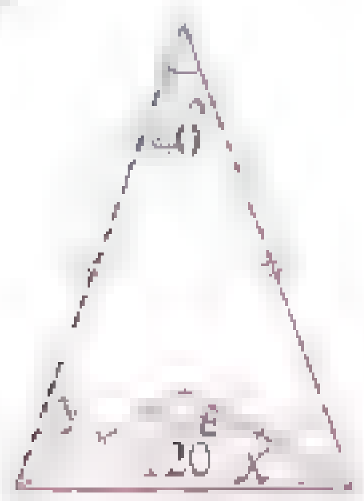
$x = \dots\dots^\circ$, $y = \dots\dots^\circ$

6



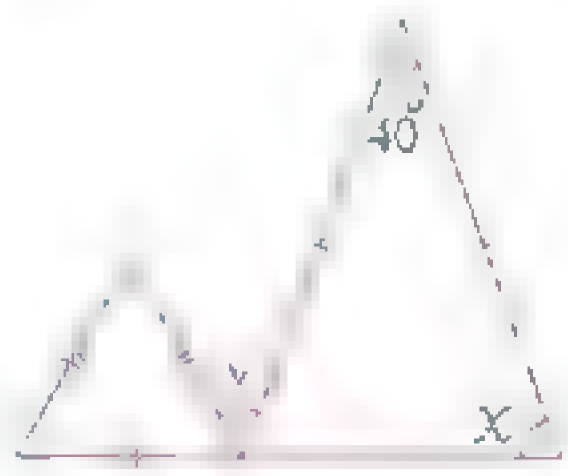
$y = \dots\dots^\circ$, $x = \dots\dots^\circ$, $z = \dots\dots^\circ$

7



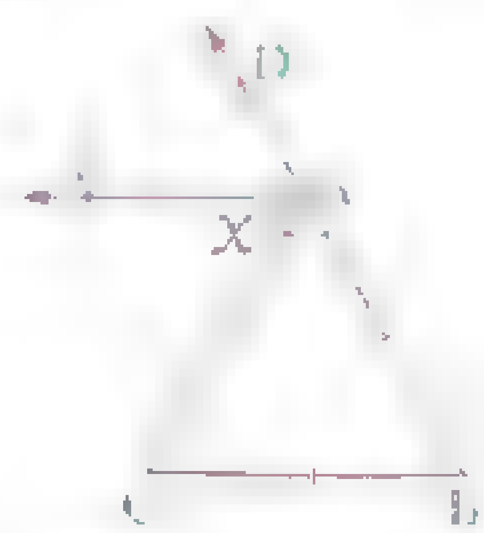
$x = \dots\dots^\circ$, $y = \dots\dots^\circ$

8



$x = \dots\dots^\circ$, $y = \dots\dots^\circ$

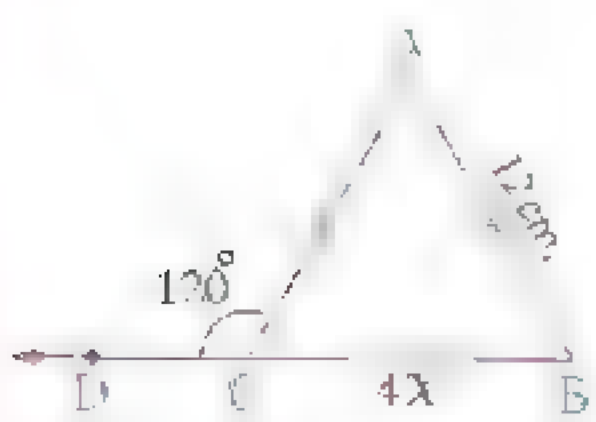
9 $\overrightarrow{AE}$ bisects $\angle$ CAD



$x = \dots\dots^\circ$

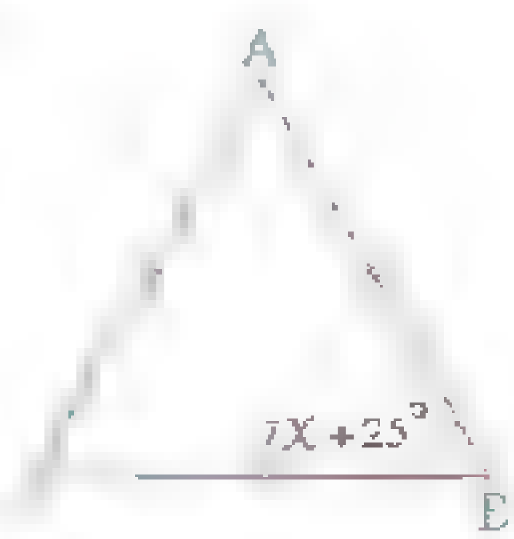
Find the value of x in each of the following figures :

1



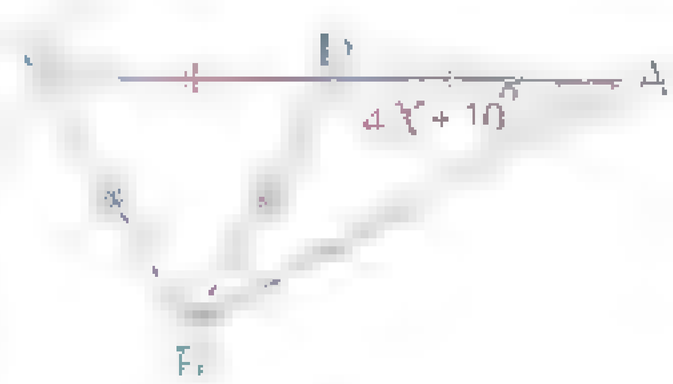
$x = \dots\dots$ cm.

2

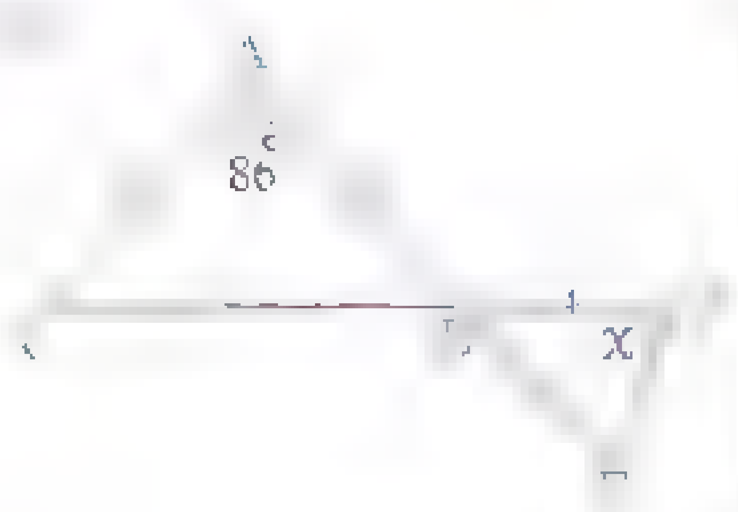


$x = \dots\dots^\circ$

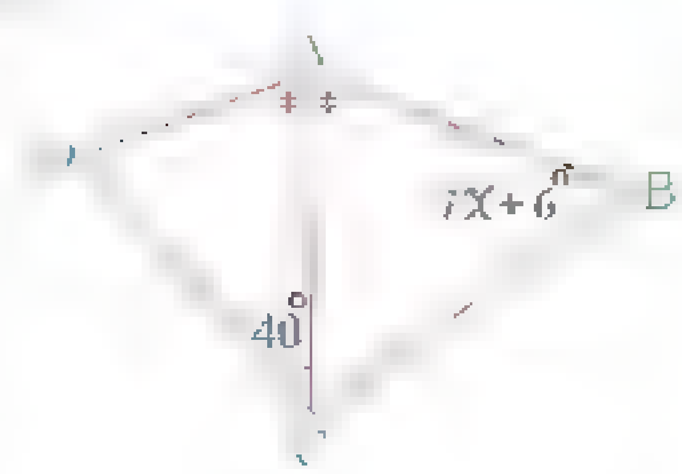
3



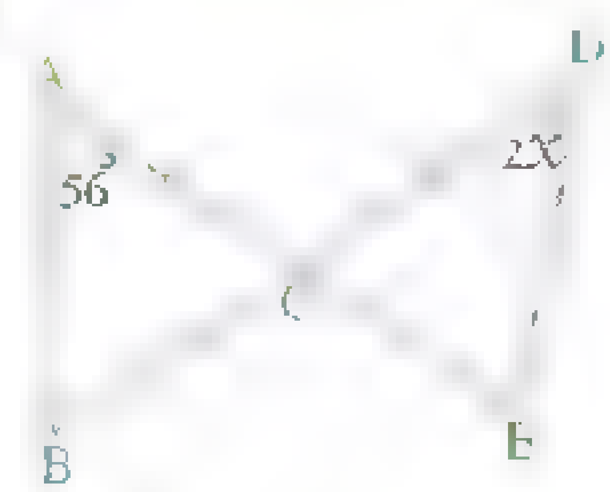
$x = \dots\dots^\circ$



$X = \ldots\ldots\ldots^\circ$



$X = \ldots\ldots\ldots^\circ$



$X = \ldots\ldots\ldots^\circ$

25 **In the opposite figure :**

$BA = BC$, $E \in \overline{AD}$

and $\overleftrightarrow{BD}$ bisects each

of $\angle CBE$ and $\angle CDE$

Prove that : $m(\angle A) + m(\angle C) = 180^\circ$

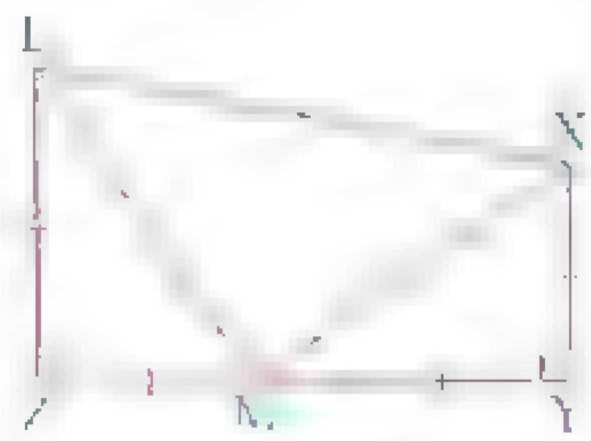
For excellent pupils

26 **In the opposite figure :**

$m(\angle Y) = m(\angle Z) = 90^\circ$

, $XY = MZ$ and $YM = ZL$

Find by proof : $m(\angle MXL)$



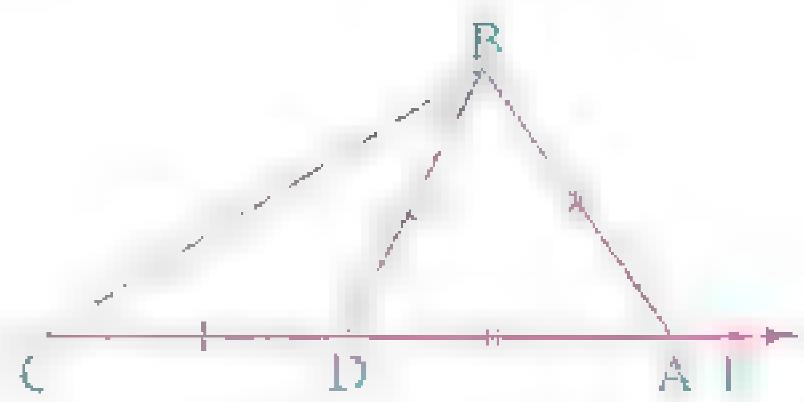
« 45° »

27 **In the opposite figure :**

ABC is a triangle , $D \in \overline{AC}$ such that $BD = DC$

$AD = AB$ and $E \in \overrightarrow{CA}$

Prove that : $m(\angle BAE) = 4\, m(\angle BCD)$

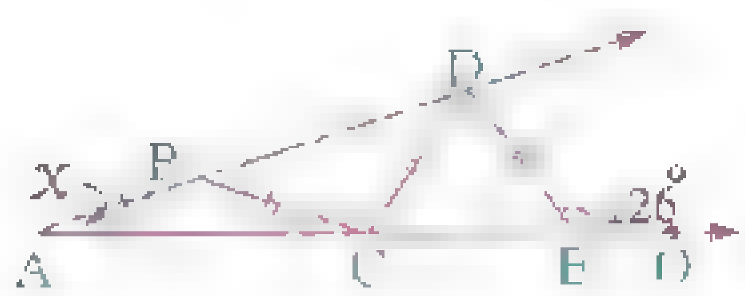


28 **In the opposite figure :**

$m(\angle A) = X^\circ$, $AB = BC = CD = DE$

and $m(\angle DEO) = 126^\circ$

Find : The value of X



« 18° »

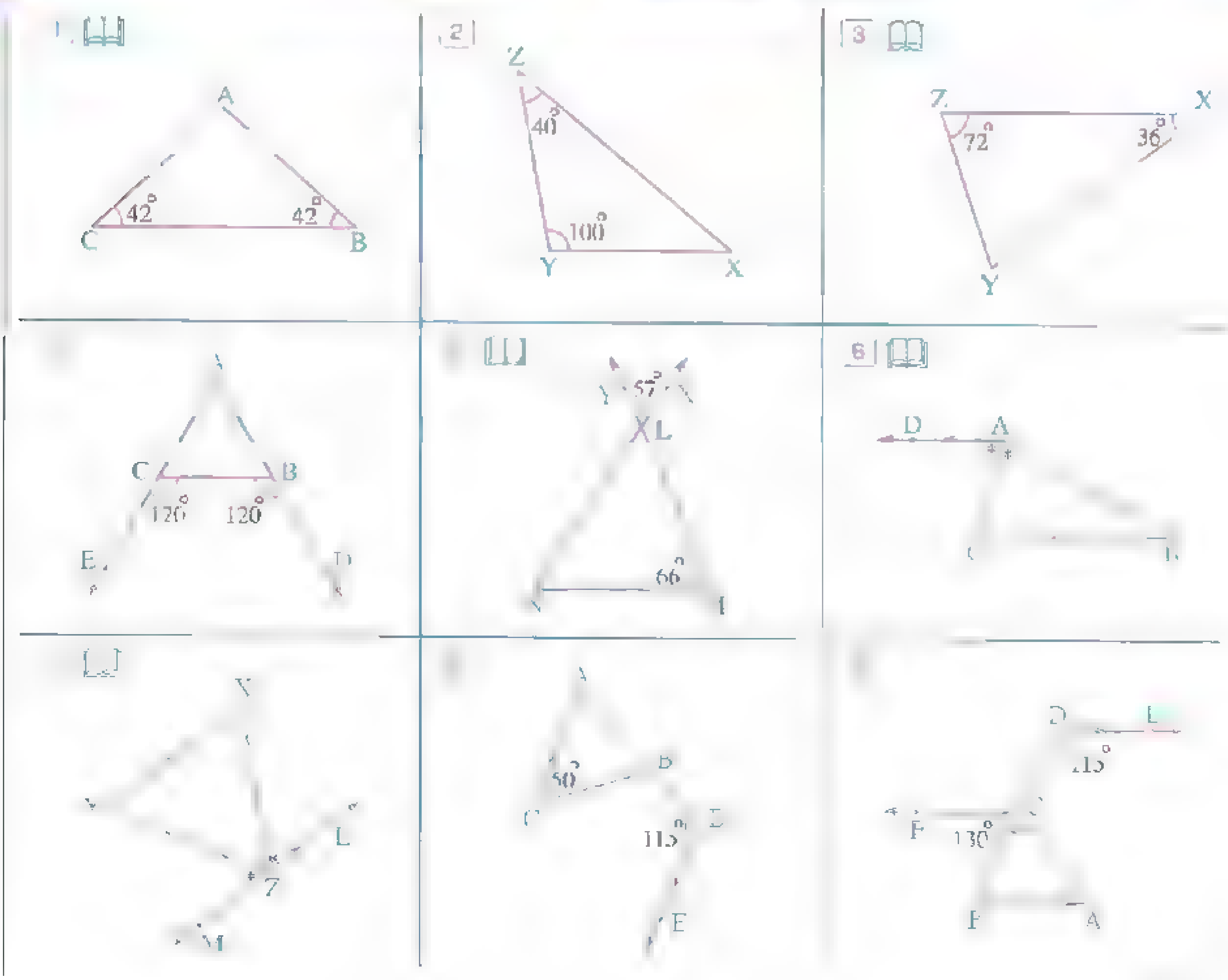
From the school book

Exercise 4 ? The converse of the isosceles triangle theorem

● Remember ● Understand ○ Apply ● Problem Solving

Interactive test

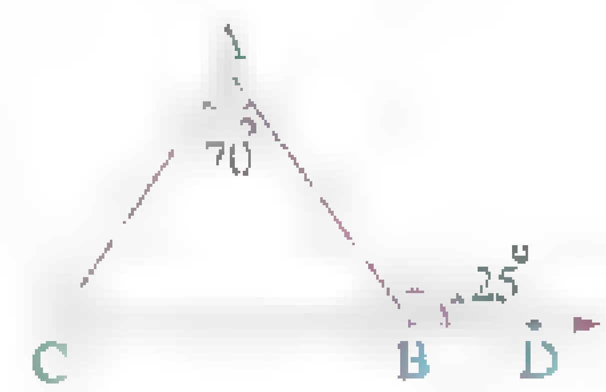
1 In each of the following figures , write the equal sides in length :



2 Complete the following :

1. If two angles in the triangle are congruent , then the two sides opposite to these two angles are .... and the triangle is .....
2. If the three angles in the triangle are congruent , then the triangle is ....
3. In Δ ABC , if m ($\angle$ A) = 50° and m ($\angle$ B) = 80° , then the triangle is ....
4. If the measure of one angle in the right angled triangle is 45° , then the triangle is ....
5. If the measure of one angle of an isosceles triangle is 60° , then the triangle is ....
6. ABC is a triangle in which AB = AC and m ($\angle$ A) = 60°
   If its perimeter = 18 cm. , then BC = ... cm.
7. In Δ ABC , CA = CB , m ($\angle$ C) = m ($\angle$ A) , then m ($\angle$ B) = ... °

3 In the opposite figure :

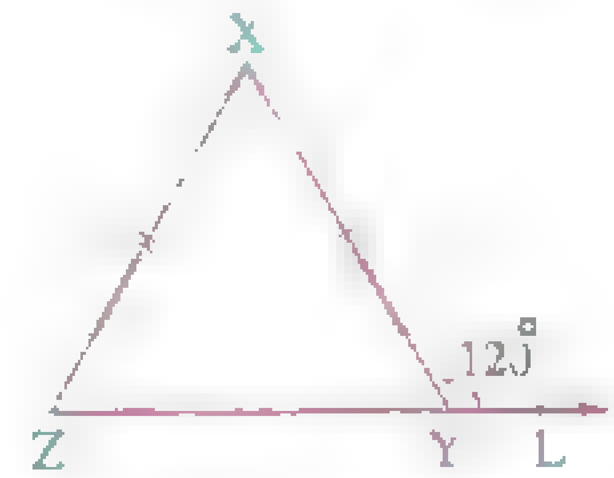


D $\in \overrightarrow{CB}$, m ($\angle$ ABD) = 125°

and m ($\angle$ A) = 70°

Prove that : Δ ABC is an isosceles triangle.

4 In the opposite figure :

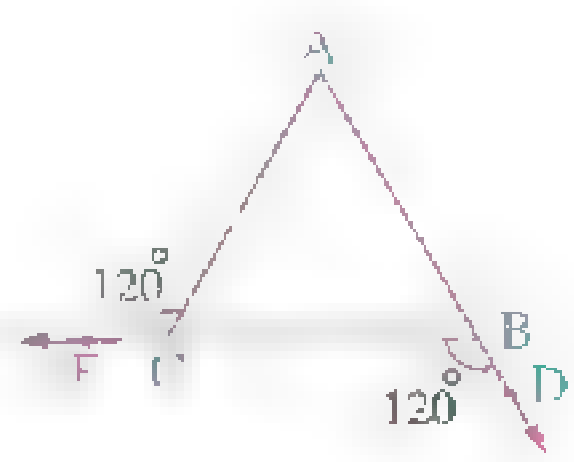


XY = XZ , m ($\angle$ XYL) = 120°

and L $\in \overrightarrow{ZY}$

Prove that : Δ XYZ is an equilateral triangle.

5 In the opposite figure :

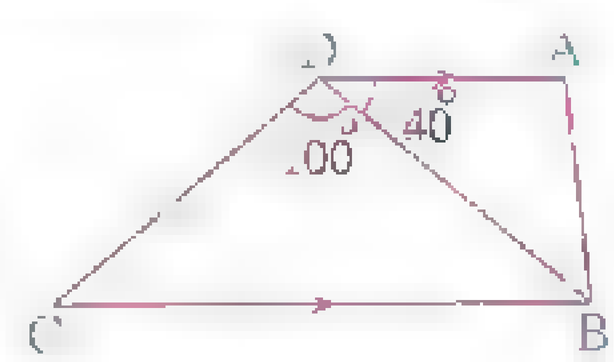


D $\in \overrightarrow{AB}$, E $\in \overrightarrow{BC}$ and

m ($\angle$ CBD) = m ($\angle$ ACE) = 120°

Prove that : Δ ABC is an equilateral triangle.

6 In the opposite figure :



$\overline{AD}$ // $\overline{BC}$, m ($\angle$ ADB) = 40°

and m ($\angle$ BDC) = 100°

Prove that : Δ DBC is an isosceles triangle.

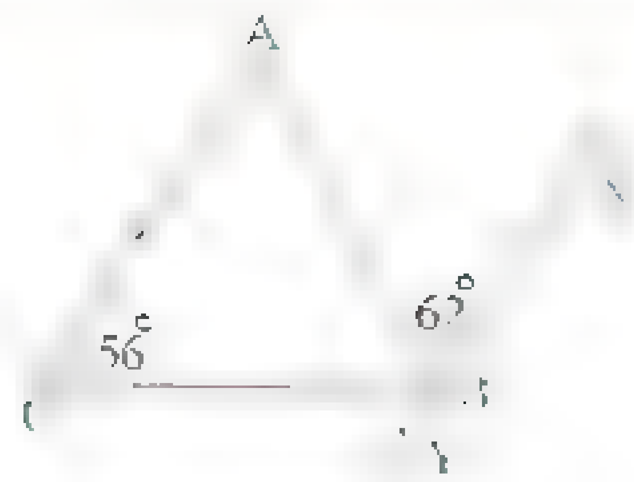
7 **In the opposite figure :**

$B \in \overleftrightarrow{XY}$, $\overleftrightarrow{XY} \parallel \overline{AC}$

, $m(\angle ABX) = 62°$ and

$m(\angle C) = 56°$

Prove that : AC = BC



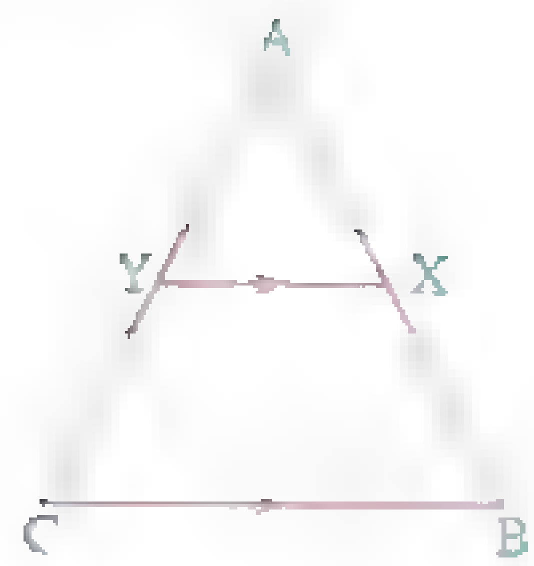
8 **In the opposite figure :**

ABC is a triangle in which AB = AC , $X \in \overline{AB}$,

$Y \in \overline{AC}$ and $\overline{XY} \parallel \overline{BC}$

Prove that : 1 Δ AXY is an isosceles triangle.

2 XB = YC



9 ABC is a triangle in which $D \in \overline{AB}$ and $E \in \overline{BC}$ such that BD = BE

So if $\overline{DE} \parallel \overline{AC}$, **prove that :** AB = BC

10 **In the opposite figure :**

$\overline{AC} \cap \overline{BD} = \{M\}$,

MB = MC and $\overline{AD} \parallel \overline{BC}$

Prove that : MA = MD

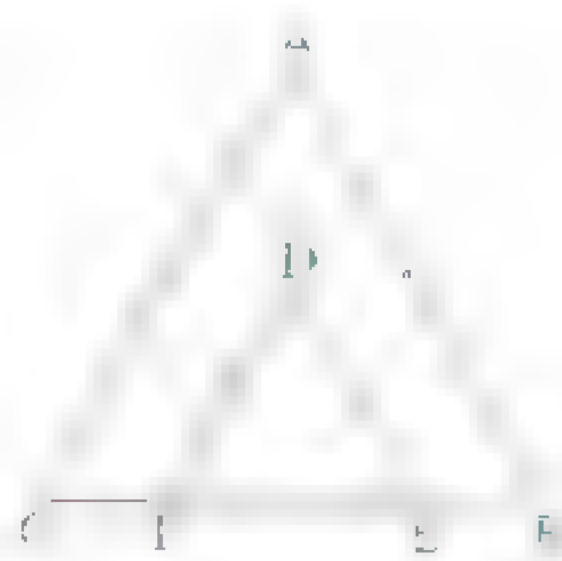
11 **In the opposite figure :**

AB = AC , $\overline{DE} \parallel \overline{AB}$ and $\overline{DF} \parallel \overline{AC}$

Prove that :

1 DE = DF

2 $m(\angle BAC) = m(\angle EDF)$

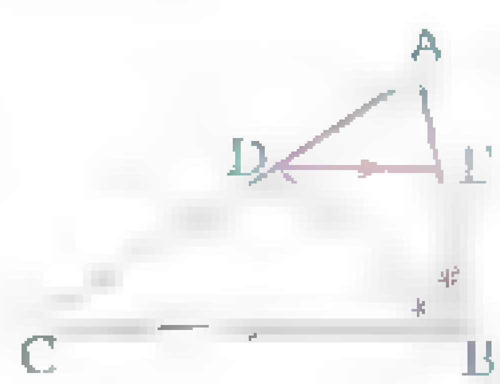


12 **In the opposite figure :**

ABC is a triangle

, $\overrightarrow{BD}$ bisects $\angle ABC$ and $\overline{ED} \parallel \overline{BC}$ where $E \in \overline{AB}$

Prove that : Δ EBD is an isosceles triangle.

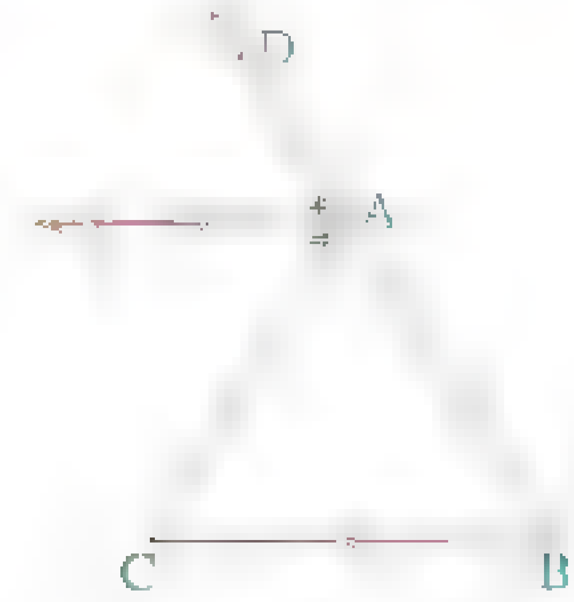


In the opposite figure :

$A \in \overrightarrow{BD}$, $\overrightarrow{AE} \parallel \overline{BC}$

and $\overrightarrow{AE}$ bisects $\angle$ CAD

Prove that : AB = AC

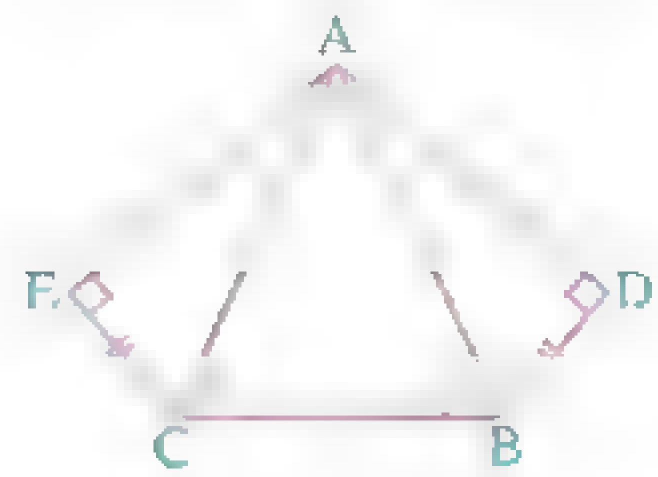


14 In the opposite figure :

BD = CE , m ($\angle$ ABC) = m ($\angle$ ACB)

and m ($\angle$ D) = m ($\angle$ E) = 90°

Prove that : m ($\angle$ DAB) = m ($\angle$ CAE)



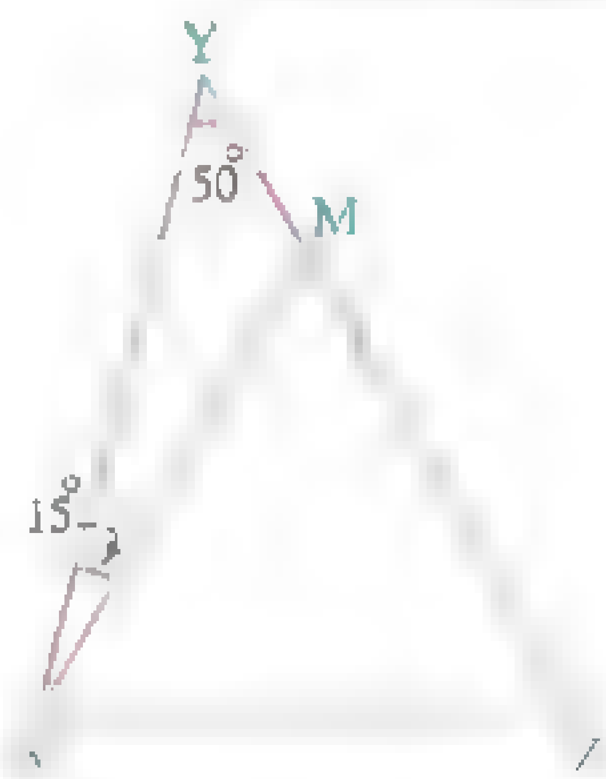
15 In the opposite figure :

YZX is a triangle in which YZ = YX

, m ($\angle$ Y) = 50°

and m ($\angle$ YXM) = 15°

Prove that : Δ MZX is an isosceles triangle.

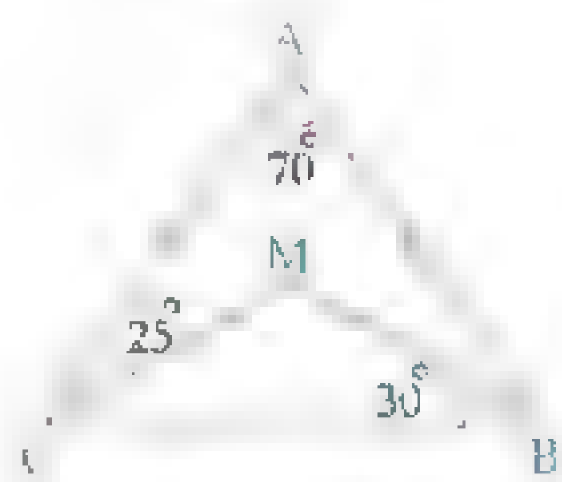


In the opposite figure :

ABC is a triangle in which AB = AC , m ($\angle$ A) = 70°

, m ($\angle$ MCA) = 25° and m ($\angle$ MBC) = 30°

Prove that : Δ MBC is an isosceles triangle.



In the opposite figure :

AD = AC , m ($\angle$ B) = 40°

and m ($\angle$ BAD) = 30°

Prove that : AB = CB

ABC is a triangle in which AB = AC , $\overrightarrow{BD}$ bisects $\angle$ ABC and $\overrightarrow{CD}$ bisects $\angle$ ACB

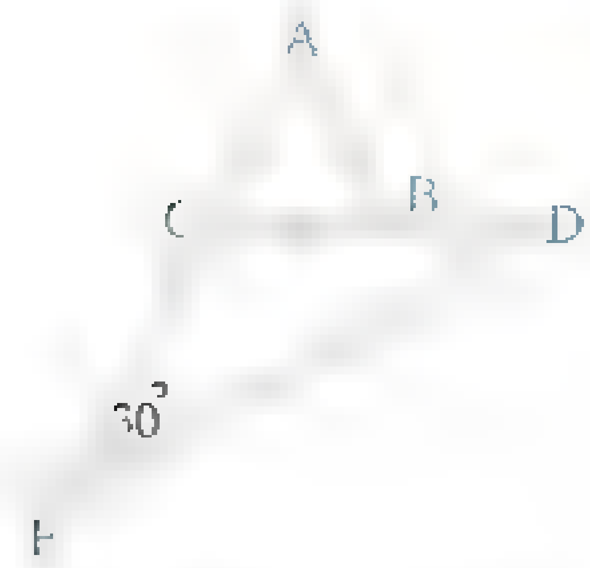
Prove that : Δ DBC is an isosceles triangle.

19 **In the opposite figure :**

ABC is an equilateral triangle , $F \in \overline{AC}$,

$D \in \overrightarrow{CB}$ and m ($\angle$ DFC) = 30°

Prove that : Δ DCF is an isosceles triangle.



20 **In the opposite figure :**

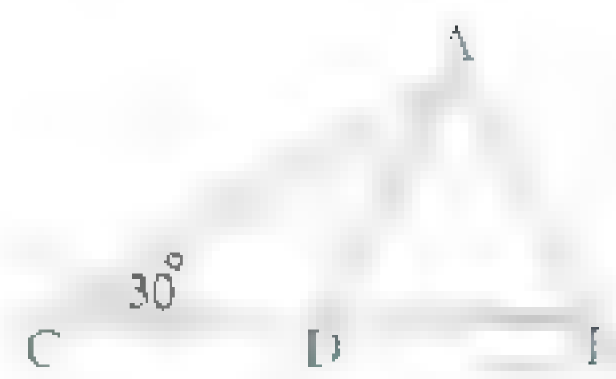
$D \in \overline{BC}$ such that DA = DB = DC

and m ($\angle$ C) = 30°

Prove that :

1] Δ ABD is an equilateral triangle.

2] Δ ABC is a right-angled triangle.



21 **In the opposite figure :**

ABC is a triangle in which $E \in \overline{AB}$,

$\overline{ED}$ // $\overline{AC}$, m ($\angle$ BED) = 60°

and $\overline{EC}$ bisects $\angle$ AED

Prove that : Δ AEC is an equilateral triangle.

22 **In the opposite figure :**

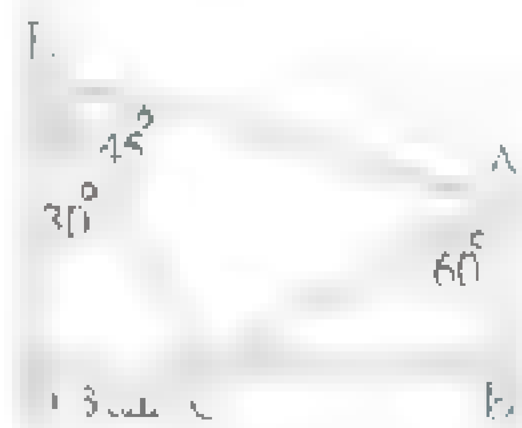
$C \in \overline{BD}$, m ($\angle$ B) = m ($\angle$ D) = 90° ,

m ($\angle$ CED) = 30°

, m ($\angle$ AEC) = 45° , m ($\angle$ BAC) = 60°

and CD = 3 cm.

Find : The length of $\overline{AC}$ « 6 cm. »



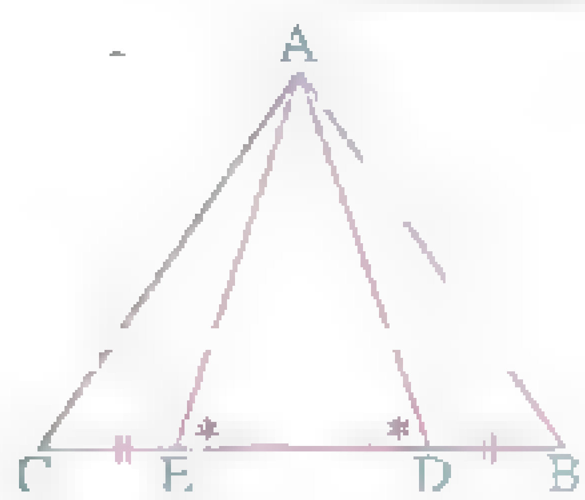
23 **In the opposite figure :**

$\angle$ ADE ≡ $\angle$ AED

, B , D , E , C are collinear

and BD = CE

Prove that : Δ ABC is an isosceles triangle.

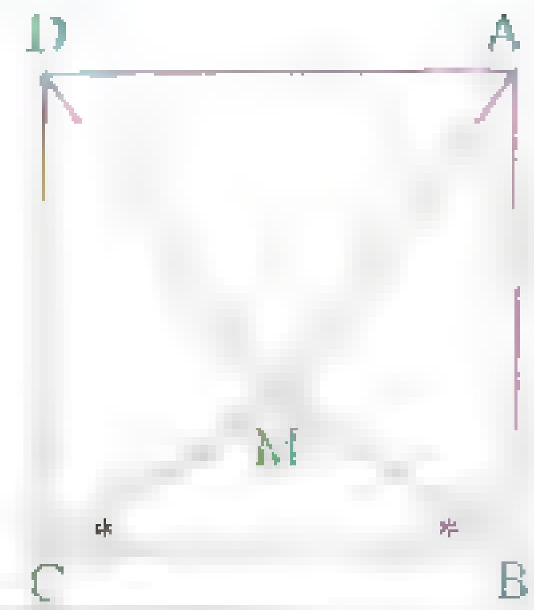


24 **In the opposite figure :**

ABCD is a square.

M is a point inside it such that : m (∠ MBC) = m (∠ MCB)

Prove that : Δ AMD is an isosceles triangle.



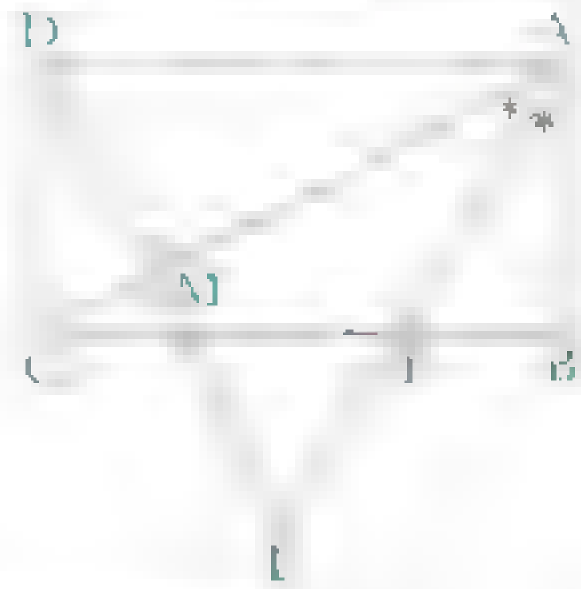
25 **In the opposite figure :**

ABCD is a rectangle in which

$\overline{AC}$ is a diagonal , $\overrightarrow{AE}$ bisects ∠ BAC

and $\overline{DE} \perp \overline{AC}$ where $\overrightarrow{AE} \cap \overrightarrow{DE} = \{E\}$, AC ∩ DE = {M}

Prove that : DA = DE

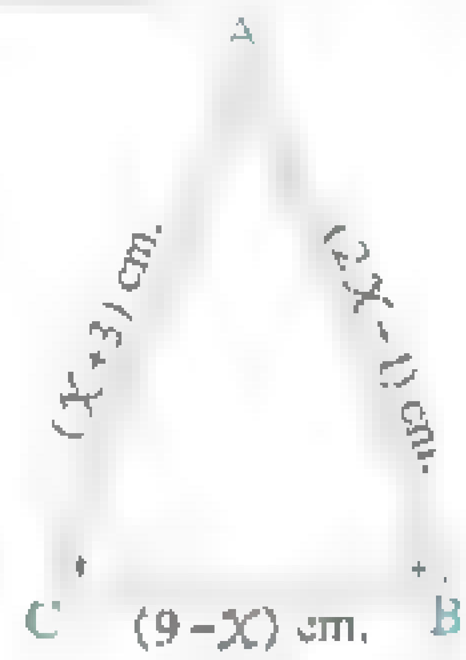


26 **In the opposite figure :**

ABC is a triangle in which :

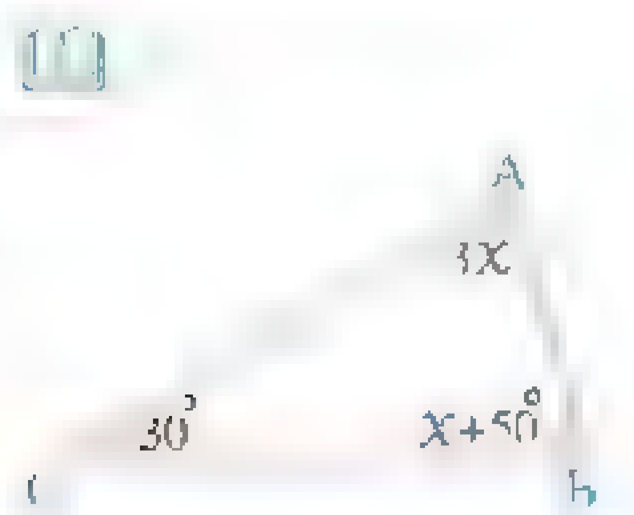
m (∠ B) = m (∠ C)

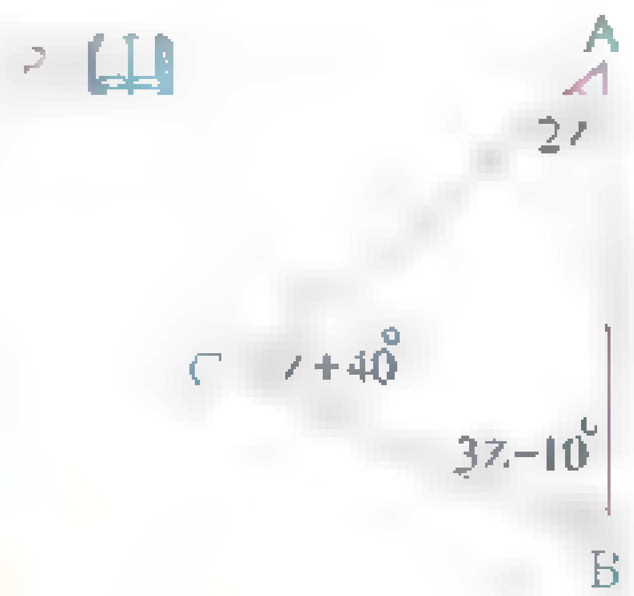
Find : The perimeter of the triangle.

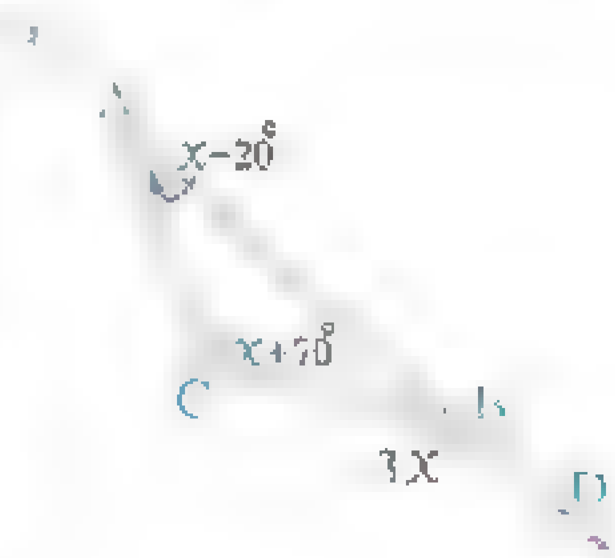


« 19 cm. »

27 **In each of the following figures , write the equal sides in length showing the steps of solution :**







For excellent pupils

28 **Choose the correct answer from those given :**

If the sum of measures of two congruent angles in a triangle = $\frac{2}{3}$ the sum of measures of its angles , then the triangle is

(a) right-angled. (b) isosceles. (c) equilateral. (d) scalene.

ABC is a triangle in which m (∠ A) = 30° and m (∠ B) : m (∠ C) = 1 : 4 , then Δ ABC is ....

(a) right-angled. (b) isosceles. (c) equilateral. (d) scalene.

Exercise 5 Corollaries of the isosceles triangle theorems

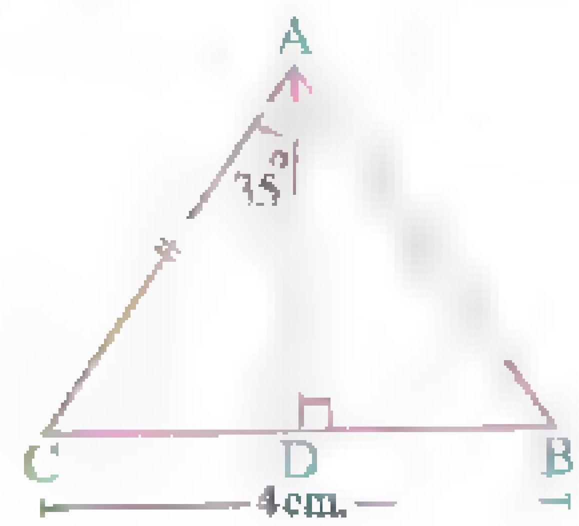
● Remember ● Understand ● Apply ● Problem Solving

Interactive test

Complete the following :

- The straight line drawn from the vertex of the isosceles triangle perpendicular to the base is called ....
- The number of axes of symmetry in the equilateral triangle equals ....
- The number of axes of symmetry in the isosceles triangle equals .
- The number of axes of symmetry in the scalene triangle equals ......
- The median of the isosceles triangle drawn from the vertex angle .
- The bisector of the vertex angle of the isosceles triangle . ...
- The straight line passing through the vertex angle of the isosceles triangle perpendicular to its base ....
- The axis of the line segment is ......
- Any point belonging to the axis of a line segment is .... from its two terminals
- In ΔABC , if $m(\angle A) = m(\angle B) = 60^\circ$, then the number of axes of symmetry of ΔABC is ....
- In ΔABC , if $m(\angle A) = m(\angle B) \neq 60^\circ$, then the number of axes of symmetry of ΔABC is ....
- In ΔABC , if $AB = AC$, $m(\angle A)$ 60° , then the number of axes of symmetry of ΔABC is ....

1 In the opposite figure :



If AB = AC , $\overline{AD} \perp \overline{BC}$, BC = 4 cm. and

m (∠ DAC) = 35° , **complete the following :**

1 m (∠ BAD) = ...°

2 m (∠ BAC) = ...°

3 m (∠ B) = ...°

4 BD = ... cm.

5 The axis of symmetry of Δ ABC is ....

2 Choose the correct answer from those given :

1 If C ∈ the axis of symmetry of $\overline{AB}$, then AC – BC = ...

(a) zero (b) 1 (c) 2 (d) 4

2 In Δ XYZ , XY = XZ , $\overline{XE}$ is a median , if m (∠ YXE) = 30°

, then m (∠ YXZ) = ...

(a) 15° (b) 30° (c) 60° (d) 90°

3 In Δ LMN , LM = LN , E ∈ $\overline{MN}$ where $\overline{LE} \perp \overline{MN}$, if ME = 4 cm.

, then MN = ... cm.

(a) 12 (b) 8 (c) 4 (d) 2

4 If the measure of one angle in the right-angled triangle is 45° , then the number of axes of symmetry of the triangle is ....

(a) zero (b) 1 (c) 2 (d) 3

5 In Δ ABC , m (∠ A) = 40° , m (∠ C) = 100° , then the number of axes of symmetry of the triangle is ....

(a) 1 (b) 2 (c) 3 (d) infinite number.

6 The triangle in which the measures of two angles in it are 45° , 65° , then the number of axes of symmetry of the triangle is ....

(a) zero (b) 3 (c) 2 (d) 1

7 An isosceles triangle , the measure of one of its angles is 60° , then the number of its axes of symmetry is ....

(a) 4 (b) 3 (c) 2 (d) 1

8 If Δ ABC has 1 axis of symmetry , m (∠ ABC) = 120° , m (∠ A) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

4 **In the opposite figure :**

ABC is a right-angled triangle at B and it is also an isosceles triangle , $\overline{BD} \perp \overline{AC}$ and AD = 20 cm. Find the length of $\overline{AC}$ and m ($\angle$ DBC) , then deduce that Δ BDC is an isosceles triangle.

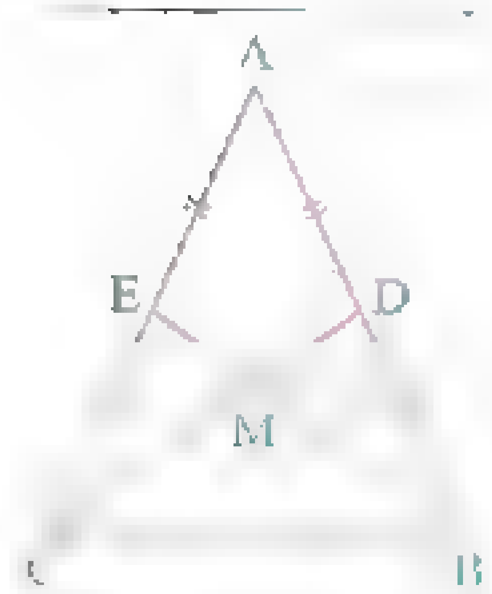
« 40 cm. , 45° »

5 **In the opposite figure :**

AB = AC , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively and $\overline{BE} \cap \overline{CD} = \{M\}$

Prove that :

1 $\overrightarrow{AM} \perp \overline{BC}$ 2 $\overrightarrow{AM}$ bisects $\angle$ BAC



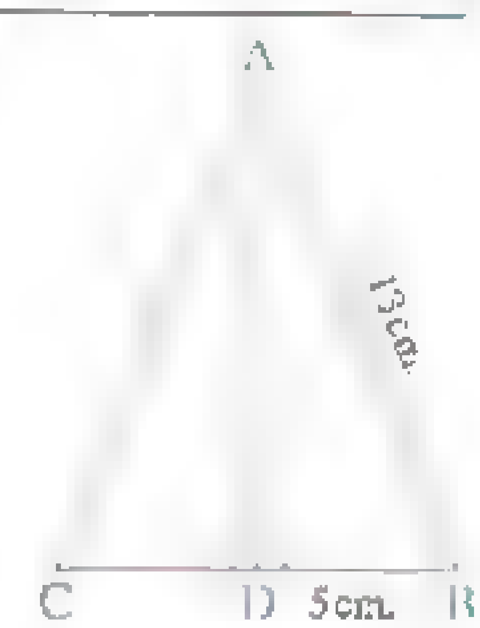
6 **In the opposite figure :**

In Δ ABC , AB = AC , $\overline{AD} \perp \overline{BC}$, AB = 13 cm. and BD = 5 cm.

Find :

1 The length of $\overline{BC}$

2 The area of Δ ABC



« 10 cm. , 60 cm^2. »

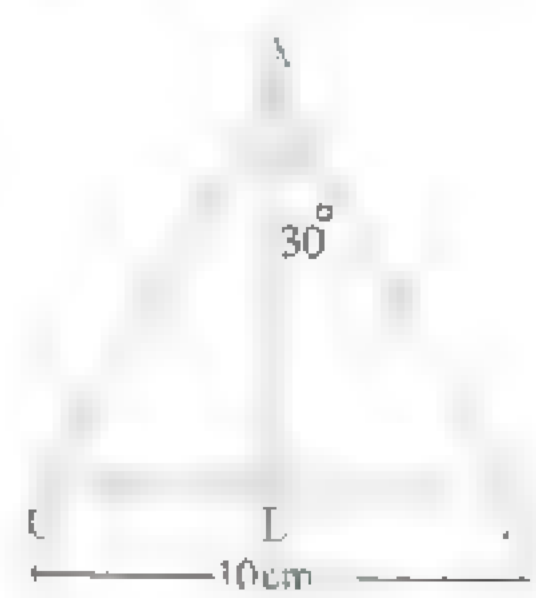
7 **In the opposite figure :**

AB = AC , BC = 10 cm. , m ($\angle$ BAD) = 30° and $\overline{AD} \perp \overline{BC}$

1 Find the length of each of : $\overline{BD}$ and $\overline{AD}$

2 How many axes of symmetry are there at Δ ABC ?

3 Find the area of Δ ABC



« 5 cm. , $5\sqrt{3}$ cm. , $25\sqrt{3}$ cm^2. »

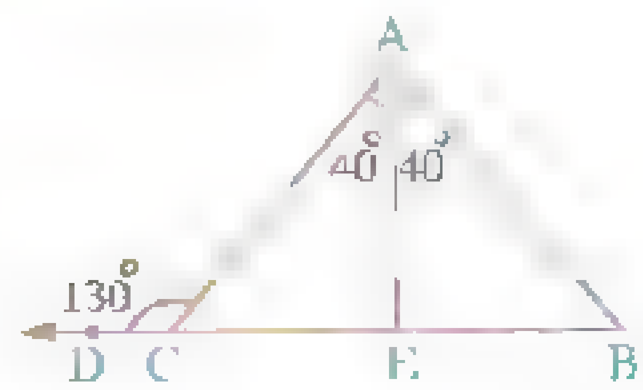
8 **In the opposite figure :**

ABC is a triangle in which AB = AC , $\overrightarrow{AE}$ bisects $\angle$ BAC , $\overrightarrow{AE} \cap \overline{BC} = \{E\}$ and D $\in \overrightarrow{AE}$

Prove that :

1 $BE = \frac{1}{2}$ BC 2 BD = CD

9 **In the opposite figure :**



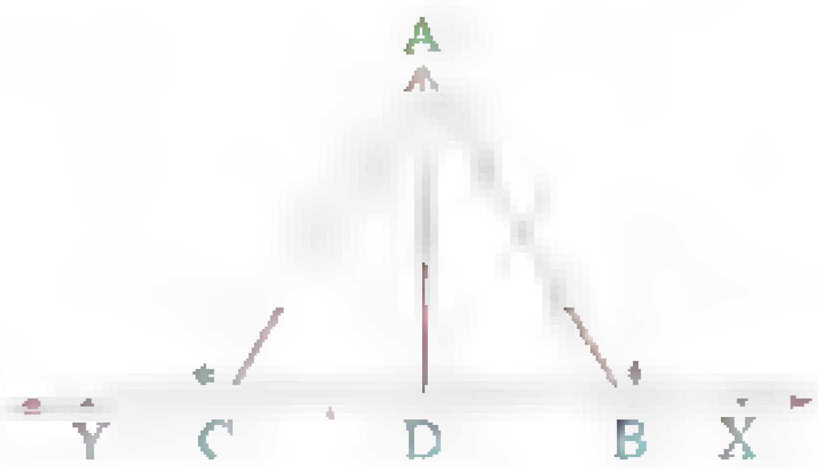
$C \in \overline{BD}$, m (∠ ACD) = 130°

and m (∠ BAE) = m (∠ CAE) = 40°

Prove that :

1 $\overline{AE} \perp \overline{BC}$ 2 E is the midpoint of $\overline{BC}$

10 **In the opposite figure :**



X , B , C , D and Y are collinear points ,

$\overline{AD}$ is a median of Δ ABC and

m (∠ ABX) = m (∠ ACY)

Prove that : $\overline{AD} \perp \overline{BC}$

11 **In the opposite figure :**

ABCD is a quadrilateral in which

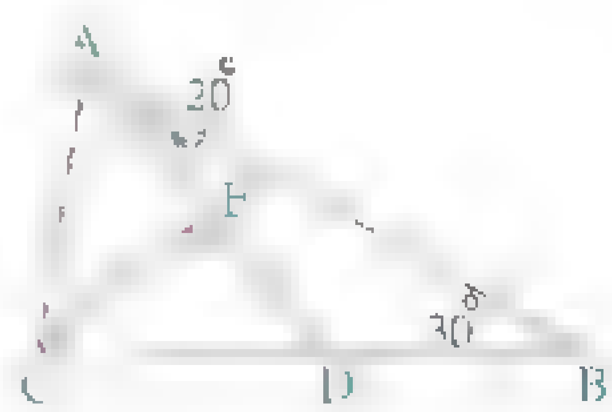
$\overline{AD}$ // $\overline{BC}$, $\overrightarrow{BD}$ bisects ∠ ABC and

$\overrightarrow{AE}$ bisects ∠ BAD

Prove that :

1 AB = AD 2 $\overline{AE} \perp \overline{BD}$ 3 BE = ED

12 **In the opposite figure :**



ABC is a triangle in which

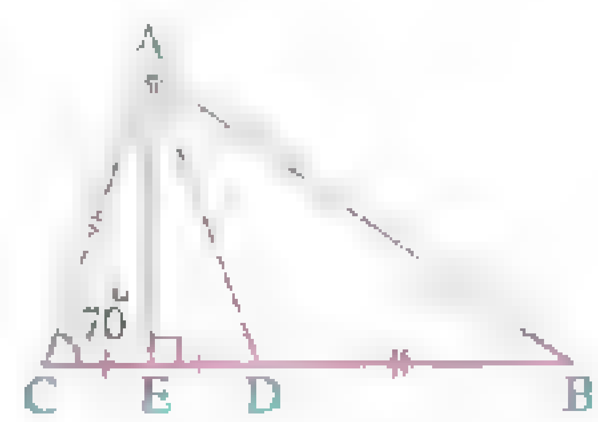
m (∠ B) = 30° , $D \in \overline{BC}$

where m (∠ BAD) = 20°

, E is the midpoint of $\overline{AD}$ and $\overline{CE} \perp \overline{AD}$

Find : m (∠ ACE) « 40° »

13 **In the opposite figure :**



ABC is a triangle in which

m (∠ C) = 70° , $D \in \overline{BC}$

where BD = AC

, E is the midpoint of $\overline{DC}$

and $\overline{AE} \perp \overline{DC}$

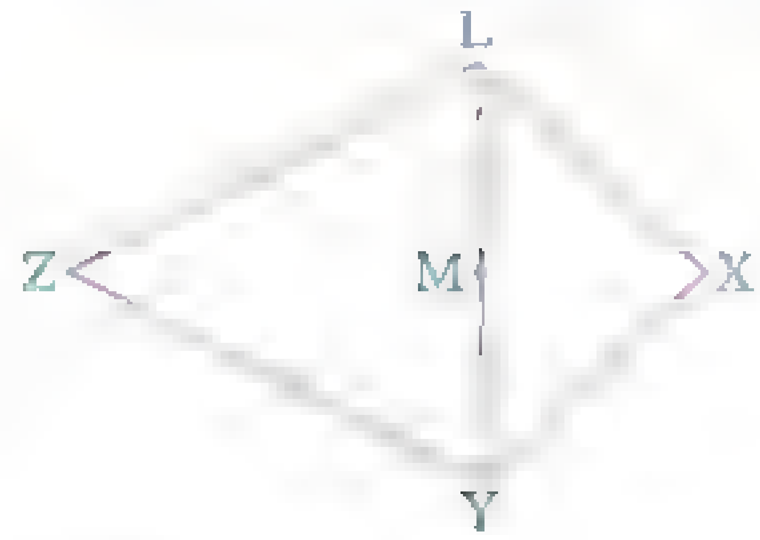
Find : m (∠ B) « 35° »

14 **In the opposite figure :**

$XY = XL$, $ZY = ZL$ and $LM = YM$

Prove that :

X , M and Z are on the same straight line.

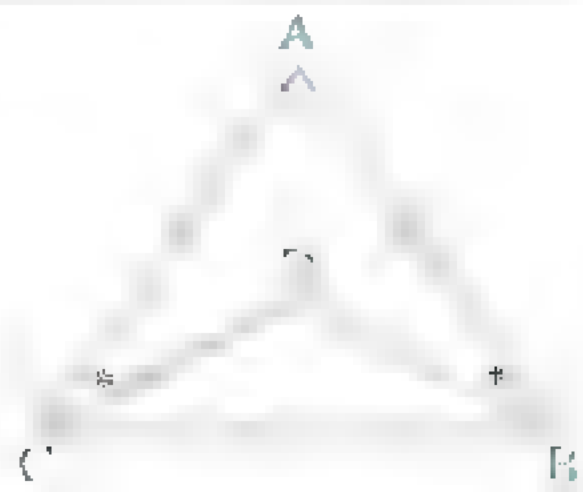


15 **In the opposite figure :**

ABC is a triangle , D is a point inside it such that

$m(\angle ABD) = m(\angle ACD)$ and $AB = AC$

Prove that : $\overrightarrow{AD}$ is the axis of symmetry of $\overline{BC}$



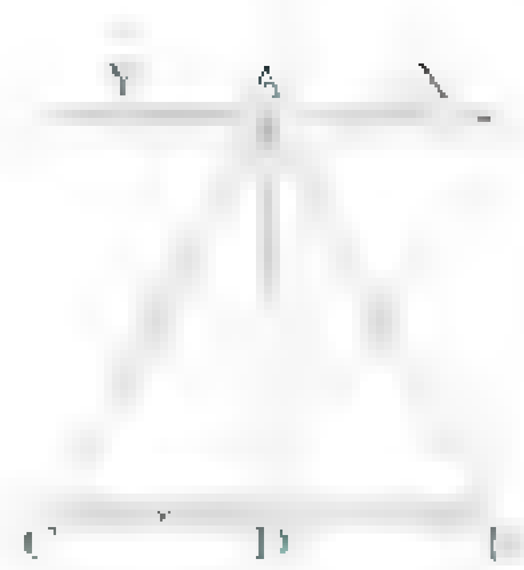
16 **In the opposite figure :**

ABC is a triangle in which $AB = AC$,

D is the midpoint of $\overline{BC}$ and $\overleftrightarrow{XY}$ passes through the vertex A

such that $\overleftrightarrow{XY} \parallel \overline{BC}$

Prove that : $\overline{AD} \perp \overleftrightarrow{XY}$

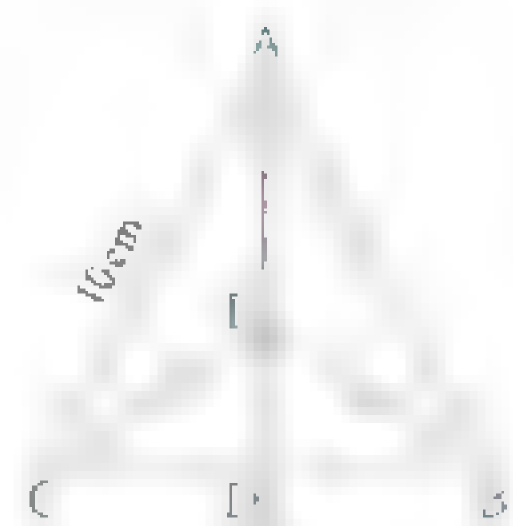


17 **In the opposite figure :**

$AB = AC = 10$ cm. , $EB = EC$ and $\overrightarrow{AE} \cap \overline{BC} = \{D\}$

Prove that : $BD = DC$ and if $BC = 6$ cm.

Find the length of each of : $\overline{CD}$ and $\overline{AD}$



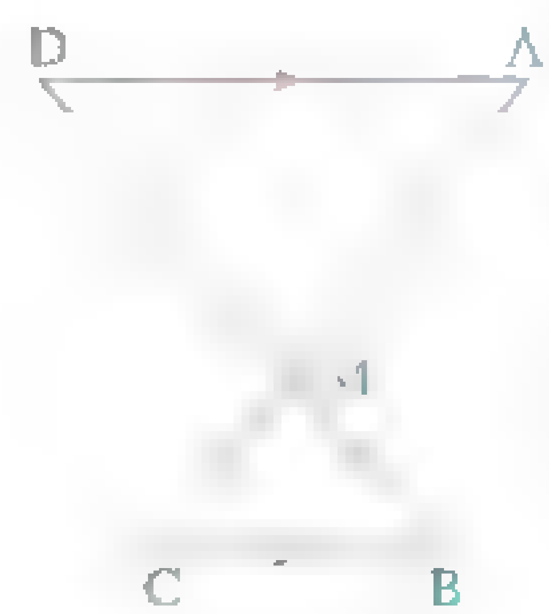
« 3 cm. , $\sqrt{91}$ cm. »

18 **In the opposite figure :**

$\overline{AC} \cap \overline{BD} = \{M\}$, $\overline{AD} \parallel \overline{BC}$ and $MB = MC$

Prove that :

1. Δ AMD is an isosceles triangle.
2. The axis of symmetry of Δ AMD is the same of Δ BMC



19 In the opposite figure :

$AB = AC$, $D \in \overrightarrow{AB}$, $E \in \overrightarrow{AC}$,

$\overrightarrow{BF}$ bisects $\angle DBC$ and $\overrightarrow{CF}$ bisects $\angle BCE$

Prove that :

1 $\triangle BFC$ is an isosceles triangle.

2 $\overleftrightarrow{AF}$ is the axis of symmetry of $\overline{BC}$

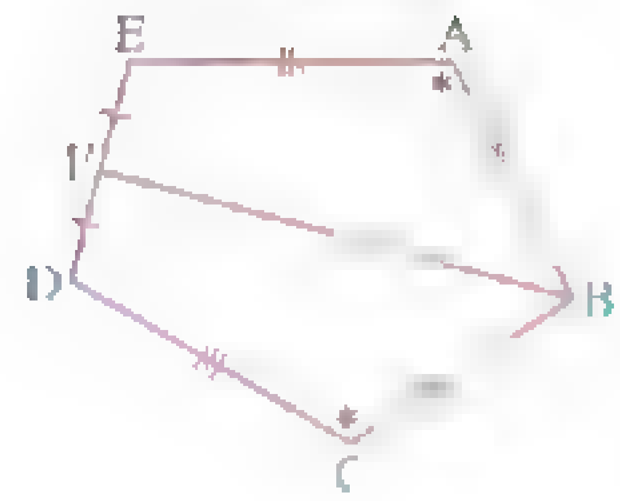
20 In the opposite figure :

$AB = BC$, $AE = CD$,

$m(\angle BAE) = m(\angle BCD)$

and F is the midpoint of $\overline{DE}$

Prove that : $\overline{BF} \perp \overline{DE}$



Choose the correct answer from those given :

1 If ABCD is a quadrilateral in which AB = AD and BC = DC , then $\overleftrightarrow{AC}$ is .... $\overline{BD}$

(a) parallel to (b) equal to

(c) the axis of symmetry of (d) congruent to

2 The triangle whose sides lengths are 2 cm. , $(X + 3)$ cm. and 5 cm. becomes an isosceles triangle when $X =$ .... cm.

(a) 1 (b) 2 (c) 3 (d) 4

3 If the length of any side in a triangle $= \frac{1}{3}$ of the perimeter of the triangle , then the number of axes of symmetry of the triangle equals ....

(a) 1 (b) 2 (c) 3 (d) zero

4 If $\overleftrightarrow{XY}$ is the axis of symmetry of $\overline{AB}$, then ....

(a) AX = BY (b) AX = BX (c) BY = XY (d) AY = BX

5 In the rhombus ABCD , the axis of symmetry of AC is ....

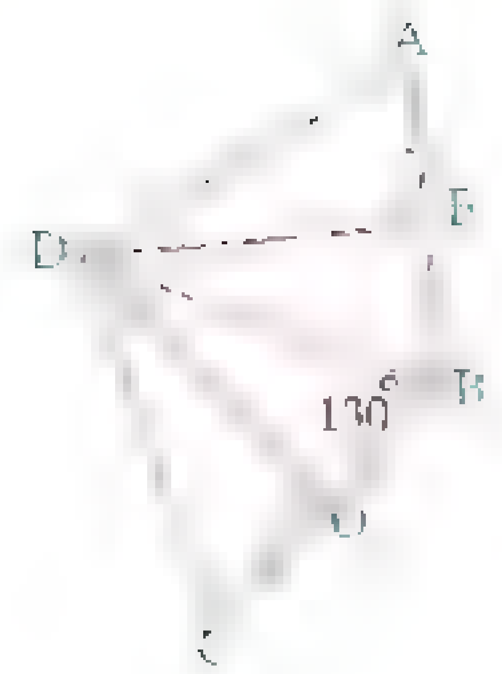
(a) $\overleftrightarrow{BD}$ (b) $\overleftrightarrow{AB}$ (c) $\overleftrightarrow{AD}$ (d) $\overleftrightarrow{CD}$

6 In the square ABCD , $\overleftrightarrow{BD}$ is the axis of symmetry of ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{AD}$ (d) $\overline{CD}$

For excellent pupils

22 **In the opposite figure :**



ABCD is a quadrilateral in which

m (∠ ABC) = 130°

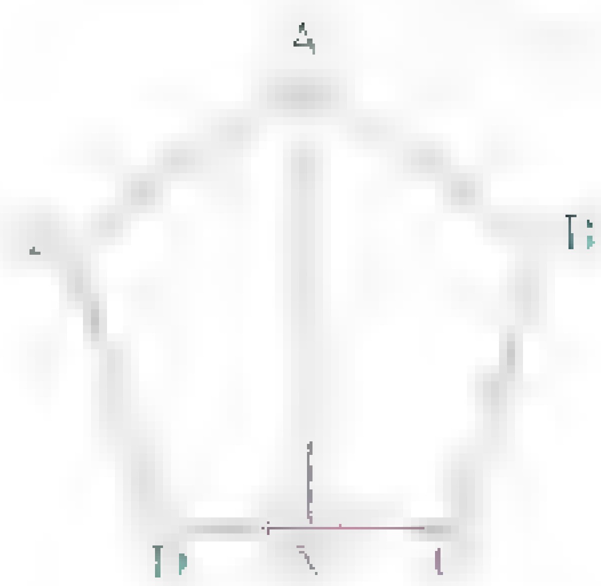
, E is the midpoint of $\overline{AB}$

, O is the midpoint of $\overline{BC}$

, $\overline{DE} \perp \overline{AB}$ and $\overline{DO} \perp \overline{BC}$

Find : m (∠ ADC)

23 **In the opposite figure :**



ABCDE is a regular pentagon and $\overline{AX} \perp \overline{CD}$

Find : m (∠ DAX)

Wonders of numbers

Choose an integer from 1 to 9, multiply it by 9, then multiply the product by 123456789.

Where do you stand ?

Inequality

Exercises of the unit :

6. Inequality.
7. Comparing the measures of angles in a triangle.
8. Comparing the lengths of sides in a triangle.
9. Triangle inequality.

Scan the QR code to solve an interactive test on each lesson

From the school book

Exercise 6 Inequality

● Remember   ○ Understand   ○ Apply   ♣ Problem Solving

Interactive test

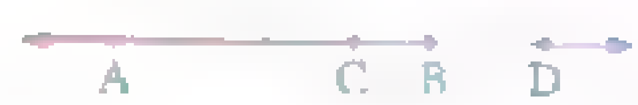
Complete each of the following using > or < :

1. **In the opposite figure :**

   If C and B belong to $\overline{AD}$ such that DC < BA , then AC .... DB

2. **In the opposite figure :**

   If B and C belong to $\overleftrightarrow{AD}$ where AB > CD , then AC .... BD



3. **In the opposite figure :**

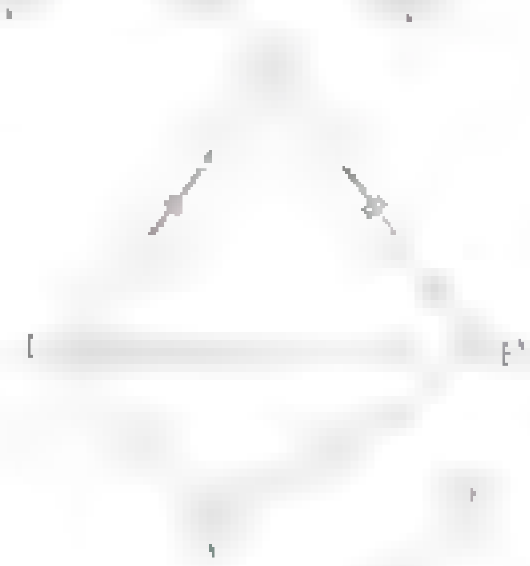
   If C ∈ $\overleftrightarrow{XY}$, m (∠ ACX) = 35°

   and m (∠ BCY) = 45° , then m (∠ XCB) .... m (∠ ACY)

4. **In the opposite figure :**

   AB = AD , m (∠ DBC) < m (∠ CDB)

   , then m (∠ ABC) .... m (∠ ADC)



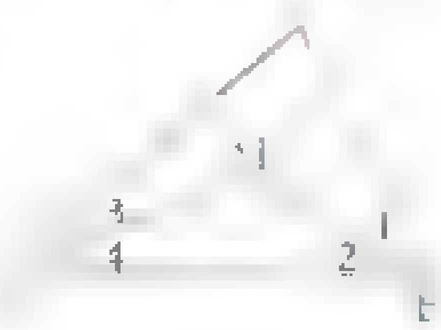
In the opposite figure :

If AB = AC and AY > AX , then BX .... CY

In the opposite figure :

m (∠ 1) > m (∠ 3) , m (∠ 2) > m (∠ 4)

, then m (∠ ABC) .... m (∠ ACB)

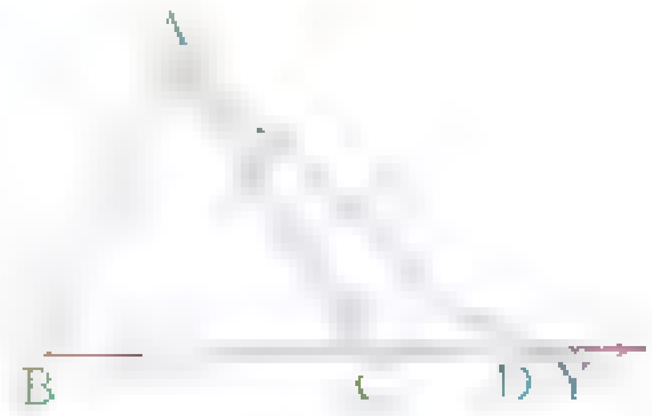


7 In the opposite figure :

ABC is a triangle , $C \in \overline{BD}$ and $Y \in \overrightarrow{CD}$

, then m (∠ ADY) .... m (∠ DAC)

, m (∠ ABC) .... m (∠ ADY)



12 Use the opposite figure to arrange the given measures ascendingly , where B , C , D and E are collinear :

1 m (∠ 1) , m (∠ 3)

2 m (∠ 2) , m (∠ 4)

3 m (∠ 5) , m (∠ 3)

4 m (∠ 2) , m (∠ 6)

5 m (∠ 3) , m (∠ 1) , m (∠ 5)

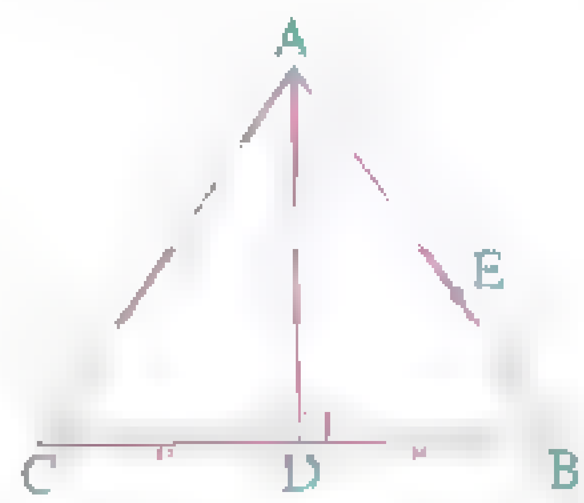
6 m (∠ 3) , m (∠ 5) , m (∠ 7)

7 m (∠ 3) , m (∠ 1) , m (∠ 7) , m (∠ 5)

13 In the opposite figure :

$E \in \overline{AB}$, $\overline{AD} \perp \overline{BC}$ and D is the midpoint of $\overline{BC}$

Prove that : AC > AE



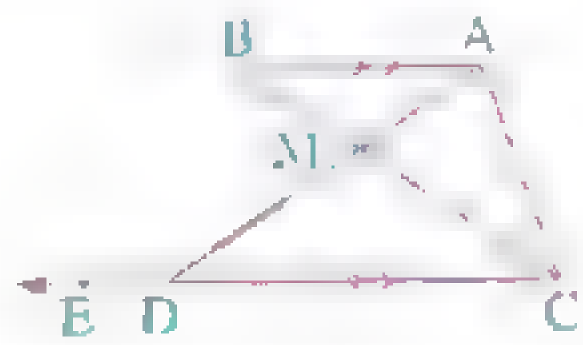
14 In the opposite figure :

$\overline{AB} \parallel \overrightarrow{CD}$, $\overline{AD} \cap \overline{BC} = \{M\}$, $E \in \overrightarrow{CD}$ and $E \notin \overline{CD}$

Prove that :

1 m (∠ ACD) > m (∠ ABC)

2 m (∠ ADE) > m (∠ ABC)



15 In the opposite figure :

$E \in \overrightarrow{CB}$ and M is the midpoint of each of $\overline{AC}$ and $\overline{BD}$

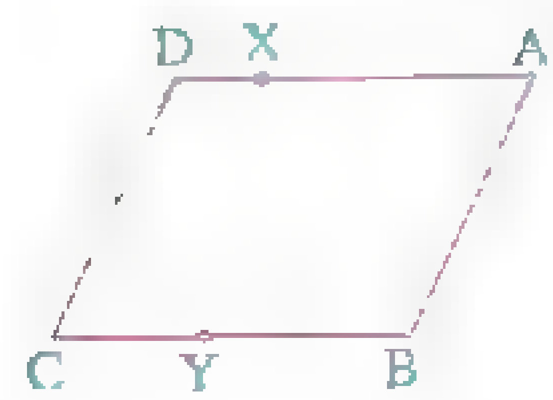
Prove that : m (∠ ABE) > m (∠ ACD)

16 In the opposite figure :

ABCD is a parallelogram , $X \in \overline{AD}$ and $Y \in \overline{BC}$

such that DX < BY

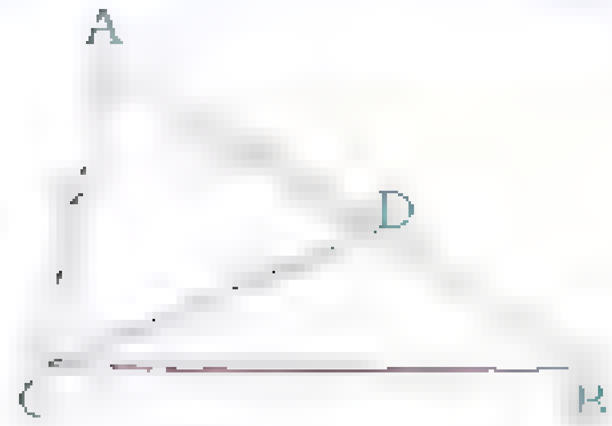
Prove that : AX + AB > CY + CD



7 **In the opposite figure :**

$D \in \overline{AB}$ where $AD = AC$

Prove that : $m(\angle ACB) > m(\angle B)$



8 **In the opposite figure :**

ABC is a triangle in which : $AC > AB$, $X \in \overline{AB}$

and $Y \in \overline{AC}$ where $m(\angle AXY) = m(\angle AYX)$

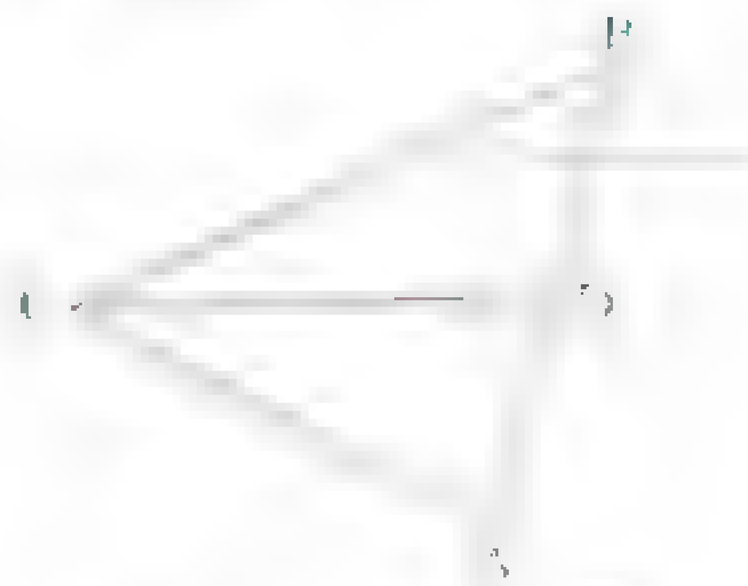
Prove that : $YC > XB$

9 **In the opposite figure :**

ABC is a triangle in which :

$\overline{AB} \equiv \overline{AC}$ and $D \in \overline{AB}$

Prove that : $m(\angle ADC) > m(\angle ACB)$



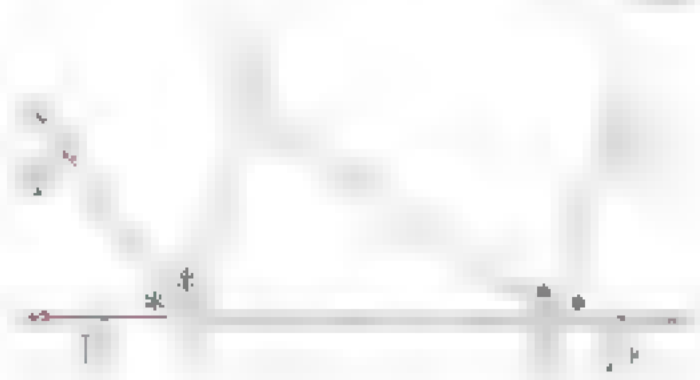
10 **In the opposite figure :**

$B \in \overleftrightarrow{DE}$, $C \in \overleftrightarrow{DE}$ such that

$m(\angle ACB) > m(\angle ABC)$

, $\overrightarrow{BX}$ bisects $\angle ABD$ and $\overrightarrow{CY}$ bisects $\angle ACE$

Prove that : $m(\angle ABX) > m(\angle ACY)$



11 M is a point inside the triangle ABC

Prove that : $m(\angle AMB) > m(\angle ACB)$

For excellent pupils

12 **In the opposite figure :**

ABC is a triangle in which : $m(\angle B) > m(\angle C)$, $D \in \overline{BC}$

such that $\overrightarrow{AD}$ bisects $\angle BAC$

Prove that : $\angle ADC$ is an obtuse angle.

13 **In the opposite figure :**

ABC is a triangle in which : $m(\angle ACB) > m(\angle ABC)$

, $A \in \overline{BD}$ such that $AC = AD$

Prove that : $\angle BCD$ is an obtuse angle.

From the school book

7 Comparing the measures of angles in a triangle

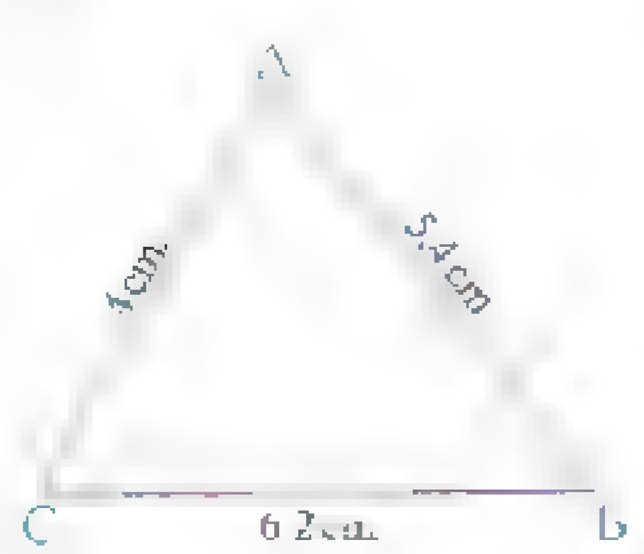
Remember · Understand · Apply · Problem Solving

Interactive test

1 Complete the following :

1. The lengths of two sides in a triangle are not equal , then the greater side in length is opposite to ....
2. In Δ ABC , AB = 7 cm. , BC = 5 cm. and AC = 6 cm. , then the smallest angle in measure is ....
3. In Δ DEF , if DE > EF , then m ($\angle$ F) > ....
4. In any triangle ABC , if AB > AC > BC , then m ($\angle$ ......) < m ($\angle$ ......) < m ($\angle$ ......)

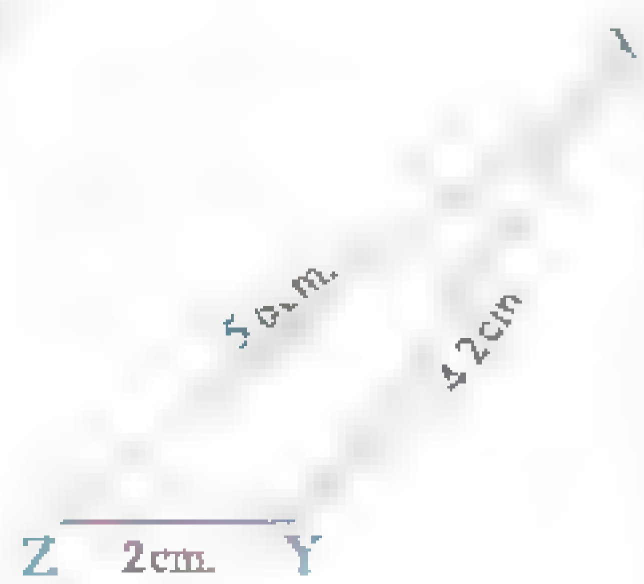
2 In each of the following figures , complete using (> or <) :



m ($\angle$ A) .... m ($\angle$ B)

m ($\angle$ A) .... m ($\angle$ C)

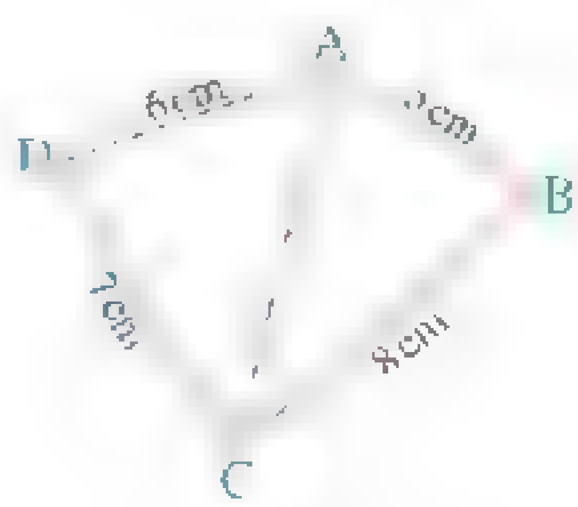
m ($\angle$ B) .... m ($\angle$ C)



m ($\angle$ Z) .... m ($\angle$ Y)

m ($\angle$ X) .... m ($\angle$ Y)

m ($\angle$ Z) .... m ($\angle$ X)



m ($\angle$ BAC) .... m ($\angle$ BCA)

m ($\angle$ DAC) .... m ($\angle$ DCA)

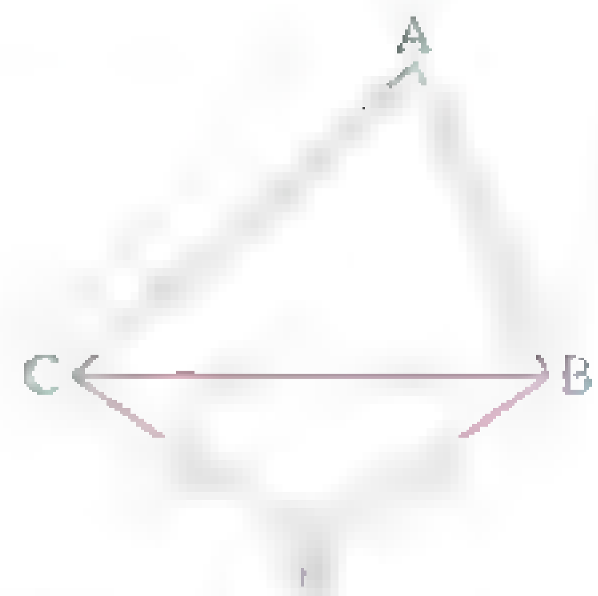
m ($\angle$ BAD) .... m ($\angle$ BCD)

3 **Arrange the measures of the angles of Δ ABC in each of the following cases ascendingly :**

1. If AB = 12 cm. , BC = 15 cm. and AC = 10 cm.
2. If AB = 5.7 cm. , BC = 8.5 cm. and AC = 6 cm.

---

4 **In the opposite figure :**



AC > AB and DB = DC

Prove that :

m ($\angle$ ABD) > m ($\angle$ ACD)

---

5 **In the opposite figure :**

XY > XL and YZ > ZL

Prove that : m ($\angle$ XLZ) > m ($\angle$ XYZ)

---

6 **In the opposite figure :**

ABCD is a quadrilateral in which :

AD = DC and BC > AB

Prove that : m ($\angle$ A) > m ($\angle$ C)

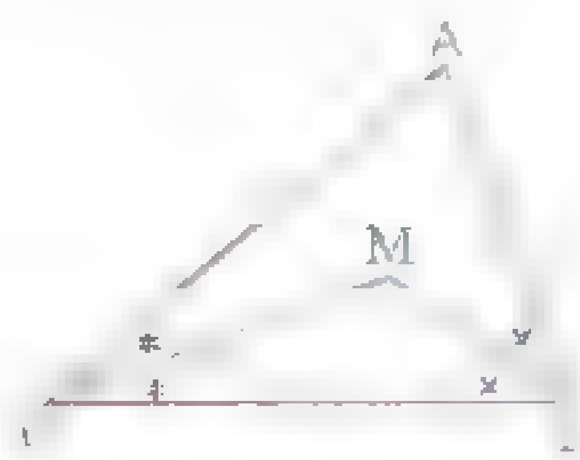
---

7 ABCD is a quadrilateral in which , $\overline{AB}$ is the longest side , $\overline{CD}$ is the shortest one

Prove that : m ($\angle$ BCD) > m ($\angle$ BAD)

---

8 **In the opposite figure :**



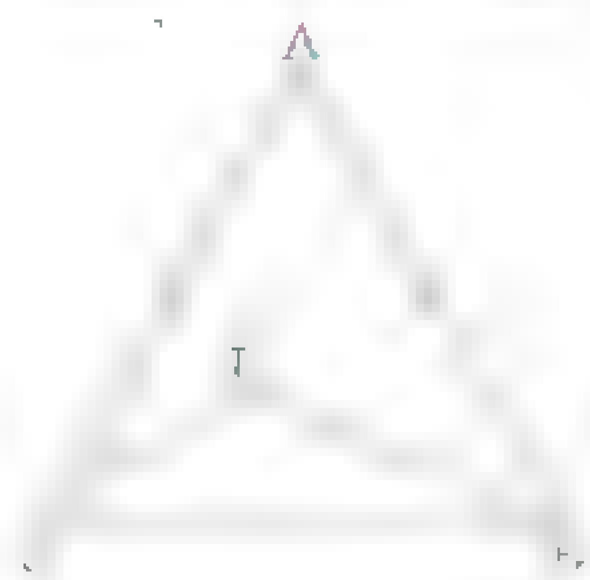
ABC is a triangle , $\overrightarrow{BM}$ bisects $\angle$ ABC and $\overrightarrow{CM}$ bisects $\angle$ ACB

If MC > MB

, prove that : m ($\angle$ ABC) > m ($\angle$ ACB)

---

9 **In the opposite figure :**



ABC is a triangle in which :

AB = AC and DB > DC

Prove that :

m ($\angle$ ABD) > m ($\angle$ ACD)

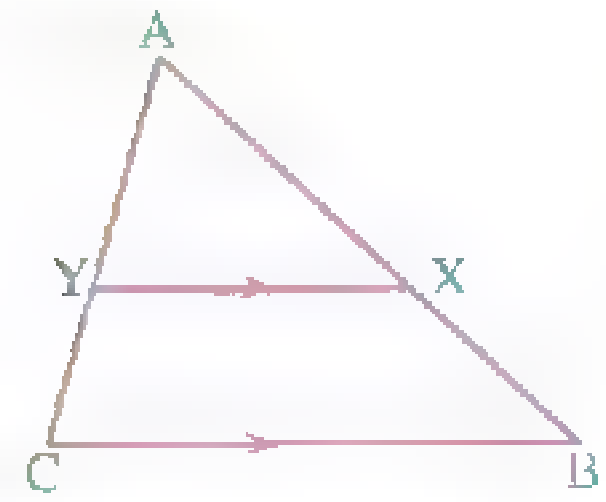
10 **In the opposite figure :**

ABC is a triangle ,

$AB > AC$ and $\overline{XY} \parallel \overline{BC}$

Prove that :

$m(\angle AYX) > m(\angle AXY)$

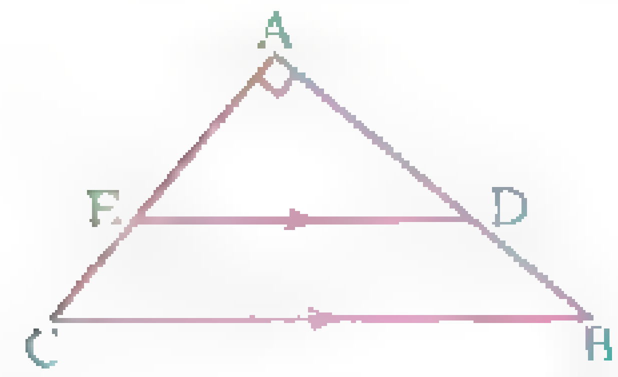


11 **In the opposite figure :**

ABC is a triangle in which : $m(\angle A) = 90^\circ$, $AB > AC$,

$D \in \overline{AB}$, $E \in \overline{AC}$ and $\overline{DE} \parallel \overline{BC}$

Prove that : $m(\angle AED) > 45^\circ$



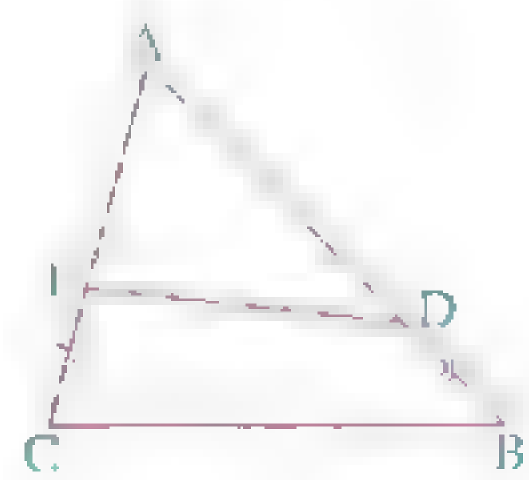
12 **In the opposite figure :**

ABC is a triangle in which :

$AB > AC$, $D \in \overline{AB}$ and

$E \in \overline{AC}$ where $BD = CE$

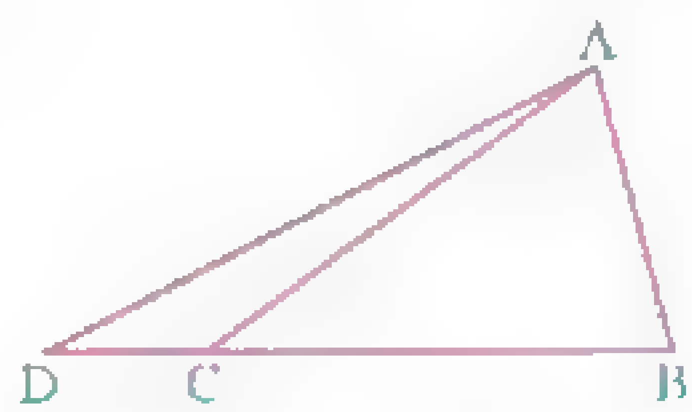
Prove that : $m(\angle AED) > m(\angle ADE)$



13 **In the opposite figure :**

$C \in \overline{BD}$ such that $AC > AB$

Prove that : $m(\angle ABD) > m(\angle D)$



14 **In the opposite figure :**

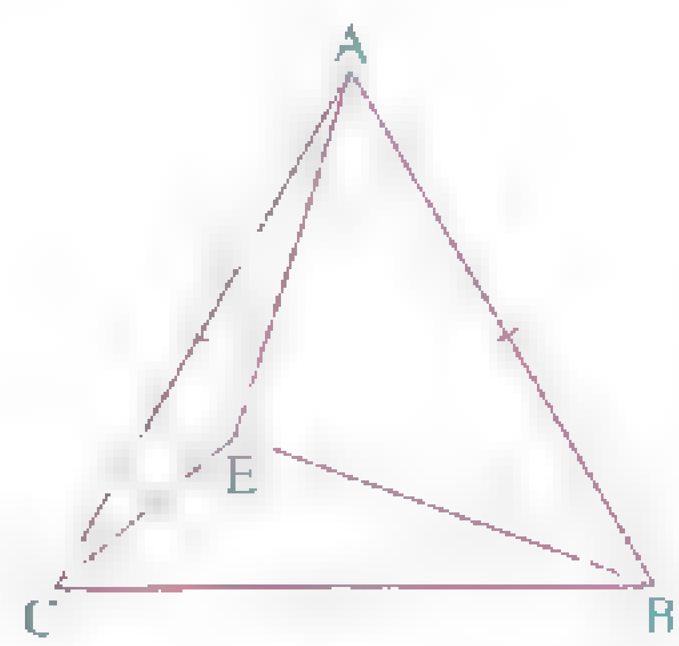
ABC is an equilateral triangle ,

E is a point inside it ,

$m(\angle ECB) > m(\angle EBC)$

Prove that : 1 $m(\angle ABE) > m(\angle ACE)$

2 $m(\angle A) > m(\angle ABE) > m(\angle ACE)$

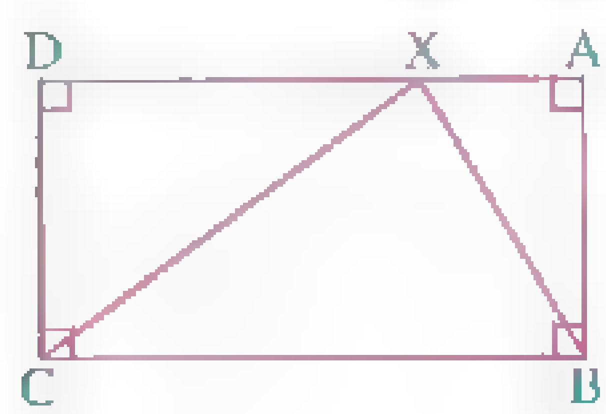


15 **In the opposite figure :**

ABCD is a rectangle , $X \in \overline{AD}$

such that $XC > XB$

Prove that : $m(\angle ABX) < m(\angle XCD)$

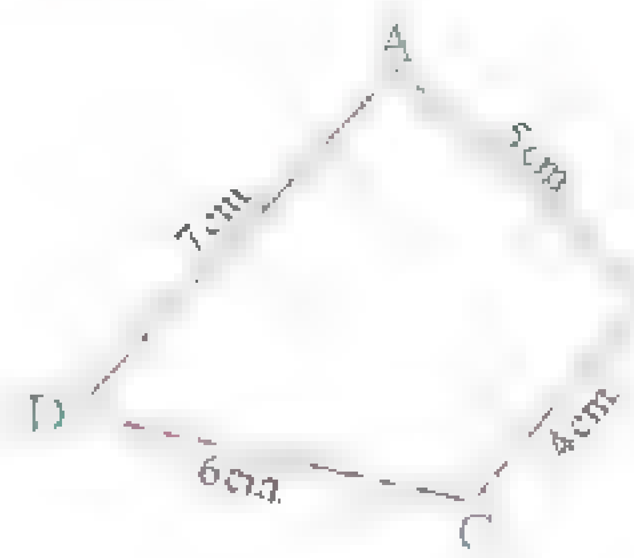


16 ABC is a triangle in which : AB > AC , D is the midpoint of $\overline{AB}$

Draw $\overrightarrow{DE}$ // $\overline{AC}$ to meet $\overline{BC}$ at E

Prove that : m (∠ CAE) > m (∠ DAE)

17 **In the opposite figure :**



ABCD is a quadrilateral in which :

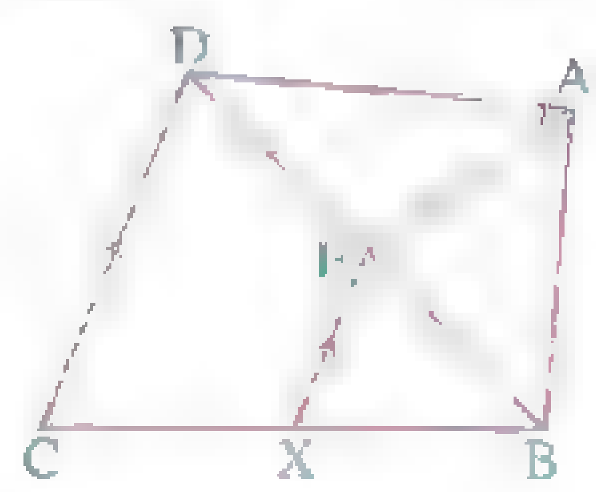
AB = 5 cm. , BC = 4 cm. , CD = 6 cm. and DA = 7 cm.

Prove that : 1 m (∠ ABC) > m (∠ ADC)

2 m (∠ BCD) > m (∠ BAD)

3 m (∠ B) + m (∠ C) > 180°

18 **In the opposite figure :**



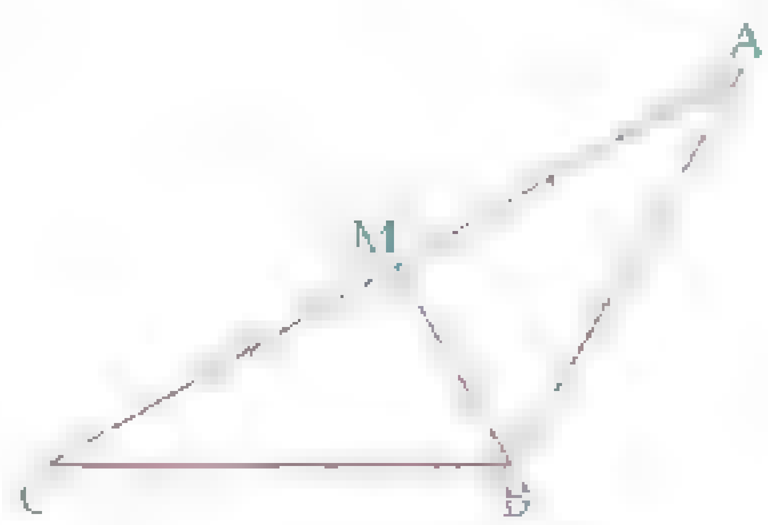
ABCD is a quadrilateral in which : m (∠ A) = 90° ,

$\overline{AE}$ is a median of Δ ABD , $\overrightarrow{EX}$ // $\overline{DC}$ and

$\overline{EX} \cap \overline{BC} = \{X\}$ If AE > EX

Prove that : m (∠ C) > m (∠ DBC)

19 **In the opposite figure :**

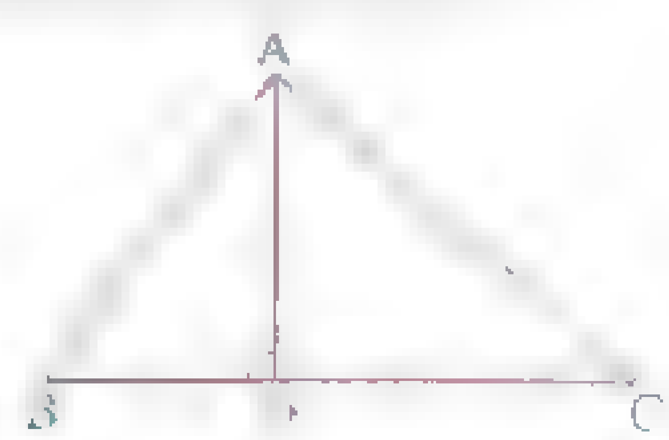


$\overline{BM}$ is a median in the

triangle ABC and BM < AM

Prove that : ∠ ABC is an obtuse angle.

20 **In the opposite figure :**



ABC is a triangle in which : AC > AB , $\overline{AD} \perp \overline{BC}$

and intersects it at D

Prove that : m (∠ BAD) < m (∠ CAD)

21 ABC is a triangle , $\overrightarrow{AD}$ bisects ∠ A and intersects $\overline{BC}$ at D , if AC > AB

Prove that : ∠ ADC is an obtuse angle.

Remember Understand Apply Problem Solving

22 ABCD is a parallelogram in which : AC > BD

Prove that : ∠ D is an obtuse angle.

For excellent pupils

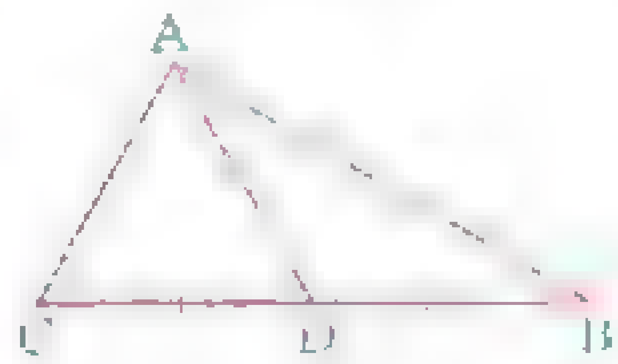
23 ABC is a triangle , D is the midpoint of $\overline{BC}$, if the perimeter of Δ ACD > the perimeter of Δ ABD

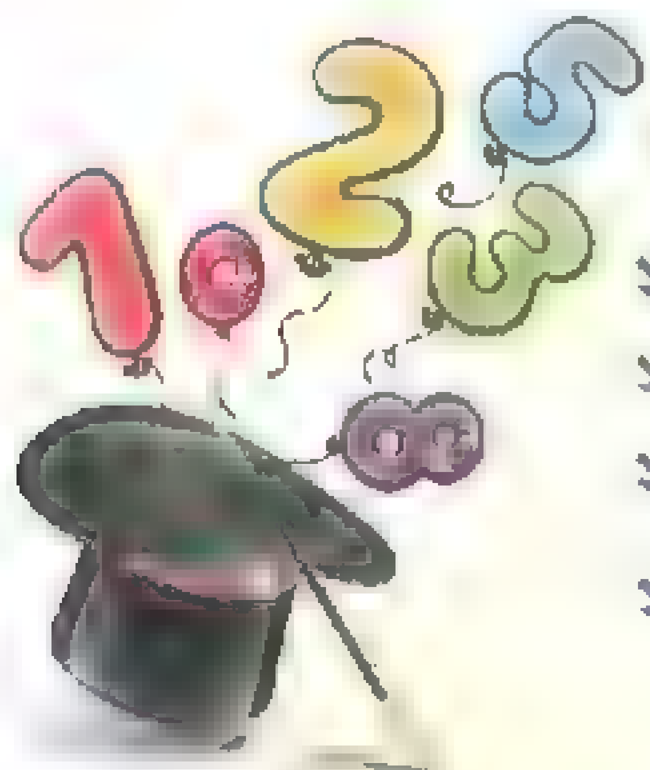
Prove that : m (∠ B) > m (∠ C)

24 **In the opposite figure :**

AB > AC and DB = DC

Prove that : m (∠ BAD) < m (∠ CAD)





Wonders of numbers

- Pick any positive 2 digit number.
- Interchange the two digits to get a new number.
- Subtract the smaller number from the bigger number.
- Is the difference divisible by 9 ?

Do the exercise again using different numbers.

[ ] From the school book

Exercise

8 Comparing the lengths of sides in a triangle

● Remember ● Understand ○ Apply ♣ Problem Solving

Interactive test

Complete the following :

1. If two angles in a triangle are unequal in measure , then the greater angle in measure is opposite to .... and if the two lengths of two sides in a triangle are unequal , then the greater side in length is opposite to the angle which is ....
2. The smallest angle of a triangle (in measure) is opposite to ....
3. The longest side in the right-angled triangle is ....
4. The shortest distance between a given point and a given straight line is ....
5. ABC is a triangle in which : m (∠ C) = 110° , then its longest side is ....
6. In Δ ABC : If m (∠ A) = 50° , m (∠ B) = 30° , then the shortest side in the triangle is ....
7. In Δ ABC : If m (∠ A) = m (∠ B) + m (∠ C) , then the longest side in the triangle is ....

Choose the correct answer from those ones :

1. In Δ ABC , if m (∠ B) > m (∠ C) , then ....

   (a) AB > AC (b) BC > AC (c) AC > AB (d) AB > BC

2. In Δ ABC , if m (∠ B) = 90° , then ....

   (a) AC > CB (b) AB > AC (c) BC > AC (d) AB = AC

3 In ΔABC , if $m(\angle A) = 40°$ and $m(\angle B) = 70°$, then ....

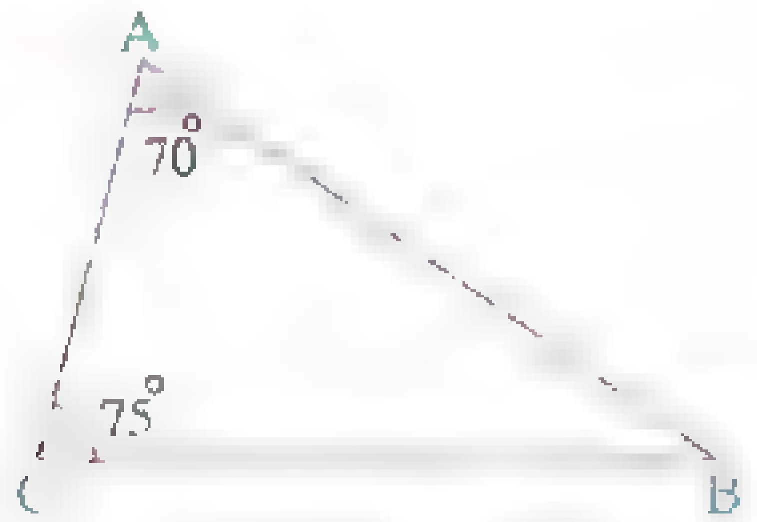
(a) $AB < AC$ (b) $AB > AC$ (c) $\overline{AB} \perp \overline{AC}$ (d) $AB = AC$

4 In ΔXYZ , if $m(\angle X) = 110°$, $m(\angle Y) = 40°$, then XY .... XZ

(a) < (b) > (c) = (d) //

In the following figures , complete using > , < or = :

1

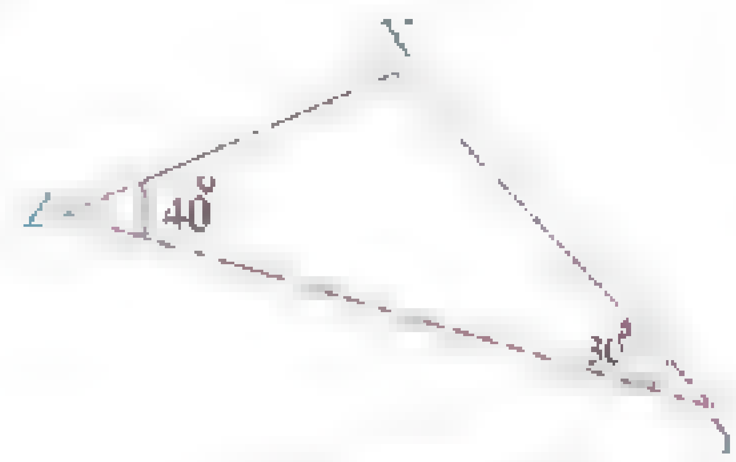


AB .... AC

AB .... BC

AC .... BC

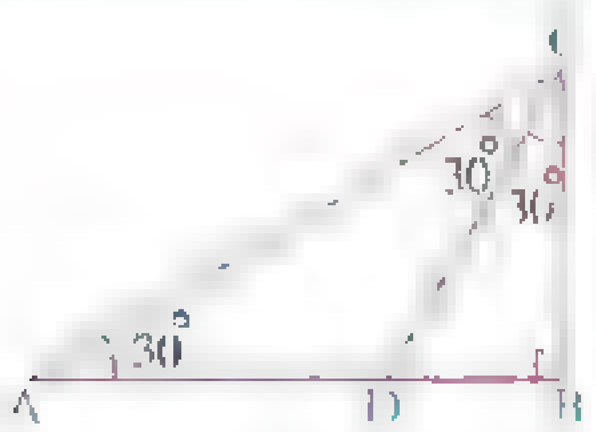
2



XY .... XZ

YZ .... XY

YZ .... XZ

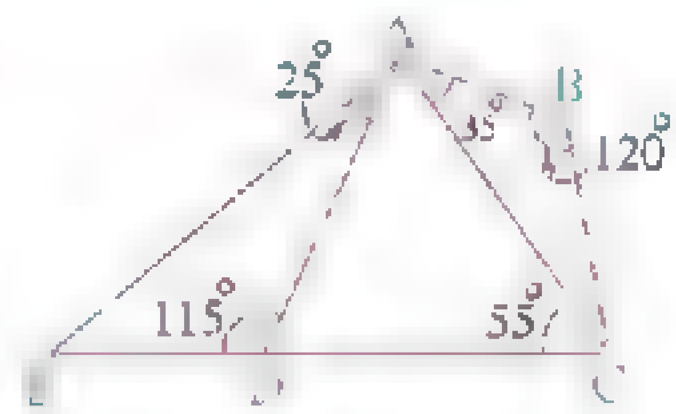


AC .... BC

BC .... DB

AC .... BD

CA .... DC



BC .... AB

CD .... CA

AD .... AE

CD .... AD

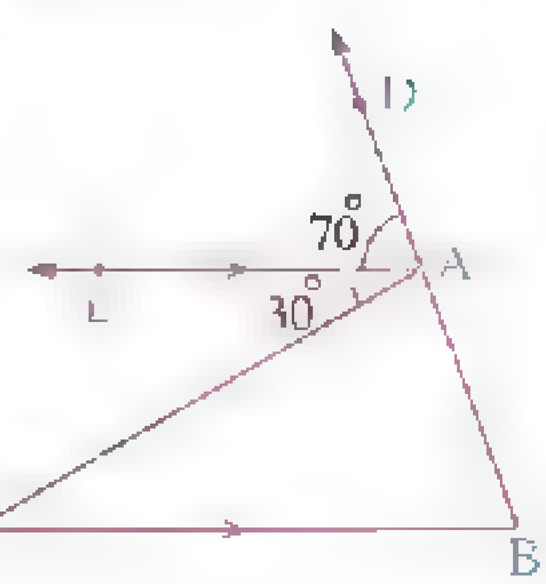
XYZ is a triangle in which : $m(\angle X) = 45°$, $m(\angle Y) = 85°$ and $m(\angle Z) = 50°$

Arrange the lengths of the sides of the triangle ascendingly.

ABC is a triangle in which : $m(\angle A) = 40°$ and $m(\angle B) = 75°$

Order the lengths of the sides of the triangle descendingly.

In the opposite figure :



$\overleftrightarrow{AE} \parallel \overline{BC}$,

$m(\angle DAE) = 70°$

and $m(\angle EAC) = 30°$

Prove that : $AC > AB$

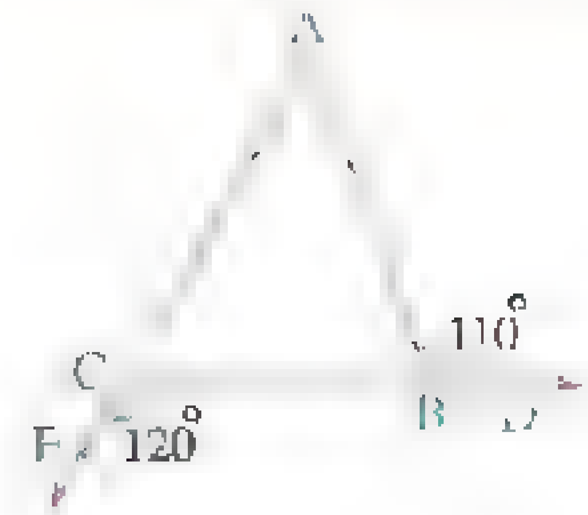
7 **In the opposite figure :**

ABC is a triangle , $D \in \overrightarrow{CB}$,

$E \in \overrightarrow{AC}$, m (∠ ABD) = 110°

and m (∠ BCE) = 120°

Prove that : AB > BC



8 **In the opposite figure :**

AB = AC , m (∠ ABC) = 65°

, m (∠ ACD) = 20° , $A \in \overline{BD}$

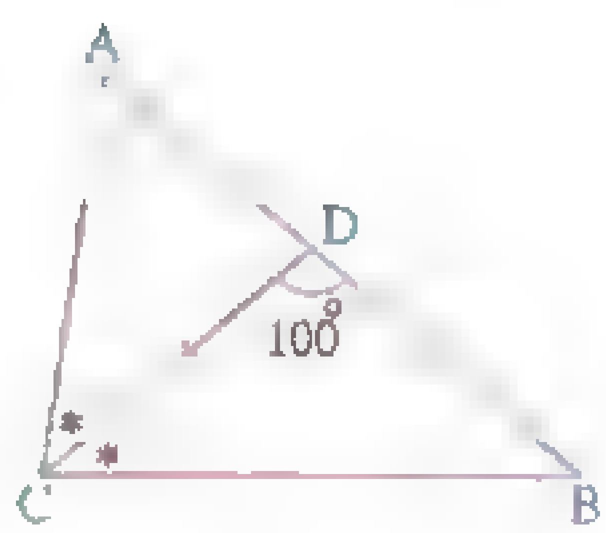
Prove that : AB > AD

9 **In the opposite figure :**

ABC is a triangle , $\overrightarrow{CD}$ bisects ∠ C and intersects $\overline{AB}$ at point D

, m (∠ BDC) = 100° and DB = DC

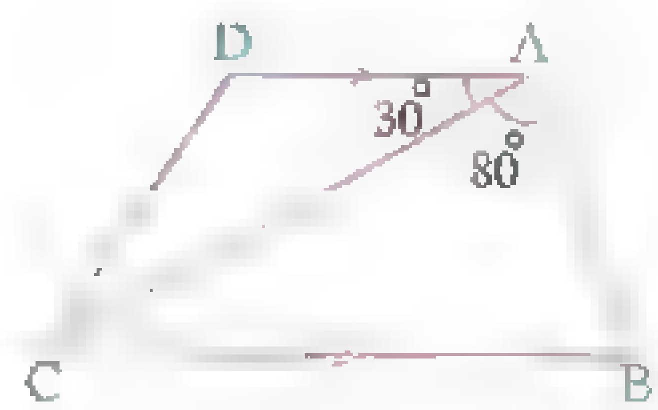
Prove that : AC > DB



10 **In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$, m (∠ BAC) = 80° and m (∠ DAC) = 30°

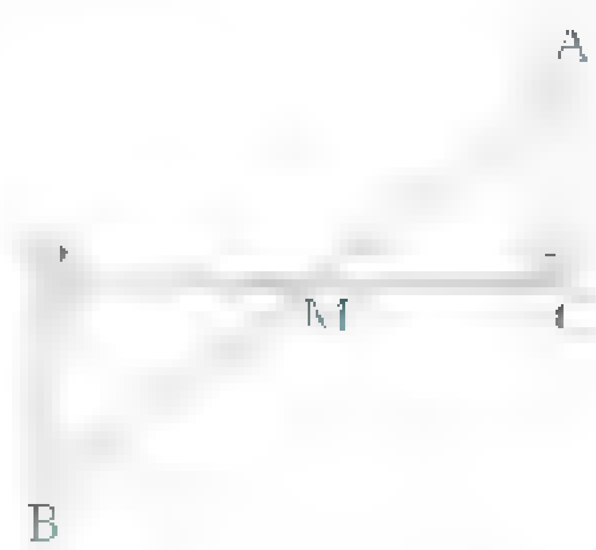
Prove that : BC > AB



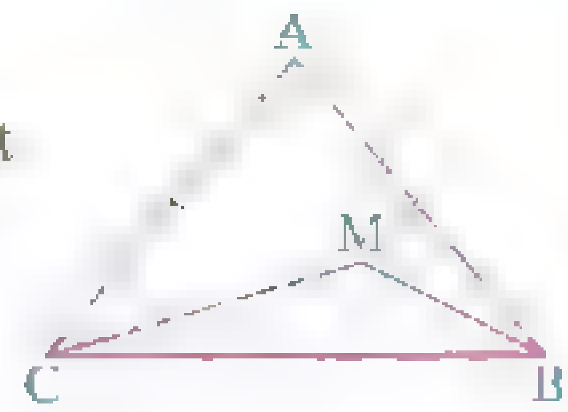
11 **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{M\}$, $\overline{AC} \perp \overline{CD}$ and $\overline{BD} \perp \overline{CD}$

Prove that : AB > CD



12 **In the opposite figure :**

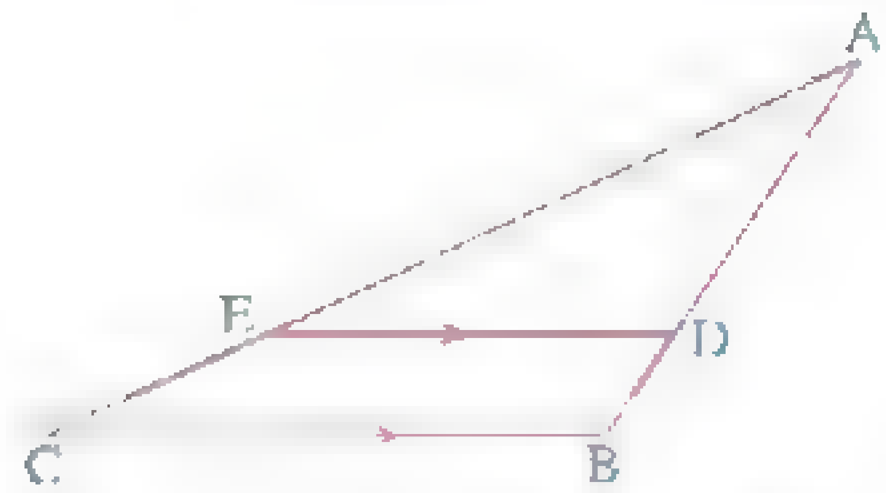


ABC is a triangle in which : AB = AC , M is a point inside it such that $m(\angle ABM) < m(\angle ACM)$

Prove that : MC > MB

---

13 **In the opposite figure :**



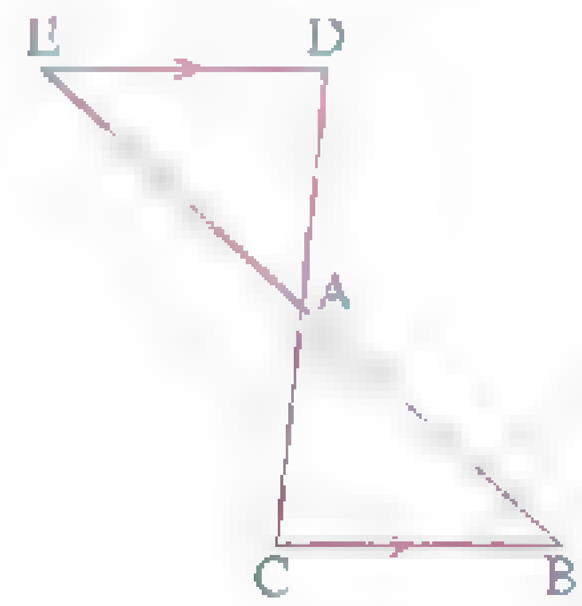
ABC is an obtuse-angled triangle at B

, $\overline{DE} \parallel \overline{BC}$

Prove that : AE > AD

---

14 **In the opposite figure :**



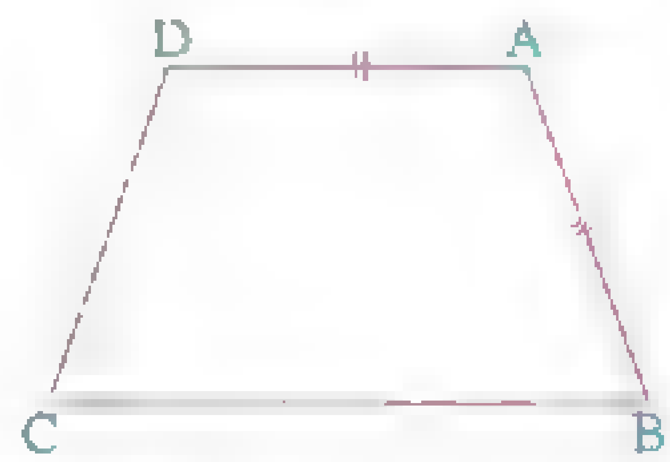
AB > AC , $\overline{DE} \parallel \overline{BC}$ and

$\overline{DC} \cap \overline{BE} = \{A\}$

Prove that : AE > AD

---

15 **In the opposite figure :**



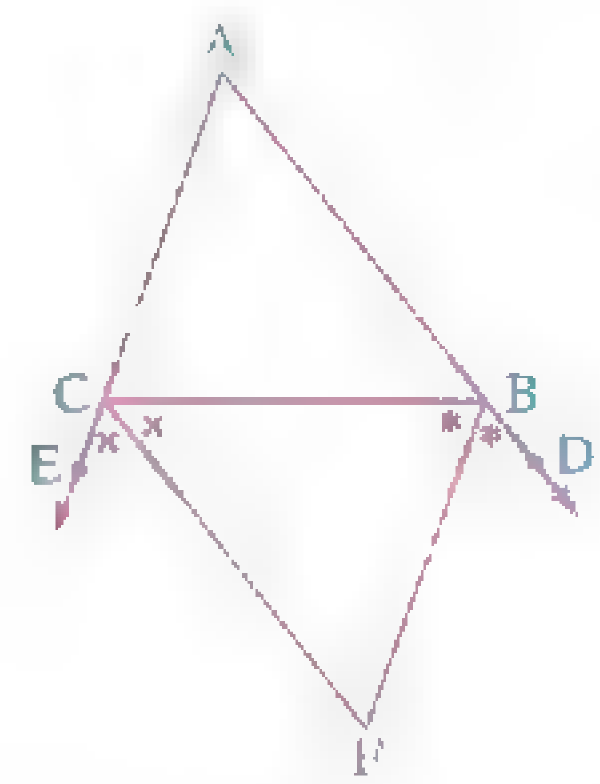
ABCD is a quadrilateral , AB = AD

and $m(\angle D) > m(\angle B)$

Prove that : BC > CD

---

16 **In the opposite figure :**



ABC is a triangle in which : AB > AC , D $\in \overrightarrow{AB}$, E $\in \overrightarrow{AC}$

, $\overrightarrow{BF}$ bisects $\angle DBC$ and $\overrightarrow{CF}$ bisects $\angle BCE$

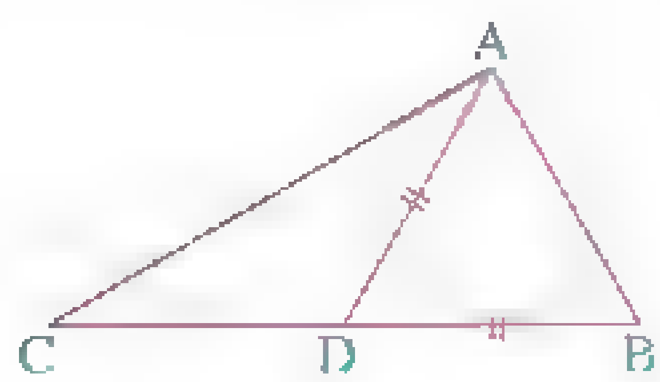
, $\overrightarrow{BF} \cap \overrightarrow{CF} = \{F\}$

Prove that :

1 $m(\angle FBC) > m(\angle BCF)$ 2 CF > BF

---

17 **In the opposite figure :**



ABC is a triangle and D $\in \overline{BC}$ where BD = AD

Prove that : BC > AC

18 **In the opposite figure :**

D is the midpoint of $\overline{AB}$, m (∠ B) = 70° and m (∠ DCB) = 50°

Prove that :

1 m (∠ A) > m (∠ ACD)

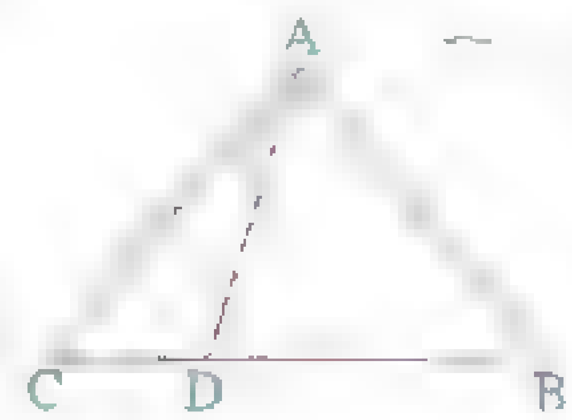
2 ∠ ACB is an acute angle.

19 **In the opposite figure :**

AF = BF = DF and m (∠ FAB) = 50°

Prove that : 1 AD > AB 2 BC > AC

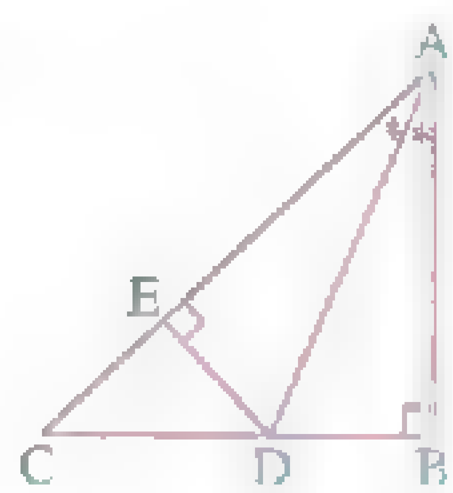
20 **In the opposite figure :**



ABC is a triangle in which : AB = AC and D ∈ $\overline{BC}$

Prove that : AB > AD

21 **In the opposite figure :**

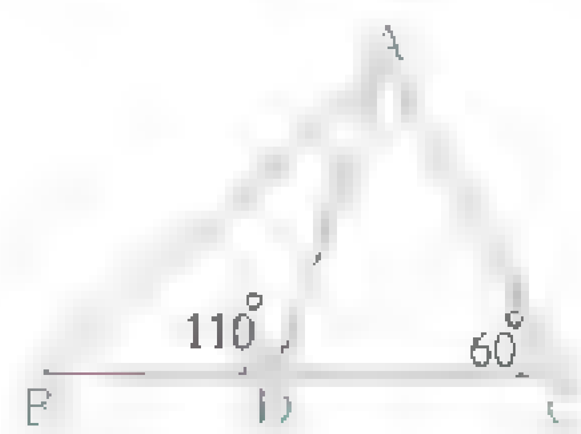


m (∠ B) = 90° , $\overline{DE} \perp \overline{AC}$ and $\overline{AD}$ bisects ∠ BAE

Prove that : 1 BD = DE

2 DC > DB

22 **In the opposite figure :**



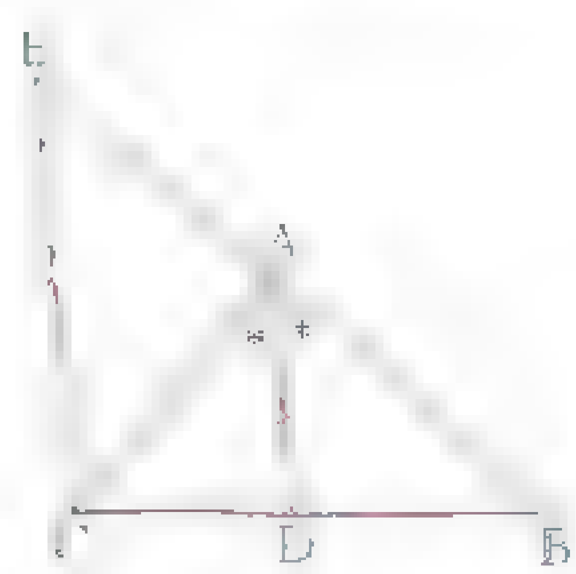
m (∠ ADB) = 110° and m (∠ C) = 60°

Prove that : AB + AC > 2 AD

23 ABC is a right-angled triangle at B

Prove that : AB + BC < 2 AC

24 **In the opposite figure :**



ABC is a triangle , $\overrightarrow{AD}$ bisects ∠ BAC

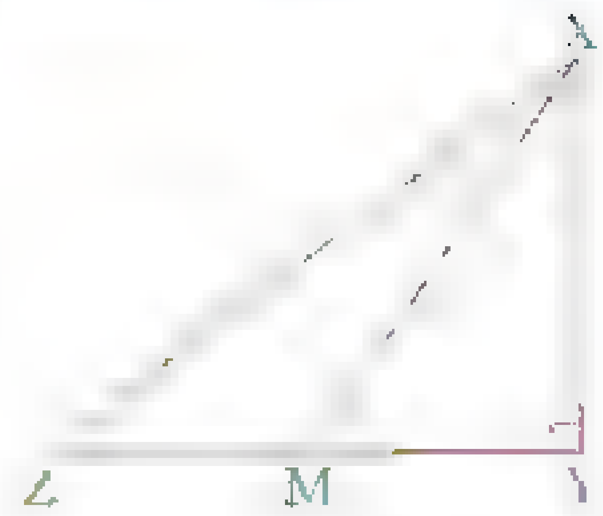
, $\overrightarrow{CE}$ // $\overrightarrow{DA}$ and cuts $\overrightarrow{BA}$ at E

Prove that : BE > BC

25 **In the opposite figure :**

XYZ is a right-angled triangle at Y and M $\in \overline{YZ}$

Prove that : XZ > XM

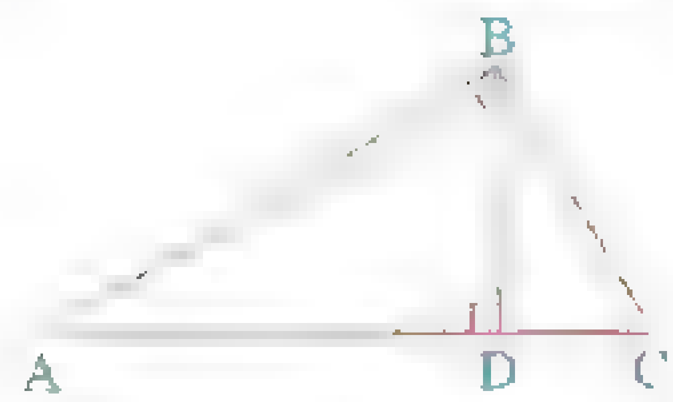


26 **In the opposite figure :**

m (∠ ABC) = 90° , $\overline{BD} \perp \overline{AC}$

and AB > BC

Prove that : AD > BD



27 ABC is a triangle , $\overrightarrow{CD}$ bisects ∠ C , $\overrightarrow{CD} \cap \overline{AB} = \{D\}$

Prove that : BC > BD

28 ABC is a right-angled triangle at B , D $\in \overline{AC}$ and E $\in \overline{BC}$ where AD = BE

Prove that : m (∠ CED) > m (∠ CDE)

29 ABC is a triangle in which : m (∠ A) = $(5X + 2)^\circ$,

m (∠ B) = $(6X - 10)^\circ$ and m (∠ C) = $(X + 20)^\circ$

Order the lengths of sides of the triangle ascendingly.

For excellent pupils

30 ABC is a triangle in which : AB = AC and X $\in \overline{AC}$, draw $\overleftrightarrow{XY}$ to cut $\overline{AB}$ at Y and cut $\overrightarrow{CB}$ at Z

Prove that : AY > AX

Exercise 9 Triangle inequality

● Remember ● Understand ○ Apply ⁂ Problem Solving

Interactive test

1 **Is it possible to draw a triangle whose side lengths are as follows ? Give reasons :**

1. 3 cm. , 4 cm. , 9 cm.
2. 📖 5 cm. , 7 cm. , 8 cm.
3. 📖 10 cm. , 6 cm. , 4 cm.
4. 13 cm. , 8 cm. , 6 cm.
5. 5 cm. , 3 cm. , 4 cm.
6. 9 cm. , 9 cm. , 19 cm.

2 **Find the interval to which the length of the third side of the triangle belongs in each of the following triangles if the lengths of the two other sides are :**

1. 📖 6 cm. , 9 cm.
2. 3 cm. , 3 cm.
3. 📖 2.9 cm. , 3.2 cm.
4. 5.7 cm. , 7.3 cm.

3 **Choose the correct answer from those given :**

● 1. The sum of lengths of any two sides in a triangle is .... the length of the third side.

(a) less than (b) greater than (c) equal (d) half

● 2. The length of any side in a triangle .... the sum of lengths of the other two sides.

(a) $>$ (b) $<$ (c) $=$ (d) twice

○ 3. Which of the following numbers cannot be the lengths of sides of a triangle ?

(a) 7 , 7 , 5 (b) 9 , 9 , 9 (c) 3 , 6 , 12 (d) 3 , 4 , 5

○ 4. If the lengths of two sides in a triangle are 7 cm. and 4 cm. , then the length of the third side can be ....

(a) 1 cm. (b) 2 cm. (c) 3 cm. (d) 4 cm.

If the lengths of two sides of an isosceles triangle are 3 cm and 7 cm , then the length of the third side is ...

(a) 7 cm. (b) 3 cm. (c) 4 cm. (d) 10 cm.

A triangle has one axis of symmetry , the lengths of two sides in it are 4 cm and 8 cm , then its perimeter = ...

(a) 16 cm. (b) 20 cm. (c) 24 cm. (d) 30 cm.

7 In Δ ABC , if AB = 3 cm. , BC = 5 cm. and AC = X cm. , then $X \in$ ....

(a)] 3 , 5 [(b)] 2 , 5 [(c)] 5 , 8 [(d)] 2 , 8 [

If the lengths of two sides of a triangle are 5 cm. and 10 cm. , then the length of the third side belongs to ....

(a) [10 , 15 [(b)] 5 , 15 [(c)] 5 , 10] (d) [10 , 15]

9 In Δ ABC : AB + BC − AC ....

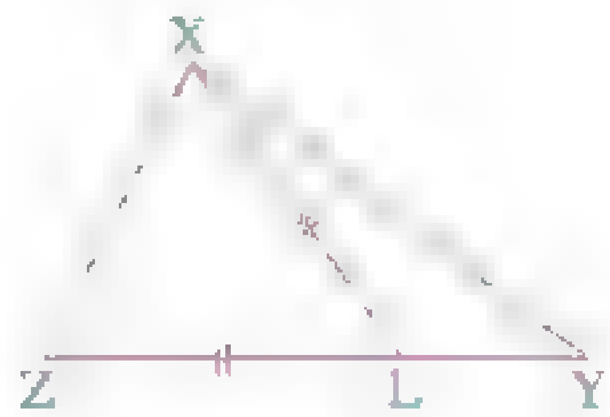
(a) > zero (b) < zero

(c) = zero (d) = the perimeter of the triangle ABC

10 In Δ ABC , $\frac{AB + BC}{AC}$ 1

(a) > (b) < (c) = (d) ≤

In the opposite figure :



XYZ is a triangle in which L ∈ YZ such that XL = LZ

Prove that : YZ > XY

ABC is a triangle in which $\overline{BC}$ is the longest side , D ∈ $\overline{BC}$ such that CD = CA

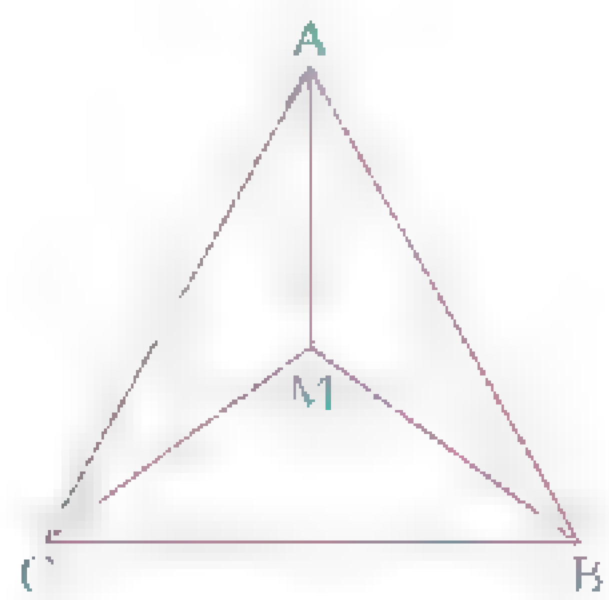
Prove that : AB > BD

---

6 ABC is a triangle , $\overrightarrow{AD}$ is drawn to cut $\overline{BC}$ at D

Prove that : BD + DC + 2 AD > AB + AC

In the opposite figure :

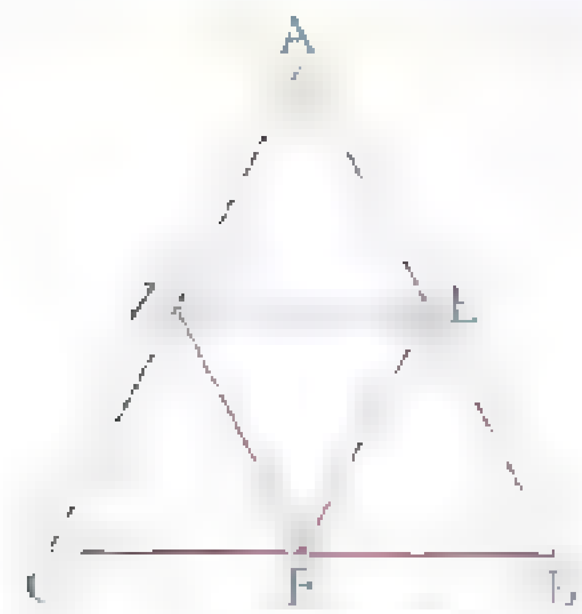


ABC is a triangle in which M is a point inside it.

Prove that :

MA + MB + MC > $\frac{1}{2}$ the perimeter of the triangle ABC

8 **In the opposite figure :**



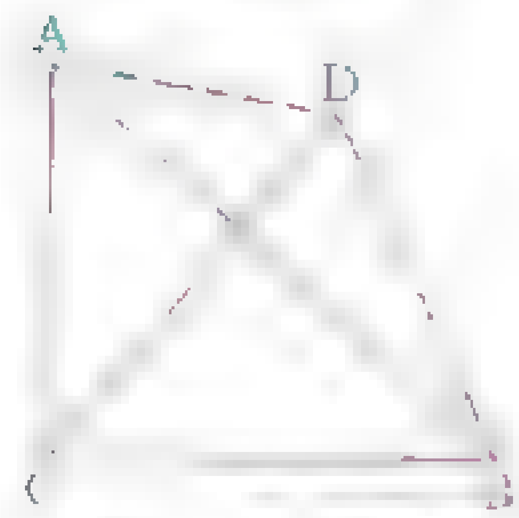
ABC is a triangle in which $E \in \overline{AB}$

, $F \in \overline{BC}$ and $Z \in \overline{AC}$

Prove that :

The perimeter of Δ ABC > the perimeter of Δ EFZ

9 **In the opposite figure :**

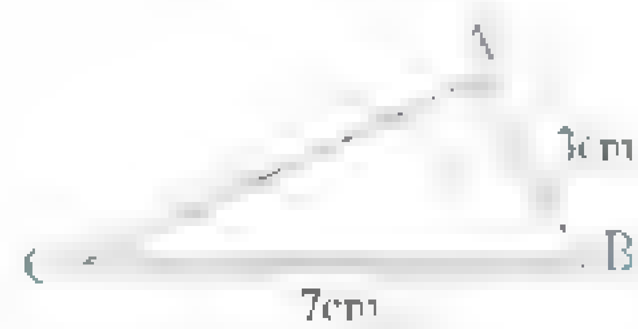


ABC is a triangle and D is a point outside it.

Prove that :

The perimeter of Δ ABC < 2 (DA + DB + DC)

10 **In the opposite figure :**



ABC is a triangle in which :

AB = 3 cm. , BC = 7 cm.

Prove that: m ($\angle$C) < m ($\angle$B)

11 Prove that the length of any side in a triangle is less than half of the perimeter.

12 ABCD is a quadrilateral.

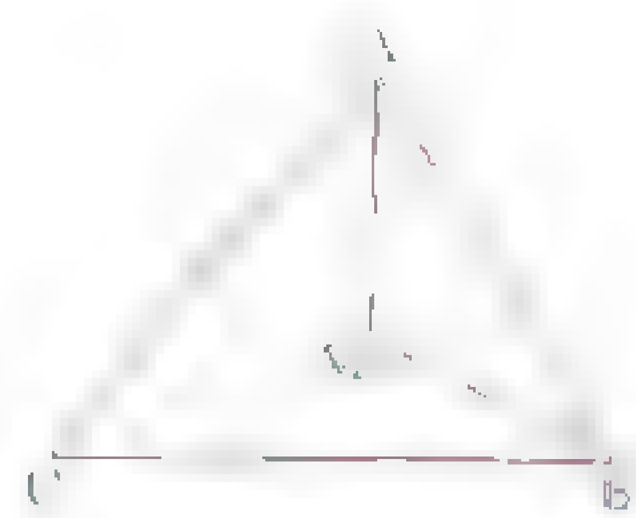
Prove that : AB + BC + CD > AD

13 Prove that the sum of the lengths of two diagonals in a convex quadrilateral is less than its perimeter.

14 Prove that the perimeter of any quadrilateral is less than twice the sum of lengths of its diagonals.

For excellent pupils

15 **In the opposite figure :**



M is a point inside the triangle ABC

Prove that : AM + MB < AC + BC

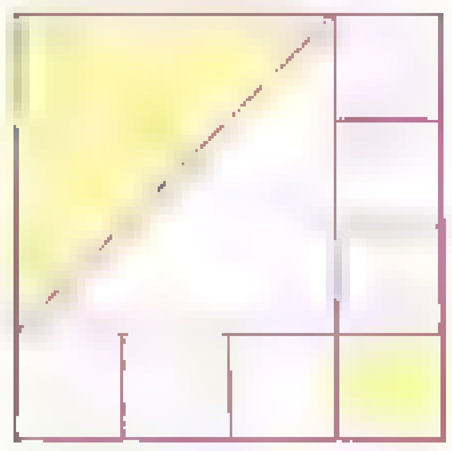
16 ABC is a triangle and F is the midpoint of $\overline{BC}$ **Prove that :**

1 AB + AC > 2 AF

2 AB + AC > AF + BF

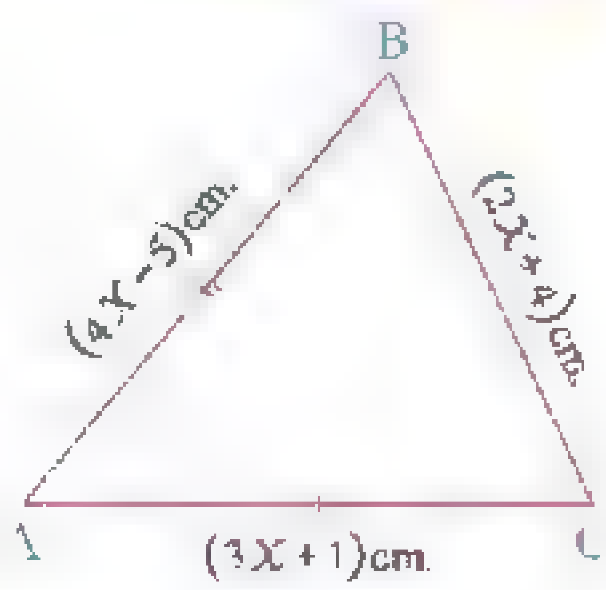
Complete the following :

1. A lamppost of height 4.5 metres is 2 metres far from a building of height 10.5 metres , then the distance between the top of the lamppost and the top of the building is .... metres.
2. The ratio between the lateral and the total areas of a cube is
3. A cuboid is of lateral area 200 cm^2. and the dimensions of its base are 8 cm. and 12 cm. , then its height equals .... cm.
4. The measure of the angle between the two hands of the clock at 7 o'clock in degrees is ....°
5. **In the opposite figure :**



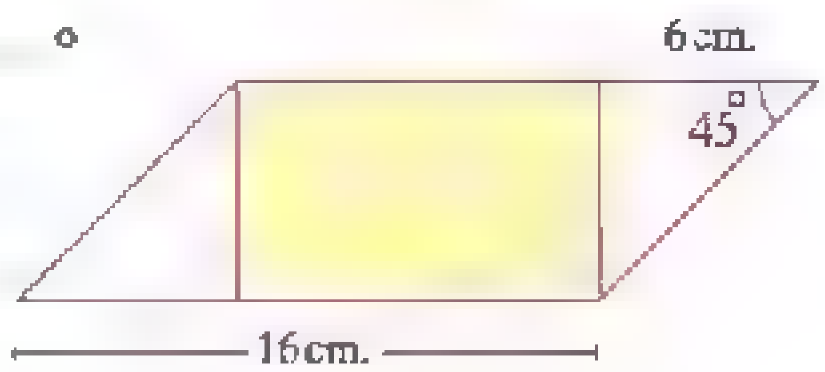
A square divided into 7 small congruent squares and two congruent triangles. If the area of the coloured square = 4 cm^2. , then the area of the coloured triangle is .... cm^2.

6. **In the opposite figure :**



ABC is a triangle in which : AB = $(4X - 5)$ cm. , BC = $(2X + 4)$ cm. , AC = $(3X + 1)$ cm. , AB = AC , then the perimeter of Δ ABC = ... cm.

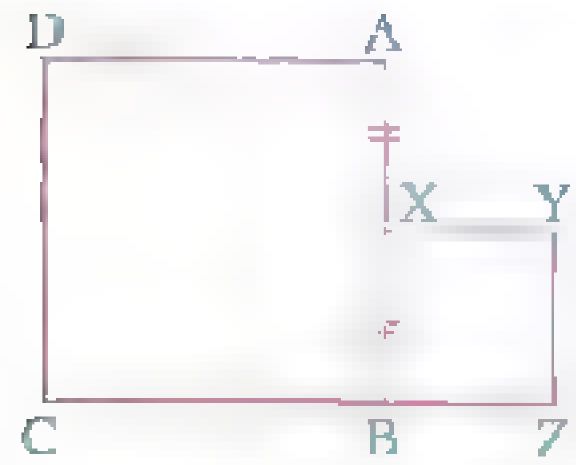
7. A rectangle its length is X cm. , its width is y cm. and its perimeter is P cm. , then the relation between X , y and P is X = ...
8. If the side length of an equilateral triangle is 10 cm. , then its height is .... cm.
9. The measure of the angle of the regular pentagon is .. °
10. The opposite figure shows a coloured rectangle inside a parallelogram , then the area of the rectangle equals .... cm^2



11 **In the opposite figure :**

If the perimeter of the square ABCD = 24 cm.

, then the area of the square XYZB is .... cm^2.



A cuboid is of total area 148 cm^2. and its lateral area is 110 cm^2. , then the area of its base is .... cm^2.

2 Choose the correct answer from the given ones :

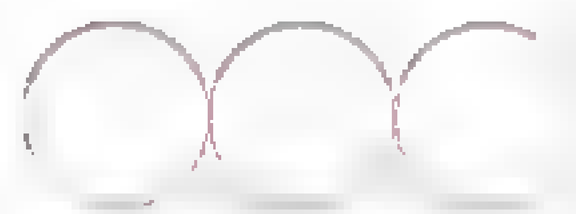
1 The acute angle supplements .... angle.

(a) an acute (b) an obtuse (c) a right (d) a reflex

2 The number of diagonals of the hexagon equals ....

(a) 3 (b) 6 (c) 9 (d) 12

3 The number of axes of symmetry of the opposite shape is ....



(a) 1 (b) 2 (c) 3 (d) 4

A wire in the shape of an equilateral triangle of side length 4 cm. is reshaped as a square, then the side length of the square is .... cm.

(a) 12 (b) 16 (c) 4 (d) 3

5 **In the opposite figure :**

A circle of radius length 2 cm. touches two sides of a square , then the area of the coloured part is .... cm^2.

(a) $4 - \pi$ (b) $\pi - 2$ (c) $\frac{\pi}{2}$ (d) 2π

The ratio between the area of a square region of side length ℓ cm. and the area of a square region of side length 2ℓ cm. is ....

(a) 1 : 2 (b) ℓ : 4 (c) 1 : 4 (d) 4 : 1

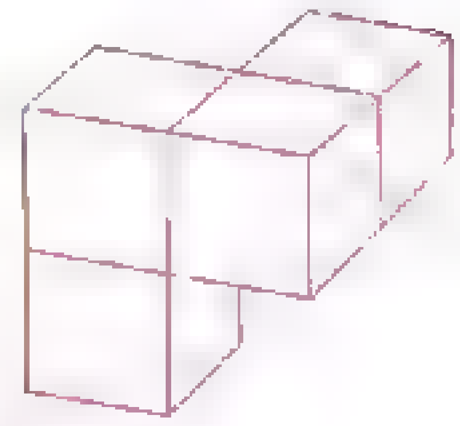
On a map , each 1 cm. represents 5 km. If the distance between two places is $\frac{1}{2}$ km. , then the distance between them on the map is ....

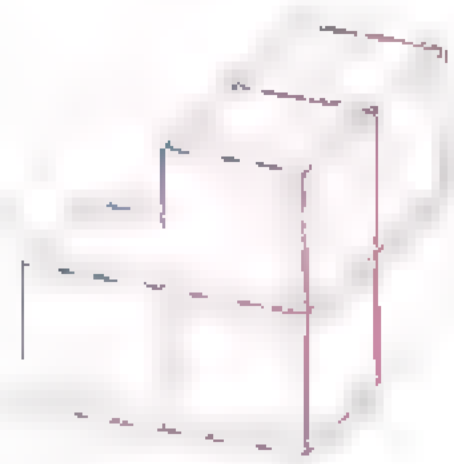
(a) 0.1 cm. (b) 10 cm. (c) 2.5 cm. (d) 0.4 cm.

If the area of the base of a cuboid is 12 cm^2. and the areas of two side faces are 6 cm^2. and 8 cm^2. , then the volume of the cuboid is .... cm^3.

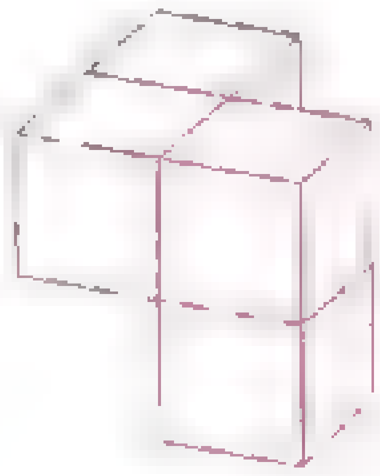
(a) 9 (b) 576 (c) 24 (d) 32

9 This solid will be rotated to another position. Which of the following may be a position of the solid after rotation ?

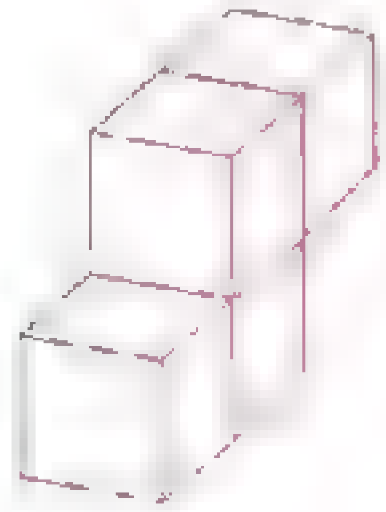




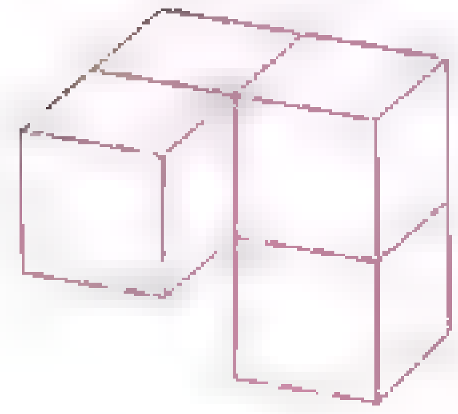
(a)



(b)



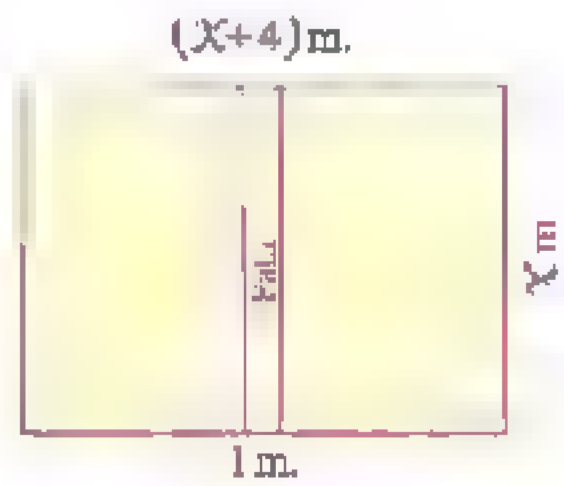
(c)



(d)

10 **In the opposite figure :**

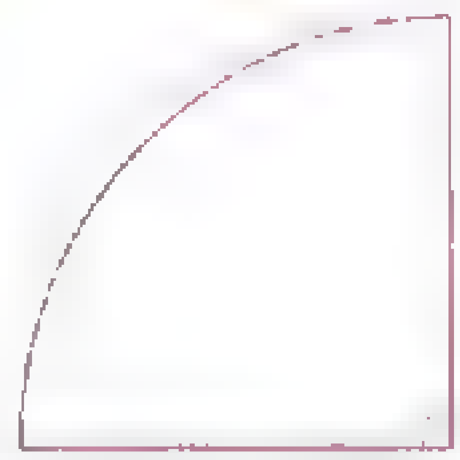
A rectangular garden with a rectangular path of width 1 metre. Which expression shows the area of the coloured part of the garden in square metres ?



(a) $x^2 + 3x$ (b) $x^2 + 4x$

(c) $x^2 + 4x - 1$ (d) $x^2 + 3x - 1$

11 The opposite figure represents a quarter of a circle of radius length 2 cm. , then the perimeter of the figure in centimetres is ......



(a) 2π (b) 5π

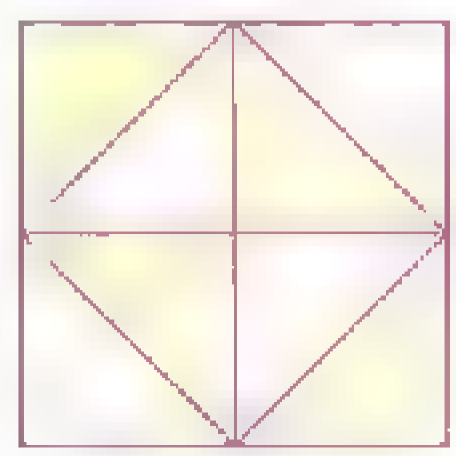
(c) $\pi + 4$ (d) $4\pi + 4$

12 The area of a square whose side length is an integer may be ...... cm^2.

(a) 600 (b) 900 (c) 800 (d) 700

13 **In the opposite figure :**

A square of perimeter 32 cm. divided into 8 congruent triangles , then the area of the coloured region is .... cm^2.



(a) 4 (b) 8

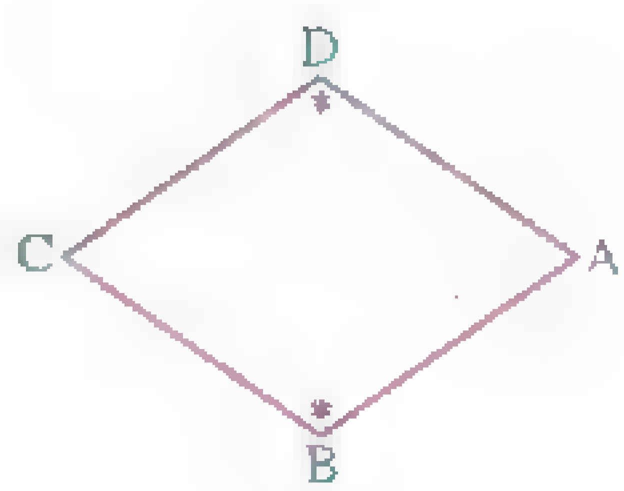
(c) 16 (d) 32

14 **In the opposite figure :**

If $m(\angle A) + m(\angle C) = 140°$

, $m(\angle B) = m(\angle D)$

, then $m(\angle B) = $ ....



(a) 50° (b) 55°

(c) 110° (d) 220°

El-MOASSER

By a group of supervisors

NOTEBOOK

- Accumulative Tests
- Monthly Tests
- Important Questions
- Final Revision
- Final Examinations

2nd PREP. 2025

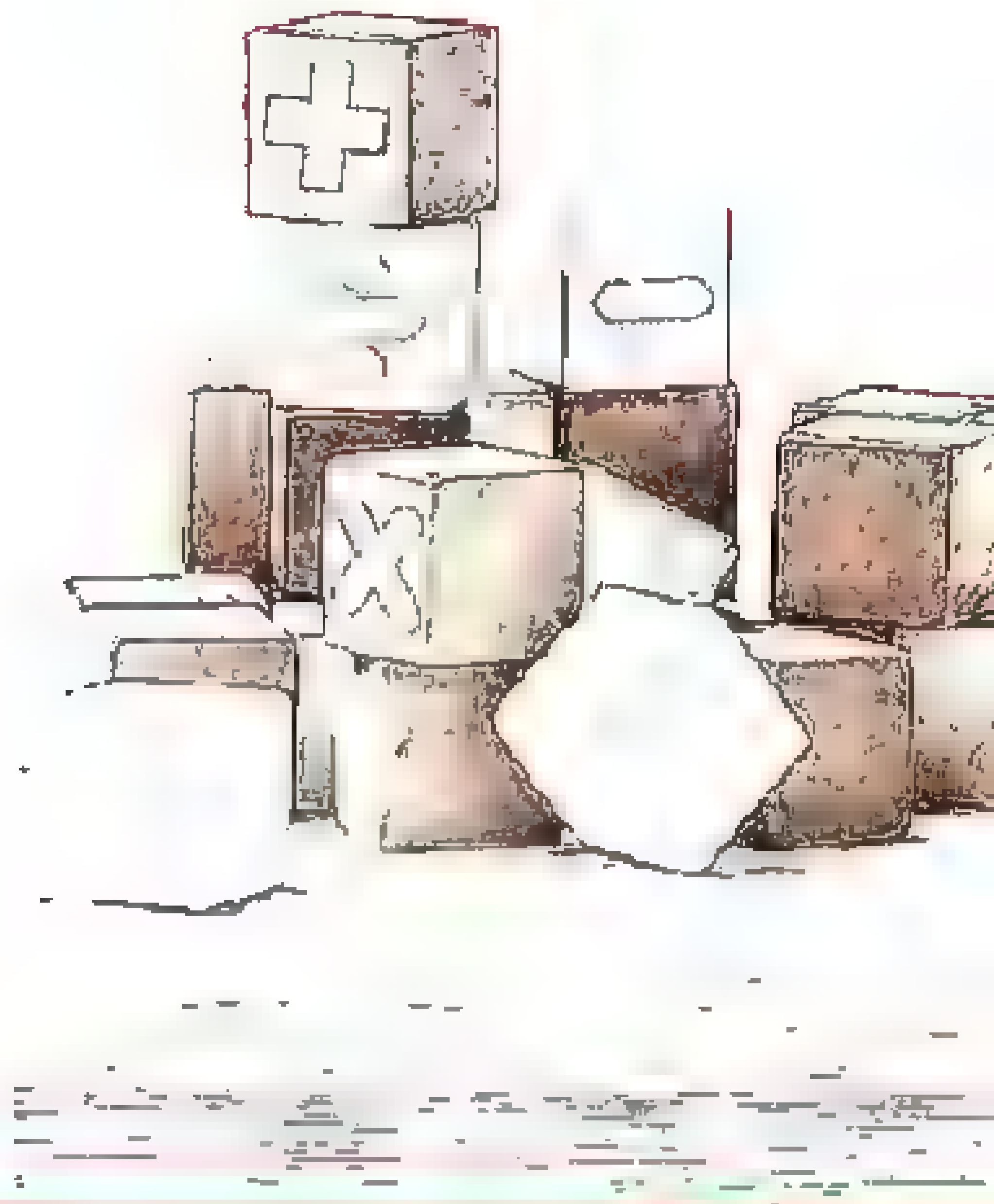
FIRST TERM

Maths

CONTENTS

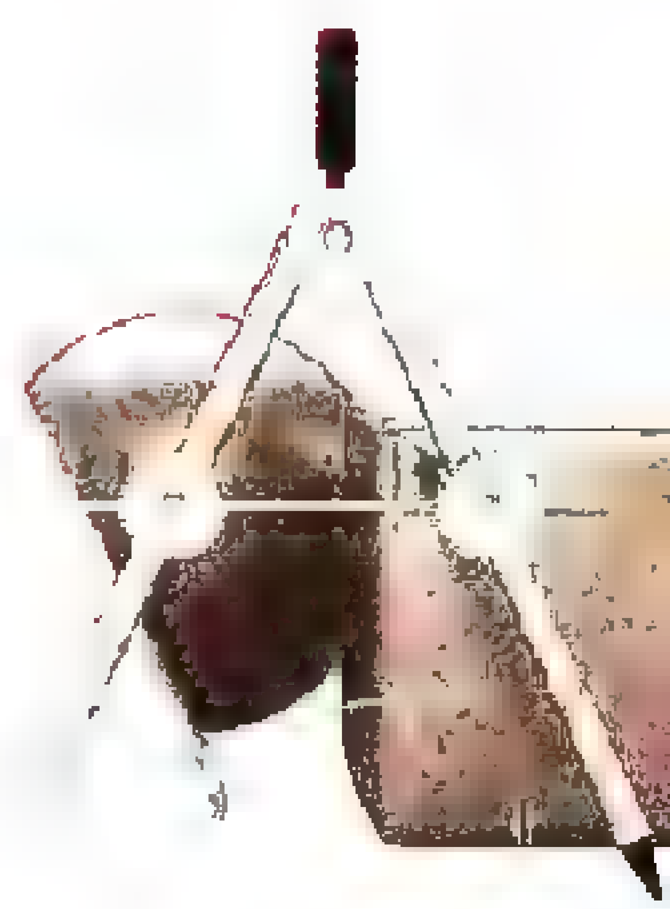
First — Algebra and Statistics

- **18 Accumulative tests**
- **Monthly tests**
- **Important questions**
- **Final revision**
- **Final examinations :**
  - School book examinations (2 models + model for the merge students)
  - 15 schools examinations



Second — Geometry

- **9 Accumulative tests**
- **Monthly tests**
- **Important questions**
- **Final revision**
- **Final examinations :**
  - School book examinations (2 models + model for the merge students)
  - 15 schools examinations



First | Algebra and Statistics

Accumulative Tests

on Algebra and Statistics

Accumulative tests on Algebra and Statistics

Accumulative test 1 on lesson 1 – unit 1

1 Choose the correct answer from the given ones :

1. $\sqrt[3]{2\frac{10}{27}} = \ldots\ldots\ldots\ldots$

(a) $\frac{3}{4}$ (b) $\frac{10}{3}$ (c) $\frac{4}{3}$ (d) $\frac{20}{27}$

2. $\sqrt[3]{\ldots\ldots} + \sqrt[3]{27} = \sqrt{64}$

(a) 25 (b) -125 (c) 125 (d) 5

3. If $\sqrt[3]{x} = \frac{1}{4}$, then $x = \ldots\ldots\ldots$

(a) $\frac{1}{2}$ (b) $\frac{1}{16}$ (c) $\frac{1}{64}$ (d) $\frac{1}{12}$

4. $\sqrt[3]{x^6} = \sqrt{\ldots\ldots\ldots}$

(a) x (b) x^2 (c) x^3 (d) x^4

2 Complete the following :

1. $\sqrt[3]{\ldots\ldots\ldots} = -\sqrt{4}$

2. $\sqrt[3]{125} - \sqrt{25} = \ldots\ldots$

3. If $\sqrt[3]{x} = 3$, then $\sqrt{x-2} = \ldots\ldots\ldots$

4. The cube whose volume is 8 cm.^{3} , then its edge length $= \ldots\ldots\ldots$ cm.

3 Find the S.S. of each of the following equations in $\mathbb{Q}$:

1. $x^3 + 1 = \text{zero}$

2. $8x^3 + 7 = 8$

Accumulative test 2 till lesson 2 – unit 1

1 Choose the correct answer from the given ones :

[1] $\sqrt{6} \in$ ....

(a) $\mathbb{N}$ (b) $\mathbb{Q}$ (c) $\mathbb{Q}'$ (d) $\mathbb{Z}$

[2] The irrational number located between 2 and 3 is ....

(a) $\sqrt{7}$ (b) $\sqrt{10}$ (c) 2.5 (d) $\sqrt{3}$

[3] The nearest integer to $\sqrt[3]{-28}$ is ....

(a) 4 (b) -30 (c) -3 (d) 3

[4] If $X = \sqrt{2}$, $y = 2$, then which of the following does not represent a rational number ?

(a) $X^2 + y$ (b) $X + y^2$ (c) $\sqrt{x^2 y}$ (d) $\sqrt{2}\, X\, y$

---

2 Complete the following :

[1] $\sqrt{4} - \sqrt[3]{-8} =$ ....

[2] If $X < \sqrt{7} < X + 1$, $X \in \mathbb{Z}$, then $X =$ ....

[3] If the volume of a cube is 125 cm^3 , then the area of one of its faces is ....

[4] The sum of the two square roots of $\frac{25}{16}$ equal ....

---

3 [a] **Prove that :** $\sqrt{5}$ is included between 2.2 and 2.3

[b] **Without using the calculator , prove that :** $\sqrt[3]{15}$ is included between 2.4 and 2.5

Accumulative test 3 till lesson 3 – unit 1

1 Choose the correct answer from the given ones :

1. $\mathbb{R}_+ \cup \mathbb{R}_- = \dots\dots$

 (a) $\mathbb{R}_+$ (b) $\mathbb{R}_-$ (c) $\mathbb{R}^*$ (d) $\mathbb{R}$

2. The irrational number located between 4 and 5 is $\dots\dots$

 (a) $\sqrt{8}$ (b) $4\sqrt{2}$ (c) $3\sqrt{2}$ (d) $\sqrt{10}$

3. $\sqrt[3]{9} \dots\dots \sqrt{4}$

 (a) $>$ (b) $<$ (c) $=$ (d) $\leq$

4. Which of the following rational numbers is located between $\frac{1}{5}$ and $\frac{2}{5}$?

 (a) $\frac{2}{10}$ (b) $\frac{1}{10}$ (c) 0.3 (d) -0.3

5. If $\frac{1}{a}$, $\frac{a}{\sqrt{5}}$ are two real numbers included between zero and 1 , then a can equal $\dots\dots$

 (a) -2 (b) 1 (c) $\sqrt{5}$ (d) 2

2 Complete the following :

1. $\mathbb{Q} \cap \mathbb{Q}' =$

2. The S.S. of $x^2 + 4 =$ zero in $\mathbb{R}$ is

3. $\sqrt{4} - \sqrt[3]{-8} = \dots\dots$

4. $\mathbb{R}_- \cap \mathbb{R}^+ = \dots\dots$

5. The S.S. of $x^3 - 8 =$ zero in $\mathbb{R}$ is $\dots\dots$

Accumulative test 4 till lesson 4 – unit 1

1 Choose the correct answer from the given ones :

1 {The multiplicative identity element , 3} .... [0 , 3]

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

2 $\mathbb{R}$ = ...

(a) $\mathbb{R}_+ \cup \mathbb{R}_-$ (b) $]-\infty , \infty[$ (c) $]-\infty , 0]$ (d) $[0 , \infty[$

3 If $\sqrt{4} - \sqrt[3]{X} = 5$, then X = ...

(a) 125 (b) 27 (c) -27 (d) 3

4 If X is a negative number , then which of the following numbers is positive ?

(a) X^3 (b) $2X$ (c) X^2 (d) $\frac{X}{2}$

2 Complete the following :

1 $[3 , 5] -]3 , 5[$ = ...

2 $]1 , \infty[\cup]-\infty , 1[$ = ...

3 The sum of the real numbers in the interval $]-4 , 4]$ equals ....

4 $\mathbb{Q} \cup \mathbb{Q}'$ = ...

3 If $X = [2 , 5]$, $Y = [0 , 3]$

1 Write X using the description method.

2 Represent X , Y on the number line.

3 Find $X - Y$ as an interval by using the number line. Is $\sqrt{29} \in X - Y$?

4 If $X = [-1 , 4]$, $Y = [3 , \infty[$

, find using the number line each of : $X \cup Y$, $X \cap Y'$, $Y - X$

Accumulative test 5 till lesson 5 unit 1

1 Choose the correct answer from the given ones :

1. $\frac{6}{\sqrt{3}} = \cdots\cdots$

(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{3}$ (d) $6\sqrt{3}$

2. $\mathbb{Q} \cup \mathbb{Q}' = \cdots\cdots$

(a) zero (b) $\mathbb{Z}$ (c) $\mathbb{R}$ (d) $\varnothing$

3. The rectangle whose dimensions are $(\sqrt{7}-1)$ cm. , $(\sqrt{7}+1)$ cm. , its area is $\cdots\cdots$ cm.^{2}

(a) 8 (b) 7 (c) 6 (d) $2\sqrt{7}$

4. If $X = \sqrt{2}+3$, $y = \sqrt{2}-3$, then $X^2 - y^2 = \cdots\cdots$

(a) $2\sqrt{3}$ (b) $12\sqrt{2}$ (c) $6\sqrt{5}$ (d) $3\sqrt{6}$

2 Complete the following :

1. $[-1 , 5] \quad \{ 1 , 5\} =$

2. The cube whose volume is 64 cm.^{3} , its total area = $\cdots\cdots$ cm.^{2}

3. The multiplicative inverse of $\frac{2\sqrt{3}}{5}$ is $\frac{\quad}{6}$

4. If $X = \sqrt[3]{3}+1$, $y = \sqrt[3]{2}-1$, then $(X+y)^3 = \cdots\cdots$

3 If $y = \sqrt{2-\sqrt{3}}$, **find the value of :** $y^4 - 2y^2 + 1$

4 If $a = 5-\sqrt{3}$, $b = 5+\sqrt{3}$, **find in the simplest form "Showing steps"**

1. $a\,b$ 2. $a^2 + b^2$

Accumulative test 6 till lesson 6 – unit 1

1 **Choose the correct answer from the given ones :**

1. The multiplicative inverse of the number $\sqrt{32}$ is ....

   (a) $4\sqrt{2}$ (b) $\frac{\sqrt{2}}{8}$ (c) $2\sqrt{5}$ (d) $\frac{\sqrt{3}}{2}$

2. $\sqrt{5}$, $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, ... "In the same pattern"

   (a) $\sqrt{75}$ (b) $\sqrt{90}$ (c) $\sqrt{112}$ (d) $\sqrt{125}$

3. $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}} =$ ....

   (a) $2\sqrt{2}$ (b) 1 (c) $\sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$

4. If $x = 2\sqrt{2} - \sqrt{7}$, $y = 2\sqrt{2} + \sqrt{7}$, then $x\,y - 1 =$ ....

   (a) 5 (b) zero (c) -4 (d) 7

2 **Complete the following :**

1. $]1 , 3] \cap [2 , 5] =$ ....
2. The S.S. of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is ....
3. The additive inverse of the number $3 - \sqrt{5}$ is ....
4. $\sqrt{18} - \sqrt{50} + \sqrt{2} =$ ....

3 If $A =]-\infty , 3[$, $B = [-2 , 5]$, **find using the number line each of :**

1. $A \cap B$ 2. $B - A$

4 **Simplify each of the following to the simplest form :**

1. $\sqrt{18} + \sqrt{54} - 3\sqrt{2} - \frac{1}{2}\sqrt{24}$
2. $\sqrt{128} - \frac{14}{\sqrt{2}} + 6\sqrt{\frac{1}{2}} - (\sqrt{2})^5$

Accumulative test 7 till lesson 7 – unit 1

1 Choose the correct answer from the given ones :

1. The multiplicative inverse of the number : $1-\sqrt{2}$ is ....
   (a) $\sqrt{2}-1$ (b) $1-\sqrt{2}$ (c) $-\sqrt{2}-1$ (d) $1+\sqrt{2}$
2. If $x=2+\sqrt{5}$, $y=2-\sqrt{5}$, then $(x-y)^2=$ ....
   (a) $2\sqrt{8}$ (b) 20 (c) $4\sqrt{5}$ (d) -1
3. $\sqrt{16}\cdot\sqrt[3]{-64}=$ ....
   (a) zero   (b) 12 (c) 8 (d) -8
4. The irrational number included between 3 and 6 is ....
   (a) $\sqrt{5}$ (b) $\sqrt{10}$ (c) $\sqrt{25}$ (d) $\sqrt[3]{27}$

---

2 Complete the following :

1. If $x\in\mathbb{R}_-$, $x^2=5$, then $\left(x+\sqrt{5}\right)^2=$ ....
2. The multiplicative inverse of the number $\frac{\sqrt{3}}{3}$ is ....
3. $\mathbb{R}^*=\mathbb{R}-$ ....
4. $\left(\sqrt{7}-\sqrt{5}\right)\left(\sqrt{7}+\sqrt{5}\right)=$ ....

---

3 [a] If $x\,y=1$, $y=2+\sqrt{3}$, **find the value of :** $x^2+\sqrt{48}$ in its simplest form.

[b] **Without using the calculator , simplify the following to the simplest form :**

$$2\sqrt{5}\left(\sqrt{5}-2\right)+\sqrt{20}-10\sqrt{\frac{1}{5}}$$

---

4 If $x=\sqrt{5}+2$, $y=$ the multiplicative inverse of x , prove that x and y are conjugate numbers , **then find the value of :** $\left(\frac{x-y}{x+y}\right)^2$

Accumulative test 8 till lesson 8 – unit 1

1 **Choose the correct answer from the given ones :**

1. $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = \dots\dots\dots$

(a) 0 (b) 1 (c) -1 (d) 2

2. The S.S. of the equation : $x^3 = 27$ in $\mathbb{R}$ is $\dots\dots\dots$

(a) $\varnothing$ (b) $\{3\}$ (c) $\{-3\}$ (d) $\{0\}$

3. $x = \sqrt[3]{3} + 1$, $y = \sqrt[3]{3} - 1$, then $x + y = \dots\dots\dots$

(a) $3\sqrt[3]{6}$ (b) $2\sqrt[3]{3}$ (c) 3 (d) 0

4. $\frac{5}{4}\sqrt[3]{5} + \frac{3}{4}\sqrt[3]{5} = \dots\dots\dots$

(a) 5 (b) $\sqrt[3]{20}$ (c) $\sqrt[3]{5}$ (d) $\sqrt[3]{40}$

2 **Complete the following :**

1. If $X = [0 , \infty[$, then $\acute{X} = \dots\dots\dots$

2. $\sqrt{8} + \sqrt{2} = \dots\dots\dots$

3. $\sqrt[3]{16} + \sqrt[3]{2} = \dots\dots\dots$ (in the simplest form)

4. $\sqrt[3]{27} - \sqrt[3]{-27} = \dots\dots\dots$

3 **Simplify each of the following to the simplest form :**

1. $\sqrt[3]{54} + 4\sqrt[3]{\frac{1}{4}} - \sqrt[3]{-2}$

2. $\sqrt[3]{32} + 4\sqrt[3]{\frac{1}{2}} - \left(2\sqrt[3]{-2}\right)^2 + \left(\sqrt{2}\right)^{\text{zero}} - \left(\frac{2}{\sqrt{2}}\right)^2$

4 **[a]** If $X = [-2 , 3]$, $Y =]-\infty , 1]$, **find using the number line each of :**

1. $X \cap Y$ 2. $X - Y$

[b] If $x = \frac{6}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2} - 1}$, **find the value of :** $\left(y - \frac{1}{3}x\right)^2$

Accumulative test 9 till lesson 9 – unit 1

1 Choose the correct answer from the given ones :

1. If the volume of a sphere = $\frac{4}{3}$ π cm^3 , then its radius = ... cm.

   (a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\left(\frac{4}{3}\right)^{zero}$ (d) π

2. The volume of a cube is 512 cm^3 , then the perimeter of one face = ... cm.

   (a) 8 (b) 64 (c) 32 (d) 16

3. $\sqrt[3]{2}+\sqrt[3]{2}=$ ....

   (a) $\sqrt[3]{16}$ (b) $\sqrt[3]{8}$ (c) $\sqrt[3]{4}$ (d) $\sqrt[3]{2}$

4. A sphere and a cylinder are equal in volume and there radii are equal in length , then the height of the cylinder = ... the radius of the sphere.

   (a) 3 (b) 4 (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

2 Complete the following :

1. If $x=\sqrt{5}-3$, $y=\sqrt{5}+3$, then $xy=$ ....
2. A right circular cylinder with volume 90 π cm^3 and its height 10 cm. , then its base radius length = ...
3. $]-2 , 2[\cap \mathbb{N}=$ ....
4. If the dimensions of a cuboid are $\sqrt{5}$ cm. , $\sqrt{5}$ cm. and $\sqrt{5}$ cm. , then its volume = ... cm^3

3 [a] Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is 64 π cm^3

[b] If $x=\frac{4}{\sqrt{7}-\sqrt{3}}$, $y=\sqrt{7}-\sqrt{3}$

, **prove that :** x , y are two conjugate numbers , **then find the value of :** xy

4 [a] The volume of a sphere is 36 π cm^3 Calculate its surface area in terms of π

[b] **Simplify to the simplest form :** $\sqrt{125}$ $\sqrt[3]{250}+\frac{1}{2}\sqrt[3]{16}+\sqrt{20}$

Accumulative test 10 till lesson 10 – unit 1

1 **Choose the correct answer from the given ones :**

1 The S.S. of the equation : $x + 5 = |-5|$ in $\mathbb{N}$ is ....

(a) $\varnothing$ (b) $\{0\}$ (c) $\{10\}$ (d) $\{-10\}$

2 The S.S. of the equation : $\sqrt{3}\,x - 1 = 2$ in $\mathbb{R}$ is ....

(a) $\{2\sqrt{3}\}$ (b) $\{\sqrt{3}\}$ (c) $\{2\}$ (d) $\{2\sqrt{2}\}$

3 If three quarters of the volume of a sphere is 8π cm^3, then its radius length is .... cm.

(a) 64 (b) 8 (c) 4 (d) 2

4 The irrational number included between 2 and 3 is ....

(a) $2\frac{1}{2}$ (b) $\sqrt{10}$ (c) $\sqrt{7}$ (d) $\sqrt{3}$

2 **Complete the following :**

1 The S.S. of the inequality : $-x \le 2$ in $\mathbb{R}$ is ....

2 The S.S. of the inequality : $3x - 2 > 1$ in $\mathbb{R}$ is ....

3 The S.S. of the equation : $(x^2 + 3)(x^3 + 1) = 0$ in $\mathbb{R}$ is ....

4 If $[3, 8]$ is the S.S. of the inequality : $a \le x - 2 \le b$, then $a + b =$ ....

3 [a] The volume of a sphere is $\frac{99000}{7}$ cm^3, calculate its radius length. $\left(\pi = \frac{22}{7}\right)$

[b] **Find the S.S. of the inequality :** $-3 \le 2x + 1 < 7$ in $\mathbb{R}$ in the form of an interval, then represent the solution on the number line.

4 [a] If $X = [-1, 4[$, $Y = [2, 6]$ **using the number line find each of the following :**

1 $X \cup Y$ 2 $X \cap Y$

[b] **Find in $\mathbb{R}$ the S.S. of the inequality :** $x - 1 < 3 - x \le x + 5$ in the form of an interval and represent it on the number line.

Accumulative test 11 till lesson 1 unit 2

1 Choose the correct answer from the given ones :

1 The relation : $2X + y = 6$ is represented by a straight line intersects the y-axis at the point

(a) $(0 , -6)$ (b) $(0 , 6)$ (c) $(6 , 0)$ (d) $(3 , 0)$

2 The relation : $2X = 3y$ is represented by a straight line passing through the point ....

(a) $(2 , 3)$ (b) $\left(0 , \frac{3}{2}\right)$ (c) $(0 , 0)$ (d) $\left(\frac{2}{3} , 0\right)$

3 The S.S. of the equation : $X + 9 = |-5|$ in $\mathbb{R}$ is ....

(a) $\{0\}$ (b) $\varnothing$ (c) $\{-4\}$ (d) $\{4\}$

4 The point $(3k , 2k)$ lies on the straight line : $X - 3y = 9$, then $k =$ ....

(a) -3 (b) 1 (c) 0 (d) 2

2 Complete the following :

1 $\left(\sqrt{8}+\sqrt{2}\right)\left(\sqrt{8}-\sqrt{2}\right) =$ ....

2 If $(3 , 6)$ satisfies the relation : $2X + y = a$, then $a =$ ....

3 $X = 4$ is represented by a straight line parallel to axis.

4 The volume of a sphere $= \frac{9}{2}\pi$ cm^3 , then its diameter length = ... cm.

3 [a] **Find four ordered pairs satisfying the relation :** $y + 2X = 5$

[b] **Without using the calculator , simplify the following to the simplest form :**

"Showing steps" $\sqrt{12}+\sqrt[3]{54}+3\sqrt{\frac{1}{3}}\quad 6\sqrt[3]{\frac{1}{4}}$

4 [a] **Find in $\mathbb{R}$ the S.S. of the inequality :**

$-2X + 5 \le X - 4$ and represent it on the number line.

[b] **Graph the relation :** $X - 4y = 4$ and if the straight line representing the relation intersects the X-axis at the point A and the y-axis at the point B , find the area of the triangle OAB where O is the origin point.

Accumulative test 12 till lesson 2 – unit 2

1 Choose the correct answer from the given ones :

1 Which of the following ordered pairs satisfies the relation : $y = -x - 3$?

(a) $(1 , -4)$ (b) $(2 , -5)$ (c) $(1 , -2)$ (d) $(3 , 0)$

2 $\sqrt{16} - \sqrt[3]{-64} = \dots\dots\dots$

(a) zero (b) 12 (c) 8 (d) -8

3 The slope of the vertical straight line is ……………

(a) zero (b) 1 (c) -1 (d) undefined.

4 If the slope of the straight line representing the relation : $x + m y = 5$ is undefined , then $m = \dots\dots\dots$

(a) 1 (b) -1 (c) 5 (d) zero

2 Complete the following :

1 The cube whose volume is 27 cm^3 , then its lateral area = …………… cm^2

2 The slope of the straight line perpendicular on y-axis is ……………

3 If the straight line which passes through the two points $(3 , k)$, $(4 , 5)$ is parallel to x-axis , then $k = \dots\dots\dots$

4 The straight line which passes through the two points $(5 , -5)$, $(5 , 5)$ is parallel to ………… axis.

3 [a] Represent graphically , then find the slope of the straight line that represents the relation : $x + y = 7$

[b] **Find the S.S. in $\mathbb{R}$ for the inequality :**

$2x + 3 \le 5x + 3 \le 2x + 9$, then represent it on the number line.

4 [a] Prove that the points A , B and C are collinear where A $(2 , -3)$, B $(4 , -5)$ and C $(0 , -1)$

[b] **Simplify to the simplest form :** $5\sqrt{8} + 4\sqrt[3]{\frac{1}{4}}\ \ 2\sqrt{50} - \sqrt[3]{16}$

Accumulative test 13 till lesson 3 unit 2

1 **Choose the correct answer from the given ones :**

1 If 100 gm. of food have 300 calories , then the number of calories that exist in 30 gm. of the same food equals .... calories.

(a) 90 (b) 100 (c) 900 (d) 9000

2 $\sqrt{9}$ .... $]-3 , \infty[$

(a) $\subset$ (b) $\not\subset$ (c) $\in$ (d) $\notin$

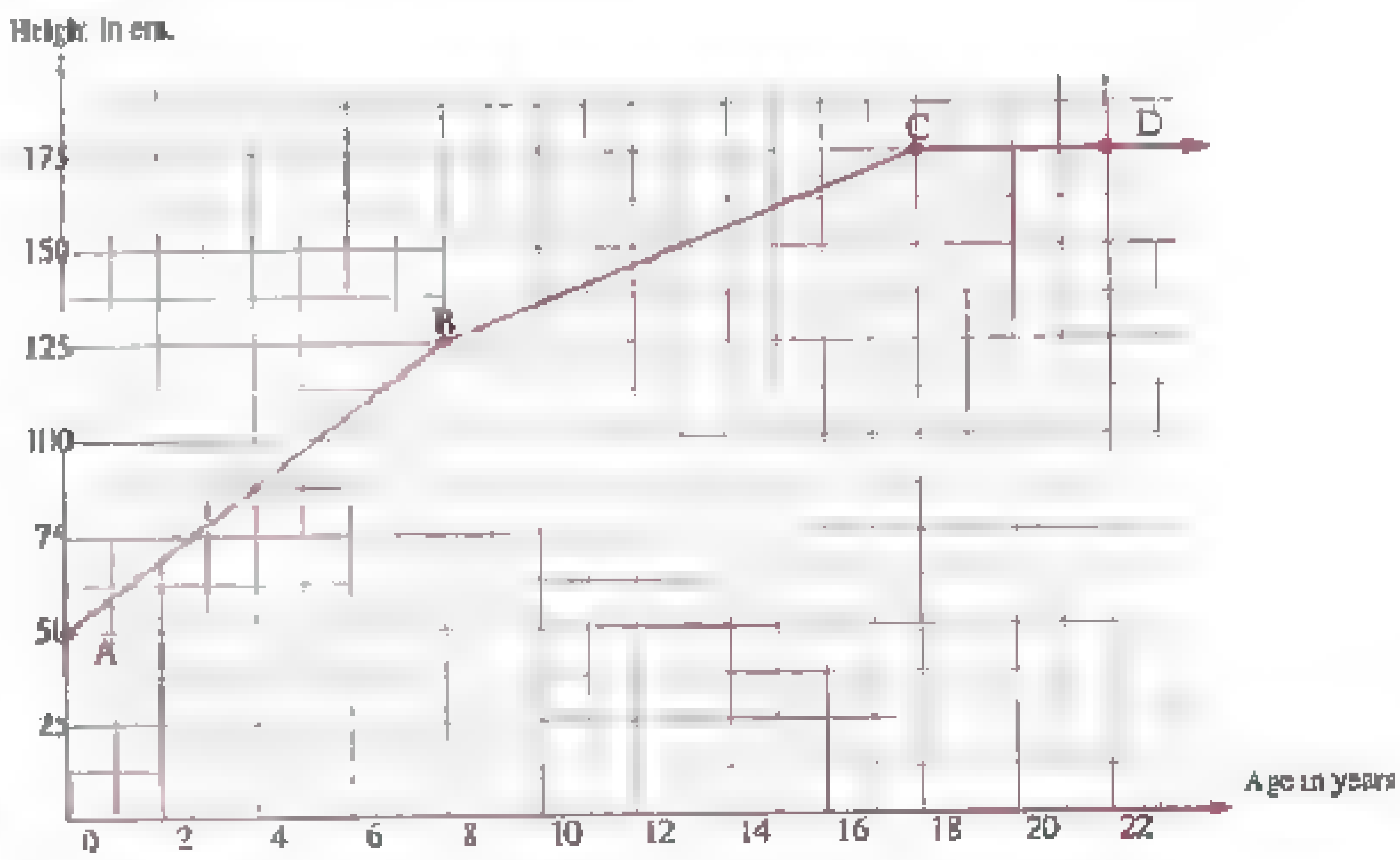
2 **Complete the following :**

1 The slope of the straight line parallel to X-axis is ....

2 The intersection point of the two straight lines $X = 2$, $y = 3$ is ....

3 If the slope of the straight line which passes through the two points $(2 , k)$, $(3 , -1)$ is 2 , then $k =$ ....

3 **The opposite figure shows the relation between the height of a person (in cm.) and his age (in years) :**



1 Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$

2 Calculate the difference between the height of this person when he was 8 years old and his height when he was 20 years old.

4 [a] A right circular cylinder , its diameter length is 14 cm. and its height is 10 cm. , find the lateral area and the volume of the cylinder. $\left(\pi = \frac{22}{7}\right)$

[b] Represent graphically the relation : $y - 2X = 1$, then find the points of intersection of the straight line with the two axes.

5 If $\mathbb{R}_+$ is the set of the positive real numbers and $Z = [-2 , 3]$, find :

1 $\mathbb{R}_+ \cap Z$ 2 $\mathbb{R}_+ \cup Z$ 3 $\overline{\mathbb{R}}_+$

Accumulative test 14 till lesson 1 – unit 3

1 Choose the correct answer from the given ones :

1. The irrational number included between 3 and 4 is ....

(a) 1.5 (b) $\sqrt{5}$ (c) $\sqrt{11}$ (d) 3.5

2. $\sqrt{50} - \sqrt{8} =$ ....

(a) $\sqrt{42}$ (b) $3\sqrt{2}$ (c) $2\sqrt{3}$ (d) $\sqrt{58}$

---

2 Complete the following :

1. If (3 , a) satisfies the relation : $2x + y = 10$, then a = ...
2. The slope of the straight line which passes through (3 , 2) and (5 , 6) is ....

---

3 The following table shows the marks obtained by 30 students in an examination :

5	9	11	4	9	9	16	7	8	12	2	10	7	12	5
8	15	13	13	9	7	14	19	3	11	14	3	12	13	7

Form the frequency table to these data.

---

4 [a] **Find the S.S. in $\mathbb{R}$ of the following inequality as an interval :** $1 < 3x - 2 < 13$

[b] **Find three ordered pairs satisfying the relation :**

$y = x + 2$, then represent it graphically.

---

5 [a] **Simplify to the simplest form :** $4\sqrt{\frac{1}{2}} + \sqrt{32} - \sqrt{72}$

[b] If $x = \sqrt{5} + 2$, $xy = 1$

, **find :** y , then prove that x and y are two conjugate numbers.

Accumulative test 15 till lesson 2 – unit 3

1 **Choose the correct answer from the given ones :**

1 The figure 

represents the interval

(a) $[3 , 7[$ (b) $[3 , 7]$ (c) $]3 , 7[$ (d) $]3 , 7]$

2 If the point $(3 , a)$ lies on the straight line : $y + 2X = 5$, then $a =$ ....

(a) 1 (b) -1 (c) 11 (d) zero

---

2 **Complete the following :**

1 The slope of the straight line passes through $(3 , 8)$ and $(-1 , 4)$ is ......

2 If $X + y = X y = \sqrt{5}$, then $X^2 y + y^2 X =$ ....

---

3 **The following table is the frequency distribution of wages of 100 workers weekly :**

Sets	50 –	60 –	70 –	80 –	90 –	Total
Frequency	5	15	30	40	10	100

1 Find the number of workers whose wages are less than 70 pounds weekly.

2 Graph the ascending cumulative frequency curve.

---

4 **Find the S.S. in $\mathbb{R}$ of each of the following :**

1 $2\sqrt{2}\, X - 1 = 3$

2 $-5 \leq 2X - 3 < 7$ and represent the S.S. on the number line.

---

5 [a] Find the slope of $\overleftrightarrow{AB}$, where : $A(-1 , 3)$ and $B(2 , 5)$, is the point $C(8 , 1) \in \overleftrightarrow{AB}$?

[b] If $X = [2 , \infty[$ and $Y =]-2 , 3[$, find using the number line :

1 $X \cap Y$ 2 $X \cup Y$

3 $X - Y$

Accumulative test 16 till lesson 3 – unit 3

1 **Choose the correct answer from the given ones :**

1 The arithmetic mean of a frequency distribution equals ....

(a) $\frac{\text{sum of } (X \times f)}{\text{sum of } f}$ (b) $\frac{\text{sum of } (X + f)}{\text{sum of } f}$

(c) $\frac{\text{sum of } f \times \text{sum of } X}{\text{sum of } f}$ (d) sum of $(f + X) \times \frac{2}{\text{sum of } f}$

2 The arithmetic mean of the values : 18 , 23 , 2 k – 1 , 29 , k is 18 , then k = ...

(a) 1 (b) 7 (c) 29 (d) 90

3 $\sqrt{a} + \sqrt{18} = 4\sqrt{2}$ if a = ...

(a) $\sqrt{2}$ (b) zero (c) 2 (d) 3

4 If the arithmetic mean of the lengths of a triangle equals 12 cm. , then its perimeter = ... cm.

(a) 4 (b) 36 (c) 24 (d) 48

2 **Complete the following :**

1 The slope of the straight line parallel to X-axis is ....

2 If the lower limit of a set is 10 and its centre is 15 , then its upper limit is ....

3 The conjugate of the number : $\sqrt{2} + 1$ is ....

4 The arithmetic mean of the two values : $\sqrt{5}$ and $\sqrt{45}$ is ....

3 **The following table shows the frequency distribution of extra wages weekly for 100 workers in a factory :**

Extra wages in pounds	20 –	30 –	40 –	50 –	X –	70 –
Number of workers	10	k	22	26	20	8

1 Calculate the value of each of X and k

2 Find the arithmetic mean of this distribution.

4 [a] A right circular cylinder of volume is 924 cm.^{3} and its height 6 cm.

Calculate the diameter length of its base $\left(\pi = \frac{22}{7}\right)$

[b] If $X = \frac{4}{\sqrt{7} - \sqrt{3}}$, $y = \frac{4}{\sqrt{7} + \sqrt{3}}$, put X and y in the simplest form , **then find the value of : $X^2 y^2$**

Accumulative test 17 till lesson 4 – unit 3

1 Choose the correct answer from the given ones :

1. The order of the median of the values : 5 , 7 , 6 , 4 and 8 is ....
   (a) third. (b) fourth. (c) fifth. (d) sixth.
2. The arithmetic mean of five numbers is 7 , then the sum of these numbers equals ....
   (a) 12 (b) 35 (c) 21 (d) 18
3. (3 , 2) does not satisfy the relation ....
   (a) $y + X = 5$ (b) $3y - X = 3$ (c) $y + X = 7$ (d) $2y - X = 1$
4. The intersection point of the ascending and descending cumulative curves is (30 , 50) , then the sum of frequencies is ....
   (a) 30 (b) 50 (c) 100 (d) 60

2 Complete the following :

1. The median of the values : 7 , 3 , 9 , 1 and 5 is ....
2. The dimensions of a cuboid are $\sqrt{2}$ cm. , $\sqrt{3}$ cm. and $\sqrt{6}$ cm , then its volume .... cm^3
3. If the order of the median of a set of values is the fourth , then the number of these values equals ....
4. The median of the values : 34 , 23 , 25 , 40 , 22 and 4 is ....

3 [a] **Graph the relation : $y = 2 - X$**

[b] **The following table shows a frequency distribution :**

Sets	20 –	30 –	40 –	50 –	60 –	70 –	Total
Frequency	10	k	22	25	20	8	100

Find : 1 The value of k

2 The median using the descending cumulative frequency.

4 [a] **Find the S.S. of the inequality : $-3 \le 2 - 5X \le 12$ where $X \in \mathbb{R}$**

[b] **If $X =]3 , 7]$, $Y = [5 , \infty[$ by using the number line find :**

1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

Accumulative test 18 till lesson 5 – unit 3

1 Choose the correct answer from the given ones :

1 The most common value or the most repeated value of a set of values is ....

(a) the arithmetic mean. (b) the median.

(c) the mode. (d) the range.

2 The arithmetic mean of the values : k , – k , 3 k equals ....

(a) 3 k (b) 2 k (c) – k (d) k

3 $\sqrt{4} - \sqrt[3]{8} = $ ....

(a) 4 (b) – 2 (c) zero (d) – 4

4 The mode of the values : 8 , $\sqrt{8}$, $\sqrt[3]{8}$, $2\sqrt{2}$ is ....

(a) 8 (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

2 Complete the following :

1 The mode of the values : 3 , 6 , 5 , 6 , 3 , 6 , 5 is ....

2 If (a , 3 a) satisfies the relation : $2X + y = 15$, then a = ...

3 If the volume of a sphere is 36π cm^3 , then its radius length = ... cm.

4 If the mode of the values : $X + 1$, 4 , 5 , 7 is 4 , then X = ...

3 [a] **Find the S.S. of the equation : $\sqrt{7}\,X + 1 = 8$ in $\mathbb{R}$**

[b] **Reduce to the simplest form : $\left(\sqrt{5} - \sqrt{2}\right)^2 + \sqrt{40}$**

4 [a] Find the value of y such that the straight line passing through the two points (3 , 4) and (2 , y) is parallel to the X-axis.

[b] **The following table shows the frequency distribution with equal range sets for the weekly wages of 100 workers in a factory :**

Sets of wages in L.E.	70–	80–	90–	100–	X–	120–	130
Number of workers	10	13	k – 4	20	16	14	11

Find : 1 The value of each of X and k

2 The mode of wages in L.E. by using the histogram.

Monthly Tests

on Algebra and Statistics

October contents

- Unit One : Real numbers.
  From lesson (1) "Cube root of a rational number" to the end of the lesson of "Ordering numbers in ℝ"

November contents

- Unit One : Real numbers.
  From "Intervals" to the end of the lesson of "Applications on the real numbers"

October tests on Algebra and Statistics

Test 1

Total mark 10

1 Choose the correct answer from the given ones : (3 marks)

1. If $-\sqrt{25} = \sqrt[3]{y}$, then y = ...

(a) 5 (b) −5 (c) 125 (d) −125

2. The irrational number included between −2 and −1 is ....

(a) −3 (b) $-1\frac{1}{3}$ (c) $-\sqrt{3}$ (d) $\sqrt{2}$

3. $\mathbb{R}_+ =$ ....

(a)]0 , ∞[(b)]−∞ , 0[(c) [0 , ∞[(d)]−∞ , 0]

2 Complete the following : (3 marks)

1. $\mathbb{R}_+ \cup \mathbb{R}_- =$ ....

2. The S.S. of the equation : $\left(x-\sqrt{5}\right)\left(x+\sqrt{3}\right)=0$ in $\mathbb{Q}$ is ....

3. $[1 , 4] - \{1 , 4\} =$ ....

3 Prove that : $\sqrt{2}$ lies between 1.4 and 1.5 (2 marks)

4 The capacity of a cube is 27 litres. Find its inner edge length. (2 marks)

Test 2

Total mark

10

1 Choose the correct answer from the given ones : (3 marks)

1 $\mathbb{R}_+ \cap \mathbb{R}_- =$ ....

(a) $\mathbb{R}^*$ (b) $\mathbb{R}$ (c) $\mathbb{Q}$ (d) $\varnothing$

2 $\sqrt[3]{0.001 \times \frac{1}{8}} =$ ....

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{20}$ (d) 20

3 The irrational number included between 2 and 3 is .... cm^2

(a) $\sqrt{3}$ (b) $\sqrt{-1}$ (c) $\sqrt{7}$ (d) $2\frac{1}{2}$

2 Complete the following : (3 marks)

1 If $x^3 = 27$, then $x =$.

2 $\mathbb{Q} \cup \mathbb{Q}' =$ ....

3 The S.S. of the equation : $x^2 + 4 = 0$ in $\mathbb{R}$ is ....

3 Find in $\mathbb{R}$ the S.S. of the equation : $2 + x^3 = 1$ (2 marks)

4 If $X = [-1, 4]$ and $Y = [3, \infty[$ (2 marks)

, find the following using the number line :

$X \cup Y$, $Y - X$

November tests on Algebra and Statistics

Total mark 10

Test 1

Choose the correct answer from the given ones : (3 marks)

1. The S.S. of the equation : $\sqrt{3}\,X - 1 = 2$ in $\mathbb{R}$ is ....

(a) $\{2\sqrt{3}\}$ (b) $\{\sqrt{2}\}$ (c) $\{\sqrt{3}\}$ (d) $\{2, \sqrt{2}\}$

2. The conjugate number of the number $\frac{1}{\sqrt{3}-\sqrt{2}}$ is ....

(a) $\sqrt{3}+\sqrt{2}$ (b) $\sqrt{3}-\sqrt{2}$ (c) $\sqrt{3}\times\sqrt{2}$ (d) $-\sqrt{3}-\sqrt{2}$

3. The volume of the sphere whose diameter length is 6 cm. equals .... cm^3

(a) 288 (b) 12π (c) 36π (d) 288π

Complete the following : (3 marks)

1. The edge length of a cube is 4 cm. , then its total area = ... cm^2

2. The multiplicative inverse of the number $\frac{\sqrt{2}}{6}$ is ....

3. If $X^2 = \frac{8}{9}$, then X in the simplest form = ...

Simplify to the simplest form : $2\sqrt{18}+\sqrt{50}+\frac{1}{3}\sqrt{162}$ (2 marks)

Find in $\mathbb{R}$ the S.S. of the inequality : (2 marks)

$-1 < 3X + 5 < 11$, then represent it on the number line.

Test 2

Total mark

10

1 **Choose the correct answer from the given ones :** (3 marks)

1 If the dimensions of a cuboid are $\sqrt{2}$ cm. , $\sqrt{3}$ cm. and $\sqrt{6}$ cm.

, then its volume = ... cm^3

(a) $\sqrt{30}$ (b) $\sqrt{6}$ (c) 6 (d) $\sqrt{18}$

2 The S.S. of the inequality : $-X < 2$ in $\mathbb{R}$ is ....

(a) $]-\infty , 2[$ (b) $]-\infty , -2[$ (c) $[-2 , \infty[$ (d) $]-2 , \infty[$

3 The additive inverse of the number $\left(\sqrt{2}-\sqrt{5}\right)$ is ....

(a) $\sqrt{2}+\sqrt{5}$ (b) $\sqrt{5}-\sqrt{2}$ (c) $\sqrt{2}-\sqrt{5}$ (d) $-\sqrt{2}-\sqrt{5}$

2 **Complete the following :** (3 marks)

1 The multiplicative neutral in $\mathbb{R}$ is .... and the additive neutral in $\mathbb{R}$ is ....

2 $\sqrt{5}$, $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, ... (in the same pattern)

3 $\sqrt[3]{2}+\sqrt[3]{2}=$ ....

3 Find the lateral area of a right circular cylinder of volume 924 cm^3 if its height equals 6 cm.

$(\pi \simeq \frac{22}{7})$ (2 marks)

4 If $a=\sqrt{3}+\sqrt{2}$ and $b=\frac{1}{\sqrt{3}+\sqrt{2}}$ (2 marks)

, find in the simplest form the value of : a^2-b^2

Important Questions

on Algebra and Statistics

Important questions on Unit One ?

Real Numbers

First — Multiple choice questions

1. $\sqrt[3]{-27}+3=$ ....

(a) zero (b) 3 (c) 6 (d) -24

2. The irrational number located between 2 and 3 is ....

(a) $\sqrt{3}$ (b) $\sqrt{-1}$ (c) $\sqrt{7}$ (d) $2\frac{1}{2}$

3. $\sqrt{8}-\sqrt{2}=$ ....

(a) $\sqrt{2}$ (b) $\sqrt{6}$ (c) $2\sqrt{2}$ (d) 2

4. $[3 , 7]-[2 , 5]=$ ....

(a) $[5 , 7]$ (b) $]5 , 7[$ (c) $\{5 , 7\}$ (d) $]5 , 7]$

5. The S.S. of the equation : $x^3=8$ in $\mathbb{Q}$ is ....

(a) $\{-2\}$ (b) $\{2\}$ (c) $\{2 , -2\}$ (d) $\{64\}$

6. The multiplicative inverse of the number $\frac{\sqrt{3}}{6}$ is ....

(a) $\frac{-\sqrt{3}}{6}$ (b) $6\sqrt{3}$ (c) $2\sqrt{3}$ (d) $-2\sqrt{3}$

7. A right circular cylinder of volume 90π cm^3. and height 10 cm. , then the diameter length of its base is ....

(a) 2 cm. (b) 4 cm. (c) 6 cm. (d) 3 cm.

8. If $\sqrt{9}=\sqrt[3]{x}$, then $x=$ ....

(a) 27 (b) 64 (c) 64 (d) -27

9. $[3 , 5]-\{5\}=$ ....

(a) $[3 , 4]$ (b) $[3 , 5[$ (c) $\{3 , 4\}$ (d) $]3 , 5]$

10. The volume of the cuboid whose dimensions are $\sqrt{2}$ cm. , $\sqrt{3}$ cm. and $\sqrt{6}$ cm. is .... cm^3.

(a) 6 (b) 36 (c) $6\sqrt{6}$ (d) $18\sqrt{2}$

11 The S.S. of the inequality : $-x < 2$ in $\mathbb{R}$ is ....

(a) $]-\infty , 2[$ (b) $]-\infty , -2[$ (c) $]-2 , \infty[$ (d) $]2 , \infty[$

12 If $x < \sqrt[3]{36} < x + 1$, $x \in \mathbb{Z}$, then $x =$ ....

(a) 2 (b) 3 (c) 4 (d) 6

13 $\mathbb{R} =$ ....

(a) $[0 , \infty[$ (b) $]-\infty , \infty[$ (c) $]-\infty , 0[$ (d) $[1 , \infty[$

14 The volume of the sphere whose diameter length is 6 cm. equals .... cm^3.

(a) 9π (b) 12π (c) 36π (d) 228π

15 If the volume of a sphere is $\frac{500}{3}\pi$ cm^3 , then its radius length is .... cm.

(a) 10 (b) $\frac{5}{3}$ (c) 5 (d) π

16 If three quarters of the volume of a sphere equals 8π cm^3 , then the length of its radius equals .... cm.

(a) 64 (b) 8 (c) 4 (d) 2

17 The volume of a cube is 64 cm^3 , then its lateral area is .... cm^2.

(a) 4 (b) 8 (c) 64 (d) 96

18 A right circular cylinder whose base radius length is r cm. and its height equals the length of its diameter , then its volume is .... cm^3.

(a) πr^3 (b) πr^2 (c) $2\pi r^3$ (d) $2 r^3$

19 $\mathbb{Q} \cap \acute{\mathbb{Q}} =$ ....

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{Q}$ (d) $\acute{\mathbb{Q}}$

20 The S.S. of the equation : $x^2 + 9 = 0$ in $\mathbb{R}$ is ....

(a) $\{-9\}$ (b) $\{-3\}$ (c) $\{3 , -3\}$ (d) $\varnothing$

21 The S.S. of the inequality : $-1 \leq -x < 1$ in $\mathbb{R}$ is ....

(a) $]-1 , 1]$ (b) $[-1 , 1[$ (c) $]-1 , 1[$ (d) $[-1 , 1]$

22 The edge length of the cube whose volume is $2\sqrt{2}$ cm^3 equals .... cm.

(a) $\sqrt{2}$ (b) 2 (c) 8 (d) 1.5

The S.S. of the equation : $(x^2+4)(x^2-9)=0$ in $\mathbb{R}$ is ....

(a) $\{3\}$ (b) $\{-3\}$ (c) $\{3, -3\}$ (d) $\varnothing$

A right circular cylinder whose radius length is 7 cm. and its height is 5 cm. , then its lateral area is .... cm^2

(a) 50π (b) 70π (c) 9π (d) 35π

$\mathbb{R}_+ \cup \mathbb{R}_- =$ ....

(a) $\varnothing$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\mathbb{R}-\{0\}$

If $x > 5$, then $\sqrt{(5-x)^2} =$ ....

(a) $5-x$ (b) $\sqrt{5}-\sqrt{x}$ (c) $x-5$ (d) $x+5$

Second Complete questions

$\mathbb{Q} \cup \mathbb{Q}' =$ ....

$\mathbb{Q} \cap \mathbb{Q}' =$ ....

If $\sqrt{a} = 2\sqrt{3}$, then a = ...

$\{-1, 0, 1\} \cap]-1, 1[=$ ....

$[3, 4] - \{3, 5\} =$ ....

The multiplicative inverse of the number $(\sqrt{3}+\sqrt{2})$ in the simplest form is ....

If $\frac{1}{x} = \sqrt{5}-2$, then the value of x in the simplest form is ....

$\mathbb{R}_+$ in an interval form is ....

$\sqrt{27} \cdot \sqrt{3} = \sqrt{\text{..............}}$

The irrational number $\sqrt{10}$ lies between the two consecutive integers .... and ....

$\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \text{..............}$ «in the same pattern»

The sum of lengths of all edges of a cube is 48 cm. , then its volume is ....

13 The volume of the sphere whose diameter length is 2 cm. is ………… π cm^3.

14 If the volume of a sphere = $\frac{9}{2}$ π cm^3, then its diameter length = ………… cm

15 The rectangle whose dimensions are $\left(3-\sqrt{5}\right)$ cm. and $\left(3+\sqrt{5}\right)$ cm. its perimeter = …………

16 The circumference of a circle is $4\sqrt{5}\,\pi$ cm., then its area is ………… cm^2.

17 $\sqrt{9+16} = 3 +$ …………

18 The additive inverse of the number $\sqrt[3]{-8}$ is …………

19 If $X = \sqrt{3}+2$, $y = \sqrt{3}-2$, then $(X y , X + y) = ($ ………… , ………… $)$

20 A right circular cylinder whose volume is $125\,\pi$ cm^3, its height = its radius length , then its radius length is ………… cm.

21 $]-2 , 3] \cap \mathbb{R} =$ …………

22 $[-1 , 2[\cap \mathbb{Z} =$ …………

23 $\mathbb{R}_- \cap [-3 , 2[=$ …………

24 $\sqrt{a} + \sqrt{18} = 4\sqrt{2}$, if a = …………

Third Essay questions

1 If $X = [-3 , 2[$, $Y = [-1 , 5]$, find using the number line :

1 $X \cup Y$ 2 $X \cap Y$ 3 $X - Y$

2 If $A =]-\infty , 3[$, $B = [2 , 5]$

, find using the number line : $B - A$, $A \cap B$, $A \cup B$ and $\acute{A}$

3 If $X = \sqrt{3}+1$, $y = \frac{2}{\sqrt{3}+1}$, find the value of : $\frac{X y}{X - y}$

4 If $X = \frac{1}{\sqrt{5}+2}$, $y = \sqrt{5}+2$, prove that : X and y are conjugate numbers

, then find the value of : $X^2 y^2$ in its simplest form.

5 If $X = \sqrt{7} - \sqrt{5}$, $y = \frac{2}{\sqrt{7}-\sqrt{5}}$, find the value of : $X^2 + 2 X y + y^2$

6 If $x=\sqrt{7}+\sqrt{5}$, $y=\frac{2}{x}$, **prove that :** x and y are conjugate numbers , **then find the value of :** x^2+xy+y^2

7 If $x=\sqrt{4+\sqrt{7}}$, $y=\sqrt{4-\sqrt{7}}$, **find :** $(x+y)^2$ in the simplest form.

8 If $x=\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$, **prove that :** $x+\frac{1}{x}=22$

9 **Find in $\mathbb{R}$ the S.S. of the inequality :** $5x-3<2x+9$

10 **Find the solution set of the inequality in $\mathbb{R}$, then represent it on the number line :** $5-3x>11$

11 **Find the solution set in $\mathbb{R}$, then represent it on the number line :** $1<2x+3\leq 9$

12 **Find the solution set in $\mathbb{R}$, then represent it on the number line :** $\frac{x}{3}+2>3$

13 **Find in $\mathbb{R}$ the solution set of the inequality :** $4x+3>5x+2>4x$, then represent it on the number line.

14 **Simplify to the simplest form :** $\sqrt[3]{2}\left(\sqrt[3]{4}+2\right)-2\left(\sqrt[3]{2}+1\right)$

15 **Simplify to the simplest form :** $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$

16 **Simplify to the simplest form :** $\sqrt[3]{81}+\sqrt[3]{24}-3\sqrt[3]{\frac{1}{9}}$

17 **Find in the simplest form :** $\sqrt{12}+\sqrt[3]{54}-2\sqrt{3}-\sqrt[3]{16}$

18 **Find in the simplest form :** $\sqrt{50}+\sqrt[3]{54}-10\sqrt{\frac{1}{2}}-\sqrt[3]{16}$

19 **Find in $\mathbb{R}$ the solution set of the equation :** $(x-2)^3=125$

20 A sphere with volume $36\pi\ cm^3$, **find :**

1 The radius length of the sphere.

2 The area of the sphere in terms of π

21 Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is $27\pi\ cm^3$

22 A right circular cylinder whose base radius length is $4\sqrt{2}$ cm. and its height is 9 cm. Find its volume in terms of π.

23 A right circular cylinder whose height is 10 cm. and its base radius length is 7 cm. Calculate its volume and its lateral area $\left(\pi = \frac{22}{7}\right)$

24 A metallic sphere , with diameter length 6 cm. has got melt and changed into a right circular cylinder with base radius length 3 cm. Find its height

25 A metallic right circular cylinder whose base radius length is 3 cm. and its height is 4 cm. has got melt and changed into a sphere. Find the radius length of this sphere.

26 Find the volume of a right circular cylinder of lateral area 440 cm^2. and height 10 cm. $\left(\pi = \frac{22}{7}\right)$

Important questions on Unit Two

Relation between Two Variables

First Multiple choice questions

1 Which of the following ordered pairs satisfies the relation : $2x + y = 5$?

(a) $(-1 , 3)$ (b) $(1 , 3)$ (c) $(3 , 1)$ (d) $(2 , 2)$

2 The ordered pair which does not satisfy the relation : $2x + y = 5$ is ....

(a) $(1 , 3)$ (b) $(-1 , 7)$ (c) $(3 , 1)$ (d) $(4 , -3)$

3 If $(5 , 2)$ satisfies the relation : $x + 2y = c$, then $c =$ ....

(a) 8 (b) 9 (c) 7 (d) 6

4 If the point $(k , 2k)$ satisfies the relation : $y + 2x = 8$, then $k =$ ....

(a) 2 (b) 3 (c) 4 (d) 5

5 If the straight line which passes through the two points $(3 , k)$ and $(4 , 5)$ is parallel to the x-axis , then $k =$ ....

(a) 3 (b) 4 (c) 5 (d) -5

6 The slope of the straight line which passes through the two points $(4 , 5)$ and $(4 , 8)$ is ....

(a) zero. (b) undefined. (c) $\frac{2}{5}$ (d) $\frac{4}{5}$

7 If $A(-3 , 1)$ and $B(1 , 3)$, then the slope of $\overleftrightarrow{AB} =$ ....

(a) -1 (b) 2 (c) $\frac{1}{2}$ (d) 1

8 The relation : $2x + 7y = 14$ is represented by a straight line intersecting the x-axis at ....

(a) $(2 , 0)$ (b) $(0 , 2)$ (c) $(7 , 0)$ (d) $(0 , 7)$

9 The slope of the horizontal straight line is ....

(a) zero. (b) 1 (c) 2 (d) undefined.

10 Which of the following relations is represented by the x-axis ?

(a) $x = 0$ (b) $y = 0$ (c) $x = y$ (d) $y = -x$

Second Complete questions

1. The slope of the straight line passing through the two points $(-1, 4)$, $(0, 5)$ is ....

2. The slope of the straight line parallel to y-axis is ....

3. The slope of the straight line perpendicular to y axis is ...

4. If the straight line passing through the two points $(3, 4)$ and $(2, y)$ is parallel to X-axis , then $y =$ ....

5. The relation : $y = 5$ is represented by a straight line parallel to .... axis and its slope is ....

6. If $(-3, 2)$ satisfies the relation : $5X - k\,y = 7$, then $k =$ ....

7. If the points A , B and C are collinear , then the slope of $\overline{AB}$ = the slope of ....

8. If the two ordered pairs $(a, 2)$ and $(3, b)$ satisfy the relation : $X + 2y = 3$, then $a =$ .... and $b =$ ....

9. If $A = (3, y)$, $B = (6, 5)$ and the slope of $\overline{AB}$ equals $\frac{2}{3}$, then $y =$ ....

10. The relation : $8X + 3y = 24$ is represented graphically by a straight line intersecting y-axis at the point ....

Third Essay questions

1. Represent the relation : $y = X + 2$ graphically.

2. Represent the relation : $X + y = 3$ graphically , then find the point of intersection with X-axis.

3. Graph the relation : $y = 2X$, then find the slope of the straight line.

4. Find the two points of intersection of the straight line : $2X + 3y = 12$ with the coordinate axes.

5. **Prove that :** The points $A(-1, 3)$, $B(2, 4)$ and $C(5, 5)$ are collinear.

6 Are the points A $(2 , -1)$, B $(-1 , 3)$ and C $(2 , 3)$ collinear ?

7 If the slope of the straight line which passes through the two points $(4 , 17)$ and $(6 , k)$ is 4 , find the value of k

8 If $(k , 2k)$ satisfies the relation : $3x - y = 30$, find the value of k

9 Find the slope of $\overleftrightarrow{AB}$, where A $(-1 , 3)$ and B $(2 , 5)$, is the point C $(8 , 1) \in \overleftrightarrow{AB}$?

10 If $A = (3 , 3)$ and $B = (3 , 5)$, **prove that :** $\overleftrightarrow{AB}$ // y-axis.

Important questions on Unit Three ?

Statistics

First Multiple choice questions

1 The mode of the values : 3 , 4 , 10 , 4 is ....

(a) 3 (b) 4 (c) 20 (d) 10

2 The arithmetic mean of the numbers : 10 , 12 , 8 is ....

(a) 5 (b) 6 (c) 9 (d) 10

3 The median of the values : 34 , 23 , 25 , 40 , 22 , 4 is ....

(a) 22 (b) 23 (c) 24 (d) 25

4 The arithmetic mean of the values : $5 - X$, 5 , $5 + X$ is ....

(a) 5 (b) 81 (c) 13 (d) 3

5 If the arithmetic mean of the values : 5 , 7 , 8 , X is 6 , then $X =$ ....

(a) 8 (b) 5 (c) 7 (d) 4

6 If the order of the median of a set of values is the fourth , then the number of these values is ....

(a) 5 (b) 6 (c) 7 (d) 8

7 If the mode of the set of values : 15 , 11 , $5X$, 4 is 15 , then $X =$ ....

(a) 3 (b) 4 (c) 5 (d) 15

8 If the mode of the set of values : 12 , 7 , $X + 1$, 7 , 12 is 7 , then $X =$ ....

(a) 4 (b) 6 (c) 8 (d) 11

9 The centre of the first set of the sets : 7 – , 13 – , 19 – , 25 – is ....

(a) 6 (b) 7 (c) 10 (d) 13

10 If the lower limit of a set is 4 and the upper limit of the same set is 8 , then its centre is ....

(a) 2 (b) 4 (c) 6 (d) 8

11 If the lower limit of a set is 10 , its upper limit is X and its centre is 15 , then $X =$ ....

(a) 10 (b) 5 (c) 20 (d) 8

12 If the arithmetic mean of five numbers is 8 , then the sum of these numbers is ....

(a) 13 (b) 16 (c) 40 (d) 64

13 The point of intersection of the ascending and descending curves determines .... on the set-axis.

(a) the arithmetic mean (b) the median

(c) the mode (d) the frequencies

14 If the point (16 , 30) is the point of intersection of the ascending and descending curves , then the median is ....

(a) 16 (b) 23 (c) 60 (d) 30

15 If the order of the median of a frequency distribution is 50 , then the sum of frequencies is ....

(a) 50 (b) 25 (c) 100 (d) 5

Second Complete questions

1 The median of the values : 5 , 3 , 11 , 7 , 2 is ....

2 The mode of the values : 3 , 5 , 7 , 3 , 8 is ....

3 If the arithmetic mean of the values : 18 , 23 , 29 , 2 k – 1 , k is 18 , then k = ...

4 The order of the median of the values : 7 , 6 , 5 , 8 , 4 is ....

5 The arithmetic mean is one of the measures of ....

6 If the mode of the values : 4 , 11 , 8 , 2 X is 8 , then X = ...

7 If the point of intersection of the ascending and descending curves is (35 , 20) , then the sum of frequencies is ....

8 The arithmetic mean of the values : $2 - X$, 4 , 5 , $3 + X$, 1 is ....

9 If the order of the median of a set of values is the fifth and sixth , then the number of these values is ....

10 The point of intersection of the ascending and descending curves determines .... on the set-axis.

Third Essay questions

1 **The following frequency distribution shows the marks of 20 students in mathematics :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

Calculate the mean.

---

2 **The following table shows the frequency distribution of bonus of 100 workers :**

Bonus	20	30	40 –	50 –	n –	70 –
Number of workers	10	k	22	25	20	8

Find : 1 The value of each of n and k

2 The mean.

---

3 **The following frequency distribution shows the marks of 35 students in mathematics :**

Sets	2 –	4 –	6 –	8 –	10 – 12	Total
Frequency	5	8	10	8	4	35

Graph the histogram of that distribution and from the graph find the mode mark.

---

4 **The following table shows the frequency distribution of the weights of 20 children in kg. :**

Sets	5 –	15 –	25	35	45	Total
Frequency	3	4	7	4	2	20

Using the ascending or descending cumulative frequency curve , find the median of this distribution.

Final Revision

of Algebra and Statistics

Revision for the important rules of ? Algebra and Statistics

First Real numbers

Remember that

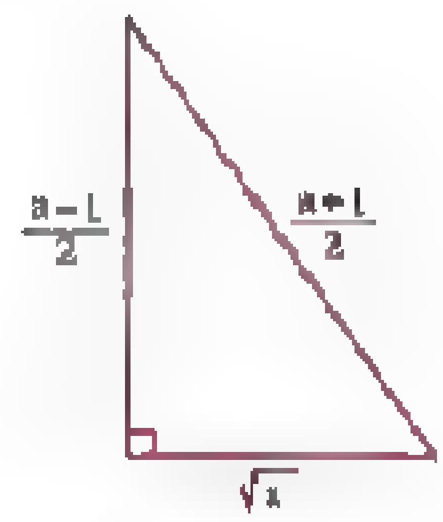
- $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$
- $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$
- $\mathbb{R} - \mathbb{Q} = \grave{\mathbb{Q}}$
- $\mathbb{R} - \grave{\mathbb{Q}} = \mathbb{Q}$
- $\mathbb{R}_+ \cap \mathbb{R}_- = \varnothing$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_-$
- $\pi \in \grave{\mathbb{Q}}$
- $\mathbb{R}^* = \mathbb{R} - \{0\}$

Remember The representing of the irrational number on the number line

Each irrational number can be represented by a point on the number line.

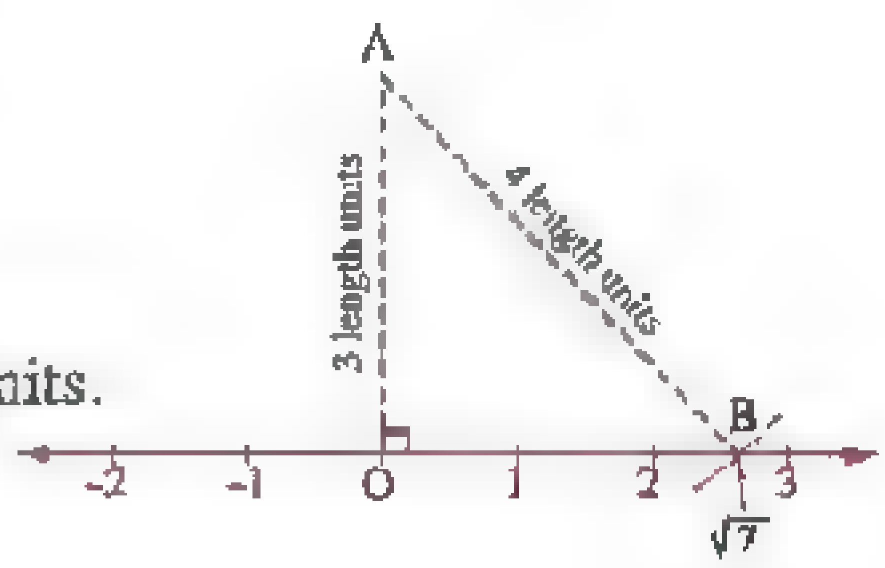
and to draw a line segment with length $= \sqrt{a}$ length unit where $a > 1$

Draw a right-angled triangle in which :



- The length of one side of the right-angle $= \frac{a-1}{2}$ length unit.
- The length of the hypotenuse $= \frac{a+1}{2}$ length unit.

and we can apply this to represent the irrational number $\sqrt{7}$ on the number line as the following :



- From the point which represents the number zero on the number line , we draw a perpendicular line segment as $\overline{OA}$ where $OA = \frac{7-1}{2} = 3$ length units.
- Using the compasses with a distance $= \frac{7+1}{2} = 4$ length units. and centre at A , draw an arc to cut the number line on the right side of the point O at the point B , then B is the point which represents $\sqrt{7}$ as in the figure.
- Notice that : To represent the number $\left(-\sqrt{7}\right)$, we draw the arc which cuts the number line on its left side , not on its right side.
- Notice that : To represent the number $\left(1+\sqrt{7}\right)$, we follow the same previous steps but we draw the perpendicular line segment $\overline{OA}$ from the point which represents the number 1 , not the number 0

Remember The operations on intervals

Intervals	Intersection	Union	Difference	Complement
$X = [-1, 5[$, $Y =]-3, 2[$	$X \cap Y = [-1, 2[$	$X \cup Y =]-3, 5[$	$X - Y = [2, 5[$, $Y - X =]-3, -1[$	$\grave{X} = \mathbb{R} - [-1, 5[$ $=]-\infty, -1[\cup [5, \infty[$
$X =]-\infty, 1]$, $Y = [-2, 1[$	$X \cap Y = [-2, 1[$	$X \cup Y =]-\infty, 1]$	$X - Y$ $=]-\infty, -2[\cup \{1\}$, $Y - X = \varnothing$	$\grave{X} =]1, \infty[$
$X = [-1, 5]$, $Y =]-1, 5[$	$X \cap Y =]-1, 5[$	$X \cup Y = [-1, 5]$	$X - Y = \{-1, 5\}$, $Y - X = \varnothing$	$\grave{Y} = \mathbb{R} -]-1, 5[$ $=]-\infty, -1] \cup [5, \infty[$
$X =]-3, 4]$, $Y = \{-3, 4\}$	$X \cap Y = \{4\}$	$X \cup Y = [-3, 4]$	$X - Y =]-3, 4[$, $Y - X = \{-3\}$	$\grave{Y} = \mathbb{R} - \{-3, 4\}$

Remember : The operations on the square roots and the cube roots

The square roots

① $\sqrt{a} \times \sqrt{b} = \sqrt{a\,b}$ *For Example :* $\sqrt{3} \times \sqrt{12} = \sqrt{3 \times 12} = \sqrt{36} = 6$

② $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ (where $b \neq 0$) *For Example :* $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$

③ $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{a\,b}}{b}$ (where $b \neq 0$) *For Example :* $\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$

The cube roots

① $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{a\,b}$ *For Example :* $\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt[3]{3 \times 9} = \sqrt[3]{27} = 3$

② $\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}}$ (where $b \neq 0$) *For Example :* $\frac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\frac{32}{4}} = \sqrt[3]{8} = 2$

Example Simplify to the simplest form :

1. $\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}}$
2. $\sqrt{18} \quad \frac{\sqrt{12}}{\sqrt{6}}$
3. $5\sqrt{2}\left(2\sqrt{2} + \sqrt{12}\right)$
4. $\sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}}$
5. $\sqrt[3]{72} + \sqrt[3]{\frac{1}{3}} + \sqrt[3]{-9}$

Solution

1. $\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}} = \sqrt{2 \times 16} - \sqrt{2 \times 36} + 3 \times 2\sqrt{\frac{1}{2}}$

$= 4\sqrt{2} - 6\sqrt{2} + 3\sqrt{\frac{1}{2} \times 4} = 4\sqrt{2} - 6\sqrt{2} + 3\sqrt{2} = \sqrt{2}$

2. $\sqrt{18} - \frac{\sqrt{12}}{\sqrt{6}} = \sqrt{2 \times 9} - \sqrt{2} = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

3. $5\sqrt{2}\left(2\sqrt{2} + \sqrt{12}\right) = 5\sqrt{2} \times 2\sqrt{2} + 5\sqrt{2} \times \sqrt{12} = 10\sqrt{4} + 5\sqrt{24} = 10 \times 2 + 5\sqrt{4 \times 6}$

$= 20 + 5 \times 2\sqrt{6} = 20 + 10\sqrt{6}$

4. $\sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}} = \sqrt[3]{2 \times 27} + 6\sqrt[3]{8 \times 2} - 3 \times 2\sqrt[3]{\frac{1}{4}}$

$= 3\sqrt[3]{2} + 6 \times 2 \times \sqrt[3]{2} - 3 \times \sqrt[3]{8 \times \frac{1}{4}} = 3\sqrt[3]{2} + 12\sqrt[3]{2} - 3\sqrt[3]{2} - 12\sqrt[3]{2}$

5. $\sqrt[3]{72} + \sqrt[3]{\frac{1}{3}} + \sqrt[3]{-9} = \sqrt[3]{8 \times 9} + \sqrt[3]{\frac{1}{3} \times \frac{9}{9}} - \sqrt[3]{9}$

$= 2\sqrt[3]{9} + \sqrt[3]{\frac{9}{27}} - \sqrt[3]{9} = 2\sqrt[3]{9} + \frac{1}{3}\sqrt[3]{9} - \sqrt[3]{9} - \frac{4}{3}\sqrt[3]{9}$

Remember **The two conjugate numbers**

If a and b are two positive rational numbers :

then each of the two numbers $\left(\sqrt{a}+\sqrt{b}\right)$ and $\left(\sqrt{a}-\sqrt{b}\right)$

is conjugate to the other one and we find that :

- Their sum $= \left(\sqrt{a}+\sqrt{b}\right)+\left(\sqrt{a}-\sqrt{b}\right) = 2\sqrt{a}$ = twice the first term
- Their product $= \left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right) = \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2 = a-b$

  = The square of the first term – the square of the second term

For example : The number $\left(\sqrt{3}-\sqrt{2}\right)$ its conjugate is $\left(\sqrt{3}+\sqrt{2}\right)$, then we find that :

- Their sum $= 2\sqrt{3}$
- Their product $= 3-2 = 1$

Remark

If we have a real number whose denominator is written in the form $\left(\sqrt{a}+\sqrt{b}\right)$ or $\left(\sqrt{a}-\sqrt{b}\right)$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

For example :

For writting the number $\frac{12}{\sqrt{6}-\sqrt{2}}$ in the simplest form , we multiply the two terms of the number by the conjugate of the denominator which is $\left(\sqrt{6}+\sqrt{2}\right)$

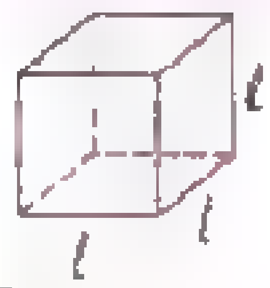
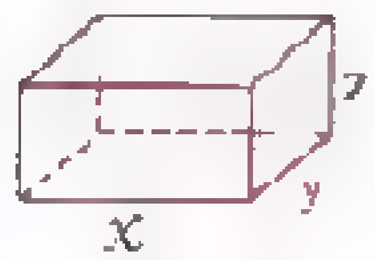
$$\therefore \frac{12}{\sqrt{6}-\sqrt{2}} \times \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \frac{12\left(\sqrt{6}+\sqrt{2}\right)}{6-2} = 3\left(\sqrt{6}+\sqrt{2}\right) = 3\sqrt{6}+3\sqrt{2}$$

Important remarks from multiplying by inspection

- **We know that :** $(x-y)(x+y) = x^2 \quad y^2$
- **And we know also :**

$(x+y)^2 = x^2+2xy+y^2$		$(x-y)^2 = x^2-2xy+y^2$
Then		Then
• $x^2+xy+y^2 = (x+y)^2 - xy$		• $x^2-xy+y^2 = (x-y)^2+xy$
• $x^2+y^2 = (x+y)^2-2xy$	or	$x^2+y^2 = (x-y)^2+2xy$

Summary of rules of areas and volumes of some solids

The solid		The lateral area	The total area	The volume
The cube		$4\,l^2$	$6\,l^2$	l^3
The cuboid		$2\,(x + y) \times z$	$2\,(x\,y + y\,z + z\,x)$	$x\,y\,z$
The cylinder		$2\,\pi\,r\,h$	$2\,\pi\,r\,h + 2\,\pi\,r^2$ $= 2\,\pi\,r\,(h + r)$	$\pi\,r^2\,h$
The sphere			$4\,\pi\,r^2$	$\frac{4}{3}\,\pi\,r^3$

Remember that : The circumference of the circle $= 2\,\pi\,r$, the area of the circle $= \pi\,r^2$

Remember Solving an equation of the first degree in one unknown in $\mathbb{R}$

- Solving the equation of the first degree in one unknown in $\mathbb{R}$ means finding the real number which satisfies this equation.

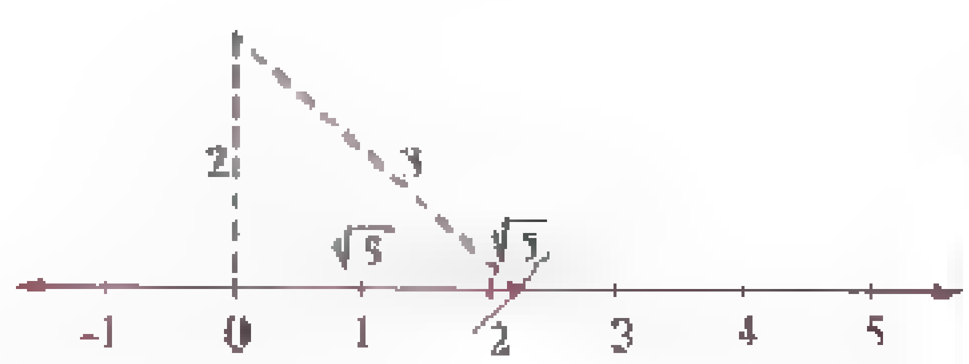
And the following example shows how to solve an equation of the first degree in one unknown

Example

Find in $\mathbb{R}$ the solution set of each of the following equations , then represent the solution on the number line :

1. $\sqrt{5}\,x - 1 = 4$
2. $x - \sqrt{3} = 2$

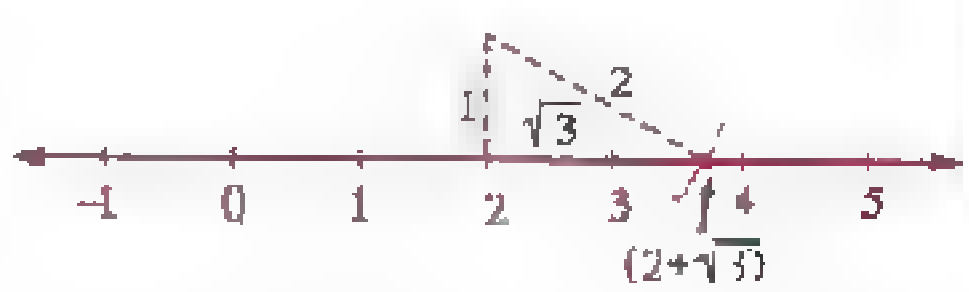
Solution



1. $\because \sqrt{5}\,x - 1 = 4 \qquad \therefore \sqrt{5}\,x = 4 + 1 = 5$

$$\therefore x = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$$

$\therefore$ The S.S. $= \{\sqrt{5}\}$



2. $\because x - \sqrt{3} = 2 \qquad \therefore x = 2 + \sqrt{3}$

$\therefore$ The S.S. $= \{2 + \sqrt{3}\}$

Solving an inequality of the first degree in one unknown in ℝ

- Solving the inequality means finding all values of the unknown which satisfy this inequality.
- The solution set of the inequality in ℝ will be written as an interval.

And the following example shows how to solve an inequality of the first degree in one unknown in ℝ

Example

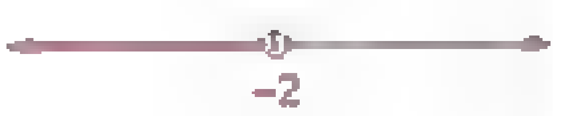
Find in ℝ the solution set of each of the following inequalities , then represent the solution on the number line :

1. $2x + 6 < 2$
2. $5 - 4x \le -3$
3. $3 < 3 - 5x < 13$
4. $x - 2 \ge 3x - 5$

Solution

1 $\because 2x + 6 < 2$ $\quad \therefore 2x < 2 - 6$

$\therefore 2x < -4$ $\quad \therefore x < \frac{-4}{2}$



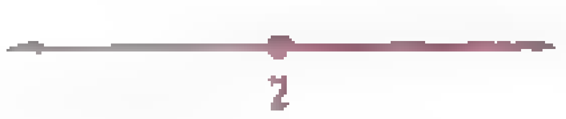
$\therefore x < -2$ $\quad \therefore$ The S.S. $=]-\infty , -2[$

2 $\because 5 - 4x \le -3$ $\quad \therefore -4x \le -8$

$\therefore x \ge \frac{-8}{-4}$

(Notice the change in the direction of the symbol of the inequality because we divided by a negative number)

$\therefore x \ge 2$ $\quad \therefore$ The S.S. $= [2 , \infty[$



3 $\because 3 < 3 - 5x < 13$ (adding -3 to all sides)

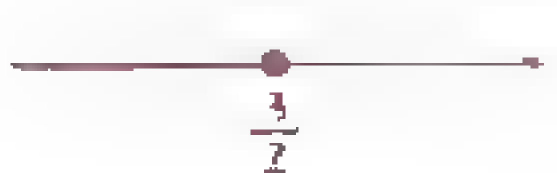
$\therefore 0 < -5x < 10$ (dividing all sides by -5) $\quad \therefore 0 > x > -2$

(Notice the change in the direction of the symbol of the inequality because we divided by a negative number)

$\therefore$ The S.S. $=]-2 , 0[$

4 $\because x - 2 \ge 3x - 5$ $\quad \therefore x - 3x \ge -5 + 2$

$\therefore -2x \ge -3$ $\quad \therefore x \le \frac{3}{2}$



$\therefore$ The S.S. $=]-\infty , \frac{3}{2}]$

Second Relation between two variables

Remember The linear relation

It is a relation of the first degree between two variables X and y , it is in the form :

$a\,X + b\,y = c$, where a , b and c are real numbers , a and b $\neq 0$ together.

And there is an infinite number of ordered pairs which satisfy this relation and it is enough to get three ordered pairs satisfying the relation at the graphical representation.

Example 1

Find three ordered pairs satisfying the relation : $3X - 2y = 6$

Solution

$\because 3X - 2y = 6$ $\quad \therefore -2y = 6 - 3X$ $\quad \therefore y = \frac{3X - 6}{2}$

- Putting $X = 0$ $\quad \therefore y = -3$ $\quad \therefore (0, -3)$ satisfies the relation.
- Putting $X = 1$ $\quad \therefore y = -\frac{3}{2}$ $\quad \therefore \left(1, -\frac{3}{2}\right)$ satisfies the relation.
- Putting $X = 2$ $\quad \therefore y = 0$ $\quad \therefore (2, 0)$ satisfies the relation.

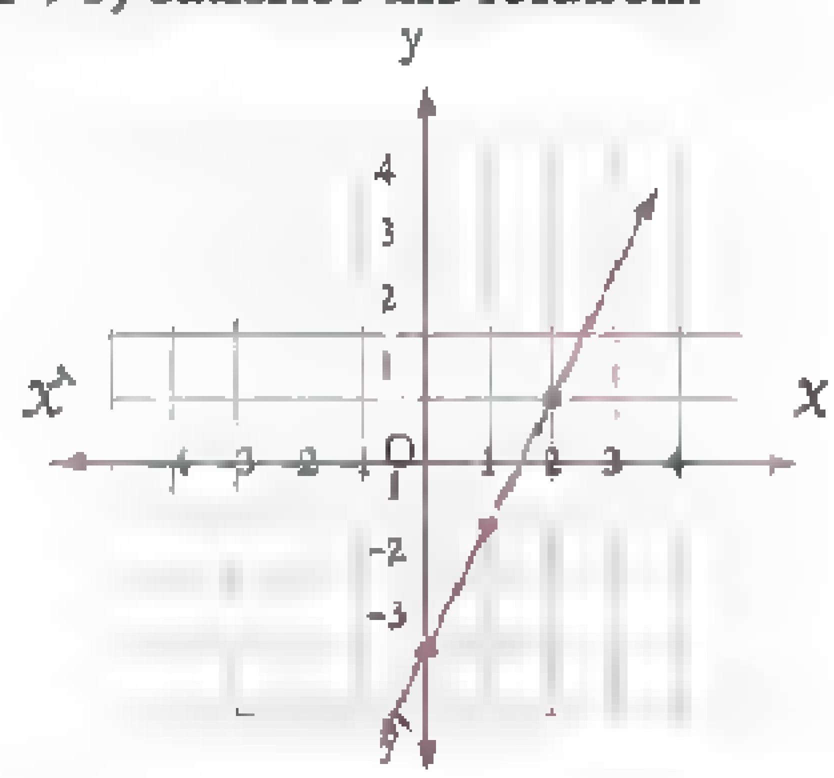
Example 2

Represent graphically the relation : $2X - y = 3$

Solution

$\because 2X - y = 3$ $\quad \therefore y = 2X - 3$

X	0	1	2
y	-3	-1	1



Remember The slope of the straight line

The slope of the straight line $= \frac{\text{the change in y-coordinates}}{\text{the change in } X\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$

i.e. $S = \frac{y_2 - y_1}{X_2 - X_1}$, where $X_1 \neq X_2$

For example : The slope of the straight line passing through the two points $(2, 3)$, $(-5, 2)$ is :

$S = \frac{2 - 3}{-5 - 2} = \frac{-1}{-7} = \frac{1}{7}$

Notice that :

- The slope of the straight line parallel to X axis $= 0$
- The slope of the straight line parallel to y-axis is undefined.

Third Statistics

Remember The tables and cumulative frequency curves

The following frequency table shows the weekly wages in pounds of 50 workers in a factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (frequency)	5	12	22	7	4	50

1 Forming the ascending cumulative frequency table and graphing the curve

The upper boundaries of sets	Frequency
Less than 54	zero
Less than 58	5
Less than 62	17
Less than 66	39
Less than 70	46
Less than 74	50

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (frequency)	5	12	22	7	4

Less than 54 = 0

Less than 58 = 5 + 0 = 5

Less than 62 = 5 + 12 = 17

Less than 66 = 5 + 12 + 22 = 39

Less than 70 = 5 + 12 + 22 + 7 = 46

Less than 74 = 5 + 12 + 22 + 7 + 4 = 50

"The ascending cumulative frequency table"

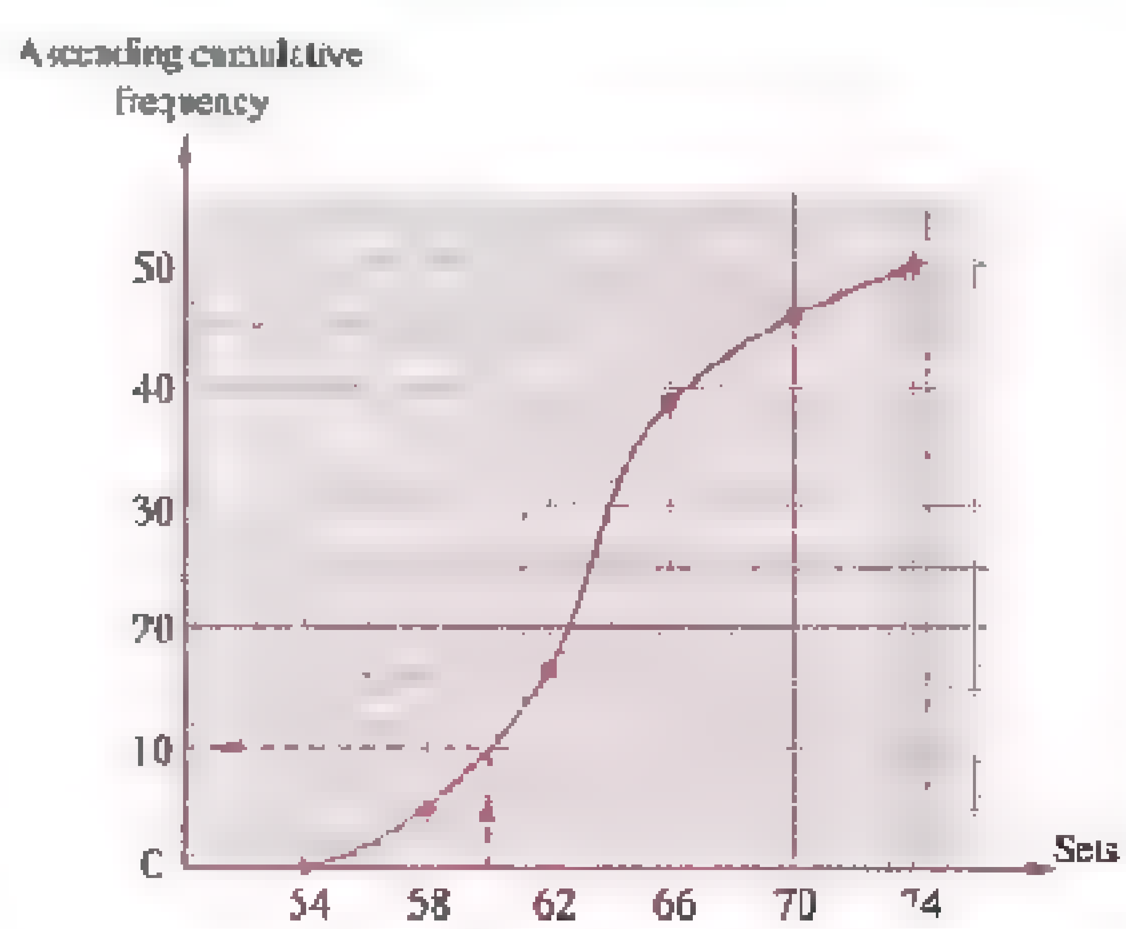
– Notice that :

The ascending cumulative frequency begins with zero and ends at the total frequency.

- From the opposite graph , we can find the number of individuals which is less than a certain value.

For Example :

The number of workers whose wages are less than 60 pounds is 10 workers



The ascending cumulative frequency curve

2 Forming the descending cumulative frequency table and graphing the curve

Sets of wages	54 –	58 –	62 –	66 -	70
Number of workers (frequency)	5	12	22	7	4

54 and more = $5 + 12 + 22 + 7 + 4 = 50$

58 and more = $12 + 22 + 7 + 4 = 45$

62 and more = $22 + 7 + 4 = 33$

66 and more = $7 + 4 = 11$

70 and more = 4

74 and more = 0

The lower boundaries of sets	Frequency
54 and more	50
58 and more	45
62 and more	33
66 and more	11
70 and more	4
74 and more	zero

"The descending cumulative frequency table"

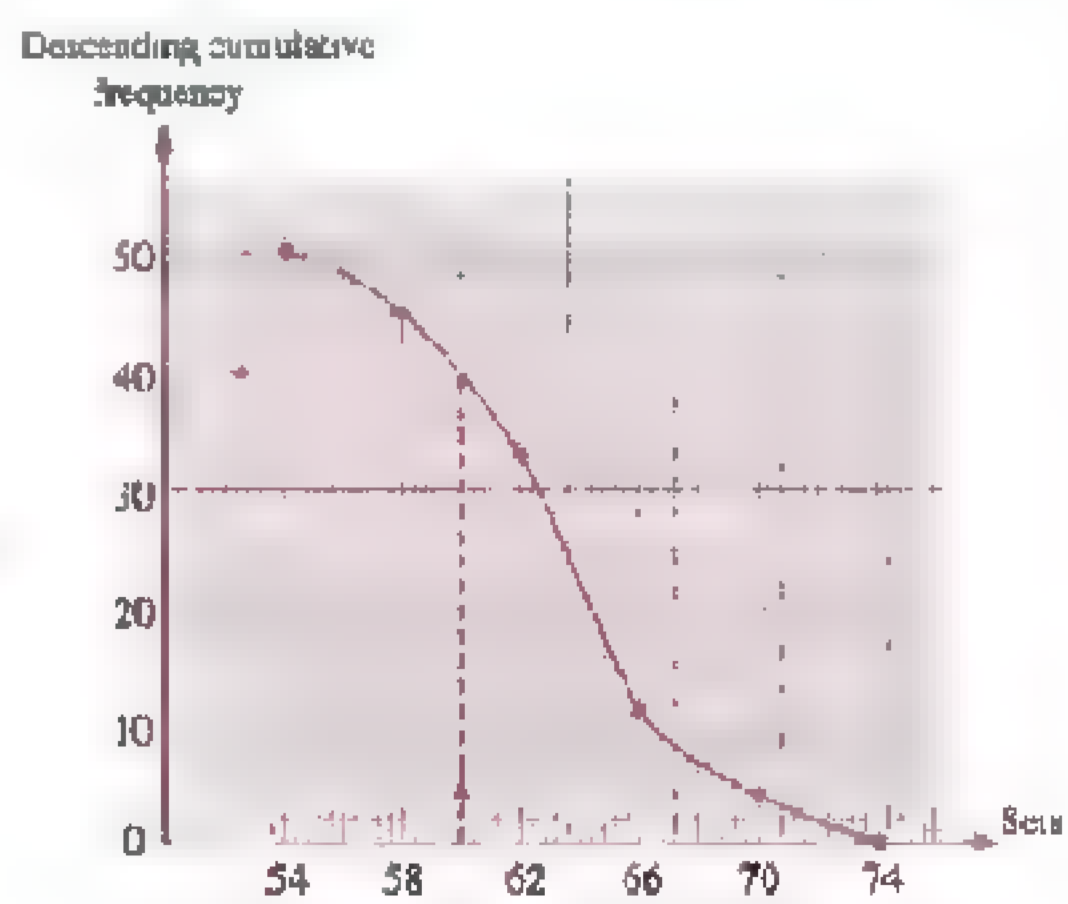
Notice that :

The descending cumulative frequency begins with the total frequency and ends with zero.

- From the opposite graph , we can find the number of individuals which is more than or equal to a certain value.

For Example :

The number of workers whose wages are 60 pounds or more is 40 workers.



The descending cumulative frequency curve.

Remember **The measures of the central tendency**

1. The mean. 2. The median. 3. The mode.

1. The mean

[a] The mean of a set of values (simple frequency distribution)

The mean of a set of values $= \dfrac{\text{The total of values}}{\text{Number of values}}$

For example : The mean of the numbers : 5 , 3 , 7 , 9 $= \dfrac{5+3+7+9}{4} = 6$

[b] The mean of a frequency distribution with sets

Example

The following table shows the distribution of the marks of 50 pupils in mathematics :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

Find the mean of these marks.

Solution

1. Determine the centres of sets according to the rule :

$$\text{The centre of a set} = \frac{\text{the lower limit} + \text{the upper limit}}{2}$$

$\therefore$ The centre of the first set $= \dfrac{10+20}{2} = 15$ ... and so on.

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

, then its centre $= \dfrac{50+60}{2} = 55$

2. Form the following table :

Set	Centre of the set « X »	Frequency « f »	$X \times f$
10	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

The mean $= \dfrac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \dfrac{1700}{50} = 34$ marks.

2 The median

[a] The median of a set of values

The median is the middle value in a set of values after arranging it ascendingly or descendingly. such that the number of values which are less than it is equal to the number of values which are greater than it.

We arrange the values ascendingly or descendingly

If the values number is odd , then

The median is the value lying in the middle exactly.

If the values number is even , then

The median

$= \frac{\text{The sum of the two values lying in the middle}}{2}$

For example :

If the values are

42 , 23 , 17 , 30 and 20

We arrange them ascendingly as follows

17 , 20 , 23 , 30 , 42

The median = 23

For example :

If the values are

27 , 13 , 23 , 24 , 13 , 21

We arrange them ascendingly as follows

13 , 13 , 21 , 23 , 24 , 27

The median = $\frac{21 + 23}{2} = 22$

[b] Finding the median of a frequency distribution with sets graphically

For finding the median of a frequency distribution with sets graphically , do the following steps :

1. Form the ascending or the descending cumulative frequency table , then draw the cumulative frequency curve of it.
2. Find the order of the median = $\frac{\text{The total of frequency}}{2}$
3. Determine the point which represents the order of the median on the vertical axis, from this point , draw a horizontal straight line to cut the curve at a point, then from this point , draw a perpendicular to the horizontal axis to intersect it at a point which represents the median.

The following example shows how to find the median using the two curves (the ascending or the descending cumulative frequency curve).

Example

The following table shows the frequency distribution of marks of 50 students in math exam :

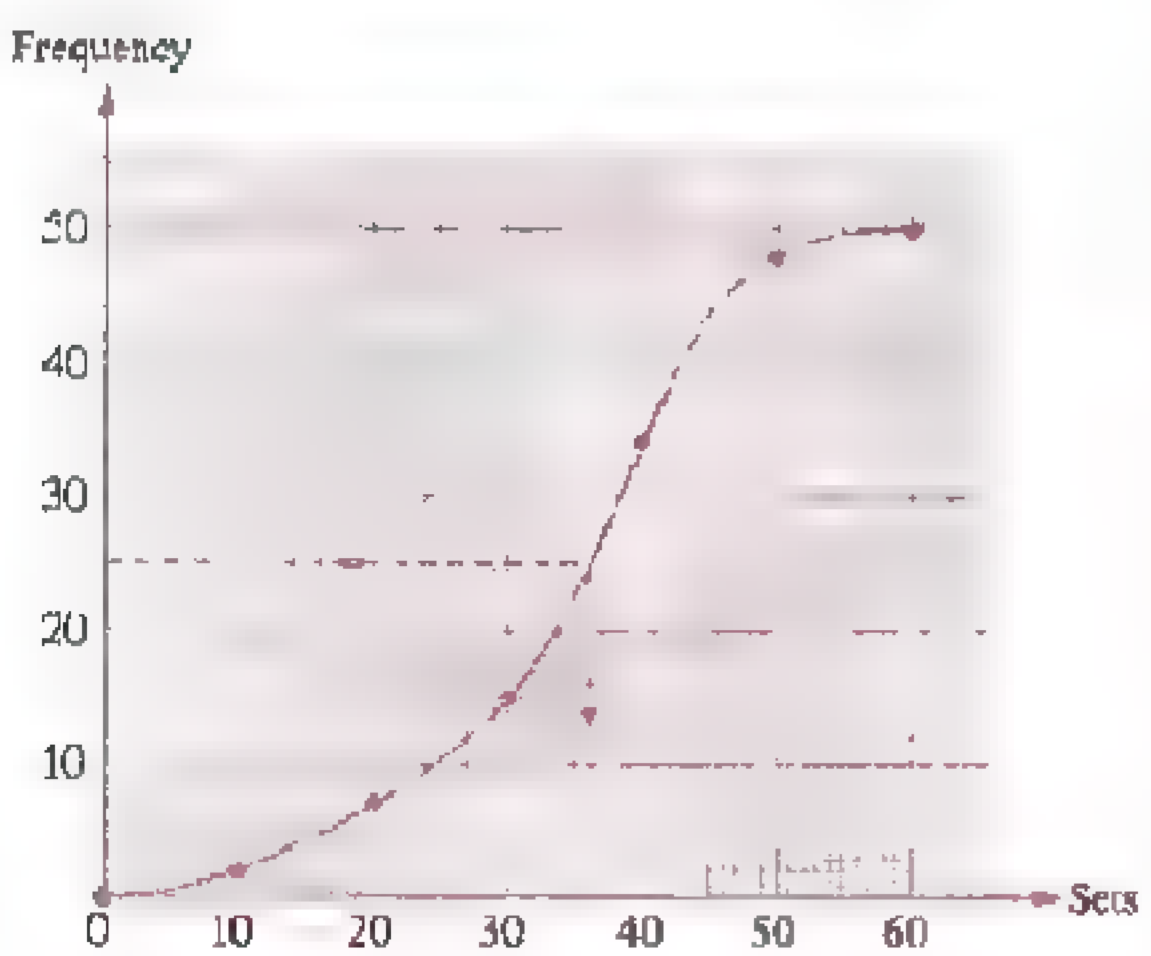
Sets of marks	0 –	10 –	20 –	30 –	40 –	50 –	Total
Number of students	2	5	8	19	14	2	**50**

Find the median mark of the student.

Solution

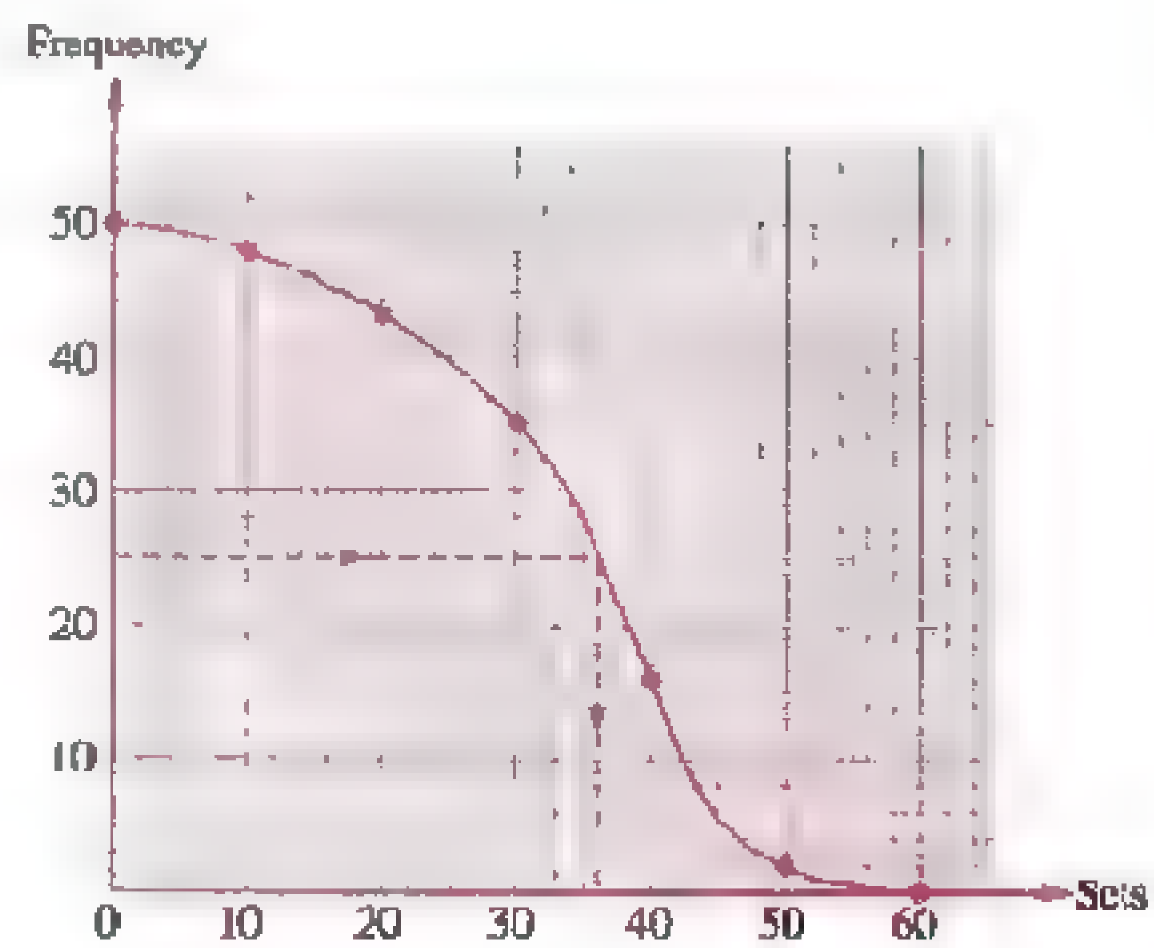
Using the ascending cumulative frequency curve :

The upper boundaries of sets	Frequency
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50



Using the descending cumulative frequency curve :

The lower boundaries of sets	Frequency
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0



$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ From the two previous graphs , the median = 36 approximately

3 The mode

[a] The mode of a set of values

The mode of a set of values is the most common value in the set, or in other words, it is the value which is repeated more than any other values.

For example : The mode of the set of the values : 7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is 7

[b] The mode of a frequency distribution with sets

Example

The following is the frequency distribution of marks of 100 pupils in one of the exams :

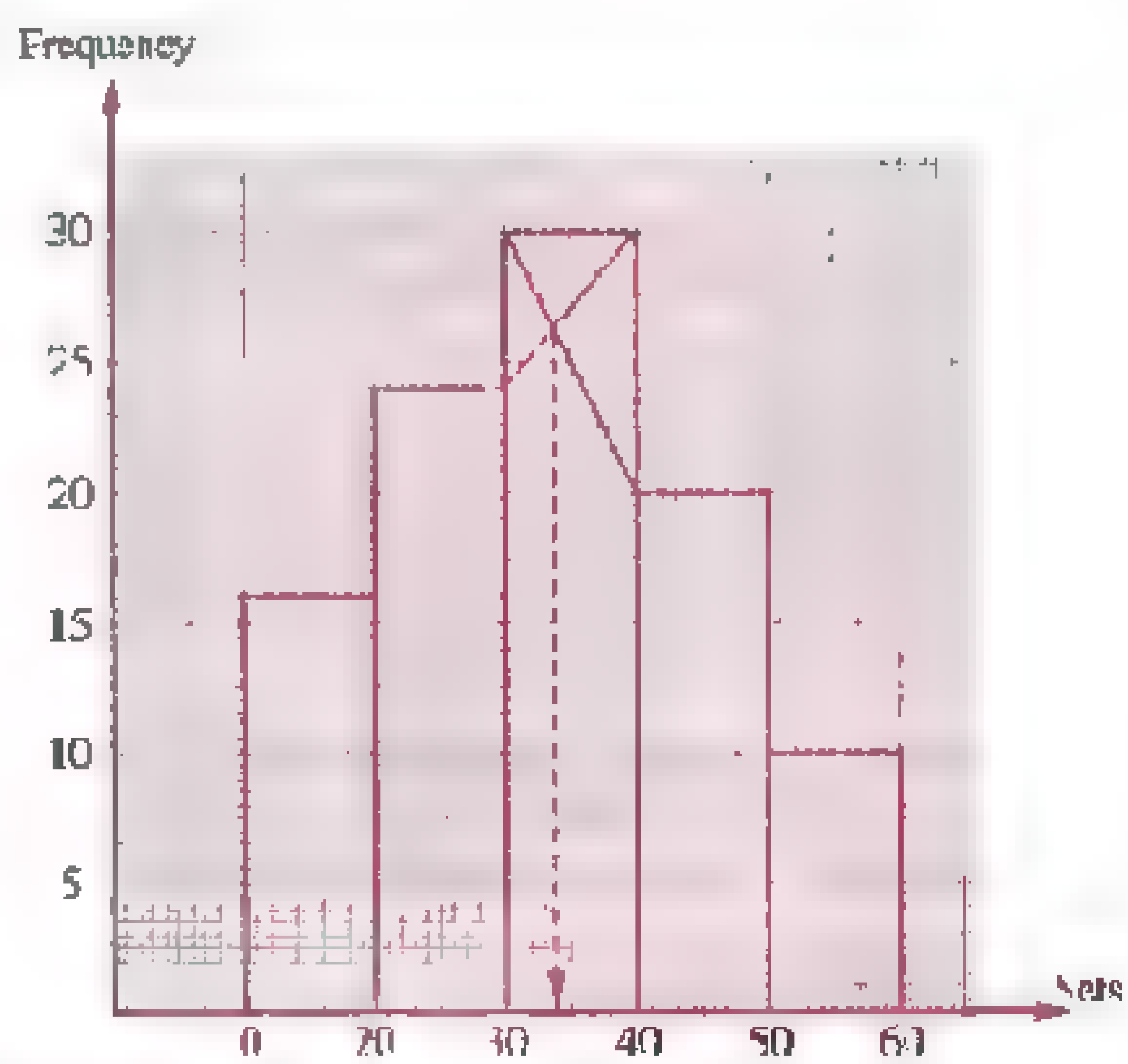
Sets of marks	10 –	20 –	30 –	40 –	50 –	Total
Number of pupils	16	24	30	20	10	100

Find the mode mark for these pupils.

Solution

You can find the mode of that distribution graphically using the histogram as follows :

1. Draw two orthogonal axes : one of them is horizontal and the other is vertical to represent the frequency of each set.
2. Divide the horizontal axis into a number of equal parts with a suitable drawing scale to represent the sets.
3. Divide the vertical axis into a number of equal parts with a suitable drawing scale to represent the greatest frequency in the sets.
4. Draw a rectangle whose base is set (10 –) and its height equals the frequency (16)
5. Draw a second rectangle adjacent to the first one whose base is set (20 –) and its height equals the frequency (24)



6. Repeat drawing the remained adjacent rectangles till the last set (50 –)
7. Determine the set which has the greatest frequency then draw two lines as shown in the histogram to intersect at a point.

   From this point , draw a vertical line to intersect the horizontal axis at a point which represents the value of the mode.

   i.e. The mode mark is 34 approximately

Final Examinations

on Algebra and Statistics

- School book examinations
- School examinations

Model 1

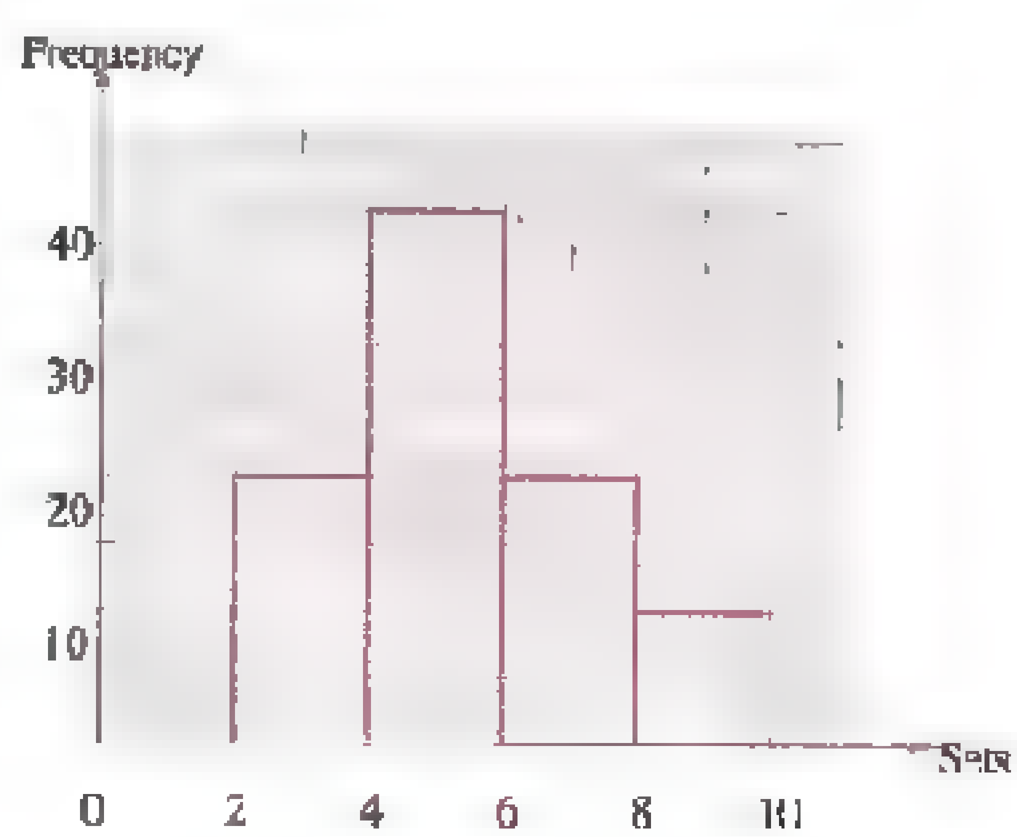
Answer the following questions :

1 Complete the following :

1. The S.S. of the equation : $(x^2 + 3)(x^3 + 1) = 0$ is .... , $x \in \mathbb{R}$
2. If the lower boundary of a set is 10 and the upper boundary is x and its centre is 15 , then $x =$ ....
3. $]-2, 2] \cup \{-2, 0\} =$ ....
4. The cube whose volume is 8 cm^3 , then the sum of all its edge lengths is .... cm.
5. The multiplicative inverse of the number $(\sqrt{3} + \sqrt{2})$ is .... in the simplest form.

2 Choose the correct answer from the given ones :

1. If the radius length of a sphere is 6 cm. , then its volume is ....
(a) 6π cm^3 (b) 36π cm^3 (c) 72π cm^3 (d) 288π cm^3
2. If the point (a , 1) satisfies the relation $x + y = 5$, then a = ...
(a) 1 (b) -4 (c) 4 (d) 5
3. $(2\sqrt[3]{2})^3 =$ ....
(a) 4 (b) 8 (c) 16 (d) 40
4. The median of the values : 34 , 23 , 25 , 40 , 22 , 4 is ....
(a) 22 (b) 23 (c) 24 (d) 25
5. If the arithmetic mean of the values : 27 , 8 , 16 , 24 , 6 , k is 14 , then k = ...
(a) 3 (b) 6 (c) 27 (d) 84
6. **In the opposite figure :**
The value of the mode = ...
(a) 4 (b) 5
(c) 6 (d) 40



3 [a] **Find the value of :** $\sqrt{18}+\sqrt[3]{54}\quad 3\sqrt{2}\quad \frac{1}{2}\sqrt[3]{16}$

[b] If $X=\frac{3}{\sqrt{5}-\sqrt{2}}$ and $y=\sqrt{5}-\sqrt{2}$

, **prove that :** X and y are two conjugate numbers.

4 [a] The area of a square is 1089 cm^2. Find the length of its diagonal.

[b] **Find the S.S. of the inequality :** $\frac{3X+1}{6}<X+1<\frac{X+4}{2}$ in $\mathbb{R}$

, then represent it on the number line.

5 [a] The radius length of the base of a right circular cylinder is $4\sqrt{2}$ cm. and its height is 9 cm Find its volume in terms of π and if its volume equals the volume of a sphere , find the radius length of the sphere.

[b] **Find the arithmetic mean of the following frequency distribution :**

The sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Model 2

Answer the following questions :

1 **Complete the following :**

1 The additive inverse of the number : $-\sqrt{3}-\sqrt{5}$ is ....

2 $(\sqrt{8}+\sqrt{2})(\sqrt{8}-\sqrt{2})=$ ....

3 The conjugate of the number $\frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}}$ is ....

4 If the volume of a sphere is $\frac{9}{2}\pi$ cm^3 , then its diameter length is .... cm.

5 $[3\ ,4]-\{3\ ,5\}=$ ....

2 **Choose the correct answer from the given ones :**

1 If the volume of a cube is 27 cm^3 , then the area of one of its faces is ....

(a) 3 cm^2 (b) 9 cm^2 (c) 36 cm^2 (d) 54 cm^2

2 If the mode of the values 4 , 11 , 8 , 2 X is 4 , then $X=$ ....

(a) 2 (b) 4 (c) 6 (d) 8

3 If the arithmetic mean of the values 18 , 23 , 29 , $2k-1$, k is 18 , then k = ...

(a) 1 (b) 7 (c) 29 (d) 90

4 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is ....

(a) 2 (b) 4 (c) 6 (d) 8

5 A right circular cylinder the radius length of its base is r cm. and its height equals its diameter length , then its volume = ... cm^3

(a) πr^3 (b) πr^2 (c) $2\pi r^3$ (d) $2r^3$

6 The solution set of the equation : $x(x^2-1)=0$, $x \in \mathbb{R}$ is ....

(a) $\{0\}$ (b) $\{1\}$ (c) $\{-1\}$ (d) $\{0, -1, 1\}$

---

3 [a] **Reduce to the simplest form :** $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

[b] **Prove that :** $\sqrt[3]{128}+\sqrt[3]{16}-2\sqrt[3]{54}=0$

---

4 [a] **Find the S.S. of the inequality :** $-2<3x+7\le 10$ in $\mathbb{R}$, then represent the interval of solution on the number line.

[b] **If** $x=\sqrt{2}+\sqrt{3}$ **, find the value of :** x^4-2x^2+1

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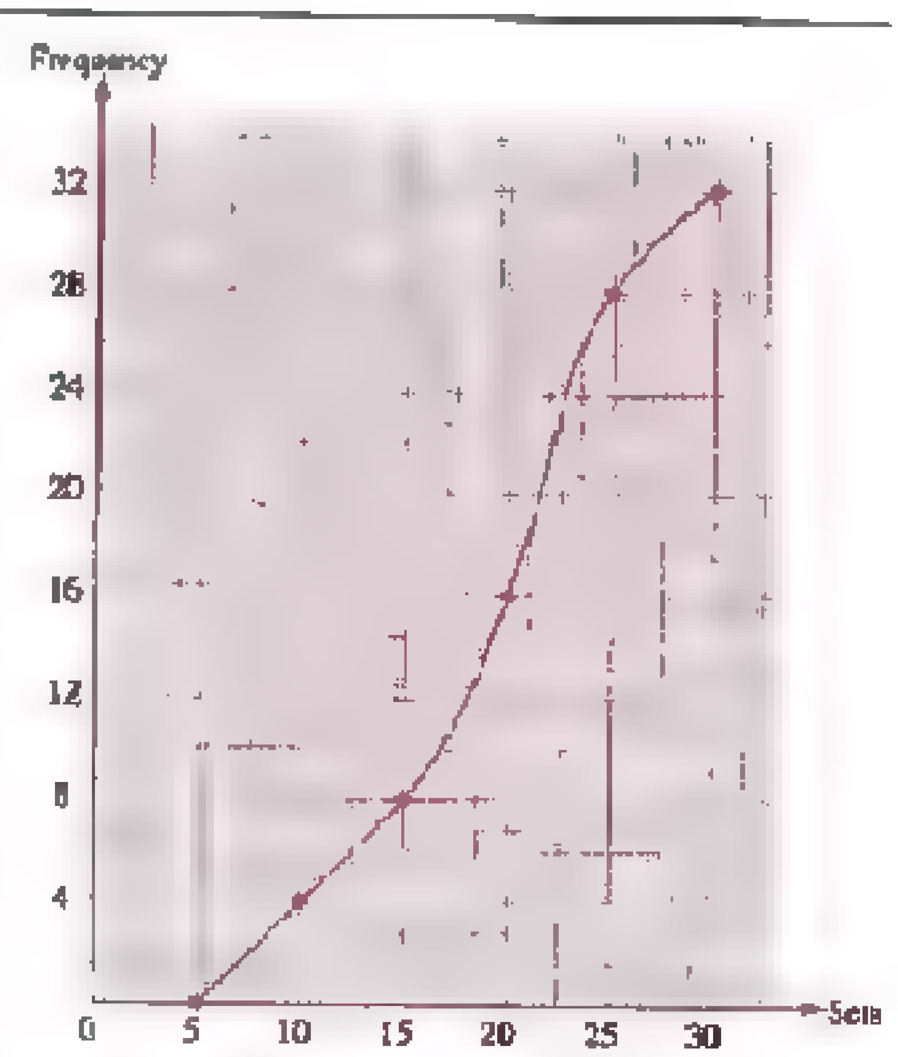
5 [a] The opposite graph represents the marks of 32 pupils in an exam.

Complete :

The median mark = ...



[b] **Find the arithmetic mean of the following frequency distribution :**

The sets	5 –	15 –	25 –	35 –	45	Total
Frequency	4	5	6	3	2	20

Model for the merge students

Answer the following questions :

1 **Complete each of the following :**

1. The conjugate of the number $\sqrt{3}+\sqrt{2}$ is ....
2. $\sqrt{18}+\sqrt{54}-3\sqrt{2}=$ ....
3. The mode for the numbers : 3 , 5 , 3 , 4 , 3 is ....
4. The median of the values : 2 , 3 , 5 , 7 , 9 is ....
5. The solution set of the equation : $x^2+9=0$ in $\mathbb{R}$ is ....

2 **Choose the correct answer from those given :**

1. The arithmetic mean for the values : 9 , 6 , 5 , 14 , 1 is ....
   (a) 7 (b) 3 (c) 5 (d) 9
2. The simplest form of the expression : $\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)$ is ....
   (a) $\sqrt{3}$ (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{3}$
3. The additive inverse of the number $-\sqrt{5}$ is ....
   (a) $\sqrt{5}$ (b) 5 (c) $\sqrt{2}$ (d) -5
4. $[3 , 5]-\{3 , 5\}=$ ....
   (a) $]3 , 5[$ (b) $[3 , 5[$ (c) $\varnothing$ (d) $]3 , 5]$
5. A cube is of volume 64 cm.^{3} , then its edge length is .... cm.
   (a) 4 (b) 8 (c) 16 (d) 64

3 **Match from the column (A) to the suitable one from the column (B) :**

(A)	(B)
1. The S.S. of the equation : $x^2-25=0$ in $\mathbb{R}$ is	$[0 , 2]$
2. $[-3 , 2]\cap[0 , 2]=$	7
3. If the order of the median is fourth , then the number of values is	$\{5 , -5\}$
4. $\sqrt{3}$ is an .... number.	
5. The S.S. of the inequality : $3\le x\le 7$ on the number line is	irrational

4 **Put (✔) for the correct statements and (✘) for the incorrect ones :**

1. The arithmetic mean of a set of values = sum of values ÷ its number. ( )
2. If $x = \sqrt{13} - \sqrt{7}$, $y = \sqrt{13} + \sqrt{7}$, then x , y are two conjugate numbers. ( )
3. The irrational number $\sqrt{7}$ lies between 2 and 3 ( )
4. $\sqrt{75} - 2\sqrt{27} = 7\sqrt{3}$ ( )
5. The simplest form of the number $\frac{1}{\sqrt{5}}$ is $\frac{\sqrt{5}}{5}$ ( )

---

5 **[a]** **Complete :** If the lower limit of a set is 4 and the upper limit is 8

, then its centre = $\frac{\text{.........} + \text{.........}}{2}$ = ...

[b] **Complete the following table to obtain the arithmetic mean of the following frequency distribution :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Sets	The centre of the set « x »	Frequency « f »	$x \times f$
5 –	10	7	10 × 7 = 70
15 –	20	10	20 × 10 = ...
25 –	....	....	.... × 12 = ...
35 –	....	....	.... × 13 = ...
45 –	....	....	.... × 8 = ...
Total		50	....

The arithmetic mean = $\frac{\Sigma (x \times f)}{\Sigma (f)} = \frac{\text{.........}}{\text{.........}}$ = ...

Some Schools Examinations on Algebra and Statistics

1 Cairo Governorate

East Nasr City Adminstration
Manart Al Salam Language School

Answer the following questions :

1 Choose the correct answer :

1. The slope of any line parallel to X-axis is .....
   (a) 1 (b) undefined. (c) -1 (d) zero.

2. $[1 , 3] - \{1 , 3\} = $ ....
   (a) $]1 , 3[$ (b) $[1 , 3[$ (c) $]-3 , -1[$ (d) $]-1 , 3[$

3. The volume of the cuboid whose dimensions are $\sqrt{2}$ cm. , $\sqrt{3}$ cm and $\sqrt{6}$ cm. is .... cm^3
   (a) 6 (b) 36 (c) $6\sqrt{6}$ (d) $18\sqrt{2}$

4. If the lower limit of a set is 6 and the upper limit is 10 , then its centre is ....
   (a) 4 (b) 6 (c) 10 (d) 8

5. $\left(2\sqrt[3]{2}\right)^3 = $ .. ...
   (a) 4 (b) 8 (c) 16 (d) 40

2 Complete :

1. The multiplicative inverse of $\frac{\sqrt{2}}{6}$ is ....
2. The S.S. of the equation : $X^2 + 9 = 0$ in $\mathbb{R}$ is ....
3. $(\sqrt{7} + 2)(\sqrt{7} - 2) = $ ....
4. Let A $(-3 , 1)$ and B $(2 , -5)$, then the slope of $\overleftrightarrow{AB} = $ ....

3 [a] **Find in the simplest form :** $\sqrt{18} + \sqrt{50} - \sqrt{54}$

[b] If $X = \frac{4}{3 - \sqrt{5}}$ and $y = 3 + \sqrt{5}$, **prove that :** X and y are conjugate numbers

, then find the value of : $(X + y)^2$

4 [a] If $X = [-1 , 4]$ and $Y = [2 , 7]$ **, then find each of :**

1. $X \cup Y$ 2. $X \cap Y$

[b] If $(3\text{ m} , 2\text{ m})$ satisfies the relation : $y = 2X - 8$ **, find the value of :** m

4 **[a] Find the S.S. of each of the following in $\mathbb{R}$:**

1 $8x^3 - 20 = 7$ 2 $3x + 7 \leq 10$

[b] Find the arithmetic mean of the following frequency distribution :

The sets	0 –	4 –	8 –	12 –	16 –	Total
Frequency	2	10	8	7	3	30

2 Cairo Governorate

Maadi Directorate
Al-Shorouk Language School

Answer the following questions :

1 **Choose the correct answer :**

1 The volume of a cube is 125 cm^3 , then its edge length is .... cm.

(a) 5   (b) 25   (c) 50   (d) 125

2 The multiplicative inverse of $\frac{\sqrt{6}}{2}$ is ....

(a) $-\frac{\sqrt{6}}{2}$ (b) $\frac{\sqrt{6}}{3}$ (c) $3\sqrt{6}$ (d) $2\sqrt{6}$

3 The slope of x-axis is ....

(a) 0   (b) $\frac{1}{2}$ (c) -1 (d) undefined.

4 If (k , 1) satisfies the relation : $x + y = 5$, then k = ...

(a) -4 (b) 1   (c) 4   (d) 5

5 If the mode of the values : 4 , 11 , 8 , $x + 1$ is 4 , then x = ...

(a) 3   (b) 4   (c) 5   (d) 8

2 **Complete each of the following :**

1 $\sqrt{12} + \frac{6}{\sqrt{3}} =$ .... (in the simplest form)

2 The slope of the line passing through (2 , 4) and (3 , – 1) equals ....

3 The mean of the values : 2 , 4 , 7 , 3 , 5 , 9 is ....

4 $\mathbb{Q} \cup \mathbb{Q}' =$ ....

3 **[a] Find the solution set of the inequality :** $-2 < 3x + 7 < 10$ in $\mathbb{R}$, then represent the S.S. on the number line.

[b] If $x = \sqrt{5} - \sqrt{3}$ and $y = \frac{2}{\sqrt{5} - \sqrt{3}}$

, prove that : x and y are conjugate numbers **, then find :** $x^2 + y^2$

[a] If $X =]-1 , 4]$ and $Y = [2 , \infty[$, **find using the number line :**

1. $X \cap Y$ 2. $X \cup Y$ 3. $X - Y$

[b] If the volume of a sphere is 36π cm^3 , find its area.

---

[a] Represent graphically the relation : $y - x = 2$

[b] From the following frequency distribution , find the mean :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

3 Giza Governorate

Al-Agoza Directorate
El-Manar Islamic Language School

Answer the following questions :

1 **Choose the correct answer :**

1. $\sqrt{6} \in$ ....

(a) $\mathbb{N}$ (b) $\mathbb{Q}$ (c) $\mathbb{Q}'$ (d) $\mathbb{Z}$

2. $\sqrt[3]{\qquad} + \sqrt[3]{27} = \sqrt{64}$

(a) 25 (b) – 125 (c) 125 (d) 5

3. $[3 , 5] -]3 , 5] =$ ....

(a) $\{3\}$ (b) $]3 , 5[$ (c) $\{3 , 5\}$ (d) $\{5\}$

4. The multiplicative inverse of $\frac{2\sqrt{3}}{6}$ is ....

(a) $\sqrt{3}$ (b) $\frac{2\sqrt{3}}{5}$ (c) $\frac{2\sqrt{3}}{6}$ (d) $\frac{5\sqrt{3}}{6}$

5. If $x = \sqrt{2} + 3$, $y = \sqrt{2} - 3$, then $x^2 - y^2 =$ ....

(a) $\sqrt{2} + 3$ (b) $12\sqrt{2}$ (c) $6\sqrt{5}$ (d) $3\sqrt{6}$

---

2 **Complete :**

1. If $(3 , n)$ satisfies the relation : $2x + y = 7$, then $n =$ ....
2. If $x = \sqrt[3]{3} + 1$, $y = \sqrt[3]{3} - 1$, then $(x - y)^3 =$ ....
3. The S.S. of : $x^2 + 25 = 0$ in $\mathbb{R}$ is ....
4. $\sqrt{7}$ lies between the two integers .... , ...

3 **[a] Simplify :** $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54}$

[b] If $x = \sqrt{5} - 3$, $y = \sqrt{5} + 3$ **, then find :** $x\,y$

[c] If the mode of the numbers : 3 , 5 , 6 , 2 x , 5 , 6 is 6 **, find :** x

4 **[a] Find using the number line :** $]-2 , 2] \cap [1 , 5]$

[b] Find the volume of the cuboid of dimensions 2 cm. , $2\sqrt{5}$ cm. , $3\sqrt{5}$ cm.

[c] Find the median of the numbers : 4 , 11 , 5 , 20 , 13

5 **[a] Find the S.S. of :** $-3 < 2x + 1 < 7$ in $\mathbb{R}$, then represent it on the number line.

[b] If $x = \sqrt{5} + 2$, $x\,y = 1$ **find :** y **and prove that :** x , y are two conjugate numbers.

[c] Find the mean of the following data :

Sets	5 –	15 –	25 –	35	45 –	Total
Frequency	20	30	15	25	10	100

4 Giza Governorate

Math Inspection

Answer the following questions :

1 **Choose the correct answer from those given :**

1. If 3 x = 6 , then 5 x = ...
   (a) $\frac{5}{2}$ (b) $\frac{2}{5}$ (c) 10 (d) 5
2. The slope of the straight line passing through (2 , 4) and (4 , 6) equals ......
   (a) – 1 (b) 1 (c) 7 (d) zero
3. If (a , 1) satisfies the relation : x + y = 5 , then a = ...
   (a) – 4 (b) 1 (c) 4 (d) 5
4. $[3 , 5] - \{3\}$ = ...
   (a) $[3 , 4]$ (b) $[3 , 5[$ (c) $\{3 , 4\}$ (d) $]3 , 5]$
5. The median of the values : 5 , 3 , 11 , 7 , 2 is ....
   (a) 3 (b) 5 (c) 7 (d) 8

2 **Complete each of the following :**

1. $2^3 \times 2^3$ = ...
2. If the radius length of a sphere is 6 cm. , then its volume is .... cm^3

3 The slope of y-axis is ....

4 The mode of the values : 3 , 5 , 3 , 4 , 3 is ....

[a] Find in the simplest form : $\sqrt[3]{24}+\sqrt{12}-2\sqrt[3]{3}-2\sqrt{3}$

[b] If $x=\sqrt{3}+\sqrt{2}$, $y=\frac{1}{\sqrt{3}+\sqrt{2}}$, **find the value of :** x^2-y^2

[a] The dimensions of a cuboid are 3 cm. , 4 cm and 5 cm

Calculate its volume and its total area.

[b] **Find in $\mathbb{R}$ the S.S. of the inequality :** $3x-1\geq 8$, then represent the solution set on the number line.

[a] **Represent the relation :** $y=x+2$ graphically.

[b] **Find the mean of the following frequency distribution :**

Sets	4 –	8 –	12 –	16 –	20 –	**Total**
Frequency	2	4	8	6	4	**24**

5 Alexandria Governorate

East Zone
Supervision of Mathematics

Answer the following questions :

1 Choose the correct answer :

1 The S.S. of $(x+2)^3=125$ in $\mathbb{R}$ is ....

(a) $\{3\}$ (b) $\{5\}$ (c) $\{7\}$ (d) $\varnothing$

2 If the order of the median of a set of values is the fourth , then the number of the values is ....

(a) 3 (b) 5 (c) 7 (d) 9

3 If A is (2 , 7) and B is (5 , – 2) , then the slope of $\overleftrightarrow{AB}=$ ....

(a) – 2 (b) 2 (c) – 3 (d) 3

4 If $5x=35$, then $2x+1=$ ....

(a) 7 (b) 15 (c) 8 (d) 71

5 The multiplicative inverse of $\frac{\sqrt{3}}{6}$ is ....

(a) $-\frac{\sqrt{3}}{6}$ (b) $6\sqrt{3}$ (c) $2\sqrt{3}$ (d) $-2\sqrt{3}$

2 Complete the following :

1. If $7^{X} = 1$, then $X =$ ....
2. If the mode of the values : 5 , 9 , 5 , $X - 2$, 9 is 9 , then $X =$ ....
3. If (– 1 , 5) satisfies the relation : $2X + y = k$, then k = ...
4. The slope of any line parallel to y-axis is ....

---

3 [a] Find in the simplest form : $\sqrt[3]{54} + 4\sqrt[3]{\frac{1}{4}} - \sqrt[3]{2}$

[b] If A =]– 1 , 3] and B = [0 , 5[, **then find :**

1. $A \cap B$
2. $B - A$

---

4 [a] If the volume of a sphere is 288π cm³ , find its area

[b] Find the S.S. of the inequality : $2 < 3X + 7 \le 10$ in $\mathbb{R}$, then represent the interval of the solution set on the number line.

---

5 [a] Represent graphically the relation : $y = 2X - 3$

[b] By using the following distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	9	12	k	4	40

1. Find the value of k
2. Find the arithmetic mean.

6 El-Kalyoubia Governorate

Maths Supervision
Official Language Schools

Answer the following questions :

1 Choose the correct answer from those given :

1. The slope of the straight line perpendicular to y-axis is ....
   (a) undefined. (b) 0 (c) 1 (d) – 1
2. The ordered pair which does not satisfy the relation : $3X + y = -7$ is ....
   (a) (– 1 , – 4) (b) (1 , 4) (c) (0 , – 7) (d) (1 , – 10)
3. A circle is of area 16π cm² , then the length of its diameter equals .... cm.
   (a) 4 (b) 8 (c) 12 (d) 16
4. If the point (30 , 50) is the point of intersection of the ascending and descending curves , then the sum of its frequency equals ....
   (a) 30 (b) 50 (c) 60 (d) 100

A cube is of volume 27 cm^3 , then the sum of its edge lengths equals ...... cm.

(a) 4 (b) 12 (c) 18 (d) 36

Complete the following :

1 $[1 , 5] - \{1 , 5\} =$ ....

2 If the mode of the values : 3 , 6 , 9 , 3 X is 6 , then $X =$ ...

3 The third of the number 3^6 is ....

The relation $3 X + 4 y \ \ 12$ is represented by a straight line which cuts X-axis at the point ......

If $X = [\ \ 1 , 2]$ and $Y = [1 , 3]$, **find using the number line :**

1 $X \cap Y$ 2 $X - Y$

Simplify to the simplest form : $\sqrt{75} - 2\sqrt{27}\ \ 6\sqrt{\frac{1}{3}}$

If the slope of the straight line which passes through the points (2 k , 3) and (3 , 1 – k) is one , **find :** the value of k

Find in $\mathbb{R}$ the solution set of the inequality : $4 X < 5 X + 2 \leq 4 X + 3$

A sphere where its radius length is 3 cm. Find its volume and surface area in terms of π

The following table represents the marks of 20 students in mathematics :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

Find the mean mark of the students marks.

1 El-Menofia Governorate

Shibeen El-Koum Directorate
Maths Supervision

Answer the following questions : *(Calculators are allowed)*

Choose the correct answer :

The slope of the straight line which is perpendicular to X-axis is ......

(a) zero (b) – 1 (c) 1 (d) undefined.

If the arithmetic mean of the numbers : 2 , 7 , X equals 4 , then $X =$ ....

(a) 2 (b) 3 (c) 4 (d) 5

3 The ordered pair which satisfies the relation : $2 X + y = 5$ is ....

(a) (– 1 , 3) (b) (1 , 3) (c) (3 , 1) (d) (2 , 2)

4 If $3^{x} = 1$, then $x =$ ....

(a) -1 (b) 1 (c) zero (d) 3

5 The median of the values : 15 , 22 , 9 , 11 , 33 is ....

(a) 15 (b) 9 (c) 18 (d) 90

2 Complete the following :

1 The conjugate of the number $\frac{3}{\sqrt{5}-\sqrt{2}}$ is ....

2 If A = (2 , y) , B = (5 , 3) and the slope of $\overleftrightarrow{AB}$ = zero , then y = ...

3 The solution set of the inequality : $-x > 3$ in $\mathbb{R}$ is ....

4 $[3 , 7] - \{3 , 8\} =$ ....

3 [a] Prove that the points A , B , C lie on the same straight line (collinear) if A (2 , 1) , B (−3 , 4) , C (−4 , 5)

[b] A right circular cylinder its height is 4 cm. Find the length of its base radius if the volume of the cylinder is 64π cm^3

4 [a] If X = [−1 , 4[, Y = [2 , ∞[, find by using the number line :

1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

[b] Find the solution set of the inequality in $\mathbb{R}$: $-2 < 5x + 3 < 13$, then represent it on the number line.

5 [a] Put in the simplest form : $3\sqrt[3]{16} + \sqrt{50} - 2\sqrt[3]{54} - 2\sqrt{8}$

[b] Find the mode of the following frequency distribution by using the histogram :

Sets	10 –	20 –	30	40 –	50 –	60	Total
Frequency	3	8	12	8	5	4	40

8 El-Gharbia Governorate

Central Mathematics Supervision
Official Language Schools

Answer the following questions :

1 Choose the correct answer from the given ones :

1 $[3 , 5] - \{5\} =$ ....

(a) {3} (b) [0 , 3] (c) [3 , 4] (d) [3 , 5[

2 If the point (m , 2) satisfies the relation : $y + 2x = 8$, then m = ...

(a) 2 (b) 3 (c) 4 (d) 5

[3] The slope of the horizontal straight line is ....

(a) 1 (b) -1 (c) zero. (d) undefined.

[4] If the lower limit of a set is 10, the upper limit is X and its centre is 15, then $X =$ ....

(a) 10 (b) 12 (c) 20 (d) 25

[5] If $2X^m + 3X^n = 5X^5$, then $m + n =$ ....

(a) 5 (b) 10 (c) 15 (d) 6

[2] Complete the following :

[1] Half the number 4^5 is $2^{\cdots}$

[2] The cube whose volume is 8 cm^3, then the sum of all edge lengths is .... cm.

[3] If the slope of the straight line passing through $(2, k)$, $(3, -1)$ is 2, then $k =$ ....

[4] If the mode of the values : 4, 5, 6, $X + 3$ is 5, then $X =$ ....

[3] [a] Simplify to the simplest form : $\sqrt{125} + 2\sqrt{80} - \sqrt{20} - \sqrt{45}$

[b] Find the S.S. of each of the following in $\mathbb{R}$:

[1] $(X - 3)^3 = 8$

[2] $8 + 3X < 14$, then represent the solution on the number line.

[4] [a] Graph the relation : $y - 2X = 4$ and find the slope of the straight line from the graph.

[b] If $X = \dfrac{3}{\sqrt{6} - \sqrt{3}}$ and $y = \sqrt{6} - \sqrt{3}$, then find with steps the value of : $\dfrac{Xy}{X - y}$

[5] [a] If $X =]-\infty, 5]$, $Y =]1, 7]$, find using the number line :

[1] $X \cap Y$ [2] $Y - X$

[b] From the following distribution find :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	k − 1	6	3	2	20

[1] The value of k [2] The arithmetic mean.

9 Ismailia Governorate

Directorate of Education
Math's Supervision

Answer the following questions :

1 Choose the correct answer :

1) $\mathbb{R}_+ \cap \mathbb{R}_- =$ ......

(a) $\mathbb{R}_+$ (b) $\mathbb{R}$ (c) $\varnothing$ (d) $\mathbb{R}_-$

2) The slope of the straight line parallel to y-axis is ....

(a) 0 (b) 1 (c) -1 (d) undefined.

3) $10x + 2x =$ ...

(a) $12x^2$ (b) $12x$ (c) $20x$ (d) $20x^2$

4) The order of the median of the values : 5 , 7 , 6 , 4 and 8 is ....

(a) third. (b) fourth. (c) fifth. (d) sixth

5) If the point (a , 1) satisfies the relation : $x + y = 5$, then a = ...

(a) 1 (b) -4 (c) 4 (d) 5

2 Complete the following :

1) The mode of the values : 3 , 6 , 5 , 6 , 3 , 6 , 5 is ....

2) If $x \in [-2 , 5]$, then $x^2 \in [$.............. , ...$]$

3) The square whose side length is $\sqrt{5}$ cm. , its area is ...... cm^2

4) If A , B and C are collinear points , then the slope of $\overleftrightarrow{BC}$ = the slope of ......

3 [a] Find the slope of $\overleftrightarrow{AB}$ where A $(-1 , 3)$ and B $(2 , 5)$

[b] **Simplify :** $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54}$

4 [a] **Find the S.S. of the equation in $\mathbb{R}$:** $(x^2 + 4)(x^2 - 9) = 0$

[b] A right circular cylinder , the length of its base radius is 7 cm. , its height is 10 cm , Find its volume

5 [a] If $x = \sqrt{3} - \sqrt{2}$ and $y = \frac{1}{\sqrt{3} - \sqrt{2}}$, find the value of : $x + y$

[b] **Find the arithmetic mean of the following table :**

Sets	5 –	15 –	25 –	35 –	45	Total
Frequency	2	1	3	3	1	10

10 Kafr El-Sheikh Governorate

Biala Directorate
Rakha Official Language School

Answer the following questions :

1 Choose the correct answer :

1. The irrational number that lies between 2 , 3 is ....
   (a) $\sqrt{3}$ (b) 2.5 (c) $\sqrt{10}$ (d) $\sqrt{7}$
2. If the point of intersection of the ascending and descending curves is (12 , 40) , then the median is ....
   (a) 12 (b) 40 (c) 56 (d) 18
3. The S.S. of the equation : $X + 7 = |-7|$ in $\mathbb{N}$ is ....
   (a) $\varnothing$ (b) $\{14\}$ (c) $\{-14\}$ (d) $\{0\}$
4. A right cylinder its volume is 90 π cm^3 and its height is 10 cm. , then the radius length of its base is .... cm.
   (a) 2 (b) 4 (c) 6 (d) 3
5. If (k , 1) satisfies the relation : $X + y = 6$, then k = ...
   (a) 1 (b) 5 (c) -5 (d) -4

---

2 Complete the following :

1. If the points A = (1 , 4) , B = (3 , 7) , then the slope of $\overleftrightarrow{AB}$ = ...
2. $\sqrt{8} + \sqrt{2} =$ ....
3. The mode of the values : 4 , 2 , 3 , 5 , 4 , 4 is ....
4. If the lower limit is 6 and the upper limit is 10 for a set of values , then the centre of this set is ....

---

3 [a] Find in $\mathbb{R}$ the S.S. of the inequality : $-1 < 3X - 7 < 5$

[b] If $X = \sqrt{5} - \sqrt{3}$, $y = \dfrac{2}{\sqrt{5} - \sqrt{3}}$, prove that : X , y are conjugate numbers

, then find the value of : $\dfrac{X + y}{X y}$

---

4 [a] If A =]-3 , 3] , B = [1 , 6[, find :

1. $A \cap B$ 2. $A \cup B$

[b] Find in the simplest form : $\sqrt{50} - \sqrt{18} - 2\sqrt{2}$

---

5 [a] Find three ordered pairs satisfying the relation : y = 5 X

[b] **The following table shows the frequency of marks of 20 students :**

Sets	5	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

Find the arithmetic mean for the marks of students.

11 El-Fayoum Governorate — Directorate of Education

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 If the mode of the values : 6 , 12 , 11 , 4 k is 12 , then k = ...

(a) 2 (b) 3 (c) 4 (d) 6

2 The arithmetic mean of the values : 7 , 8 , 16 , 9 is ....

(a) 10 (b) 11 (c) 8 (d) 17

3 The straight line representing the relation : $2x + 5y = 10$ intersects the x-axis at the point ....

(a) (2 , 0) (b) (5 , 0) (c) (0 , 2) (d) (0 , 5)

4 $\sqrt[3]{8k^6} =$ ....

(a) $2k^2$ (b) $3k^3$ (c) $4k^2$ (d) $8k$

5 If four times a number = 60 , then third of this number = ...

(a) 8 (b) 9 (c) 15 (d) 5

2 Complete the following :

1 $3m^2n^3 \times$ .... $= 15m^3n^5$

2 If (k , 2 k) satisfies the relation : $x + 2y + k = 42$, then k = ...

3 The slope of the straight line passing through the two points (0 , 4) , (4 , 5) is ....

4 A sphere its diameter length = 6 cm. , then its volume = ... π cm.^{3}

3 [a] If A =]-1 , 2] , B = [0 , 3] , **find using the number line each of the following :**

1 $A \cup B$ 2 $A \cap B$

[b] If $x = \frac{5}{\sqrt{7}-\sqrt{2}}$, $y = \sqrt{7}-\sqrt{2}$

, **prove that :** x , y are two conjugate numbers **and find the value of :** $(x + y)^2$

[a] A cube its lateral area = 64 cm^2 **Find :**

1 its edge length. 2 its total area.

3 its volume.

[b] Find with steps in the simplest form the value of : $2\sqrt{27} + 3\sqrt{12} - 4\sqrt{3} + \frac{1}{3}\sqrt{27}$

[a] Represent graphically the relation : $2X + 3y = 12$

[b] Find the values of A , B , then find the arithmetic mean of the following frequency distribution :

Sets	0 –	4 –	8	A –	16 –	Total
Frequency	5	10	14	6	B	40

12 Souhag Governorate

Directorate of Education
Math Supervision

Answer the following questions :

1 Choose the correct answer :

1 $\mathbb{R}_+ \cup \mathbb{R}_- =$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\varnothing$ (d) $\{0\}$

2 The slope of the horizontal line is ..

(a) 1 (b) undefined. (c) zero (d) 2

3 $]8 , 10[\cup \{8 , 9 , 10\} =$ ......

(a) $[8 , 10]$ (b) $]8 , 10[$ (c) $[8 , 10[$ (d) $\{8 , 10\}$

4 The lower limit of a set is 10 and its centre is 15 , then its upper limit is .....

(a) 10 (b) 15 (c) 20 (d) 30

5 $\sqrt{9} = \sqrt[3]{\quad}$

(a) 64 (b) 9 (c) 36 (d) 27

6 If the intersection point of the ascending and descending curves is (15 , 9) , then the median equals

(a) 6 (b) 15 (c) 9 (d) 30

Complete each of the following :

1 If the mode of the values : 15 , 11 , 5 a , 12 is 15 , then a = ...

2 If the point (5 , 2) satisfies the relation : $2X + y = c$, then c = ...

3 If the total area of a cube is 36 cm^2 , then its volume is .... cm^3

4 If $\frac{1}{x} = \sqrt{5} - 2$, then $x =$ .... (in the simplest form)

5 The slope of the straight line passing through the points (3 , 5) and (1 , 1) is

6 The S.S. in $\mathbb{R}$ of : $(x^2 + 4)(x^2 - 9) = 0$ is

[a] Find in $\mathbb{R}$ the S.S. of the inequality : $-2 < 3x + 4 < 16$ and represent it on the number line.

[b] Simplify : $\sqrt{75} - 2\sqrt{27} + 3\sqrt{\frac{1}{3}}$

[a] Find the base radius length of a right circular cylinder whose height is equal to its base radius length if its volume is 27π cm.^{3}

[b] If $x = \sqrt{7} - \sqrt{5}$, $y = \frac{2}{\sqrt{7} - \sqrt{5}}$ **, then find the value of :** $(x + y)^2$

[a] If $X = [-2 , 6]$, $Y = [-3 , 4]$ **, find using the number line :**

1 $X \cup Y$ 2 $X - Y$

[b] Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35	45 –	Total
Frequency	7	10	12	13	8	**50**

Second | Geometry

Accumulative Tests

on Geometry

Accumulative tests on Geometry

Accumulative test 1 on lesson 1 – unit 4

1 Choose the correct answer from the given ones :

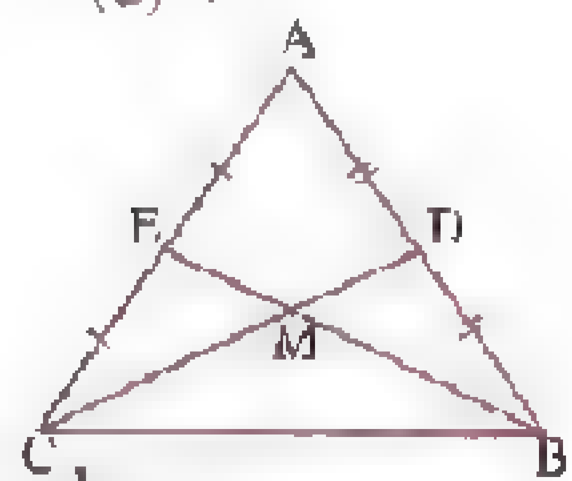
1. If M is the point of intersection of the medians of ΔABC , $\overline{AD}$ is a median , then AD = ...

   (a) 2 AM (b) $\frac{2}{3}$ MD (c) $\frac{3}{2}$ AM (d) 4 MD

2. The point of intersection of medians of the triangle divides each of them in the ratio 4 : .... from the base.

   (a) 2 (b) 8 (c) 1 (d) 4

3. **In the opposite figure :**

   BM = 6 cm. , then ME = ... cm.

   (a) 3 (b) 6

   (c) 7 (d) 9

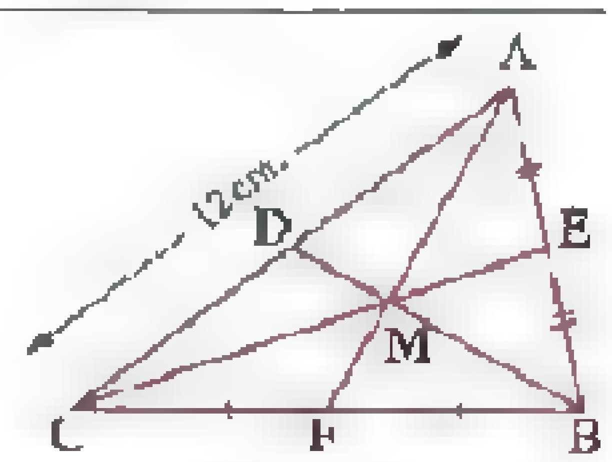
4. In ΔABC , $\overline{AD}$ is a median , M is the point of intersection of its medians , then $(AM)^2$ = ... $(AD)^2$

   (a) 2 (b) $\frac{3}{2}$ (c) $\frac{4}{9}$ (d) $\frac{1}{2}$

2 Complete the following :

1. The point of concurrence of the medians of the triangle divides each median in the ratio .... from the vertex.
2. If $\overline{AD}$ is a median in ΔABC , M is the point of intersection of the medians , MD = 2 cm. , then AM = ... cm.
3. The number of medians of the scalene triangle is ....
4. The medians of the triangle intersect at ....

3 In the opposite figure :

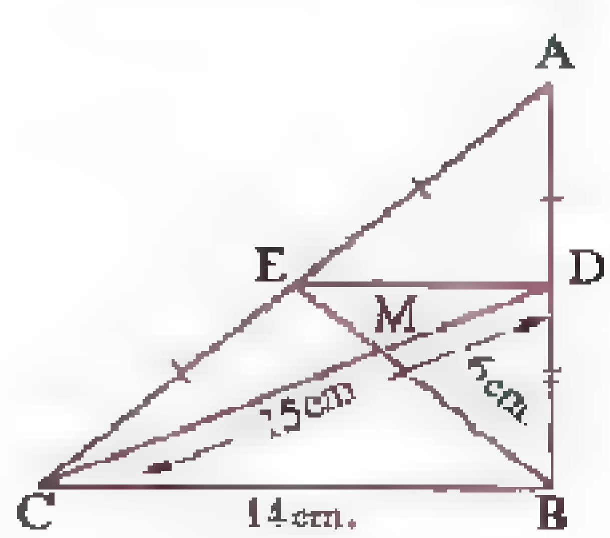


E is the midpoint of $\overline{AB}$

, F is the midpoint of $\overline{BC}$, AC = 12 cm.

Find with proof : The length of $\overline{AD}$

4 In the opposite figure :



M is the point of intersection of the medians of ΔABC

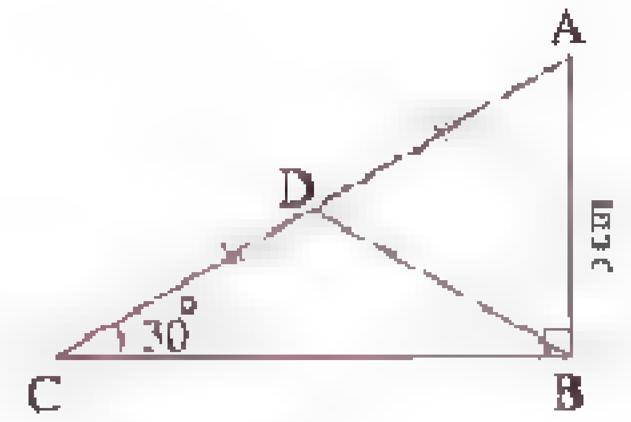
, BM = 6 cm. , BC = 14 cm. , DC = 15 cm.

Find : The perimeter of ΔMDE

Accumulative test 2 till lesson 2 – unit 4

1 Choose the correct answer from the given ones :

1 In the opposite figure :



ABC is a right-angled triangle at B

, D is the midpoint of $\overline{AC}$, m (∠ ACB) = 30°

, AB = 5 cm. , then BD = ... cm.

(a) 5 (b) 10 (c) 2.5 (d) 15

2 If $\overline{BD}$ is a median in △ ABC , BD = $\frac{1}{2}$ AC , then ....

(a) m (∠ ABC) = 90° (b) m (∠ BAC) = 90°

(c) m (∠ ABC) = 30° (d) m (∠ ACB) = 90°

3 If M is the point of intersection of the medians of △ ABC , D is the midpoint of $\overline{BC}$, then MD : AD = ...

(a) 1 : 2 (b) 2 : 3 (c) 1 : 3 (d) 3 : 2

4 A rectangle , its diagonals intersect at M , the length of its diagonal is 6 cm. , then the length of the median $\overline{AM}$ is ....

(a) 1 cm. (b) 2 cm. (c) 3 cm. (d) 4 cm.

2 Complete the following :

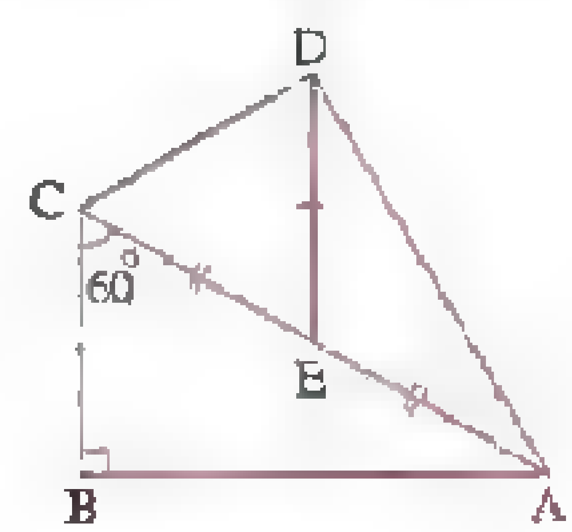
1 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ....

2 The point of intersection of the medians of the triangle divides each median in the ratio 2 : .... from the base.

3 If M is the point of intersection of the medians of △ ABC , $\overline{AD}$ is a median its length is 6 cm. , then AM = ... cm.

4 If ABC is a right-angled triangle at B , AB = 3 cm. , BC = 4 cm. , then the length of the median drawn from B to $\overline{AC}$ = ...

3 In the opposite figure :



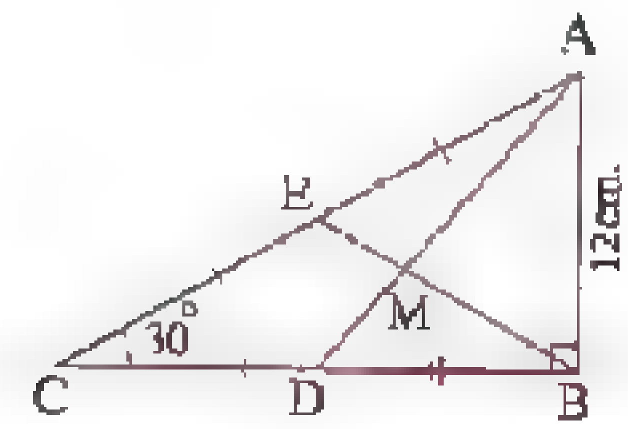
ABC is a right-angled triangle at B

, m (∠ ACB) = 60°

, E is the midpoint of $\overline{AC}$, DE = BC

Prove that : m (∠ ADC) = 90°

4 In the opposite figure :



ABC is a right-angled triangle at B

, m ($\angle$ C) = 30° , D is the midpoint of $\overline{BC}$

, E is the midpoint $\overline{AC}$, $\overline{AD} \cap \overline{BE} = \{M\}$

, if AB = 12 cm. , AD = 15 cm.

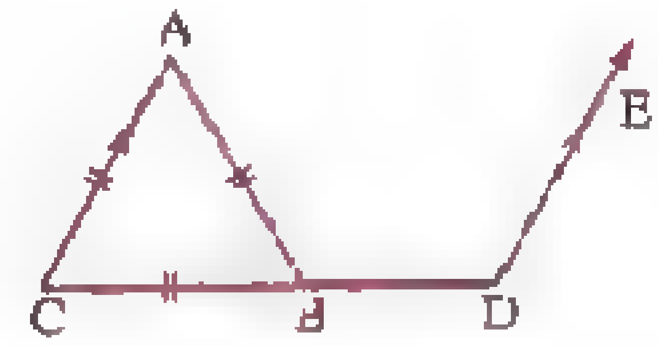
Find with proof :

1. The length of $\overline{AE}$
2. The length of $\overline{ME}$
3. The perimeter of Δ AME

Accumulative test 3 till lesson 3 unit 4

1 **Choose the correct answer from the given ones :**

1 **In the opposite figure :**



ABC is an equilateral triangle

, $\overline{DE}$ // $\overline{CA}$, then m (∠ D) = ...

(a) 100° (b) 60°

(c) 120° (d) 150°

2 The point of intersection of the medians of the triangle divides each median in the ratio .... from the base.

(a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 1 : 3

3 ABC is a right-angled triangle at B , AC = 20 cm. , D is the midpoint of $\overline{AC}$, then BD = ... cm.

(a) 10 (b) 8 (c) 6 (d) 5

4 In Δ ABC , if AB = AC , m (∠ A) = 2 m (∠ B) , then m (∠ C) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

2 **Complete the following :**

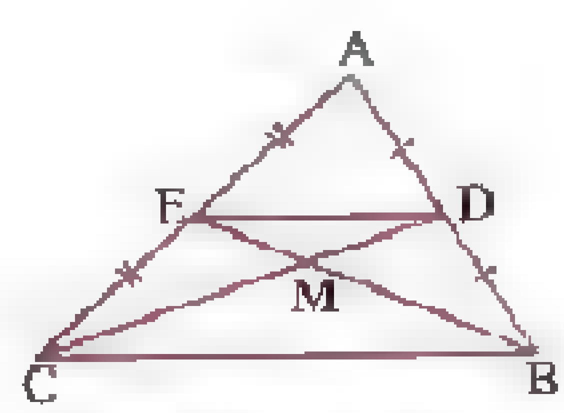
1 The two base angles of the isosceles triangle are ....

2 If ABC is a right-angled triangle at B , m (∠ C) = 30° , AC = 8 cm. , then AB = ... cm.

3 If the measure of the vertex angle of an isosceles triangle is 80° , then the measure of its base angles = ... °

4 The measure of the exterior angle of the equilateral triangle is .... °

3 **In the opposite figure :**

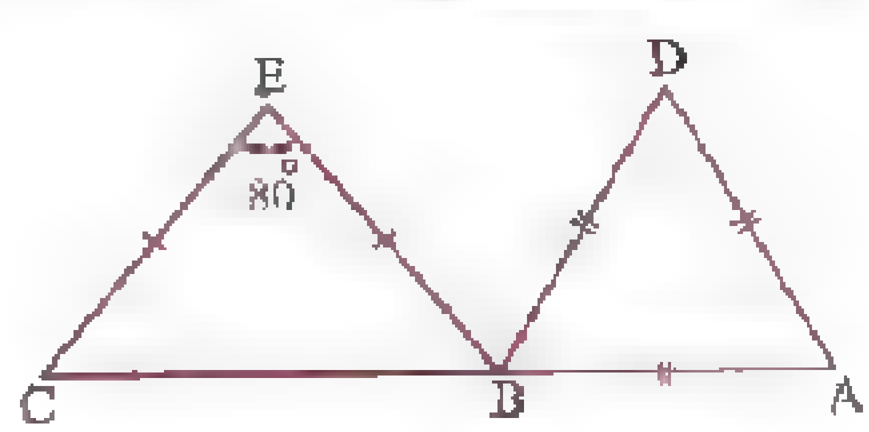


$\overline{BE}$, $\overline{CD}$ are two medians in Δ ABC intersect at point M

, the perimeter of Δ MDE = 12 cm

Find : The perimeter of Δ MBC

4 **In the opposite figure :**



B ∈ $\overline{AC}$, Δ ABD is equilateral

, EB = EC , m (∠ E) = 80°

Find : m (∠ DBE)

Accumulative test 4 till lesson 4 – unit 4

1 Choose the correct answer from the given ones :

1 If the measures of two angles in a triangle are 42° , 69° , then the triangle is ....

(a) isosceles. (b) scalene. (c) equilateral. (d) otherwise.

2 If the length of the median drawn from the vertex of the right angle in the right-angled triangle equals .... the hypotenuse.

(a) half (b) double (c) quarter (d) third

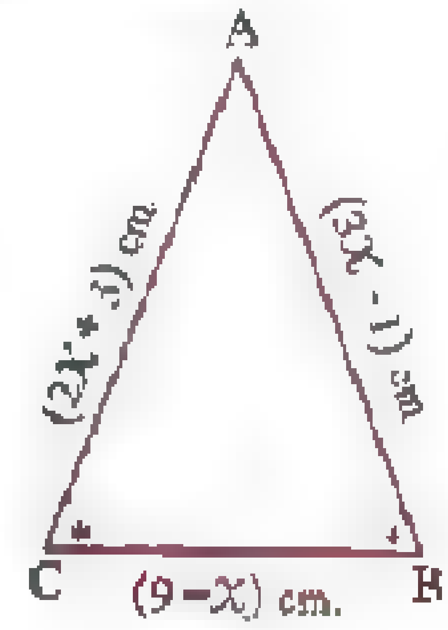
3 **In the opposite figure :**

ABC is a triangle in which

m (∠ B) = m (∠ C) , then X = ...

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) 2 (d) 4



4 If ABC is a triangle , AB = BC , then ∠ C is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

2 Complete the following :

1 An isosceles triangle , the measure of one of its angles is 60° , then the triangle is ....

2 The point of intersection of the medians of the triangle divides each median in the ratio .... : .... from the base.

3 A right-angled triangle , the measure of one of its angles is 45° , then the triangle is ....

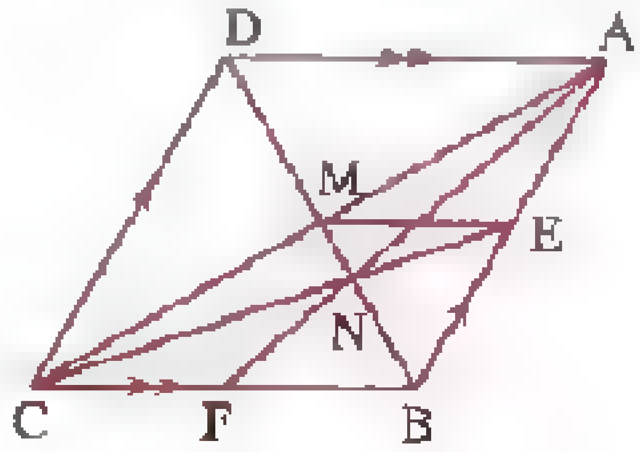
4 In Δ ABC , if $\overline{AD}$ is a median , M is the point of intersection of its medians , AM = 6 cm. , then MD = ... cm.

3 In the opposite figure :

ABCD is a parallelogram its diagonals intersect at M

, if N ∈ $\overline{BM}$ where BN = 2 NM , $\overrightarrow{CN} \cap \overline{AB} = \{E\}$

Prove that : EM = $\frac{1}{2}$ BC

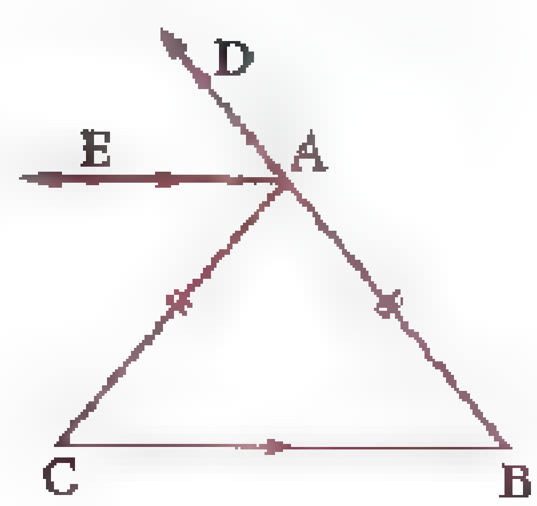


4 In the opposite figure :

D ∈ $\overrightarrow{BA}$, AB = AC

, $\overrightarrow{AE}$ // $\overline{BC}$

Prove that : $\overrightarrow{AE}$ bisects ∠ DAC



Accumulative test 5 till lesson 5 – unit 4

1 Choose the correct answer from the given ones :

1 The number of medians of the isosceles triangle is ......

(a) zero (b) 1 (c) 2 (d) 3

2 The triangle which has no axes of symmetry is ....

(a) the isosceles triangle. (b) the scalene triangle.

(c) the equilateral triangle. (d) the right-angled triangle.

3 If $\overleftrightarrow{AB}$ is the axis of symmetry of $\overline{FD}$, then $\frac{AD}{AF}$ = ...

(a) zero (b) 1 (c) $\frac{1}{2}$ (d) 2

4 ABC is an equilateral triangle , X is the point of intersection of its axes of symmetry , $\overrightarrow{AX}$ cuts $\overline{BC}$ at D , if DX = 5 cm. , then AX = ...

(a) 10 cm. (b) 15 cm. (c) 2.5 cm. (d) 7.5 cm

2 Complete the following :

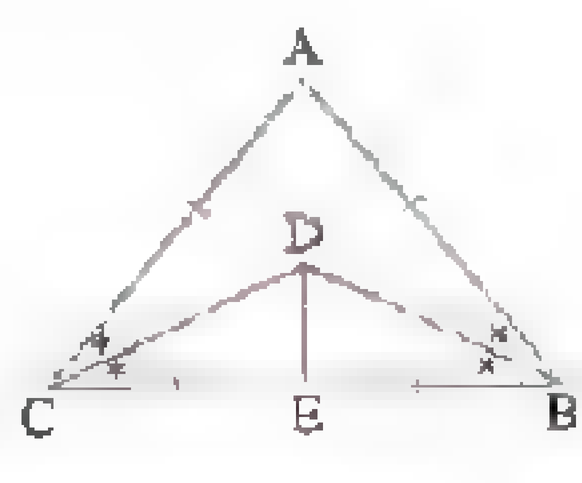
1 If the length of the median drawn from a vertex in a triangle equals half the length of the opposite side , then ....

2 Δ ABC has one axis of symmetry , m ($\angle$ A) = 120° , then m ($\angle$ B) = ...°

3 Δ ABC , if AB = AC , m ($\angle$ A) = 3 X , m ($\angle$ B) = 6 X , then X = ...°

4 The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and ....

3 In the opposite figure :



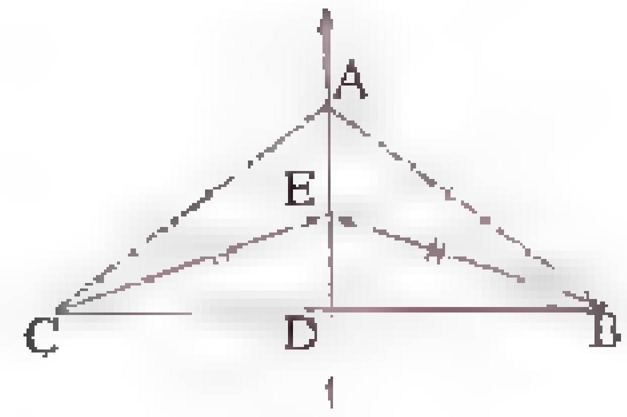
AB = AC , $\overrightarrow{BD}$ bisects $\angle$ ABC

, $\overrightarrow{CD}$ bisects $\angle$ ACB

, E is the midpoint of $\overline{BC}$

Prove that : $\overline{DE} \perp \overline{BC}$

4 In the opposite figure :



ABC is a triangle in which AB = AC = 10 cm.

, BE = EC , BC = 16 cm.

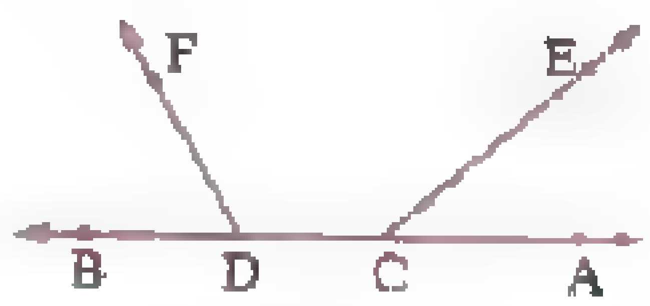
, $\overrightarrow{AE} \cap \overline{BC}$ = {D}

Find : The length of $\overline{AD}$

Accumulative test 6 till lesson 1 – unit 5

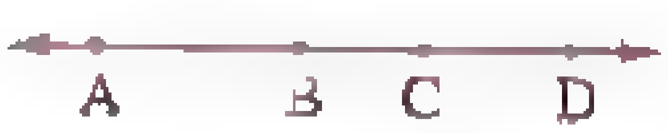
1 **Choose the correct answer from the given ones :**

1. If $x - z > y - z$, then x .... y
   (a) = (b) > (c) < (d) ≤
2. If $C \in$ the axis of symmetry of $\overline{AB}$, then AC – BC = ...
   (a) zero (b) 1 (c) 3 (d) 2
3. If Δ ABC is right angled at B , AB = $\frac{1}{2}$ AC , then m (∠ A) = ...
   (a) 45° (b) 30° (c) 90° (d) 60°
4. **In the opposite figure :**
   $C \in \overleftrightarrow{AB}$, $D \in \overleftrightarrow{AB}$
   , m (∠ ACE) < m (∠ BDF)
   , then m (∠ ECD) .... m (∠ FDC)
   (a) > (b) < (c) = (d) ≤



2 **Complete the following :**

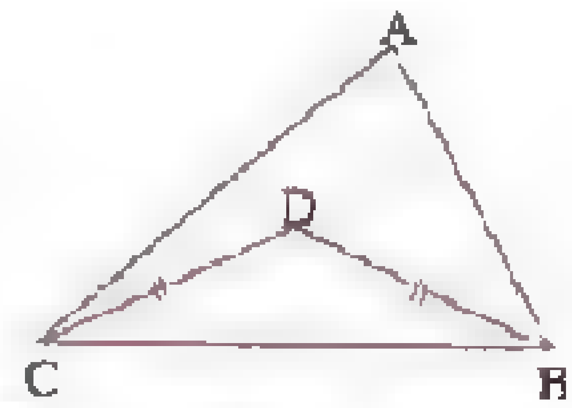
1. If the measure of the vertex angle of an isosceles triangle is 80° , then the measure of one of its base angles is ....
2. **In the opposite figure :**
   $C \in \overleftrightarrow{AB}$, $D \in \overleftrightarrow{AB}$, If AC > BD , then AB .... CD



3. The bisector of the vertex angle of an isosceles triangle ....
4. In Δ XYZ , if m (∠ X) = 40° , m (∠ Z) = 100° , then the number of its axes of symmetry is ....

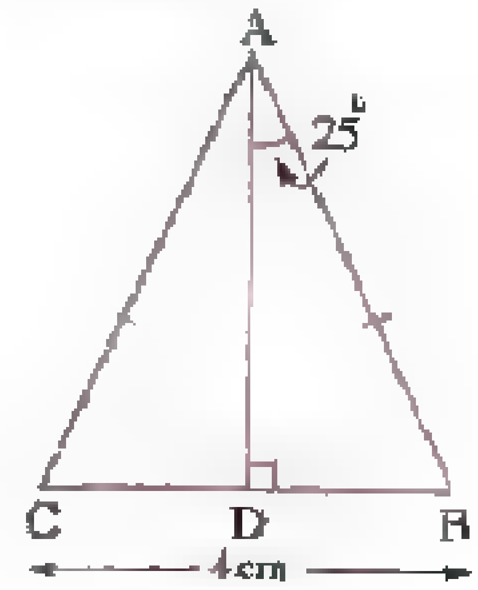
3 **In the opposite figure :**

m (∠ ABC) > m (∠ ACB)
, BD = CD
Prove that : m (∠ ABD) > m (∠ ACD)



4 **In the opposite figure :**

ABC is a triangle , AB = AC , $\overline{AD} \perp \overline{BC}$
, m (∠ BAD) = 25° , BC = 4 cm.
Find :
1. m (∠ DAC)
2. The length of $\overline{DC}$



Accumulative test 7 till lesson 2 – unit 5

1 **Choose the correct answer from the given ones :**

1 In Δ XYZ , $\overline{XY}$ is the shortest side , then the angle of the smallest measure is ....

(a) X (b) Y (c) Z (d) otherwise.

2 In Δ ABC , if AB = AC , then the exterior angle at the vertex C is ......

(a) straight. (b) acute. (c) obtuse. (d) reflex.

3 A triangle has 3 axes of symmetry , then the measure of the exterior angle at one of its vertices equals ....

(a) 90° (b) 80° (c) 120° (d) 60°

4 In Δ ABC , AB = 7 cm. , BC = 5 cm. , AC = 6 cm. , then m ($\angle$ B) .... m ($\angle$ C)

(a) > (b) < (c) = (d) ≡

2 **Complete the following :**

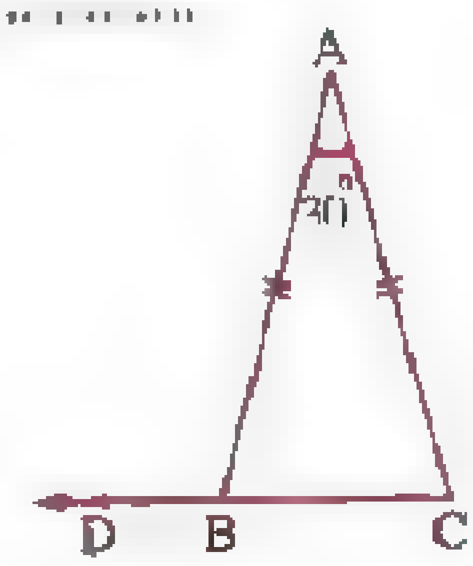
1 In a triangle , if two sides have unequal lengths , then the longer is opposite to ....

2 If A lies on the axis of symmetry $\overleftrightarrow{XY}$, then AX .... AY

3 ABC is a triangle in which : m ($\angle$ A) = 100° , then its longest side is ....

4 **In the opposite figure :**

m ($\angle$ ABD) = ... °

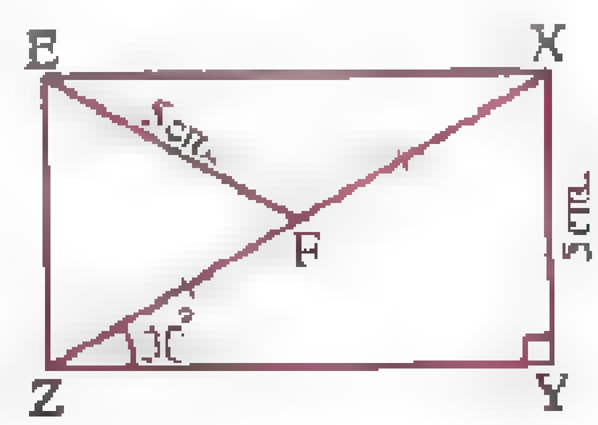


3 **In the opposite figure :**

m ($\angle$ Y) = 90° , m ($\angle$ XZY) = 30°

, XY = EF = 5 cm. , F is the midpoint of $\overline{XZ}$

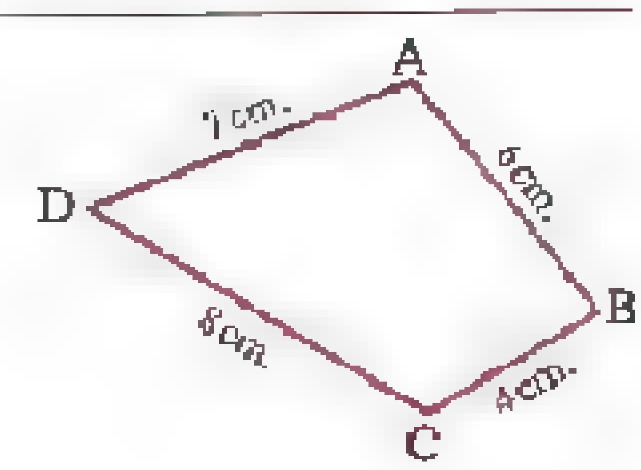
Prove that : m ($\angle$ XEZ) = 90°



4 **In the opposite figure :**

From the data on the figure.

Prove that : m ($\angle$ ABC) > m ($\angle$ ADC)



Accumulative test 8 till lesson 3 – unit 5

1 Choose the correct answer from the given ones :

1 The ratio between the length of the hypotenuse and the median drawn from the vertex of the right angle in the right angled triangle is ....

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

2 If the measures of two angles in a triangle are 48° , 84° , then its type is ....

(a) isosceles. (b) equilateral. (c) scalene. (d) right-angled.

3 If ABC is an obtuse-angled triangle at C , then BC .... AB

(a) > (b) < (c) = (d) ≥

4 The longest side in Δ XYZ where m ($\angle$ Y) = m ($\angle$ X) + m ($\angle$ Z) is ....

(a) $\overline{XY}$ (b) $\overline{XZ}$ (c) $\overline{YZ}$ (d) otherwise.

2 Complete the following :

1 The axis of symmetry of a line segment is ....

2 In Δ ABC , if AB < BC , AB > AC , then the smallest angle in measure is ....

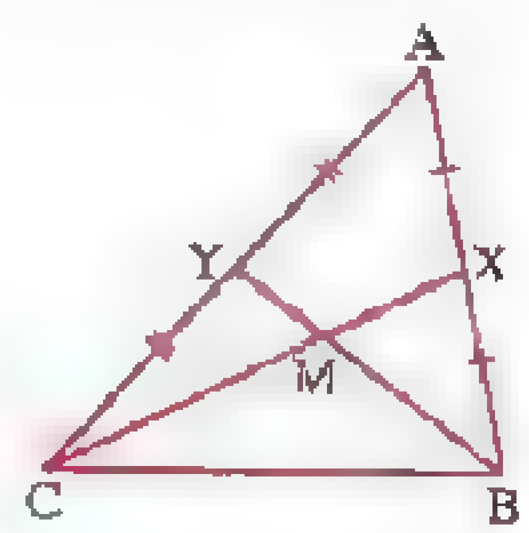
3 If XYZ is a triangle in which m ($\angle$ X) = 50° , m ($\angle$ Y) = 70° , then XY .... YZ

4 In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to ....

3 In Δ ABC , m ($\angle$ A) = $(5X + 2)^\circ$, m ($\angle$ B) = $(6X - 10)^\circ$, m ($\angle$ C) = $(X + 20)^\circ$

Order the lengths of the sides of the triangle ascendingly.

4 In the opposite figure :



X , Y are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively , XM > YM

Prove that : m ($\angle$ MBC) > m ($\angle$ MCB)

Accumulative test 9 till lesson 4 – unit 5

1 Choose the correct answer from the given ones :

1 If z , 12 , 2 z are the lengths of sides of a triangle , then the greatest value of z = ...

(a) 12 (b) 11 (c) 4 (d) 3

2 If ABC is a right-angled triangle at B , then ....

(a) AC < AB (b) AC < BC (c) AB < AC (d) BC > AB

3 ABC is a right-angled triangle at B , if AC = 20 cm. , then the length of the median drawn from B equals .... cm.

(a) 10 (b) 8 (c) 6 (d) 5

4 A triangle has one axis of symmetry and the lengths of two sides in it are 3 cm. , 8 cm , then its perimeter = ... cm.

(a) 14 (b) 19 (c) 11 (d) 24

2 Complete the following :

1 The distance between a point and a given straight line is the length of .... drawn from the point to this straight line.

2 The length of any side in a triangle .... the sum of the lengths of the other two sides.

3 In a triangle , if the lengths of two sides of a triangle are 4 cm. and 9 cm. , then the length of the third side $\in$]............ , ...[

4 The exterior angle at any vertex of the equilateral triangle is .... angle.

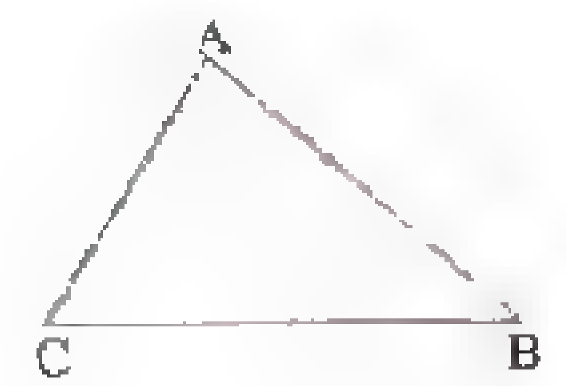
3 In Δ ABC , m ($\angle$ A) = 50° , m ($\angle$ B) = 70°

Order the lengths of the sides of the triangle descendingly.

4 In the opposite figure :

ABC is a triangle.

Prove that : AB < $\frac{1}{2}$ the perimeter of Δ ABC



Monthly Tests

on Geometry

October contents

November contents

October tests on Geometry

Total mark 10

Test 1

1 Choose the correct answer from the given ones : (3 marks)

1 The number of medians of the right-angled triangle is ....

(a) zero (b) 1 (c) 2 (d) 3

2 If ABC is a right-angled triangle at B and D is the midpoint of $\overline{AC}$, then BD = ...

(a) $\frac{1}{2}$ AC (b) AC (c) $\frac{1}{2}$ BC (d) AB

3 Δ XYZ is an isosceles triangle in which m ($\angle$ Y) = 100° , then m ($\angle$ Z) = ...

(a) 100° (b) 80° (c) 50° (d) 40°

2 Complete the following : (3 marks)

1 The length of the hypotenuse in the right-angled triangle equals .... the length of the side opposite to the angle whose measure is 30°

2 The measure of the exterior angle of the equilateral triangle equals ....°

3 The point of intersection of medians of the triangle divides each of them in the ratio .... : 2 from the base.

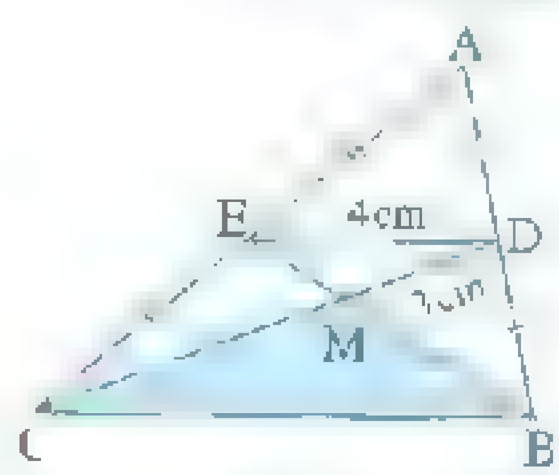
3 In the opposite figure : (2 marks)

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, $\overline{BE} \cap \overline{DC} = \{M\}$, DE = 4 cm.

, DM = 3 cm. , BE = 6 cm.

Find : The perimeter of Δ BMC



4 In the opposite figure : (2 marks)

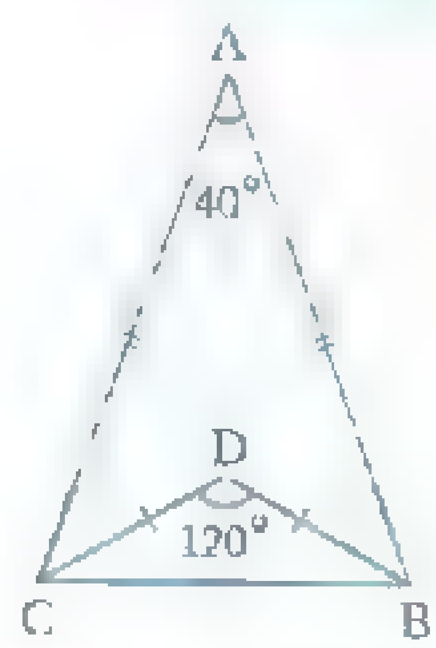
AB = AC

, BD = CD

, m ($\angle$ A) = 40°

, m ($\angle$ D) = 120°

Find : m ($\angle$ ABD)



Test 2

Total mark

10

1 Choose the correct answer from the given ones : (3 marks)

1 If M is the point of concurrence of medians of Δ ABC and $\overline{BD}$ is a median , then BM : MD = ...

(a) 2 : 3 (b) 2 : 1 (c) 3 : 1 (d) 1 : 2

2 In Δ ABC , if m ($\angle$ B) = 90° and m ($\angle$ C) = 30° , then AB = ... AC

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) twice (d) $\frac{1}{4}$

3 If the measure of one of the base angles of an isosceles triangle is 45° , then the triangle is .... triangle.

(a) obtuse-angled (b) acute-angled (c) right-angled (d) equilateral

2 Complete the following : (3 marks)

1 The medians of the triangle intersect at ....

2 If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then ....

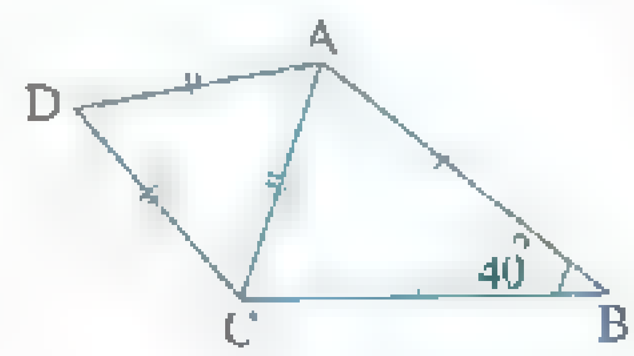
3 In an isosceles triangle , if the measure of one of its base angles equals 65° , then the measure of the vertex angle is ....

3 In the opposite figure : (2 marks)

AD = DC = AC , AB = BC

, m ($\angle$ ABC) = 40°

Find : m ($\angle$ BAD)



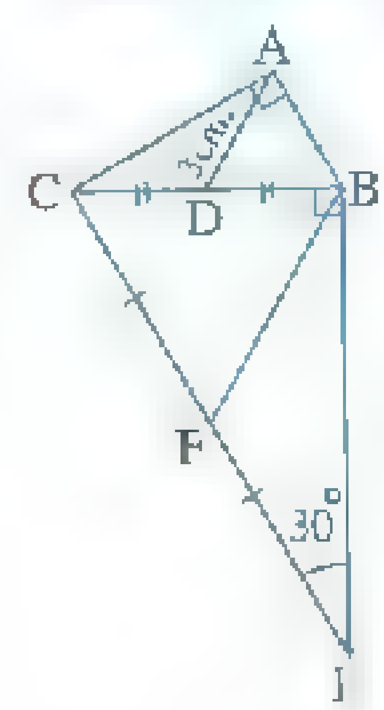
4 In the opposite figure : (2 marks)

m ($\angle$ BAC) = m ($\angle$ CBE) = 90° , m ($\angle$ BEC) = 30°

, D and F are the midpoints of $\overline{BC}$ and $\overline{CE}$ respectively

, AD = 3 cm.

Find : the length of $\overline{BF}$



November tests on Geometry

Test 1

Total mark 10

1 Choose the correct answer from the given ones : (3 marks)

1 If the measure of an interior angle in an isosceles triangle is 60°, then the number of axes of symmetry of this triangle is ....

(a) 4 (b) 3 (c) 2 (d) 1

2 **In the opposite figure :**

If $C \in \overleftrightarrow{AD}$ and $B \in \overleftrightarrow{AD}$ where $AB > DC$

, then DB .... AC

(a) > (b) < (c) = (d) ≥

3 If △ ABC has one axis of symmetry and m (∠ ABC) = 120°, then m (∠ A) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

2 Complete the following : (3 marks)

1 The straight line passing through the vertex angle of an isosceles triangle perpendicular to its base ....

2 In △ ABC, if AB = AC and m (∠ B) = m (∠ A), then m (∠ C) = ... °

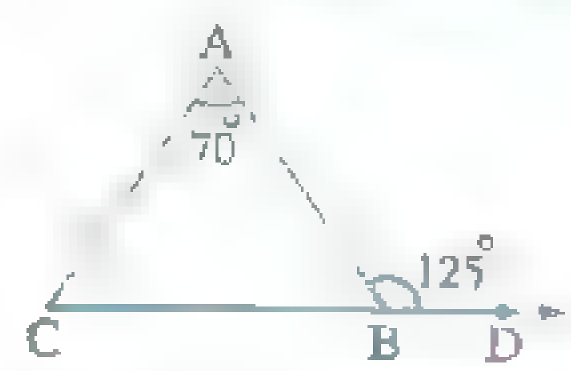
3 If M ∈ the axis of $\overline{XY}$, then $\frac{5\ MX}{4\ MY}$ = ...

3 In the opposite figure : (2 marks)

$D \in \overrightarrow{CB}$, m (∠ ABD) = 125°

and m (∠ A) = 70°

Prove that : △ ABC is an isosceles triangle.

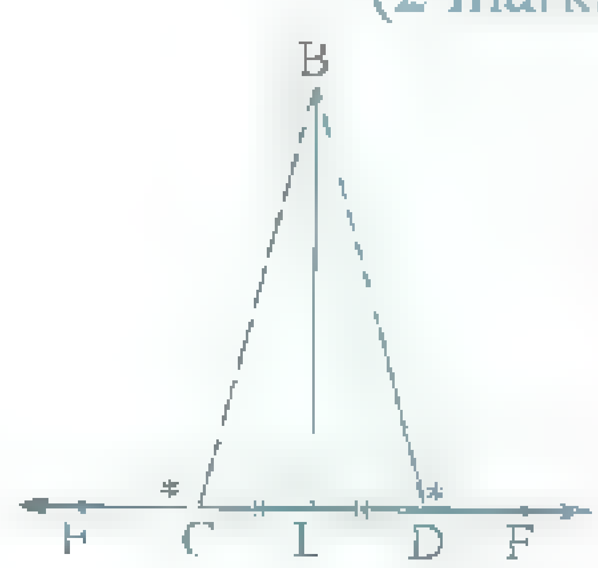


4 In the opposite figure : (2 marks)

CL = DL

, m (∠ BCE) = m (∠ BDF)

Prove that : $\overline{BL} \perp \overline{CD}$



Test 2

Total mark: 10

1 Choose the correct answer from the given ones : (3 marks)

1 **In the opposite figure :**

If $C \in \overleftrightarrow{XY}$, m $(\angle ACX) = 35°$

and m $(\angle BCY) = 45°$

, then m $(\angle XCB)$ .... m $(\angle ACY)$

(a) > (b) < (c) = (d) otherwise

2 If C ∈ the axis of symmetry of $\overline{AB}$, then AC – BC = ...

(a) zero (b) 1 (c) 2 (d) 4

3 In the square ABCD , $\overline{BD}$ is the axis of symmetry of ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{AD}$ (d) $\overline{CD}$

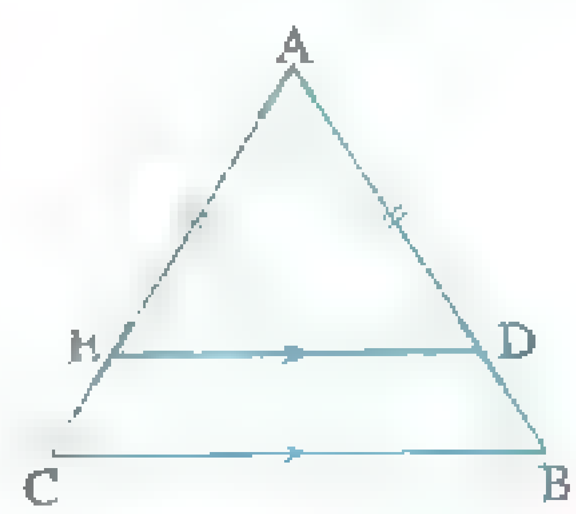
2 **Complete the following :** (3 marks)

1 The bisector of the vertex angle of an isosceles triangle ....

2 If the measure of one angle of the right-angled triangle is 45° , then the triangle is ....

3 The number of axes of symmetry of the scalene triangle equals ....

3 **In the opposite figure :** (2 marks)

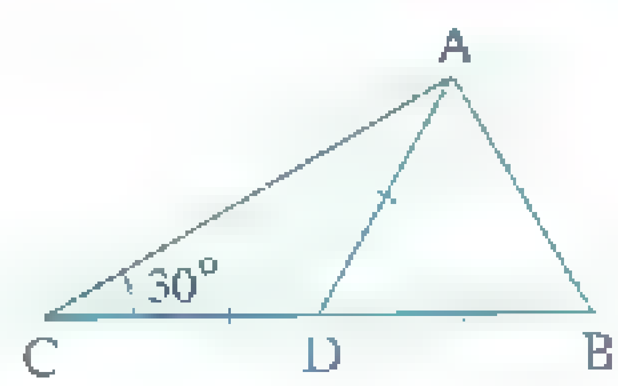


$\overline{DE} \parallel \overline{BC}$

, AD = AE

Prove that : Δ ABC is an isosceles triangle.

4 **In the opposite figure :** (2 marks)



D ∈ $\overline{BC}$ such that DA = DB = DC

and m $(\angle C) = 30°$

Prove that : Δ ABD is an equilateral triangle.

Important Questions

on Geometry

Important questions on Unit Four

Medians of Triangle – Isosceles Triangle

First Multiple choice questions

1. The number of medians of the right-angled triangle equals ....
   (a) one. (b) two. (c) three. (d) four.

2. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.
   (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{2}{4}$ (d) 2

3. The length of the median drawn from the vertex of the right angle in the right-angled triangle equals .... the length of the hypotenuse.
   (a) third (b) quarter (c) half (d) double

4. The measure of the exterior angle of the equilateral triangle equals ....°
   (a) 60 (b) 90 (c) 120 (d) 180

5. If the measure of the vertex angle of an isosceles triangle is 50° , then the measure of one of its base angles is ....°
   (a) 65 (b) 45 (c) 55 (d) 70

6. If the measure of one of the base angles of an isosceles triangle is 40° , then the measure of its vertex angle equals ....°
   (a) 40 (b) 50 (c) 80 (d) 100

7. An isosceles triangle , the measure of one of its angles is 60° , then the number of its axes of symmetry is ....
   (a) 1 (b) zero. (c) 3 (d) 2

8. If the measures of two angles of a triangle are 42° , 69° , then its type is ....
   (a) scalene. (b) isosceles. (c) equilateral. (d) right-angled.

9. The point of concurrence of the medians of the triangle divides each of them in the ratio of .... from the base.
   (a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 3 : 1

10. The point of concurrence of the medians of the triangle divides each median in the ratio of 5 : .... from the vertex.
    (a) 2.5 (b) 5 (c) 6 (d) 10

If M is the point of intersection of the medians of Δ ABC , AD is a median , then AD = ... AM

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

In Δ ABC , $\overline{AD}$ is a median , M is the point of intersection of its medians. If AD = 9 cm. , then AM = ... cm.

(a) 3 (b) 6 (c) 9 (d) 12

The triangle in which the measures of two angles are 48° , 84° , then the number of its axes of symmetry is ....

(a) 1 (b) 2 (c) 3 (d) zero

If X lies on the axis of symmetry of $\overline{AB}$, then XA .... XB

(a) // (b) ⊥ (c) = (d) ≡

Δ XYZ is an isosceles triangle in which m ($\angle$ X) = 100° , then m ($\angle$ Y) = ... °

(a) 100 (b) 80 (c) 60 (d) 40

Δ ABC is right-angled at B , AC = 12 cm. , m ($\angle$ A) = 60° , then AB = ... cm.

(a) 12 (b) 6 (c) 4 (d) 3

A right angled isosceles triangle , then the measure of one of its base angles is . . .. °

(a) 30 (b) 45 (c) 60 (d) 90

18 **In the opposite figure :**

$x + y$ = ...

(a) 100° (b) 140°

(c) 180° (d) 280°

Complete questions

The medians of a triangle intersect at ....

The base angles of the isosceles triangle are ... .

The bisector of the vertex angle of an isosceles triangle is . . .. to the base and . ...

The median drawn from the vertex angle of an isosceles triangle is .... and .. ..

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of .... and ....

6 The straight line perpendicular to a line segment at its midpoint is ....

7 The isosceles triangle in which the measure of one of its angles equals 60° is ....

8 If the measure of one angle in a right-angled triangle is 45° , then the triangle is ....

9 If $C \in$ the axis of symmetry of $\overline{AB}$, then AC – BC = ...

10 If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is ....

11 In Δ ABC , AB = AC , m ($\angle$ A) = 3 m ($\angle$ B) , then m ($\angle$ C) = ...°

Third Essay questions

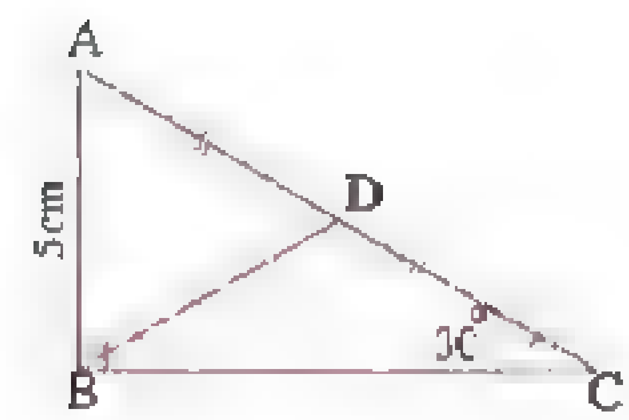
1 **In the opposite figure :**

m ($\angle$ ABC) = 90°

, m ($\angle$ C) = 30° , AD = DC

, AB = 5 cm.

Calculate : The length of each of $\overline{CA}$ and $\overline{BD}$

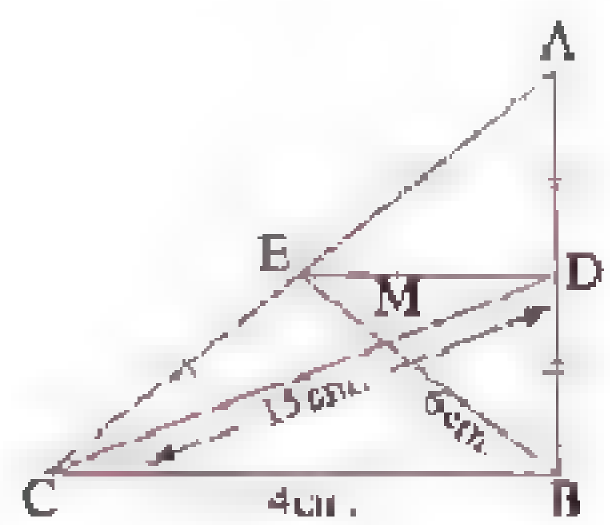


2 **In the opposite figure :**

M is the intersection point of the medians of the triangle ABC

, BM = 6 cm. , BC = 14 cm. , DC = 15 cm.

Find : The perimeter of Δ MDE



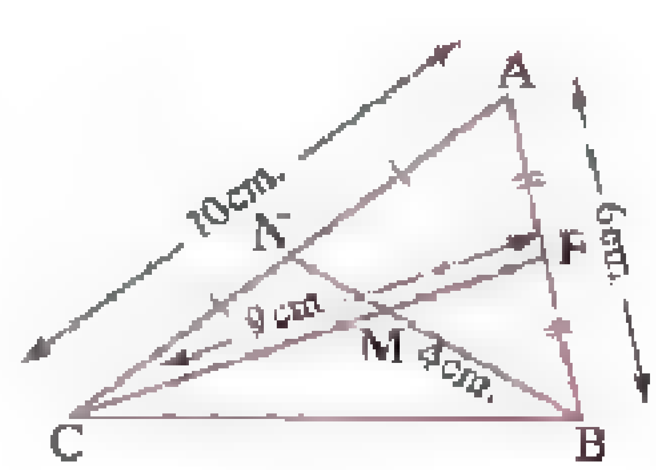
3 **In the opposite figure :**

F and N are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, AB = 6 cm. , AC = 10 cm.

, BM = 4 cm , CF = 9 cm.

Find : The perimeter of the figure AFMN



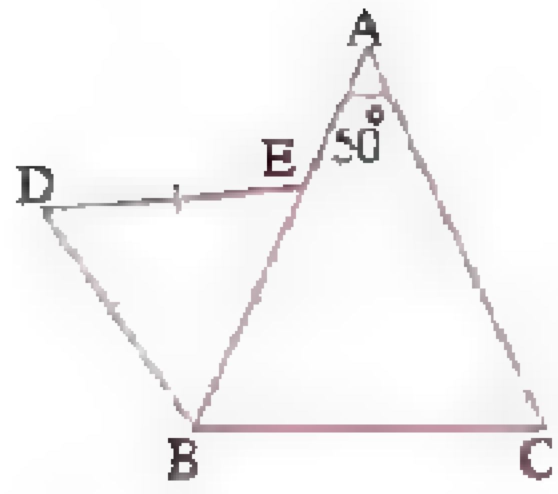
4 **In the opposite figure :**

$AB = AC$

, $m(\angle A) = 50^\circ$

, $BD = BE = DE$

Find : $m(\angle DBC)$



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5 **In the opposite figure :**

$\overrightarrow{AD} \parallel \overline{CB}$, $m(\angle DAB) = 30^\circ$

, $BA = BC$

Find : The measures of the angles of ΔABC

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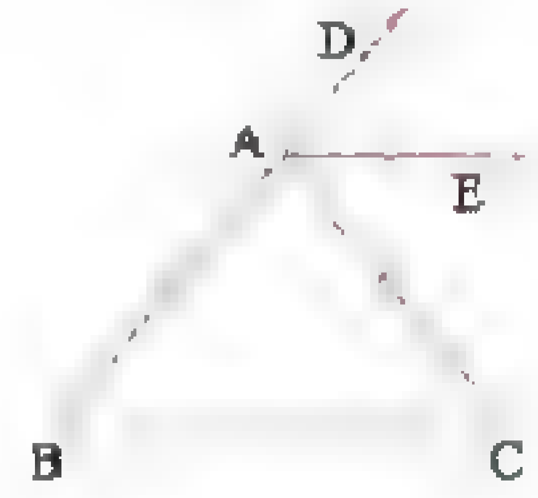
6 **In the opposite figure :**

$AB = AC$

, $\overrightarrow{AE} \parallel \overline{BC}$

Prove that : $\overrightarrow{AE}$ bisects $\angle DAC$



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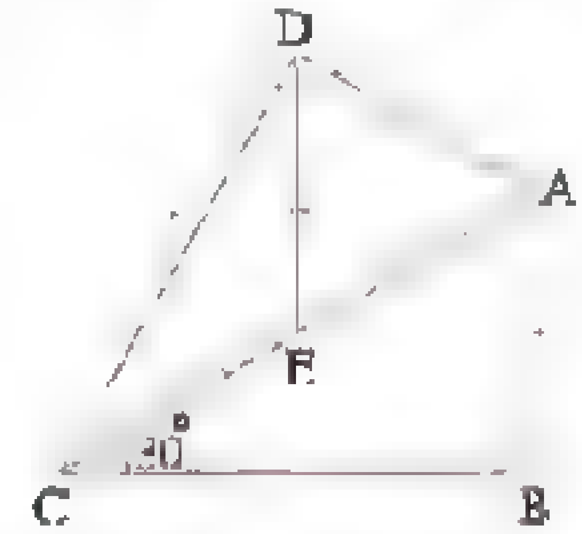
7 **In the opposite figure :**

$AB = DE$, E is the midpoint of $\overline{AC}$

, $m(\angle B) = 90^\circ$

, $m(\angle ACB) = 30^\circ$

Prove that : $m(\angle ADC) = 90^\circ$



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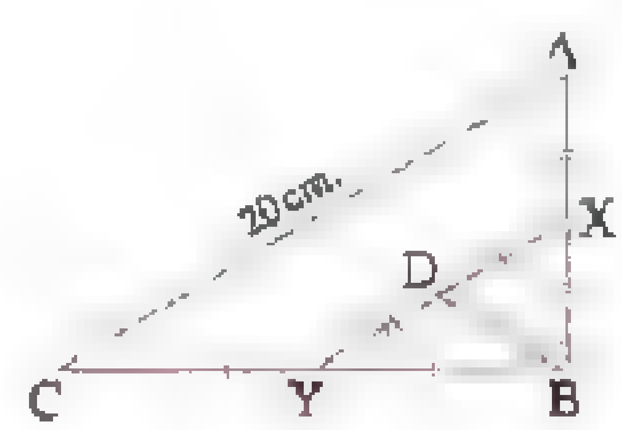
8 **In the opposite figure :**

$m(\angle ABC) = 90^\circ$, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

, D is the midpoint of $\overline{XY}$, $AC = 20$ cm.

Find : The length of $\overline{BD}$



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9 **In the opposite figure :**

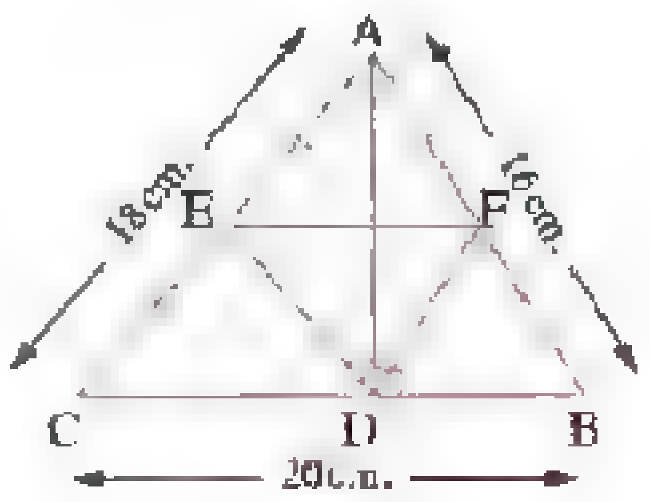
$AB = 16$ cm. , $AC = 18$ cm.

, $BC = 20$ cm.

, F and E are the midpoints of $\overline{AB}$ and $\overline{CA}$ respectively

, $\overline{AD} \perp \overline{CB}$

Find : The perimeter of ΔDEF



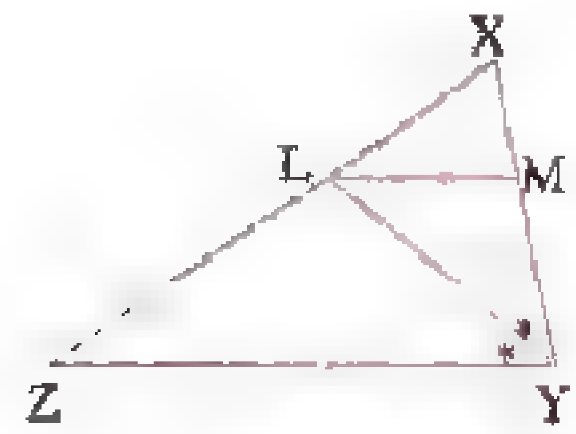
10 **In the opposite figure :**

XYZ is a triangle , $\overrightarrow{YL}$ bisects $\angle$ XYZ

and intersects $\overline{XZ}$ at L

, $\overline{LM}$ // $\overline{YZ}$ and intersects $\overline{XY}$ at M

Prove that : Δ LMY is an isosceles triangle.



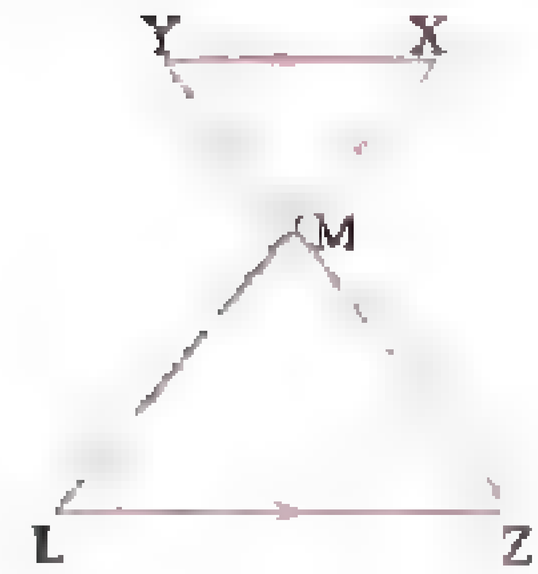
11 **In the opposite figure :**

XM = YM

, $\overline{XY}$ // $\overline{ZL}$

Prove that :

Δ MLZ is an isosceles triangle.



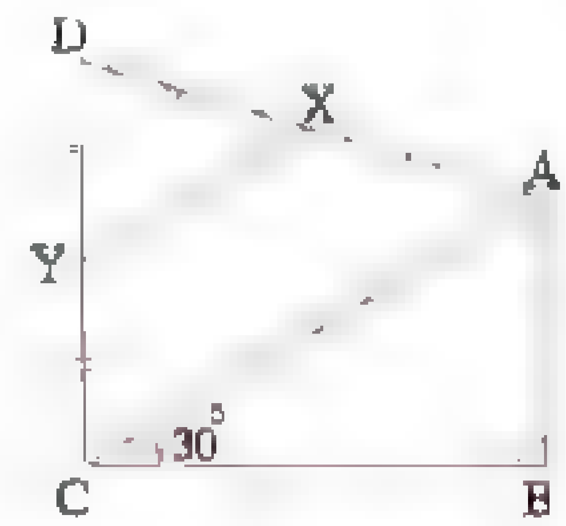
12 **In the opposite figure :**

m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

, X and Y are the midpoints

of $\overline{AD}$ and $\overline{CD}$ respectively.

Prove that : AB = XY

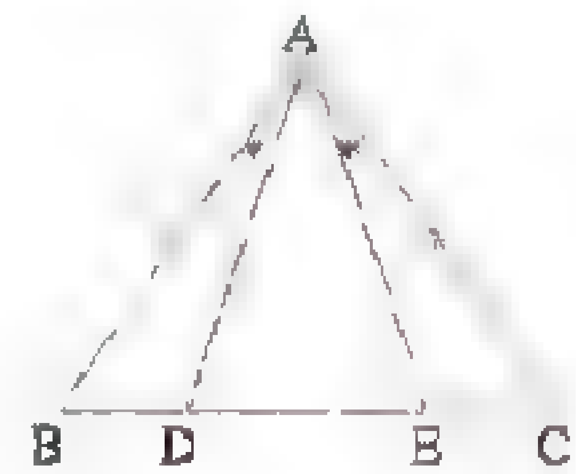


13 **In the opposite figure :**

AB = AC

, m ($\angle$ BAD) = m ($\angle$ CAE)

Prove that : AD = AE

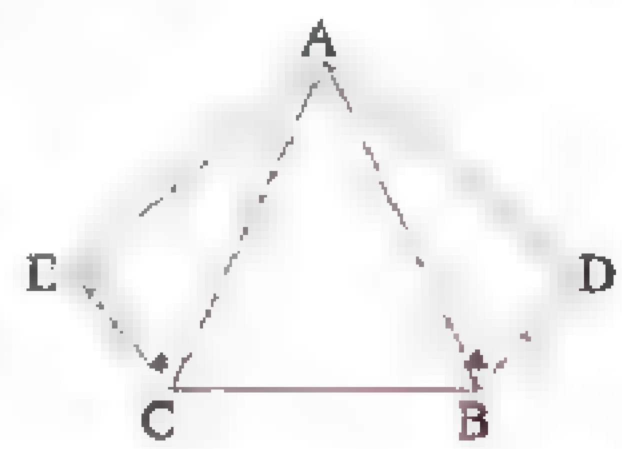


14 **In the opposite figure :**

BD = CE , m ($\angle$ D) = m ($\angle$ E) = 90°

, m ($\angle$ ABD) = m ($\angle$ ACE)

Prove that : m ($\angle$ ABC) = m ($\angle$ ACB)



15 **In the opposite figure :**

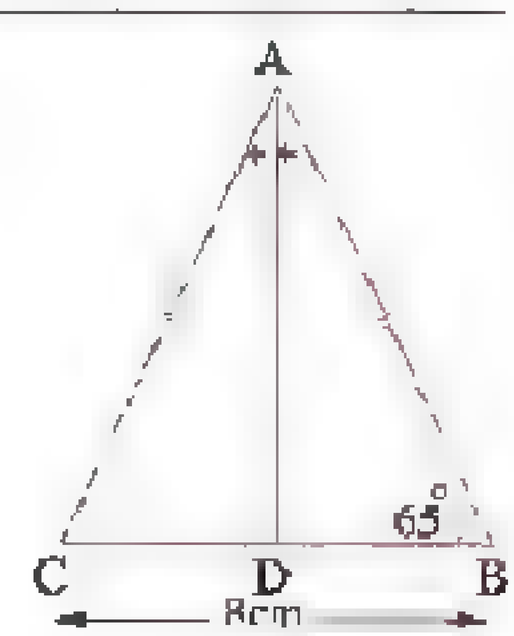
AB = AC , $\overrightarrow{AD}$ bisects $\angle$ BAC

, BC = 8 cm.

, m ($\angle$ B) = 65°

Prove that : $\overline{AD} \perp \overline{BC}$

and find : The length of $\overline{DC}$, m ($\angle$ DAC)

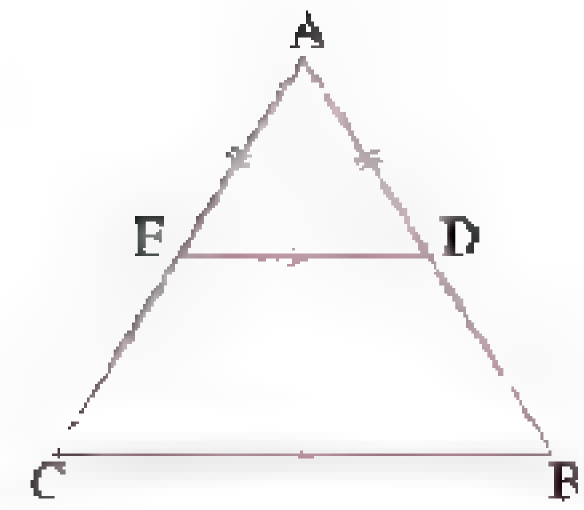


16 **In the opposite figure :**

$\overline{DE} \parallel \overline{BC}$

, $AD = AE$

Prove that : ΔABC is an isosceles triangle.



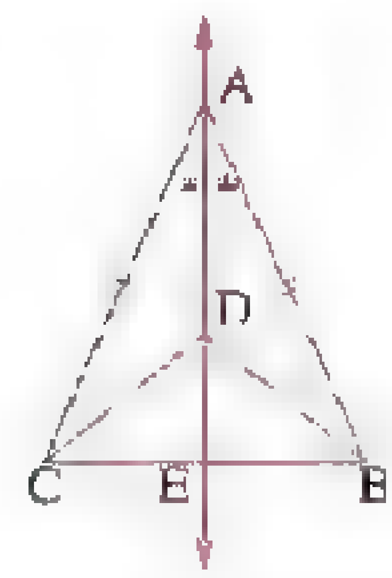
17 **In the opposite figure :**

$AB = AC$, $\overrightarrow{AE}$ bisects $\angle BAC$

, $\overrightarrow{AE} \cap \overline{BC} = \{E\}$, $D \in \overline{AE}$

Prove that : 1 $BE = \frac{1}{2} BC$

2 $BD = CD$

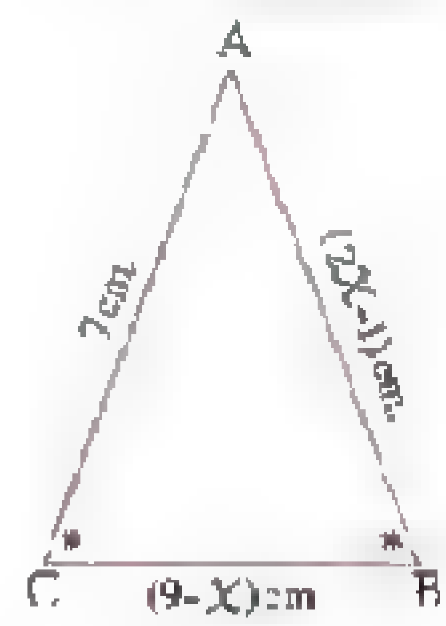


18 **In the opposite figure :**

$m(\angle B) = m(\angle C)$

Find : 1 The value of X

2 The perimeter of ΔABC



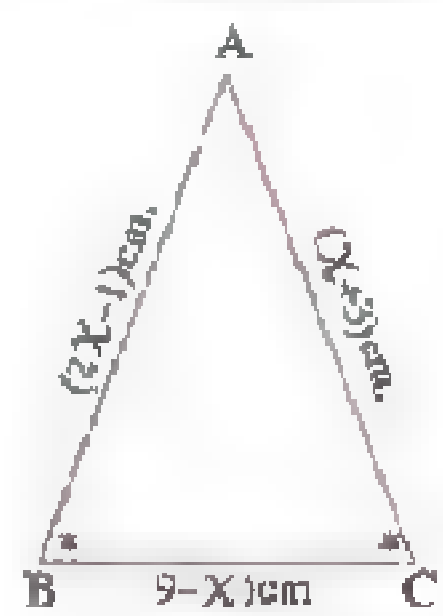
19 **In the opposite figure :**

$AB = (2X - 1)$ cm. , $AC = (X + 3)$ cm.

, $BC = (9 - X)$ cm.

, $m(\angle B) = m(\angle C)$

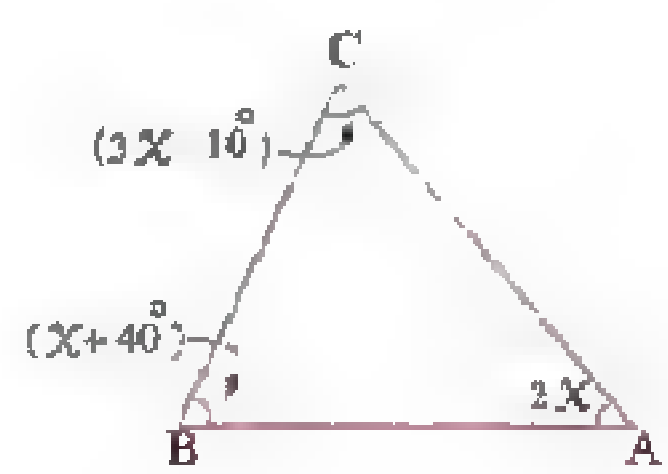
Find : The perimeter of ΔABC



20 **In the opposite figure :**

Prove that :

ΔABC is an isosceles triangle.



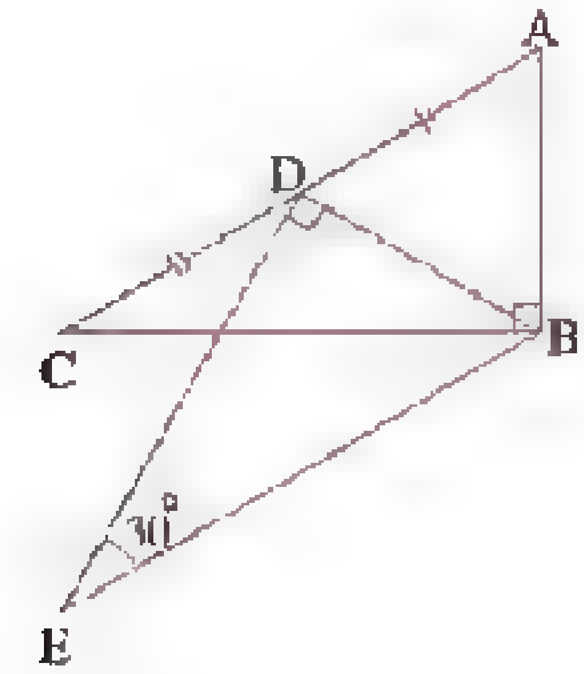
21 **In the opposite figure :**

$m(\angle ABC) = m(\angle BDE) = 90°$,

, D is the midpoint of $\overline{AC}$

, $m(\angle BED) = 30°$

Prove that : AC = BE



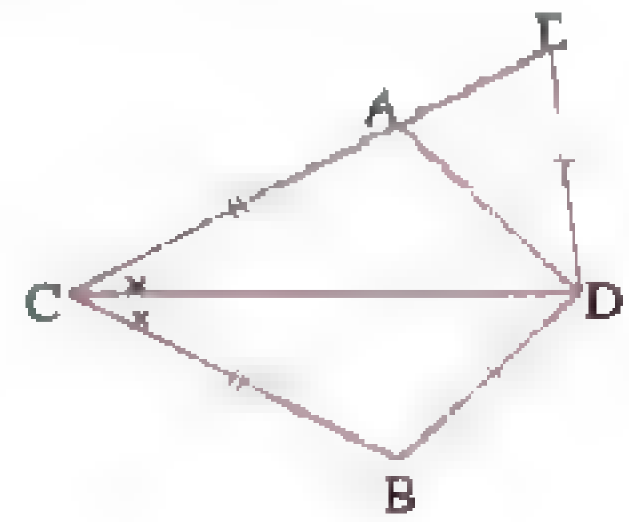
22 **In the opposite figure :**

$m(\angle ACD) = m(\angle BCD)$

, AC = BC

, DB = DE

Prove that : $m(\angle E) = m(\angle EAD)$



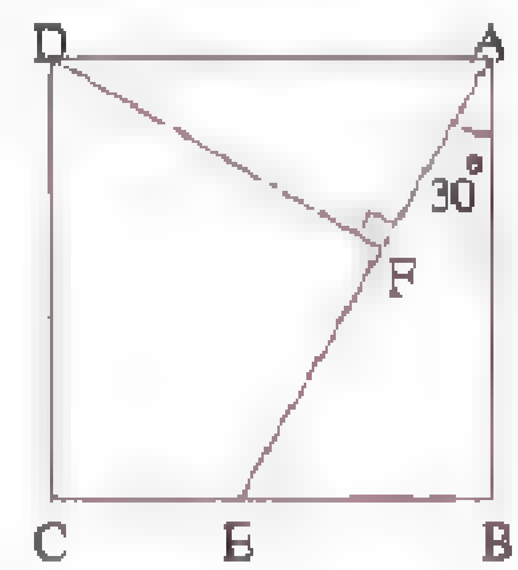
23 **In the opposite figure :**

ABCD is a square

, $m(\angle BAE) = 30°$

, $\overline{DF} \perp \overline{AE}$, AF = 4 cm.

Calculate : The area of the square.



24 **In the opposite figure :**

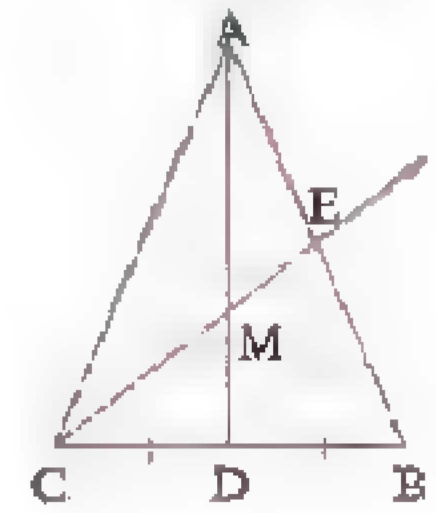
ABC is a triangle

, D is the midpoint of $\overline{BC}$

, $M \in \overline{AD}$ where AM = 2 MD

, $\overrightarrow{CM} \cap \overline{AB} = \{E\}$, EC = 12 cm.

Find : The length of EM



Important questions on Unit Five

Inequality

Multiple choice questions

1. In ΔABC , if $AB < AC$, then $m(\angle B)$ .... $m(\angle C)$

(a) $<$ (b) $\leq$ (c) $>$ (d) $=$

2. If the lengths of two sides of a triangle are 3 cm. , 6 cm. , then the length of the third side belongs to ....

(a) $[3 , 9[$ (b) $[3 , 9]$ (c) $]3 , 9]$ (d) $]3 , 9[$

3. ABC is a triangle in which $m(\angle B) = 70^\circ$, $m(\angle C) = 50^\circ$, then AC .... AB

(a) $<$ (b) $\leq$ (c) $>$ (d) $=$

4. In ΔABC , if $m(\angle A) > m(\angle B)$, then BC .... AC

(a) $>$ (b) $=$ (c) $<$ (d) $\geq$

5. The lengths 4 cm. , 9 cm. and .... cm. can be the lengths of sides of a triangle.

(a) 3 (b) 4 (c) 5 (d) 6

6. XYZ is a right-angled triangle at Y , then XZ .... YZ

(a) $<$ (b) $>$ (c) $=$ (d) $\leq$

7. In ΔABC , if $m(\angle C) = 100^\circ$, then its longest side is ....

(a) $\overline{AC}$ (b) $\overline{AB}$ (c) $\overline{BC}$

8. The numbers that can be the lengths of sides of a triangle are ....

(a) 7 , 7 , 4 (b) 3 , 4 , 9 (c) 4 , 5 , 12 (d) 5 , 5 , 15

9. In ΔABC , $AB + BC - AC >$ ....

(a) 2 AC (b) -2 AC (c) 2 (d) zero.

10. If the lengths of two sides of an isosceles triangle are 2 cm. and 5 cm. , then the length of the third side is .... cm.

(a) 2 (b) 5 (c) 4 (d) 3

11. The triangle whose side lengths are 2 cm. , $(X + 3)$ cm. and 7 cm. is an isosceles triangle if $X =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

12. The length of any side in a triangle .... the sum of the lengths of the other two sides.

(a) $>$ (b) $<$ (c) $=$ (d) $\leq$

Second Complete questions

1. In a triangle , if two angles are unequal in measure , then the greater angle in measure is ....

2. In a triangle , if two sides have unequal lengths , then the longer side is opposite to ....

3. In Δ ABC , if m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then the longest side is ....

4. In Δ ABC , if m ($\angle$ A) = 100° , then the longest side is ....

5. In Δ ABC , AB + BC .... AC

6. In a triangle , if the lengths of two sides are 5 cm. , 9 cm. , then the length of the third side $\in$]............. , ...[

7. The distance between any point and a given straight line is the length of .... drawn from this point to the given line.

8. In the right-angled triangle , ... is the longest side.

9. In Δ ABC , if m ($\angle$ A) = 2 m ($\angle$ B) = 80° , then AB > ....

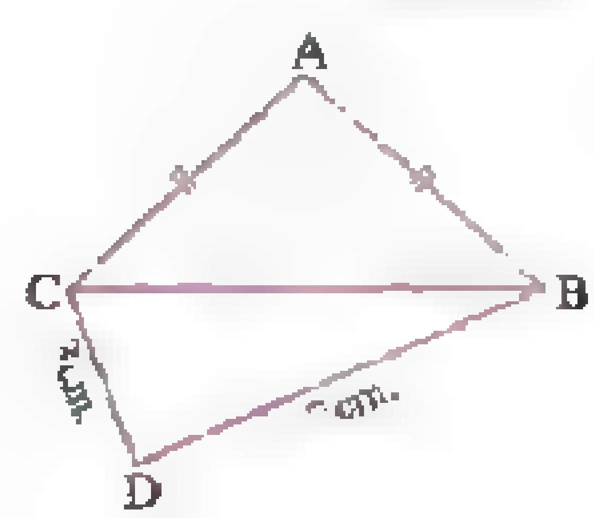
10. If X > Y , A > B , then Y + B < ....

Third Essay questions

1. In Δ ABC , m ($\angle$ A) = 40° , m ($\angle$ B) = 80° arrange the lengths of the sides of Δ ABC descendingly.

2. ABC is a triangle in which : AB = 6 cm. , AC = 8 cm. and BC = 7 cm. Arrange the measures of the angles of the triangle ABC ascendingly.

3. **In the opposite figure :**



AB = AC

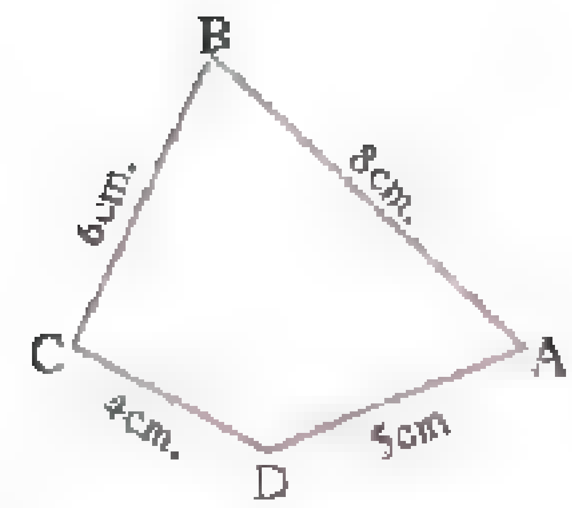
, BD = 7 cm.

, DC = 3 cm.

Prove that : m ($\angle$ ACD) > m ($\angle$ ABD)

4 In the opposite figure :

Prove that : m (∠ BCD) > m (∠ BAD)

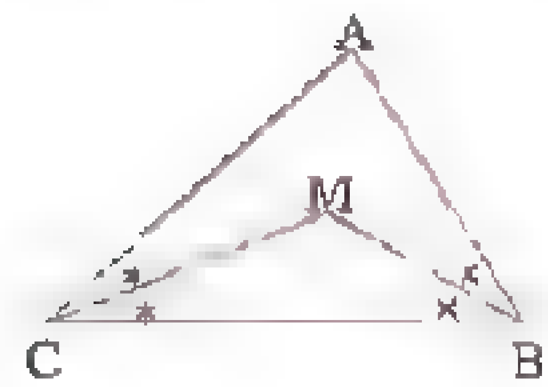


5 In the opposite figure :

ABC is a triangle in which : AB < AC

, $\overrightarrow{BM}$ bisects ∠ ABC , $\overrightarrow{CM}$ bisects ∠ ACB

Prove that : BM < CM

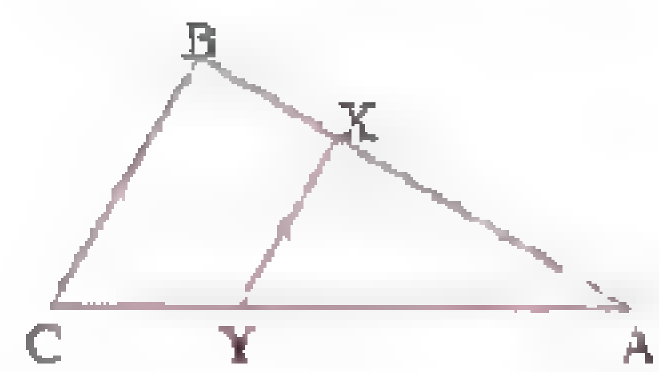


6 In the opposite figure :

ABC is a triangle in which :

AB > BC , $\overline{XY}$ // $\overline{BC}$

Prove that : AX > XY

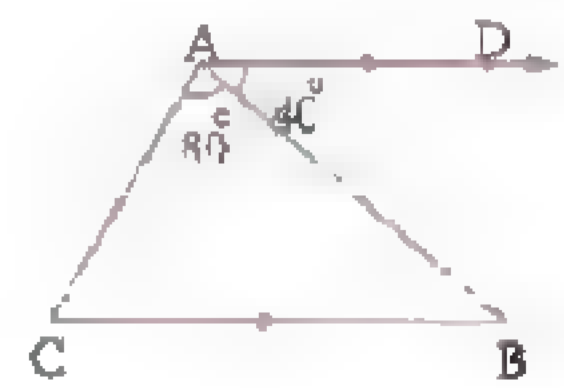


7 In the opposite figure :

$\overline{AD}$ // $\overline{CB}$, m (∠ DAB) = 40°

, m (∠ BAC) = 80°

Prove that : AB > AC



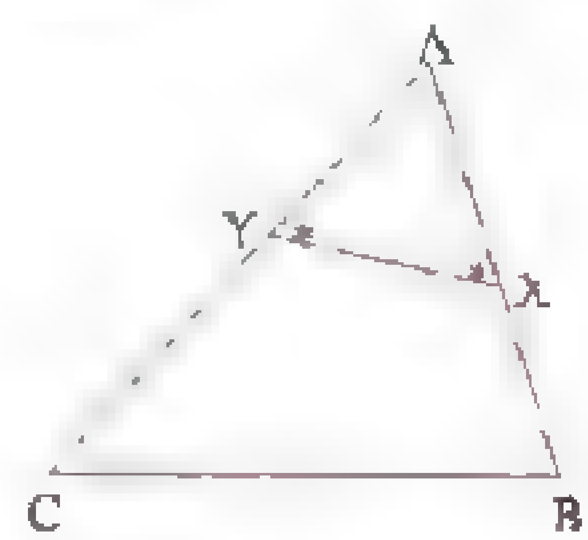
8 In the opposite figure :

ABC is a triangle in which :

AC > AB

, m (∠ AXY) = m (∠ AYX)

Prove that : YC > XB



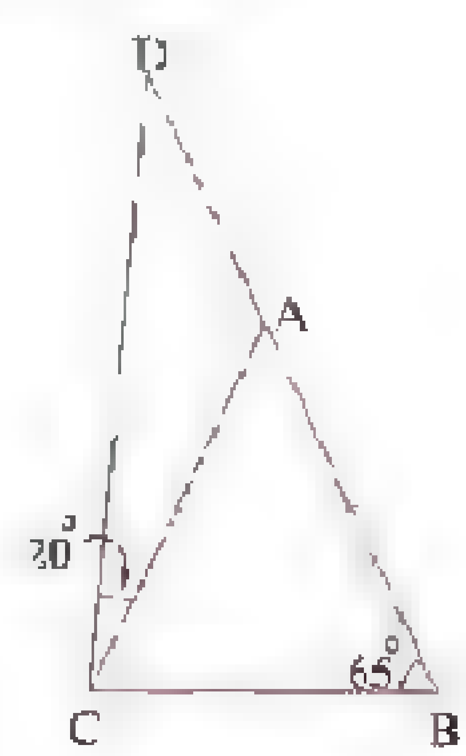
9 In the opposite figure :

AB = AC

, m (∠ ABC) = 65°

, m (∠ ACD) = 20°

Prove that : AB > AD

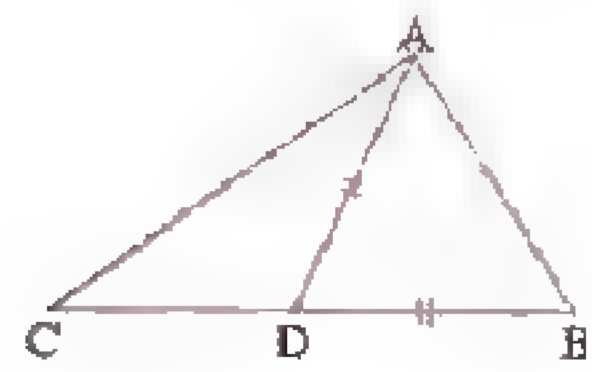


10 **In the opposite figure :**

ABC is a triangle , $D \in \overline{BC}$

, AD = BD

Prove that : BC > AC

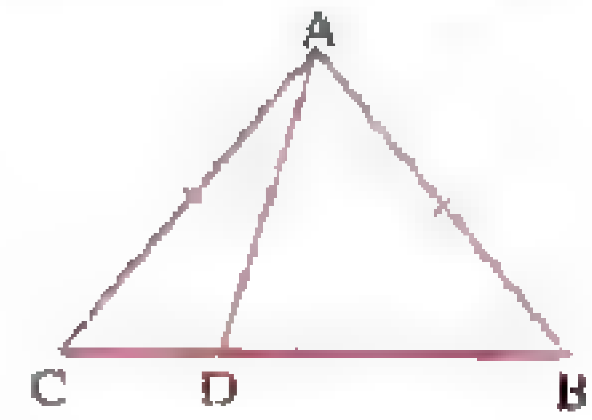


11 **In the opposite figure :**

ABC is a triangle in which :

AB = AC , $D \in \overline{BC}$

Prove that : AC > AD



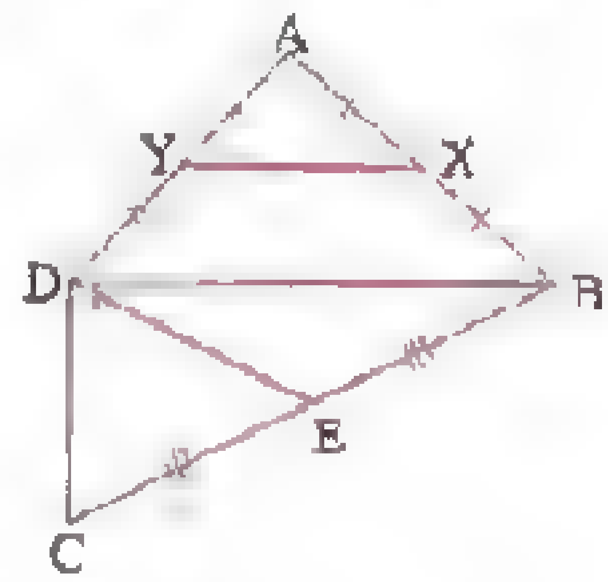
12 **In the opposite figure :**

ABCD is a quadrilateral in which : m (∠ BDC) = 90°

, X , Y and E are the midpoints of $\overline{AB}$, $\overline{AD}$

and $\overline{BC}$ respectively.

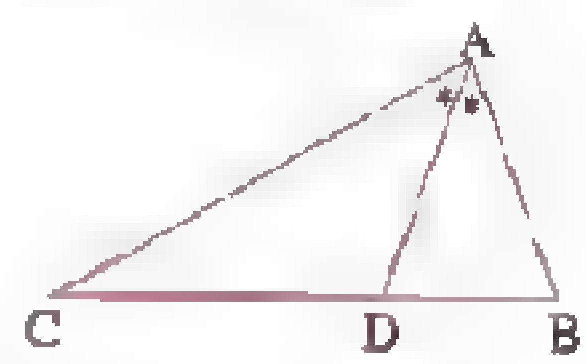
Prove that : XY < DE



13 **In the opposite figure :**

$\overrightarrow{AD}$ bisects ∠ BAC

Prove that : AC > DC

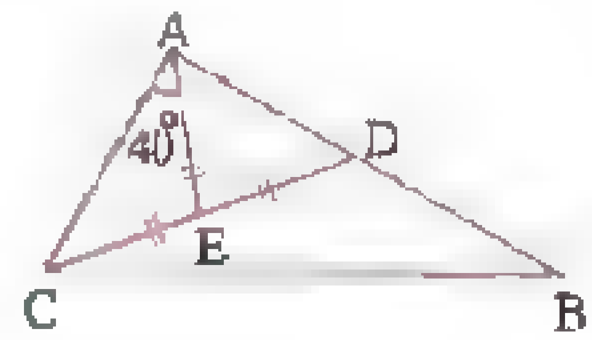


14 **In the opposite figure :**

DE = EC = AE , m (∠ CAE) = 40°

Prove that : 1 AC > AE

2 BC > AC



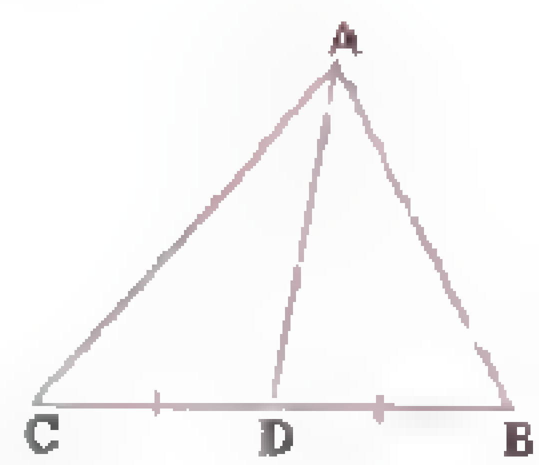
Final Revision

of Geometry

Revision for the important theorems , corollaries and rules of ? Geometry

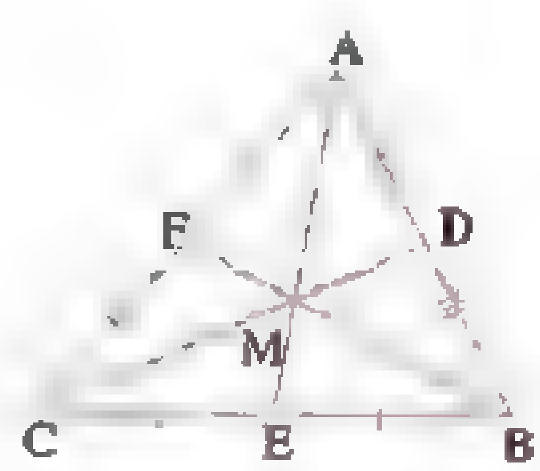
Medians of triangle

The median of the triangle is the line segment drawn from any vertex of the triangle vertices to the midpoint of the opposite side of this vertex.



If D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is a median in Δ ABC

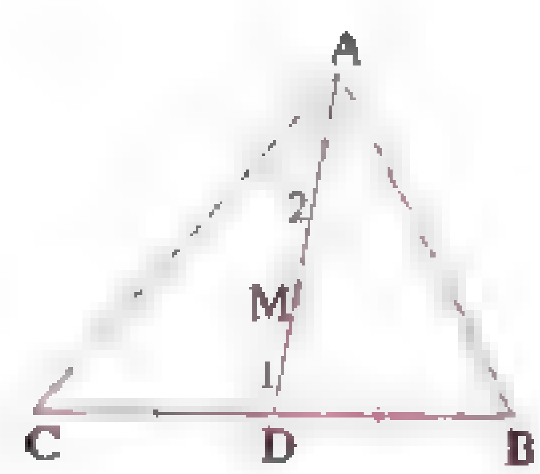
The medians of a triangle are concurrent.



If $\overline{CD}$, $\overline{BF}$ and $\overline{AE}$ are the medians of Δ ABC where $\overline{CD} \cap \overline{BF} \cap \overline{AE} = \{M\}$, then M is the intersection point of the medians of Δ ABC

The point of concurrence of the medians of the triangle divides each median in the ratio of :

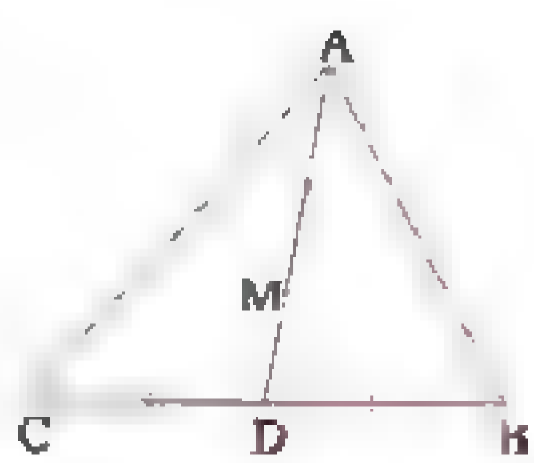
- 1 : 2 from the base.
- 2 : 1 from the vertex.



If M is the intersection point of the medians of Δ ABC , then :

- $DM = \frac{1}{2} AM$
- $AM = 2 DM$
- $DM = \frac{1}{3} AD$
- $AM = \frac{2}{3} AD$

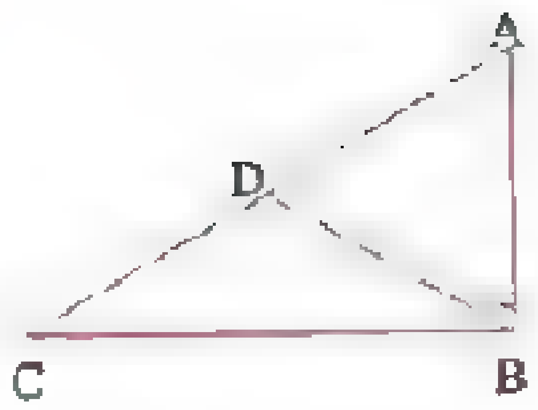
The point which divides the median of a triangle by the ratio 1 : 2 from the base is the point of the intersection of the medians of the triangle.



If DM : MA = 1 : 2 , then M is the intersection point of the medians of Δ ABC

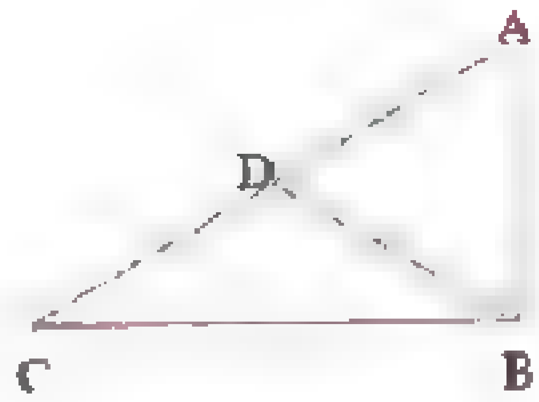
Right-angled triangle

The length of the median from the vertex of the right angle equals half the length of the hypotenuse.



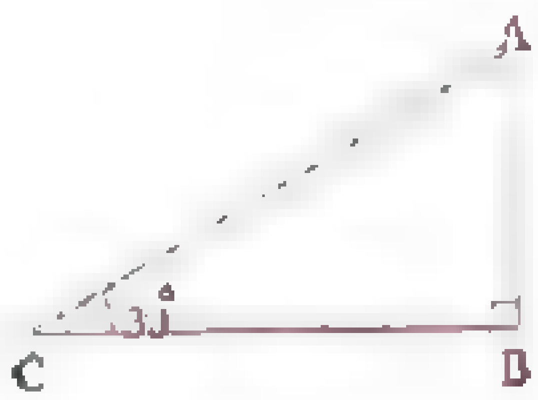
If ΔABC is right-angled at B , $\overline{BD}$ is a median in it , then $BD = \frac{1}{2} AC$

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.



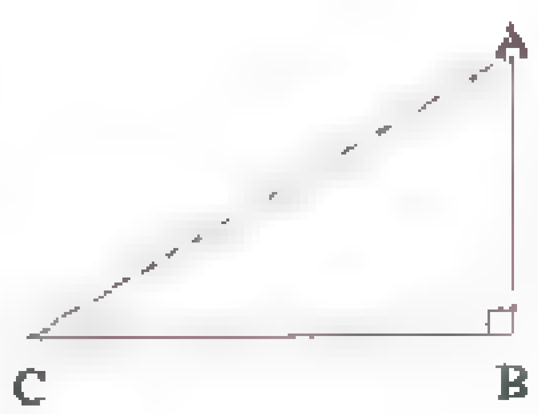
If $\overline{BD}$ is a median of ΔABC , $BD = \frac{1}{2} AC$, then $m(\angle ABC) = 90^\circ$

The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.



If ΔABC is right angled at B in which : $m(\angle C) = 30^\circ$, then $AB = \frac{1}{2} AC$

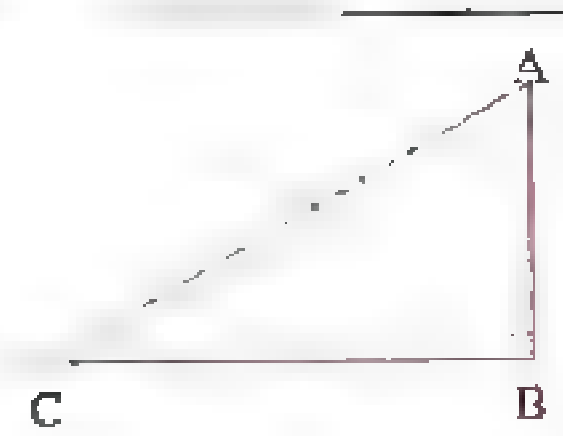
In the right-angled triangle, the hypotenuse is the longest side of the triangle.



If ΔABC is right angled at B , then $AC > AB$, $AC > BC$

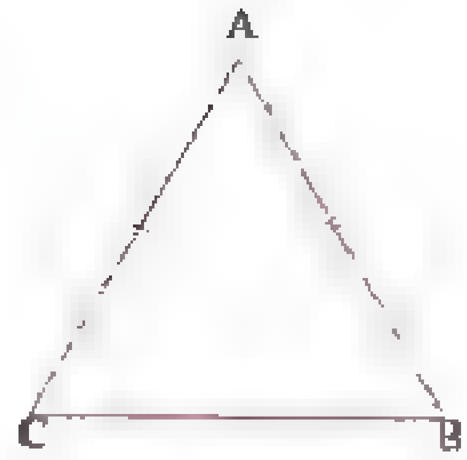
If ΔABC is right-angled at B , then :

- $(AC)^2 = (AB)^2 + (BC)^2$
- $(AB)^2 = (AC)^2 - (BC)^2$
- $(BC)^2 = (AC)^2 - (AB)^2$



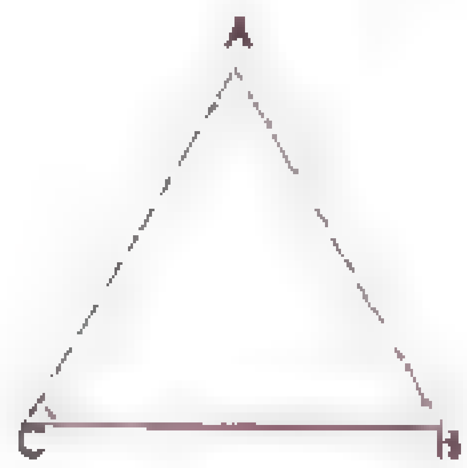
The isosceles triangle

The base angles of the isosceles triangle are congruent.



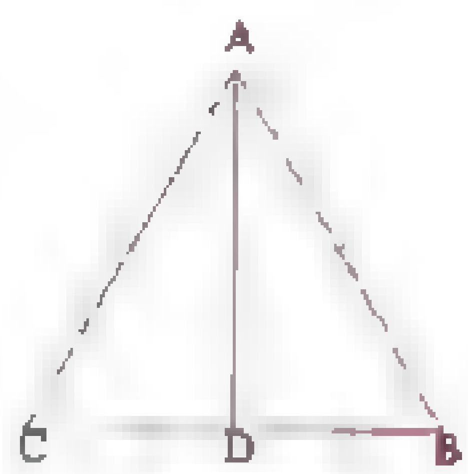
If Δ ABC in which :
AB = AC , then
m ($\angle$ B) = m ($\angle$ C)

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.



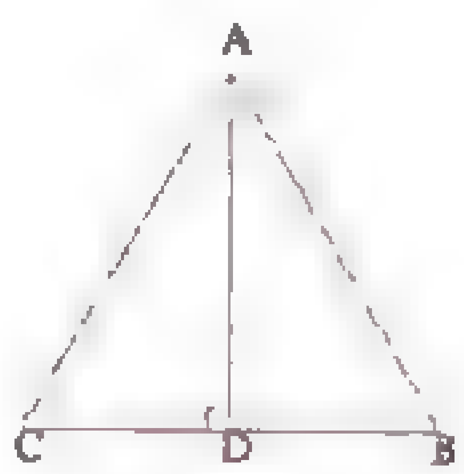
If Δ ABC in which :
m ($\angle$ B) = m ($\angle$ C)
, then AB = AC

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.



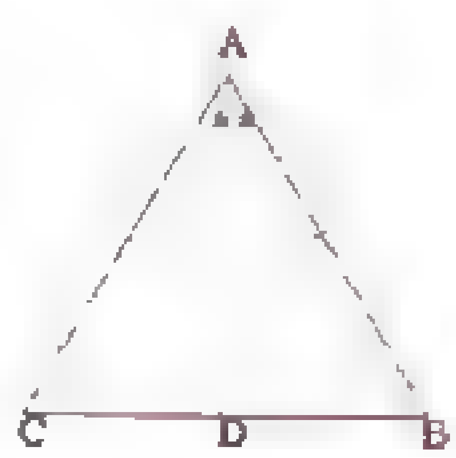
If Δ ABC in which :
AB = AC , $\overline{AD}$ is a median
, then $\overrightarrow{AD}$ bisects $\angle$ BAC
, $\overline{AD} \perp \overline{BC}$

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.



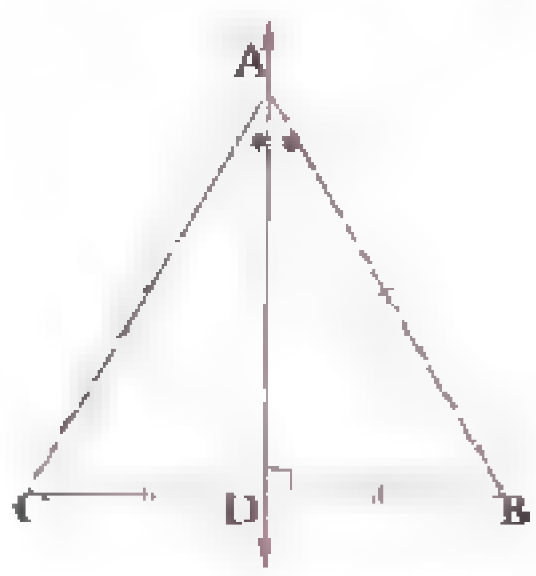
If Δ ABC in which :
AB = AC , $\overline{AD} \perp \overline{BC}$
, then D is the midpoint of $\overline{BC}$,
$\overrightarrow{AD}$ bisects $\angle$ BAC

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.



If Δ ABC in which :
AB = AC , $\overline{AD}$ bisects $\angle$ BAC , then D is the midpoint of $\overline{BC}$, $\overline{AD} \perp \overline{BC}$

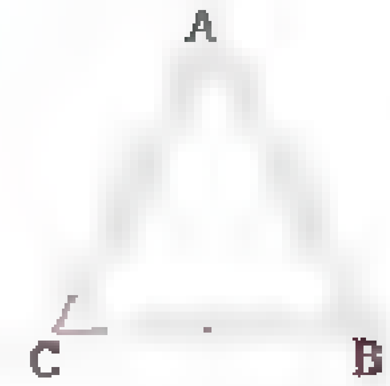
The number of axes of symmetry of the isosceles triangle equals 1



If Δ ABC in which :
AB = AC , $\overleftrightarrow{AD} \perp \overline{BC}$ and intersect it at D
, then $\overleftrightarrow{AD}$ is the axis of symmetry of the triangle ABC

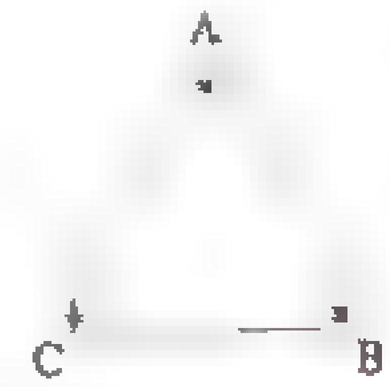
The equilateral triangle

If the triangle is an equilateral , then it is equiangular where each angle measure is 60°



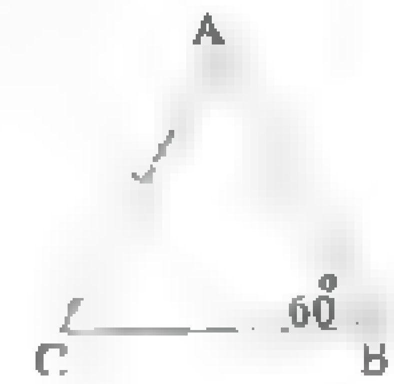
If Δ ABC in which :
AB = BC = CA , then
m ($\angle$ A) = m ($\angle$ B) = m ($\angle$ C) = 60°

If the angles of a triangle are congruent , then the triangle is equilateral.



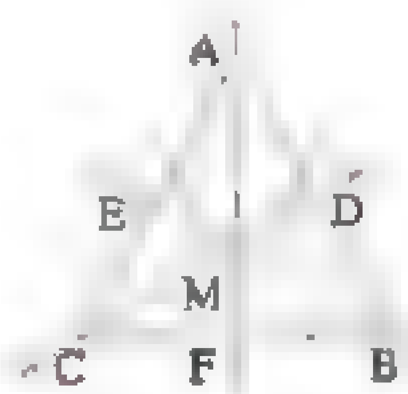
If Δ ABC in which :
m ($\angle$ A) = m ($\angle$ B) = m ($\angle$ C)
, then AB = BC = CA

The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.



If Δ ABC in which :
AB = AC , m ($\angle$ B) = 60°
, then Δ ABC is an equilateral triangle.

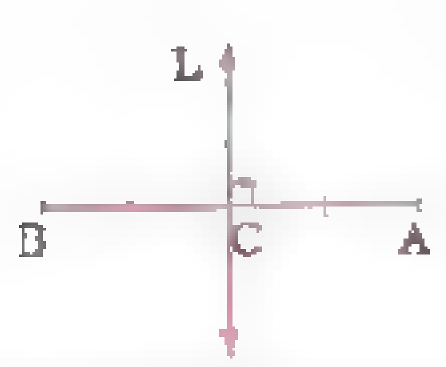
The equilateral triangle has three axes of symmetry.



If Δ ABC is an equilateral triangle
, $\overleftrightarrow{AF} \perp \overline{BC}$, $\overleftrightarrow{CD} \perp \overline{AB}$, $\overleftrightarrow{BE} \perp \overline{AC}$
, then $\overleftrightarrow{AF}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{BE}$ are the axes of symmetry of the triangle ABC

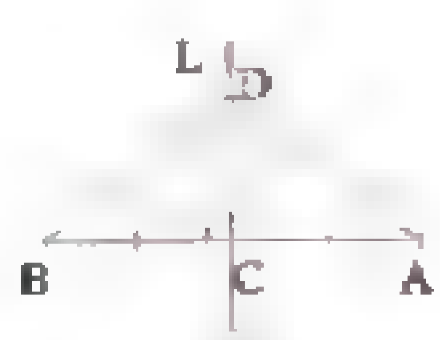
The axis of symmetry

1. The axis of symmetry of a line segment is the straight line perpendicular to it from its middle.



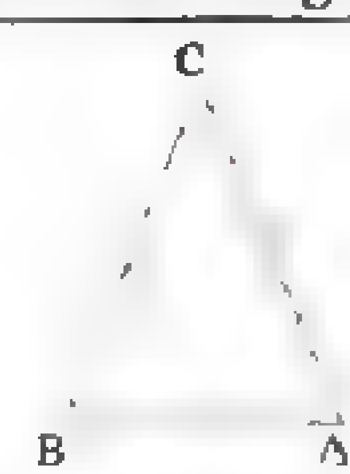
If the straight line L $\perp \overline{AB}$,
C $\in \overline{AB}$ where CA = CB
, C $\in$ the straight line L
, then L is the axis of $\overline{AB}$

2. Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).



If the straight line L is the axis of $\overline{AB}$, D $\in$ the straight line L , then DA = DB

3. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

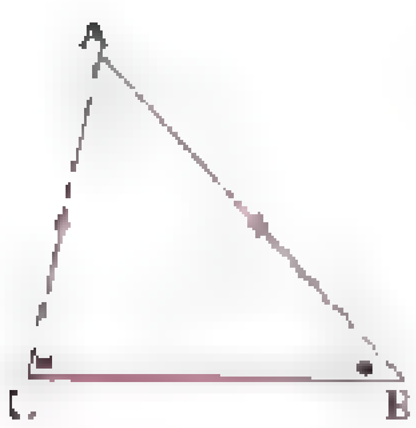


If CA = CB , then C lies on the axis of $\overline{AB}$

Inequality relations in the triangle

Comparing the measures of angles in a triangle

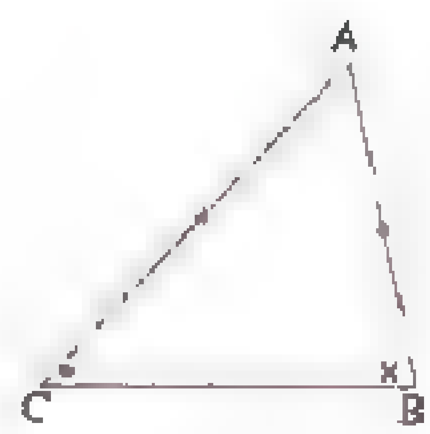
If two sides have unequal lengths , the longer is opposite to the angle of the greater measure



If AB > AC , then m (∠ C) > m (∠ B)

Comparing the lengths of sides in a triangle

If two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.



If m (∠ B) > m (∠ C) , then AC > AB

Triangle inequality

In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.

AB + BC > AC

, BC + CA > AB

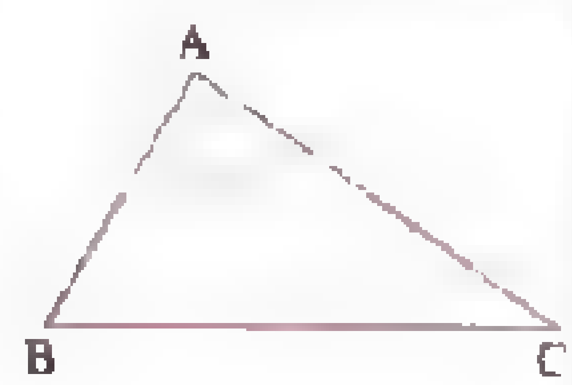
, CA + AB > BC

Notice that :

- The length of any side of a triangle is greater than the difference between the lengths of the two other sides and less than their sum.

  In Δ ABC :

  AC − AB < BC < AC + AB

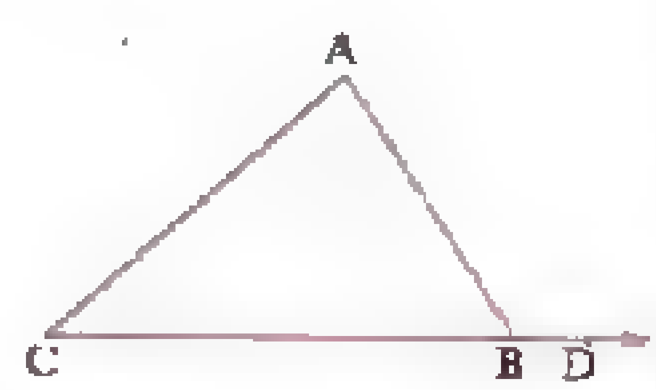


- The measure of any exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

  In Δ ABC :

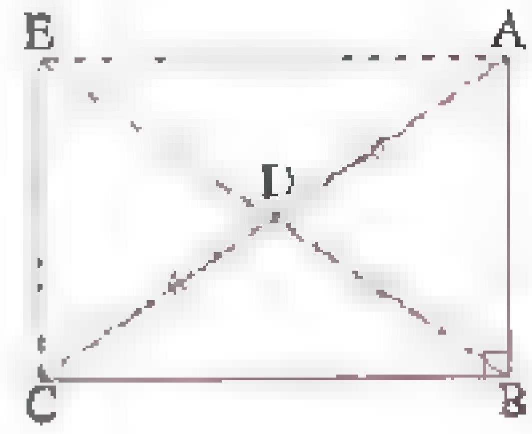
  m (∠ ABD) > m (∠ A)

  , m (∠ ABD) > m (∠ C)



Proofs of the important theorems

In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.



Given ABC is a triangle in which m ($\angle$ ABC) = 90° , $\overline{BD}$ is a median in the triangle ABC

R.T.P. $BD = \frac{1}{2} AC$

Construction Draw $\overrightarrow{BD}$ and take the point $E \in \overrightarrow{BD}$ such that BD = DE

Proof In the figure ABCE : $\because$ $\overline{AC}$ and $\overline{BE}$ bisect each other

$\therefore$ The figure ABCE is a parallelogram.

$\because$ m ($\angle$ ABC) = 90°

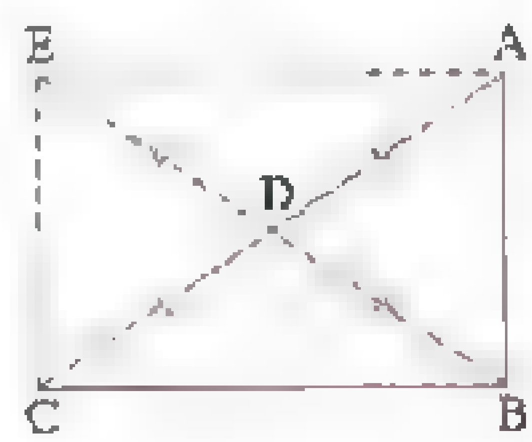
$\therefore$ The figure ABCE is a rectangle.

$\therefore$ BE = AC

, $\because$ $BD = \frac{1}{2} BE$

$\therefore$ $BD = \frac{1}{2} AC$ (Q.E.D.)

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.



Given In Δ ABC , $\overline{BD}$ is a median and DA = DB = DC

R.T.P. m ($\angle$ ABC) = 90°

Construction Draw $\overrightarrow{BD}$, then take the point $E \in \overrightarrow{BD}$ such that BD = DE

Proof $\because$ $BD = \frac{1}{2} BE = \frac{1}{2} AC$ $\therefore$ BE = AC

$\therefore$ In the figure ABCE :

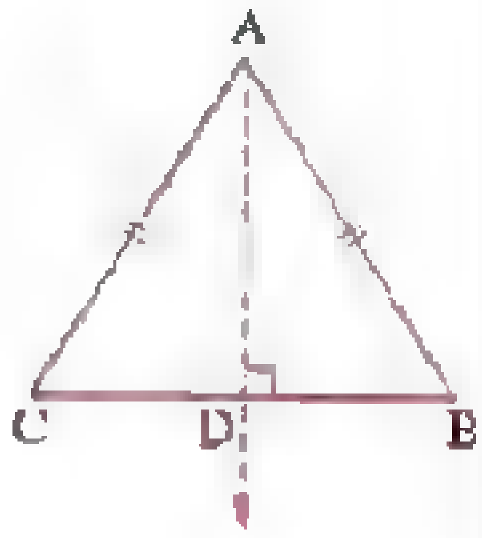
$\overline{AC}$ and $\overline{BE}$ are equal in length and bisect each other.

$\therefore$ The figure ABCE is a rectangle.

$\therefore$ m ($\angle$ ABC) = 90° (Q.E.D.)

Theorem

The base angles of the isosceles triangle are congruent.



Given | ABC is a triangle in which $\overline{AB} \equiv \overline{AC}$

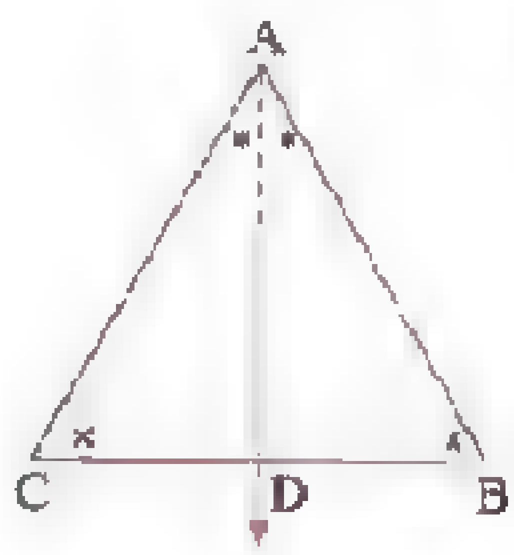
R.T.P. | $\angle B \equiv \angle C$

Construction | Draw $\overrightarrow{AD} \perp \overline{BC}$ where $\overrightarrow{AD} \cap \overline{BC} = \{D\}$

Proof | $\because \Delta\Delta$ ADB , ADC in which :

$$\left\{\begin{array}{ll} m(\angle ADB) = m(\angle ADC) = 90^\circ & \text{(const.)} \\ \overline{AB} \equiv \overline{AC} & \text{(given)} \\ \overline{AD} \text{ is a common side} & \end{array}\right.$$

$\therefore \Delta ADB \equiv \Delta ADC$, then we deduce that $\angle B \equiv \angle C$ (Q.E.D.)



If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.

Given | ABC is a triangle in which $\angle B \equiv \angle C$

R.T.P. | $\overline{AB} \equiv \overline{AC}$

Construction | bisect $\angle BAC$ by $\overrightarrow{AD}$ to intersect $\overline{BC}$ at D

Proof | $\because \angle B \equiv \angle C$

$\therefore m(\angle B) = m(\angle C)$

$\because \overrightarrow{AD}$ bisects $\angle BAC$ $\quad\therefore m(\angle BAD) = m(\angle CAD)$

$\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore m(\angle ADB) = m(\angle ADC)$

$\therefore$ In $\Delta\Delta$ ABD and ACD :

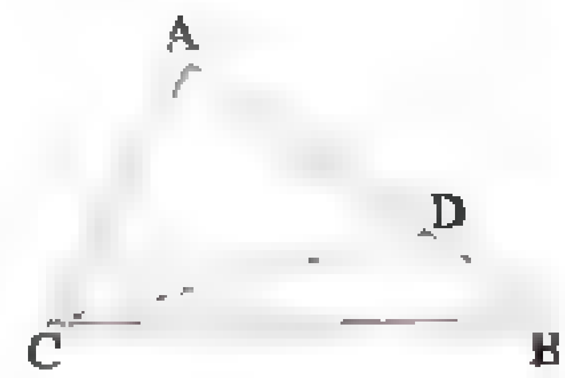
$$\left\{\begin{array}{ll} \overline{AD} \text{ is a common side} & \\ m(\angle BAD) = m(\angle CAD) & \text{(const.)} \\ m(\angle ADB) = m(\angle ADC) & \text{(by proof)} \end{array}\right.$$

$\therefore \Delta ABD \equiv \Delta ACD$, then we deduce that

$\overline{AB} \equiv \overline{AC}$, then ΔABC is an isosceles triangle. (Q.E.D.)

Theorem

In a triangle , if two sides have unequal lengths , the longer is opposite to the angle of the greater measure.



Given ABC is a triangle in which AB > AC

R.T.P. m (∠ ACB) > m (∠ ABC)

Construction Take D $\in \overline{AB}$ such that AD = AC

Proof In Δ ACD : ∵ AD = AC ∴ m (∠ ADC) = m (∠ ACD) (1)

∵ ∠ ADC is an exterior angle of Δ DBC

∴ m (∠ ADC) > m (∠ B) (2)

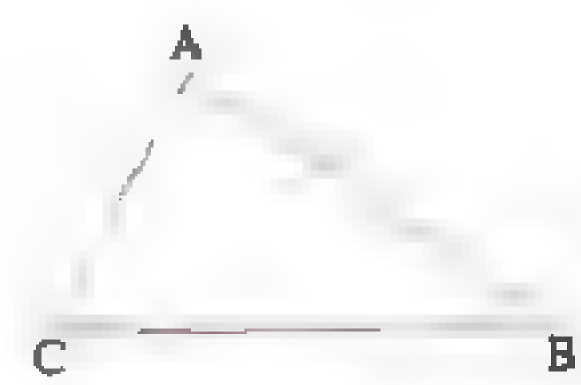
From (1) and (2) : ∴ m (∠ ACD) > m (∠ B)

, ∵ m (∠ ACB) > m (∠ ACD)

∴ m (∠ ACB) > m (∠ ABC) (Q.E.D.)

Theorem

In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.



Given ABC is a triangle in which m (∠ C) > m (∠ B)

R.T.P. AB > AC

Proof ∵ $\overline{AB}$ and $\overline{AC}$ are two line segments.

∴ One of the following cases should be verified.

1 AB > AC 2 AB = AC 3 AB < AC

Unless AB > AC , then either AB = AC or AB < AC

• If : AB = AC , then m (∠ C) = m (∠ B) and this contradicts the given where m (∠ C) > m (∠ B)

• If : AB < AC , then m (∠ C) < m (∠ B) according to the preceding theorem. Again this contradicts the given , where m (∠ C) > m (∠ B)

∴ It should be that AB > AC (Q.E.D.)

Final Examinations

on Geometry

- [illegible]
- School examinations

Model Examinations of the School Book

Model 1

Answer the following questions :

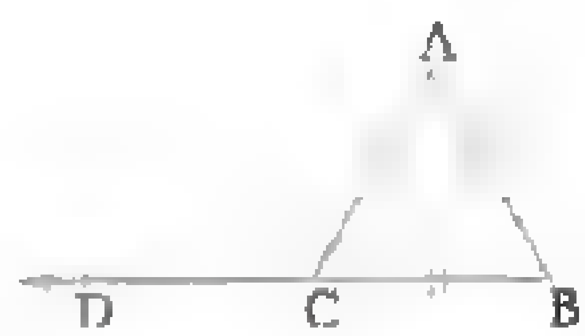
1 Complete the following :

1. The longest side in the right-angled triangle is ....
2. If the lengths of two sides in a triangle are 2 cm. and 7 cm. , then .... < the length of the third side < ....
3. If the measures of two angles in a triangle are different , then the greater in measure of them is opposite to ....
4. If the length of the median drawn from a vertex of a triangle equals half the opposite side to this vertex in length , then ....
5. If the measure of an angle in the isosceles triangle equals 60° , then the triangle is ....

---

2 Choose the correct answer from those given :

1. **In the opposite figure :**



Δ ABC is equilateral , then m ($\angle$ ACD) = ...

(a) 45° (b) 60°
(c) 120° (d) 135°

2. In Δ ABC which is right-angled at B , if AC = 20 cm. , then the length of the median of the triangle drawn from B equals ....

(a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.

3. XYZ is a triangle in which : m ($\angle$ Z) = 70° and m ($\angle$ Y) = 60° , then YZ .... XY

(a) > (b) < (c) = (d) twice

4. The lengths which can be lengths of sides of a triangle are ....

(a) 0 , 3 , 5 (b) 3 , 3 , 5 (c) 3 , 3 , 6 (d) 3 , 3 , 7

5. The triangle in which the measures of two angles of it are 42° and 69° is ....

(a) an isosceles triangle. (b) an equilateral triangle.
(c) a scalene triangle. (d) a right-angled triangle.

6. **In the opposite figure :**

m ($\angle$ C) = 2 m ($\angle$ A)
, BC = 6 cm.
, then AC = ... cm.

(a) 3 (b) 6
(c) 9 (d) 12

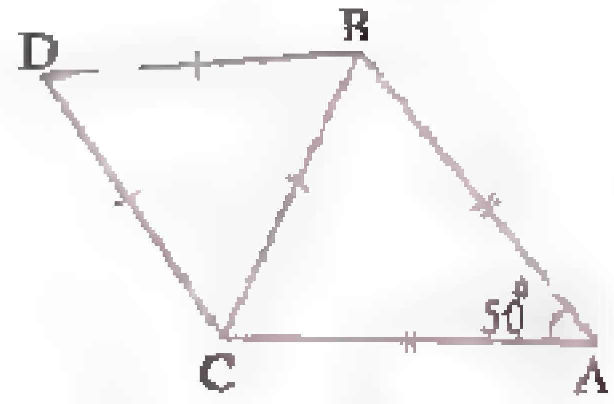
3 [a] **Complete :** ABC is a triangle in which AB > AC , then m (∠ C) .... m (∠ B)

[b] **In the opposite figure :**

m (∠ A) = 50° , AB = AC

and Δ DBC is equilateral

Find : m (∠ ABD)



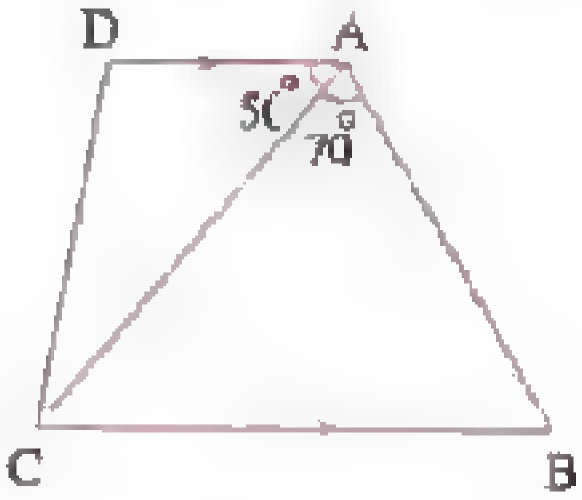
[c] **In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$

, m (∠ BAC) = 70°

and m (∠ DAC) = 50°

Prove that : BC > AC



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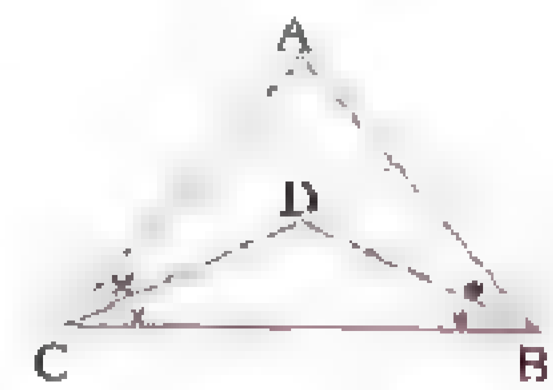
4 [a] **Prove that :** The two base angles of the isosceles triangle are congruent.

[b] **In the opposite figure :**

AB = AC , $\overrightarrow{BD}$ bisects ∠ B

and $\overrightarrow{CD}$ bisects ∠ C

Prove that : Δ DBC is isosceles.



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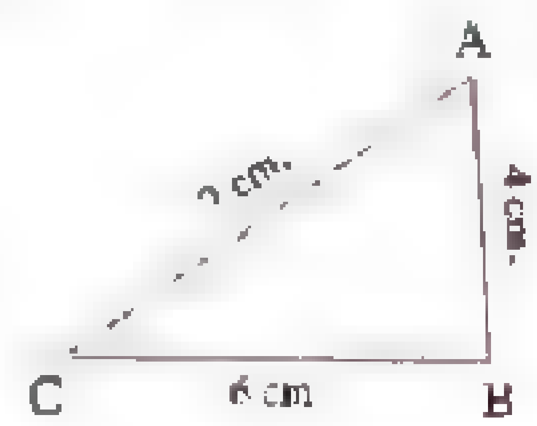
5 [a] **In the opposite figure :**

Arrange the angles

of Δ ABC descendingly

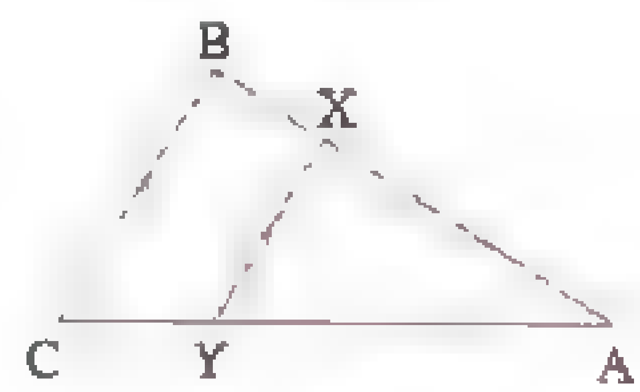
due to their measures.



[b] **In the opposite figure :**

AB > BC , $\overline{XY}$ // $\overline{BC}$

Prove that : AX > XY



Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The triangle which has three axes of symmetry is ....

(a) scalene. (b) isosceles. (c) right angled. (d) equilateral.

2 The sum of lengths of two sides in a triangle is .... the length of the third side.

(a) greater than (b) smaller than (c) equal to (d) twice

3 If the lengths of two sides in an isosceles triangle are 8 cm. and 4 cm. , then the length of the third side is .... cm.

(a) 4 (b) 8 (c) 3 (d) 12

[4] In ΔABC , if m ($\angle$ B) = 130° , then the longest side of it is ....

(a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) its median.

[5] ΔXYZ is an isosceles triangle in which : m ($\angle$ X) = 100° , then m ($\angle$ Y) = ...

(a) 100° (b) 80° (c) 60° (d) 40°

[6] **In the opposite figure :**

$x + y =$ ....

(a) 100° (b) 140°

(c) 180° (d) 280°

[2] Complete the following :

[1] If the measure of an angle in a right-angled triangle is 45° , then the triangle is ....

[2] The length of any side in a triangle .... the sum of lengths of the two other sides.

[3] If $\overline{AB} \equiv \overline{XY}$, then AB = ...

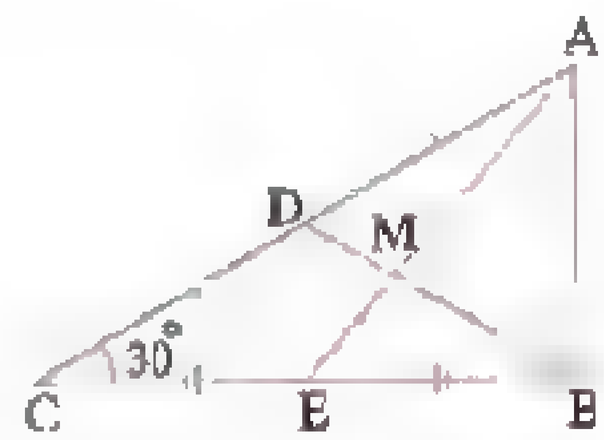
[4] In ΔABC , if m ($\angle$ A) = 30° and m ($\angle$ B) = 90° , then BC = ... AC

[5] The axis of symmetry of a line segment is the straight line which .... at its midpoint.

[3] [a] In ΔABC , AB = 7 cm. , BC = 5 cm. and AC = 6 cm.

Arrange its angles ascendingly due to their measures.

[b] **In the opposite figure :**



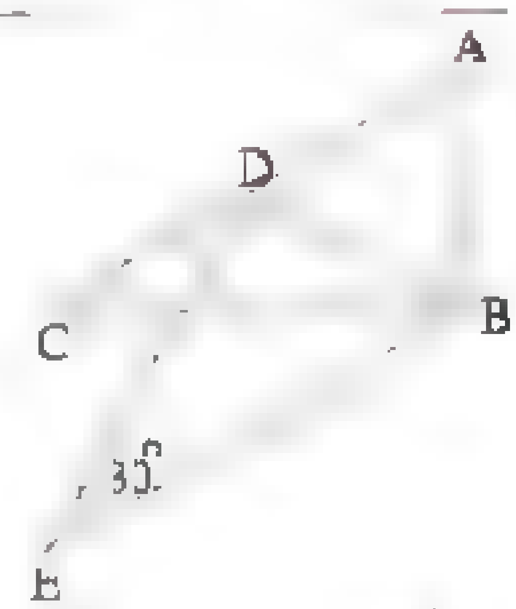
ΔABC is right-angled at B

, m ($\angle$ C) = 30° , D is the midpoint of $\overline{AC}$

, E is the midpoint of $\overline{BC}$, AC = 9 cm.

Find : The length of each of $\overline{BD}$, $\overline{BM}$ and $\overline{AB}$

[4] [a] **In the opposite figure :**



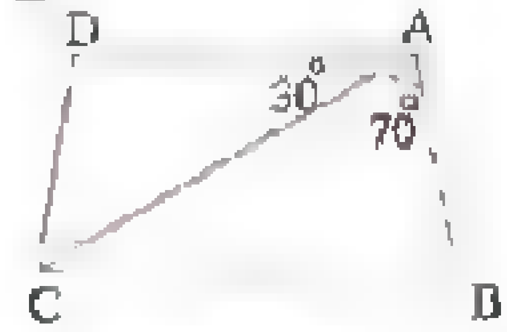
m ($\angle$ ABC) = m ($\angle$ BDE) = 90°

, m ($\angle$ E) = 30°

, D is the midpoint of $\overline{AC}$

Prove that : AC = BE

[b] **In the opposite figure :**



$\overleftrightarrow{AD}$ // $\overline{BC}$, m ($\angle$ BAC) = 70°

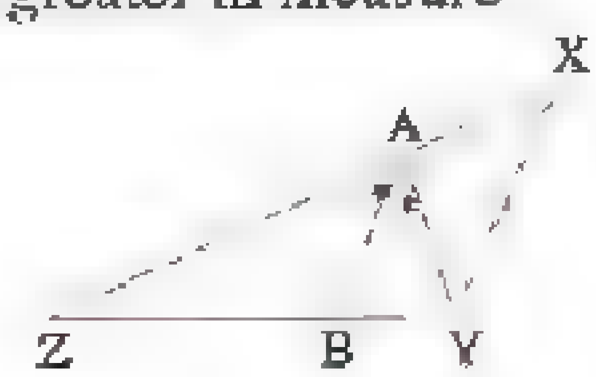
, m ($\angle$ DAC) = 30°

Prove that : AC > BC

[5] [a] **Complete :**

If the measures of two angles of a triangle are different , then the greater in measure is opposite to ....

[b] **In the opposite figure :**



$\overleftrightarrow{AB}$ // $\overleftrightarrow{XY}$ and $\overrightarrow{AB}$ bisects $\angle$ YAZ

Prove that : XZ > YZ

Model for the merge students

Answer the following questions :

1 Complete each of the following :

1 The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... from the base.

2 In the right angled triangle , the length of the median drawn from the vertex of the right angle equals ....

3 The base angles of the isosceles triangle are ....

4 In Δ ABC , if m ($\angle$ B) = 70° , m ($\angle$ C) = 50° , then AC .... AB

5 The median of the isosceles triangle from the vertex angle .... , ...

2 Choose the correct answer from those given :

1 If ABC is an equilateral triangle , then m ($\angle$ B) = ...

(a) 30° (b) 60° (c) 70° (d) 90°

2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 2

3 If the measure of the vertex angle of an isosceles triangle is 80° , then the measure of one of the base angles equals ....

(a) 60° (b) 40° (c) 30° (d) 50°

4 The number of axes of symmetry of the isosceles triangle is ....

(a) 1 (b) 2 (c) 3 (d) zero

5 In Δ ABC , if m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then the longest side is ....

(a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$

3 In the opposite figure :

Δ ABC is a right-angled triangle at B , m ($\angle$ C) = 30° , AB = 5 cm.

Find : The length of $\overline{AC}$

$\because$ m ($\angle$ B) = ...° , m ($\angle$ C) = ...°

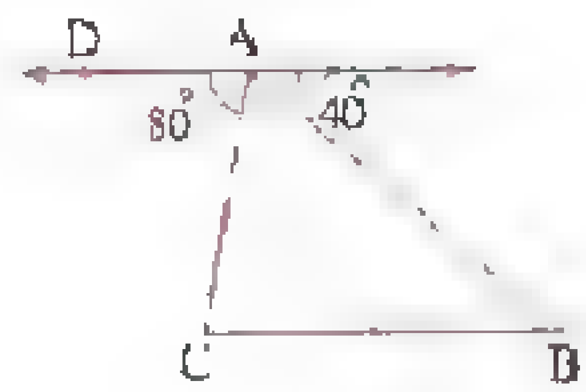
$\therefore$ AB = $\frac{1}{2}$ x .... $\therefore$ AC = ... cm.

8 [a] In Δ ABC , m ($\angle$ A) = 40° , m ($\angle$ B) = 75° , m ($\angle$ C) = 65°

Arrange the lengths of the sides of the triangle descendingly.

The order is : .... , ... , ...

[b] **In the opposite figure :**



$\overleftrightarrow{AD}$ // $\overline{BC}$

Complete :

1 m ($\angle$ B) = ...°

2 The side .... is the longest side of Δ ABC

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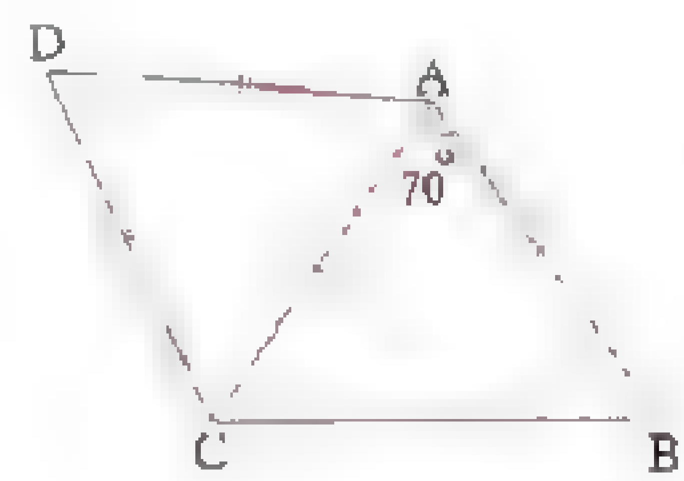
9 **In the opposite figure :**



AB = AC = CD = AD = 10 cm.

, m ($\angle$ BAC) = 70°

Put (✔) or (✘) :

1 m ($\angle$ B) = 55° ( )

2 m ($\angle$ D) = 70° ( )

3 m ($\angle$ DCB) = 120° ( )

4 AB + AD = 20 cm. ( )

5 AB + BC = BC + CD ( )

Some Schools Examinations on Geometry

1 Cairo Governorate

El-Nozha Directorate
Sunrise Language School

Answer the following questions :

1 Choose the correct answer :

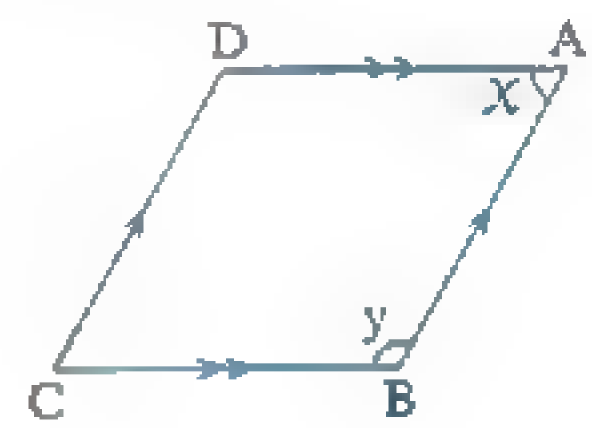
1 The sum of lengths of any two sides in any triangle .... the length of the third side.

(a) is less than (b) is greater than (c) equals (d) otherwise

2 In Δ ABC , if AB = 3 cm. and BC = 5 cm. , then AC $\in$ ....

(a) $]3 , 8]$ (b) $[2 , 8]$ (c) $]2 , 8[$ (d) $]2 , 5[$

3 **In the opposite figure :**



If ABCD is a parallelogram and $X : y = 1 : 2$

, then m ($\angle$ C) = ...

(a) 60° (b) 120°

(c) 180° (d) 360°

4 The right-angled triangle has .... median(s).

(a) 0 (b) 1 (c) 2 (d) 3

5 If Δ ABC has one axis of symmetry and m ($\angle$ ABC) = 140° , then m ($\angle$ A) = ...

(a) 30° (b) 20° (c) 40° (d) 60°

2 Complete :

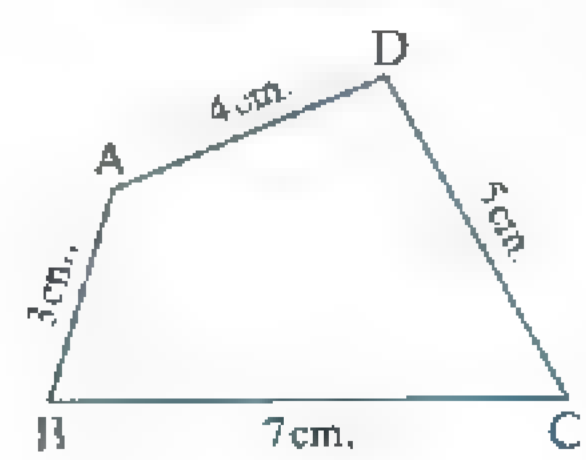
1 If the lengths of two sides of an isosceles triangle are 4 cm. and 10 cm. , then the length of the third side is ....

2 In an isosceles triangle , if any angle has a measure of 60° , then the triangle is ....

3 If ABCD is a square , then m ($\angle$ ACB) = ...°

4 In Δ ABC , m ($\angle$ C) = 60° , m ($\angle$ B) = 90° and AC = 8 cm. , then BC = ... cm.

3 [a] In the opposite figure :



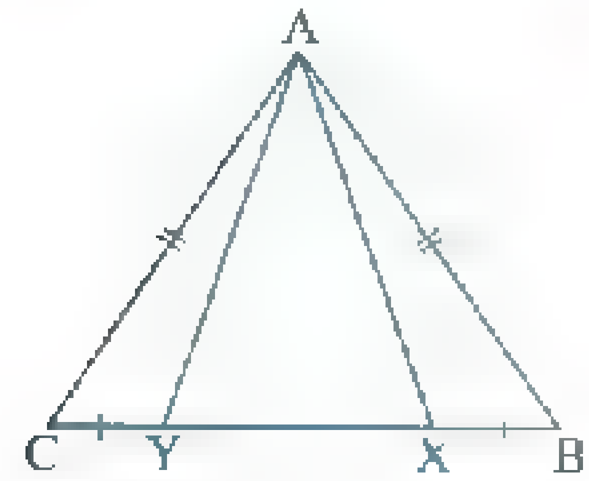
ABCD is a quadrilateral in which :

AB = 3 cm. , BC = 7 cm.

, CD = 5 cm. and DA = 4 cm.

Prove that : m ($\angle$ BAD) > m ($\angle$ BCD)

[b] In the opposite figure :



AB = AC

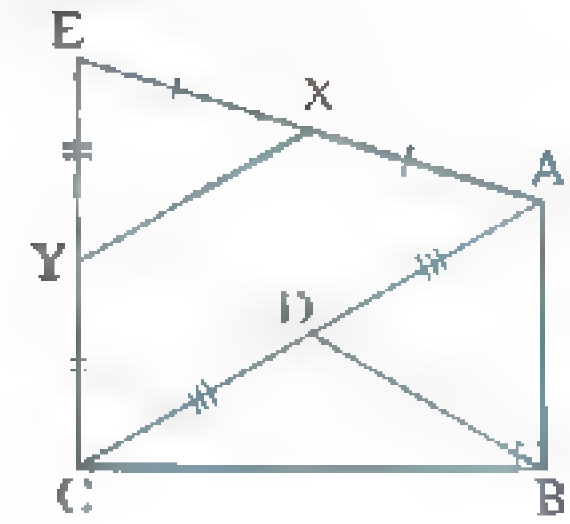
, BX = CY

Prove that :

AX = AY

---

4 [a] **In the opposite figure :**



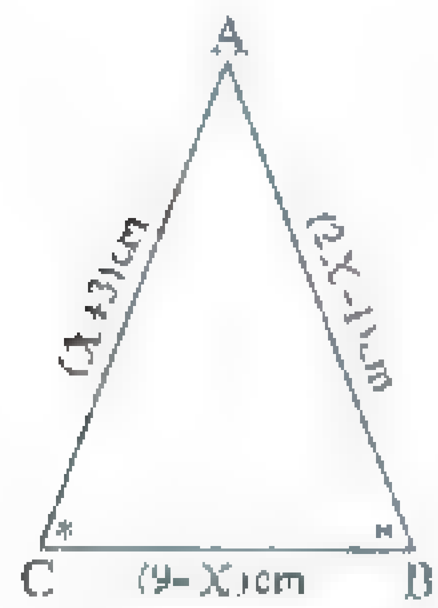
X , Y and D are the midpoints of $\overline{EA}$

, $\overline{EC}$ and $\overline{AC}$ respectively

, m (∠ ABC) = 90°

Prove that : BD = YX

[b] **In the opposite figure :**



m (∠ B) = m (∠ C)

, AB = $(2X - 1)$ cm.

, AC = $(X + 3)$ cm.

, BC = $(9 - X)$ cm.

Find with proof : The perimeter of Δ ABC

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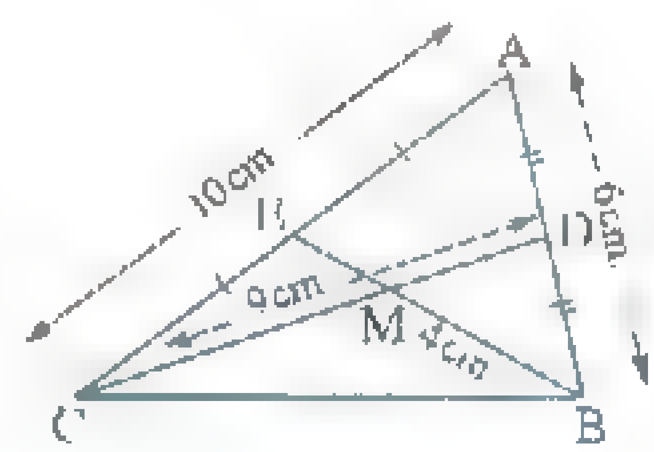
5 [a] **In the opposite figure :**



AB = 6 cm. , AC = 10 cm. , BM = 4 cm. , CD = 9 cm.

, D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, $\overline{BE} \cap \overline{CD} = \{M\}$

Find : The perimeter of the figure ADME

[b] Prove that the length of any side in a triangle is less than half of the perimeter.

2 Cairo Governorate

El-Zeitoun Zone
Math's Inspection

Answer the following questions :

1 **Choose the correct answer :**

1 The measure of the exterior angle of an equilateral triangle equals ..

(a) 45° (b) 90° (c) 60° (d) 120°

2 The number of axes of symmetry of a triangle whose measures of two angles are 40° and 70° is ....

(a) zero (b) 1 (c) 2 (d) 3

3 If Δ ABC is a right-angled triangle at B , then AC .... AB

(a) < (b) > (c) = (d) <

4 If Δ XYZ is an isosceles triangle and m ($\angle$ X) = 110° , then m ($\angle$ Y) = ...

(a) 70° (b) 35° (c) 60° (d) 110°

5 If A $\in$ the axis of symmetry of $\overline{XY}$, then $\overline{AX}$ .... $\overline{AY}$

(a) // (b) $\equiv$ (c) = (d) $\perp$

2 **Complete each of the following :**

1 If Δ ABC is a right-angled triangle at B , m ($\angle$ C) = 30° and AC = 12 cm. , then AB = ...

2 The length of any side of a triangle .... the sum of the lengths of the other two sides.

3 The length of the median from the right angle in the right-angled triangle equals ...

4 The measure of the supplementary angle of the angle of measure 70° is ....

3 [a] Arrange ascendingly the measures of the angles of Δ XYZ , if XY = 5 cm. , YZ = 7 cm. and XZ = 6 cm.

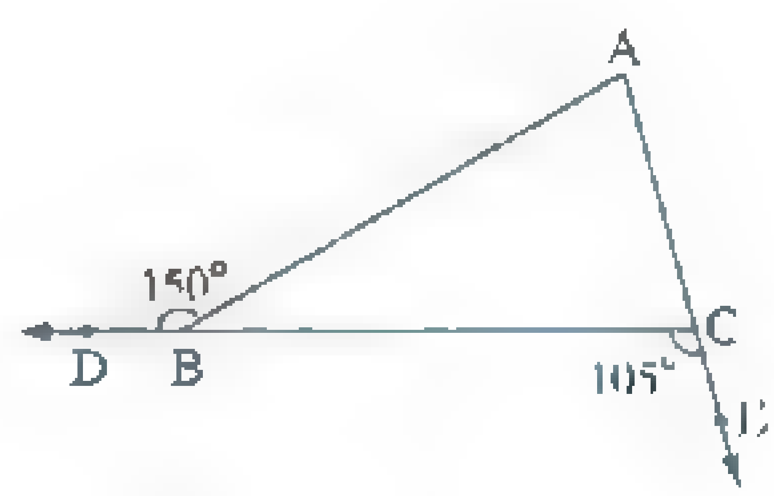
[b] **In the opposite figure :**

B $\in \overrightarrow{CD}$, C $\in \overline{AE}$

, m ($\angle$ ABD) = 150°

, m ($\angle$ BCE) = 105°

Prove that : AB > AC

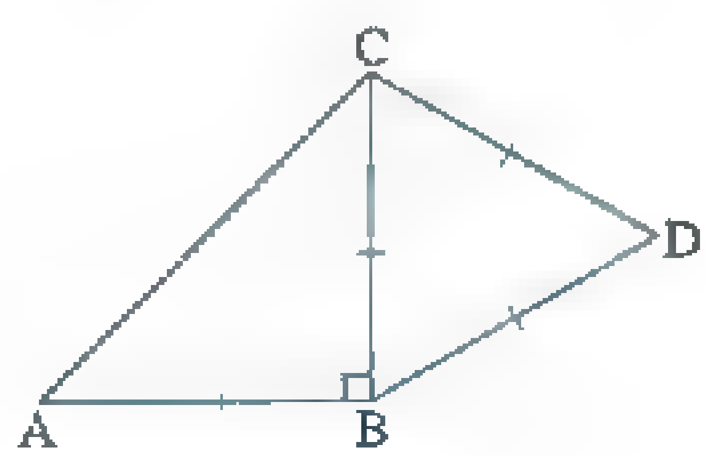


4 [a] **In the opposite figure :**

AB = BC = CD = DB

, m ($\angle$ ABC) = 90°

Find : m ($\angle$ ACD)

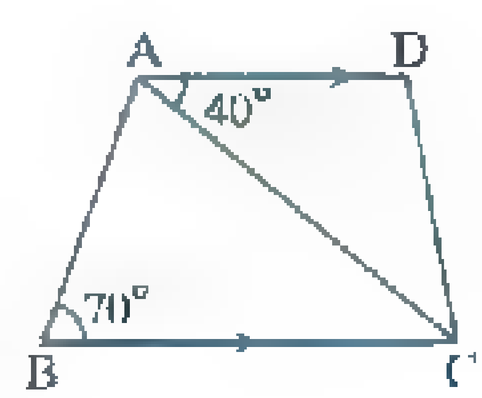


[b] **In the opposite figure :**

ABCD is a quadrilateral , $\overline{AD}$ // $\overline{BC}$

, m ($\angle$ CAD) = 40° , m ($\angle$ ABC) = 70°

Prove that : Δ ABC is an isosceles triangle.



5 **[a] In the opposite figure :**

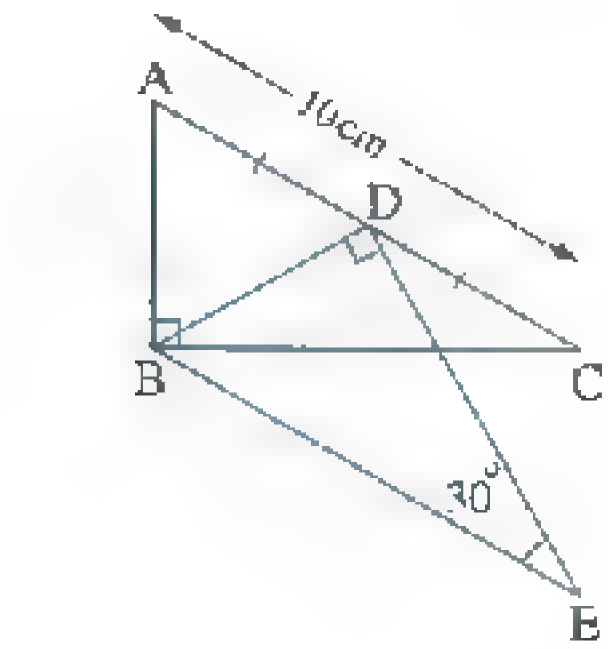
m (∠ ABC) = m (∠ BDE) = 90°

, m (∠ E) = 30°

, AC = 10 cm.

, D is the midpoint of $\overline{AC}$

Find : The length of $\overline{BE}$

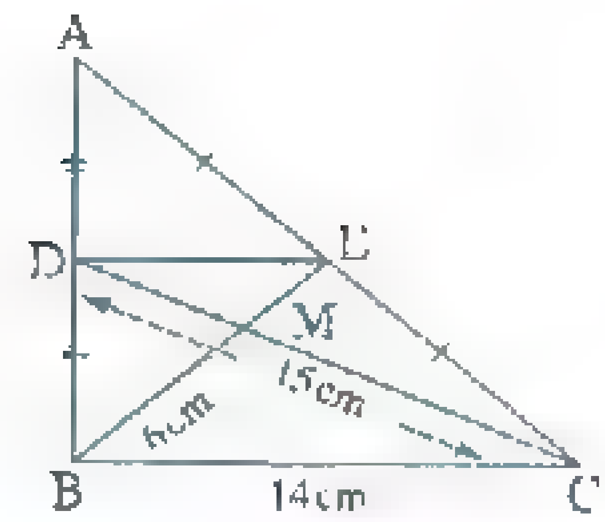


[b] In the opposite figure :

ABC is a triangle , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , $\overline{BE} \cap \overline{CD} = \{M\}$, BC = 14 cm.

, CD = 15 cm. , BM = 6 cm.

Find : The perimeter of Δ MDE



3 Giza Governorate

Math Inspection

Answer the following questions :

1 **Choose the correct answer from those given :**

1. The supplementary angle of the angle whose measure is 30° is an angle of measure ……………

(a) 60° (b) 180° (c) 150° (d) 90°

2. The triangle which has three axes of symmetry is ……………

(a) scalene. (b) isosceles. (c) right-angled. (d) equilateral.

3. If the lengths of two sides of a triangle are 5 cm. and 10 cm. , then the length of the third side belongs to ……………

(a) [10 , 15[(b)]5 , 15[(c)]5 , 10[(d) [10 , 15]

4. In Δ ABC , if AC = 4 cm. , BC = 3 cm. , then m (∠ B) …………… m (∠ A)

(a) > (b) < (c) = (d) ≤

5. The point of concurrence of the medians of the triangle divides each median in the ratio …………… from the base.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 2 : 3

2 **Complete each of the following :**

1. The longest side in the right angled triangle is ……………

2. The base angles of the isosceles triangle are ……………

3. If m (∠ A) = 150° , then m (reflex ∠ A) = ……………°

4. The measure of the exterior angle of the equilateral triangle equals

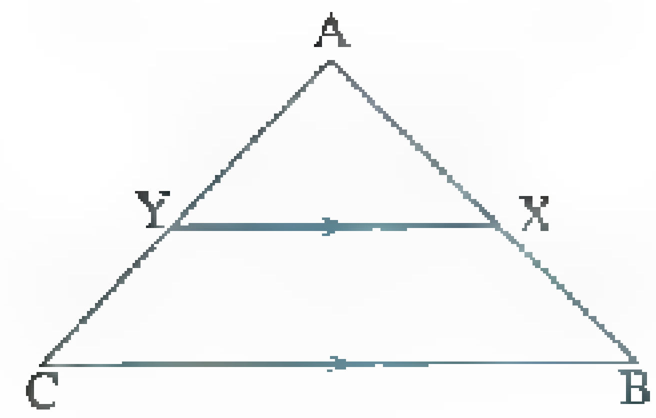
3 [a] **In the opposite figure :**

ABC is a triangle in which AB = AC

, $X \in \overline{AB}$

, $Y \in \overline{AC}$ and $\overline{XY} \parallel \overline{BC}$

Prove that : Δ AXY is an isosceles triangle.



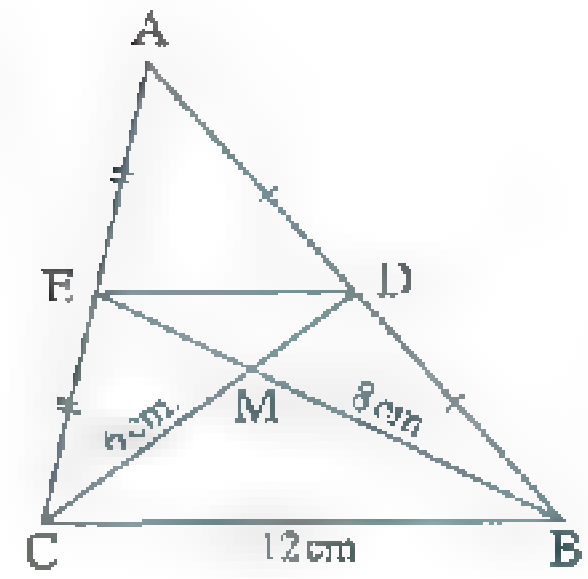
[b] **In the opposite figure :**

$\overline{CD}$ and $\overline{BE}$ are two medians in Δ ABC intersecting at M

, CB = 12 cm. , BM = 8 cm.

and CM = 6 cm.

Find : The perimeter of Δ MDE



4 [a] In Δ ABC , AB = 6 cm. , BC = 7 cm. and AC = 8 cm.

Arrange its angles ascendingly due to their measures.

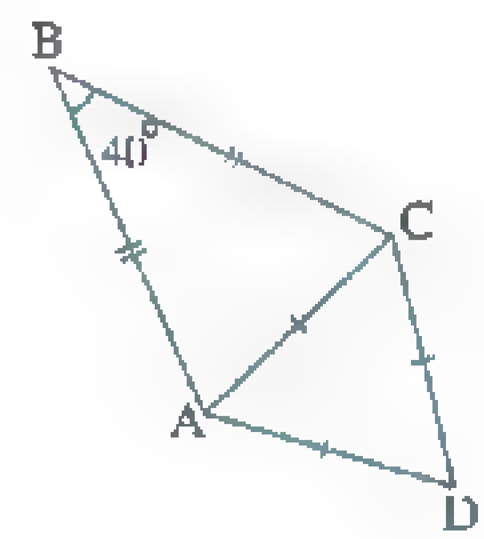
[b] **In the opposite figure :**

AB = BC

, AD = DC = AC

and m ($\angle$ ABC) = 40°

Find : m ($\angle$ BAD)

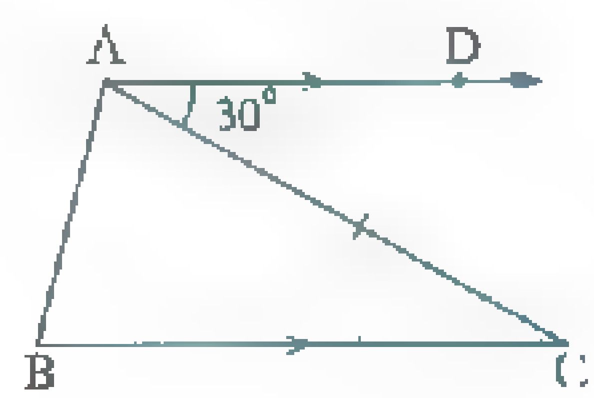


5 [a] **In the opposite figure :**

ABC is a triangle in which AC = BC

, $\overrightarrow{AD} \parallel \overline{BC}$ and m ($\angle$ DAC) = 30°

Find : The measures of the angles of Δ ABC

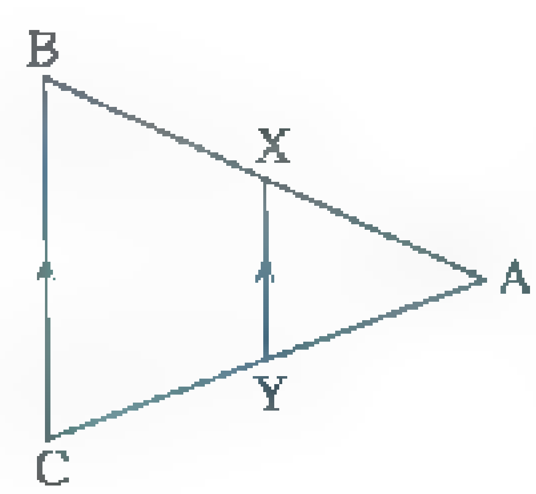


[b] **In the opposite figure :**

AB > BC

, $\overline{XY} \parallel \overline{BC}$

Prove that : AX > XY



4 Giza Governorate — Awseem Directorate

Answer the following questions :

1 Choose the correct answer :

1 The number of axes of symmetry of an equilateral triangle is ....

(a) 0 (b) 1 (c) 2 (d) 3

2 In Δ ABC , if BC > AB , then m ($\angle$ A) .... m ($\angle$ C)

(a) > (b) = (c) < (d) $\leq$

3 If A lies on the axis of symmetry of $\overline{XY}$, then $\overline{AX}$ .... $\overline{AY}$

(a) // (b) $\perp$ (c) $\equiv$ (d) =

4 If $\overline{AD}$ is a median of Δ ABC , and M is the point of intersection of the medians , then AM = ... AD

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

5 ABC is an isosceles triangle , its side lengths are 4 cm. , 9 cm. and X cm. , then X = ...

(a) 4 (b) 9 (c) 5 (d) 13

2 Complete :

1 ABC is a triangle in which : m ($\angle$ C) = 112° , then the longest side is ....

2 The bisector of the vertex angle of an isosceles triangle .... and ....

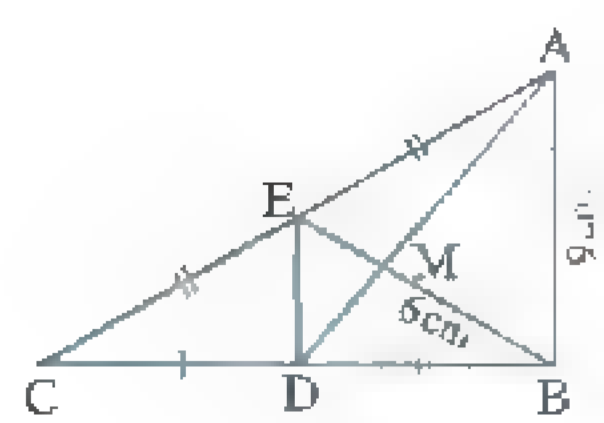
3 The lengths of two sides of a triangle are not equal , then the greater side in length is opposite to ....

4 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

3 [a] In the opposite figure :

ABC is a triangle , E and D are the midpoints of $\overline{AC}$ and $\overline{BC}$ respectively , $\overline{AD} \cap \overline{BE} = \{M\}$, AD = 12 cm. , MB = 6 cm. , AB = 9 cm.

Find : The perimeter of Δ EMD



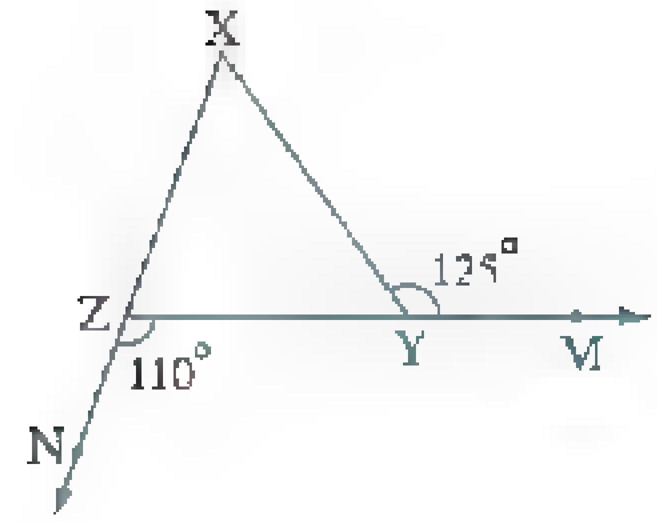
[b] In the opposite figure :

$Y \in \overrightarrow{ZM}$, $Z \in \overrightarrow{XN}$

, m ($\angle$ XYM) = 125°

, m ($\angle$ MZN) = 110°

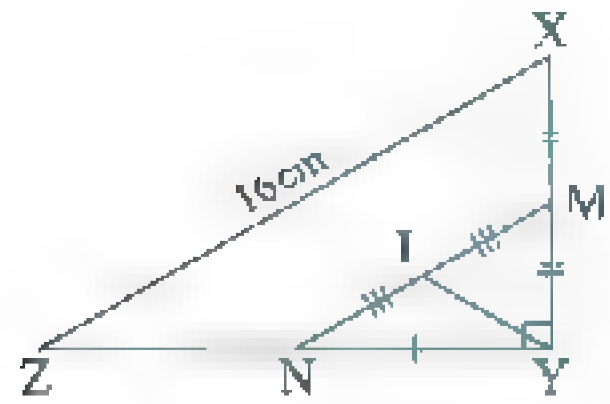
Prove that : XY > ZY



4 [a] In the opposite figure :

m ($\angle$ XYZ) = 90° , M , N and L are the midpoints of $\overline{XY}$, $\overline{ZY}$ and $\overline{MN}$ respectively , XZ = 16 cm.

Find : The length of $\overline{YL}$

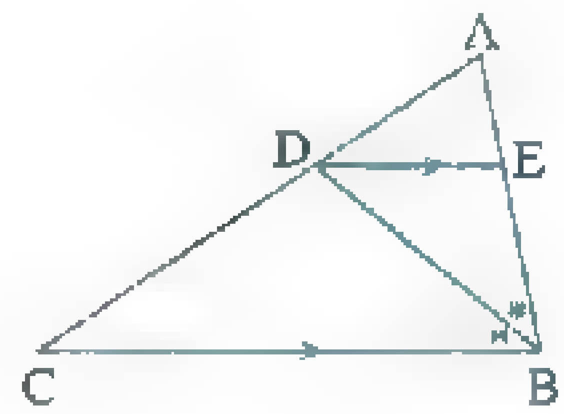


[b] In the opposite figure :

$\overrightarrow{BD}$ bisects $\angle$ ABC and intersects $\overline{AC}$ at D

, $\overline{DE}$ // $\overline{BC}$ where $E \in \overline{AB}$

Prove that : Δ EBD is an isosceles triangle.



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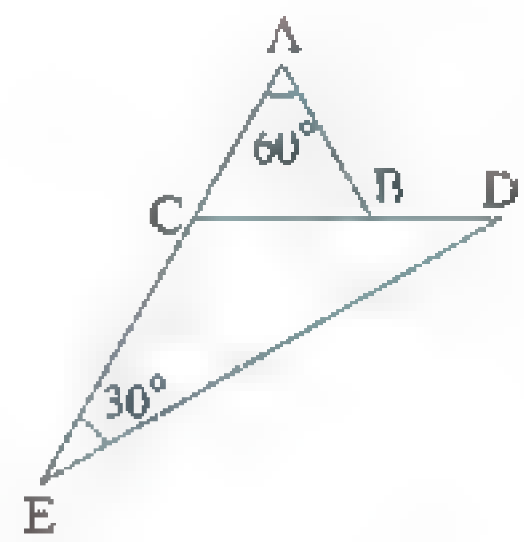
5 [a] In the opposite figure :

Δ CDE is an isosceles triangle in which EC = DC

, $E \in \overrightarrow{AC}$, m ($\angle$ DEC) = 30°

, m ($\angle$ CAB) = 60°

Prove that : Δ ABC is an equilateral triangle.



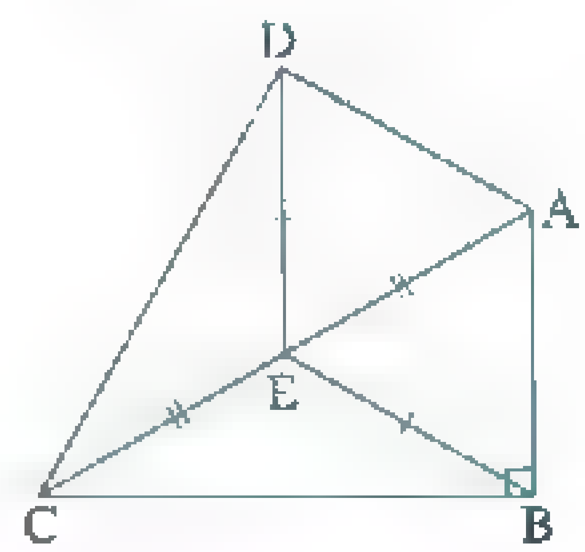
[b] In the opposite figure :

m ($\angle$ ABC) = 90°

, AE = CE

, BE = DE

Prove that : m ($\angle$ ADC) = 90°



5 Alexandria Governorate — West Administration

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The axis of symmetry of a line segment is the .... bisector of this line segment.

(a) parallel (b) perpendicular (c) inclined (d) skew

2 In Δ ABC , if $\overline{AD}$ is a median , M is the point of intersection of its medians , then AM = ... AD

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

3 The isosceles triangle has .... axis of symmetry.

(a) one (b) two (c) three (d) four

4 In Δ XYZ , if m ($\angle$ Z) = 70° and m ($\angle$ Y) = 60° , then YZ .... XY

(a) > (b) < (c) ≥ (d) =

5 The set of numbers can be the lengths of the sides of a triangle is ......

(a) {4 , 6 , 10} (b) {4 , 6 , 8} (c) {2 , 3 , 6} (d) {4 , 5 , 10}

2 Complete each of the following :

1 If AB = XY , then AB − XY = ...

2 The measure of each exterior angle of an equilateral triangle is ....°

3 The longest side in a right-angled triangle is ....

4 In Δ LMN , if m ($\angle$ L) = 42° and m ($\angle$ M) = 69° , then the type of Δ LMN according to its sides is .... triangle.

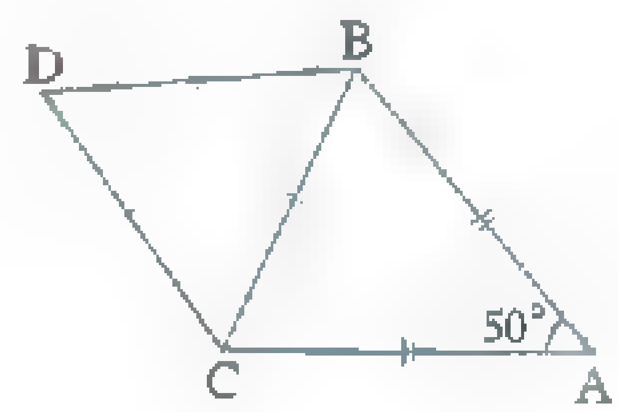
3 [a] In the opposite figure :

m ($\angle$ A) = 50°

, AB = AC

, Δ DBC is an equilateral triangle

Find : m ($\angle$ ABD)



[b] In Δ ABC , AB = 5 cm. , BC = 6 cm. and CA = 7 cm.

Arrange the angles of Δ ABC in an ascending order.

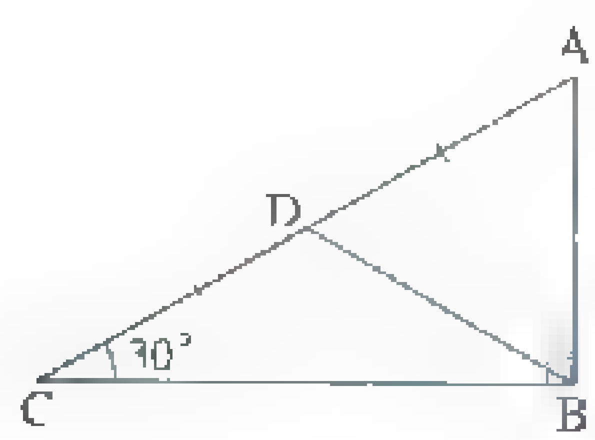
4 [a] In the opposite figure :

Δ ABC is right-angled at B

, m ($\angle$ C) = 30°

, D is the midpoint of $\overline{AC}$

Prove that : AB = DB



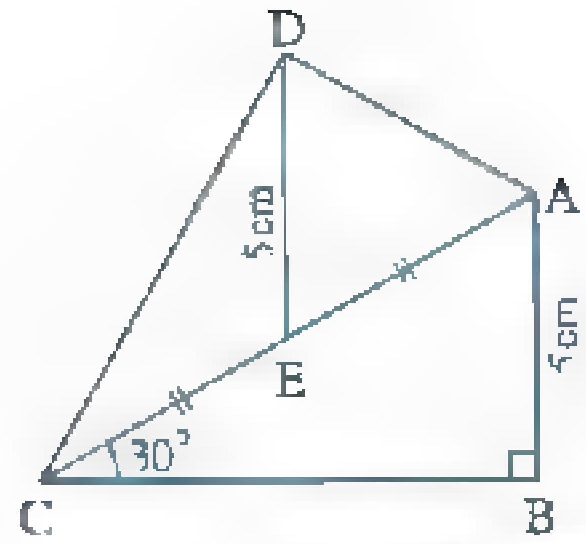
[b] In the opposite figure :

ABC is a right-angled triangle at B

, m (∠ ACB) = 30° , AB = 5 cm.

, E is the midpoint of $\overline{AC}$, DE = 5 cm.

Prove that : m (∠ ADC) = 90°

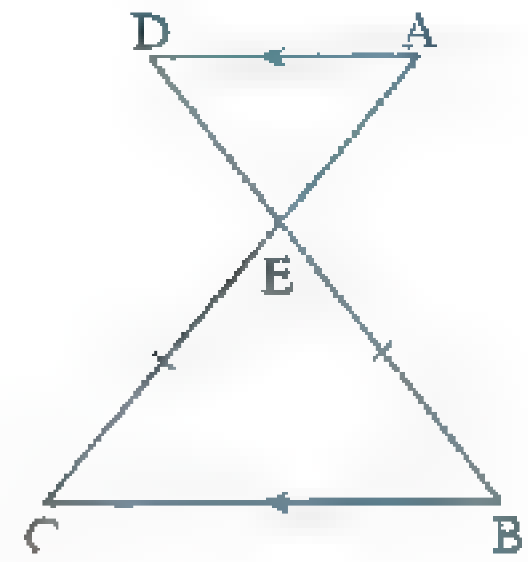


5 **[a] In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$

, EB = EC

Prove that : EA = ED

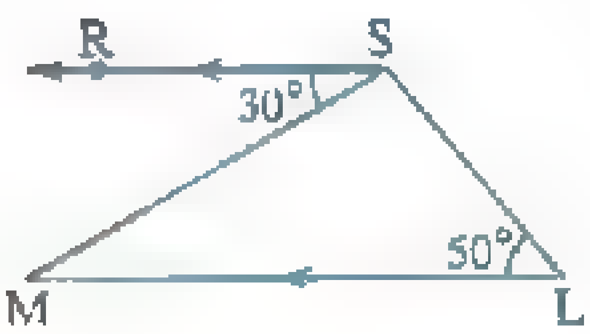


[b] In the opposite figure :

$\overrightarrow{SR}$ // $\overline{LM}$, m (∠ L) = 50°

, m (∠ RSM) = 30°

Prove that : ML > SL



6 El-Kalyoubia Governorate

Maths Supervision
Official Language Schools

Answer the following questions :

1 **Choose the correct answer from those given :**

[1] In Δ ABC , if AB = 6 cm. and AC = 7 cm. , then BC ∈ ....

(a)]6 , 13] (b) [6 , 7] (c)]1 , 13[(d) [1 , 7[

2 The point of intersection of the medians of the triangle divides each of them in the ratio of .... from the vertex.

(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 2 : 3

3 Δ XYZ is an isosceles triangle in which m (∠ X) = 110° , then m (∠ Y) = ...

(a) 110° (b) 35° (c) 60° (d) 45°

4 XYZ is a triangle in which m (∠ Z) = 70° , m (∠ Y) = 60° , then YZ .... XY

(a) > (b) < (c) = (d) ≥

[5] If 4 cm. , (X + 3) cm. and 8 cm. are side lengths of an isosceles triangle , then X = ...

(a) 3 (b) 4 (c) 5 (d) 6

2 **Complete the following :**

[1] In Δ ABC , if m ($\angle$ A) = m ($\angle$ B) + m ($\angle$ C) , then the longest side is ....

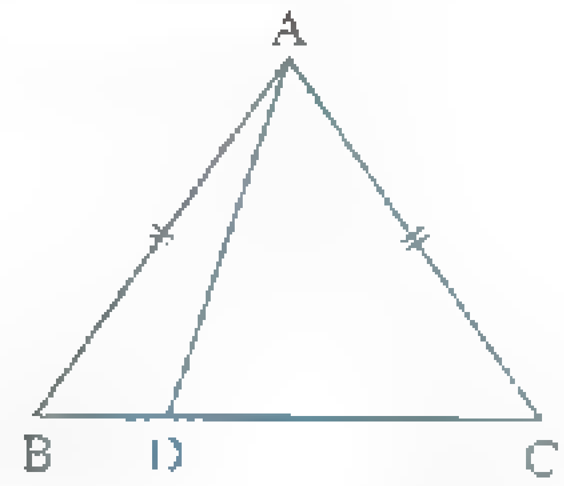
[2] If $\angle$ X and $\angle$ Y are two supplementary angles , $\angle$ X $\equiv$ $\angle$ Y , then m ($\angle$ X) = ... °

[3] The number of axes of symmetry of the rectangle equals ....

[4] If Δ ABC $\equiv$ Δ XYZ , then AC XZ = ...

---

3 [a] **In the opposite figure :**

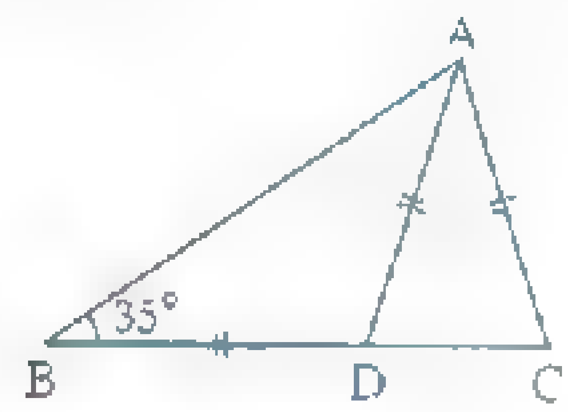


AC = AB , D $\in \overline{BC}$

Prove that :

AB > AD

[b] **In the opposite figure :**



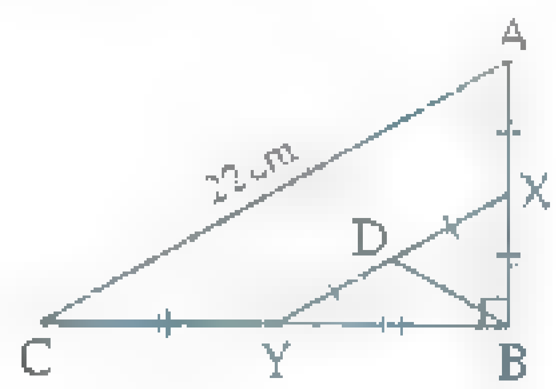
AC = AD = BD

, m ($\angle$ B) = 35°

Find : m ($\angle$ BAC)

---

4 [a] **In the opposite figure :**

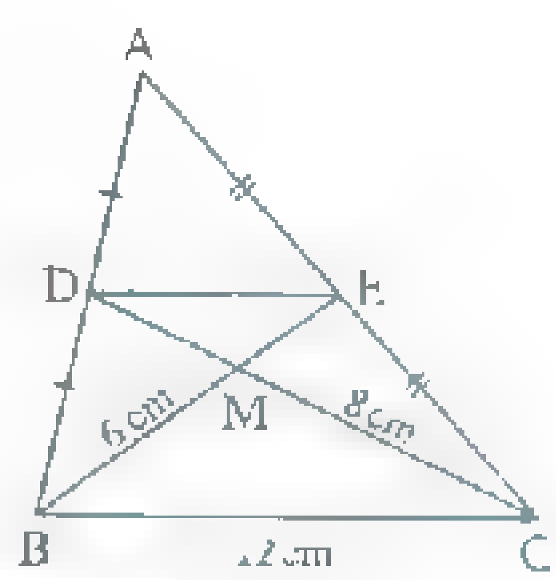


m ($\angle$ ABC) = 90° , X , Y and D are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{XY}$ respectively

, AC = 22 cm.

Find : BD

[b] **In the opposite figure :**



$\overline{BE}$ and $\overline{CD}$ are medians in Δ ABC

, $\overline{BE} \cap \overline{CD} = \{M\}$, MB = 6 cm.

, MC = 8 cm. , BC = 12 cm.

Find : The perimeter of Δ MDE

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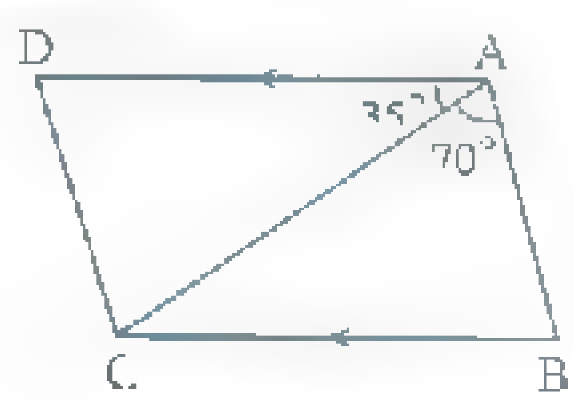
5 [a] **In the opposite figure :**

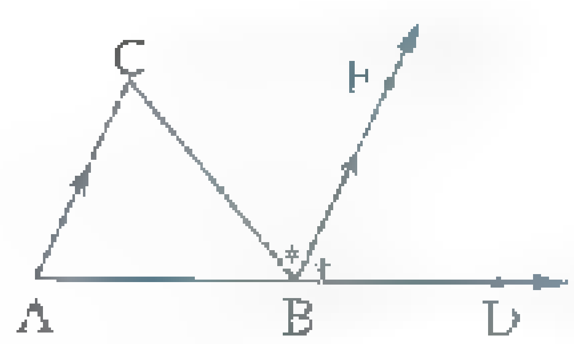


$\overline{AD}$ // $\overline{BC}$, m ($\angle$ BAC) = 70°

and m ($\angle$ DAC) = 35°

Prove that : AC > BC

[b] **In the opposite figure :**



D $\in \overrightarrow{AB}$, $\overrightarrow{BE}$ bisects $\angle$ CBD and $\overrightarrow{BE}$ // AC

Prove that : Δ ABC is an isosceles triangle.

7 El-Sharkia Governorate

G.L.S. Department
Supervision of Math

Answer the following questions :

1 Choose the correct answer :

1 The measure of the exterior angle of the equilateral triangle equals ....

(a) $30°$ (b) $60°$ (c) $90°$ (d) $120°$

2 In Δ ABC, if BC = 9 cm. and AB = 7 cm., then m ($\angle$ C) .... m ($\angle$ A)

(a) = (b) < (c) > (d) ≥

3 In Δ ABC, if m ($\angle$ A) = $40°$ and m ($\angle$ B) = $70°$, then the number of axes of symmetry of Δ ABC equals ....

(a) 1 (b) 2 (c) 3 (d) 4

4 If Δ XYZ is a right-angled triangle at Y, then XZ .... YZ

(a) = (b) < (c) > (d) ≤

5 In Δ ABC, if m ($\angle$ C) = $60°$ and m ($\angle$ B) = $90°$, then AC = ...

(a) 2 BC (b) $\frac{1}{2}$ AB (c) 2 AB (d) $\frac{1}{2}$ BC

2 Complete the following :

1 In the triangle ABC, if AB = 3 cm. and BC = 5 cm., then AC ∈] .... , ... [

2 The intersection point of the medians of the triangle divides each median by the ratio .... : .... from its base.

3 The bisector of the vertex angle of an isosceles triangle bisects the base and ....

4 The lengths 6 cm., 3 cm. and .... cm. can be lengths of sides of an isosceles triangle.

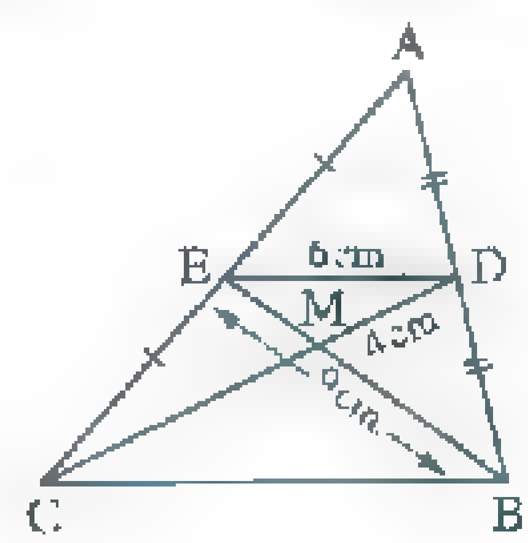
3 [a] In the opposite figure :

$\overline{BE}$ and $\overline{CD}$ are two medians in Δ ABC

, $\overline{BE} \cap \overline{CD}$ = {M}, BE = 9 cm.

, MD = 4 cm., DE = 6 cm.

Find : The perimeter of Δ BMC



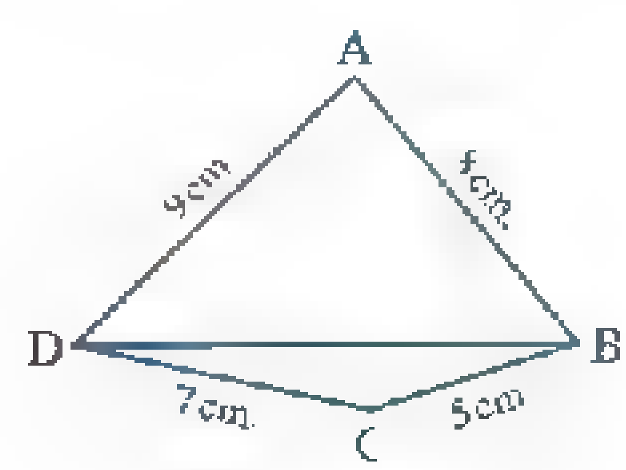
[b] In the opposite figure :

ABCD is a quadrilateral where

, AB = 8 cm., AD = 9 cm.

, BC = 5 cm. and CD = 7 cm.

Prove that : m ($\angle$ ABC) > m ($\angle$ ADC)

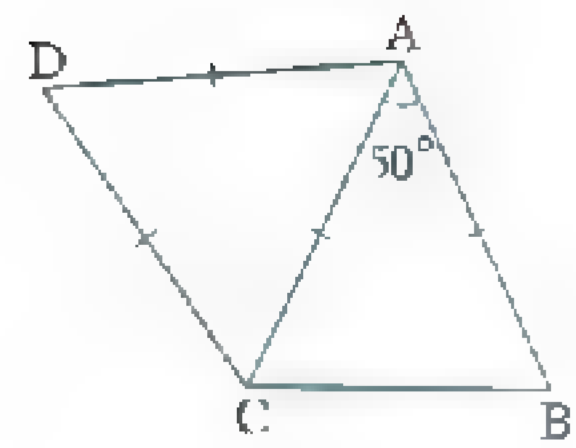


4 [a] **In the opposite figure :**

AB = AC = CD = DA

, m (∠ BAC) = 50°

Find with proof : m (∠ BCD)

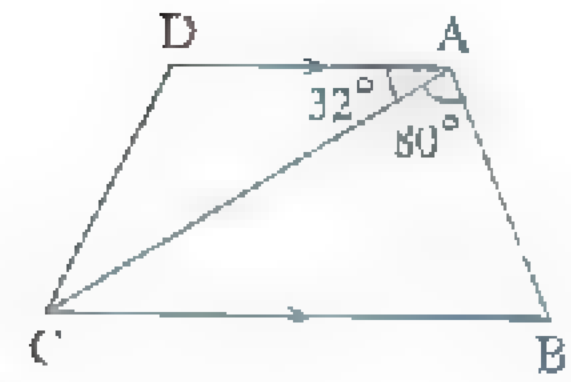


[b] **In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$, m (∠ BAC) = 80°

, m (∠ CAD) = 32°

Prove that : BC > AB



5 [a] **In the opposite figure :**

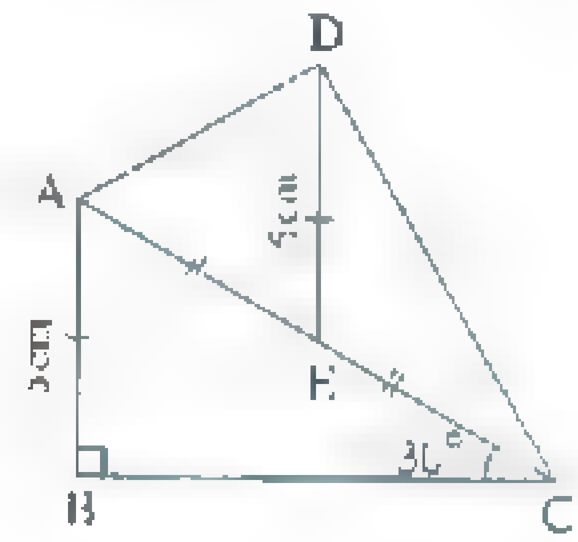
Δ ABC is right-angled at B

, m (∠ ACB) = 30° , AB = 5 cm.

, DE = 5 cm.

and E is the midpoint of $\overline{AC}$

Prove that : m (∠ ADC) = 90°



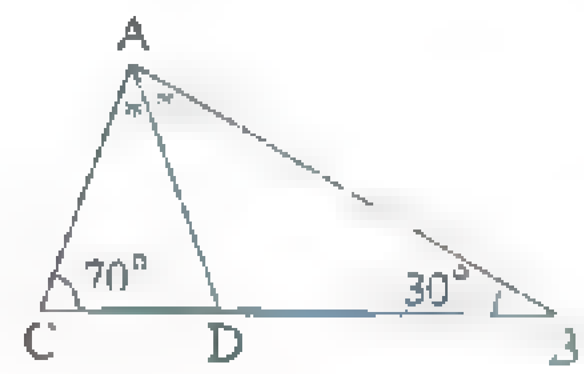
[b] **In the opposite figure :**

ABC is a triangle in which

m (∠ C) = 70° , m (∠ B) = 30°

, $\overline{AD}$ bisects ∠ BAC

Prove that : AD = AC



8 El-Dakahlia Governorate

Maths Supervision

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The length of the median drawn from the vertex of the right angle in the right-angled triangle equals .... the length of the hypotenuse.

(a) half (b) twice (c) third (d) quarter

2 The point of intersection of the medians of the triangle divides each median in the ratio .... from the vertex.

(a) 1 : 2 (b) 2 : 1 (c) 2 : 3 (d) 1 : 3

3 The measure of the exterior angle of an equilateral triangle equals .....

(a) 60° (b) 90° (c) 120° (d) 360°

4 If the measures of two angles in a triangle are $55°$, $70°$, then the triangle is ......

(a) isosceles. (b) equilateral. (c) scalene. (d) obtuse.

5 The lengths of two sides in an isosceles triangle are 4 cm , 9 cm. , then the length of the third side is .... cm.

(a) 4 (b) 5 (c) 9 (d) 13

2 Complete each of the following :

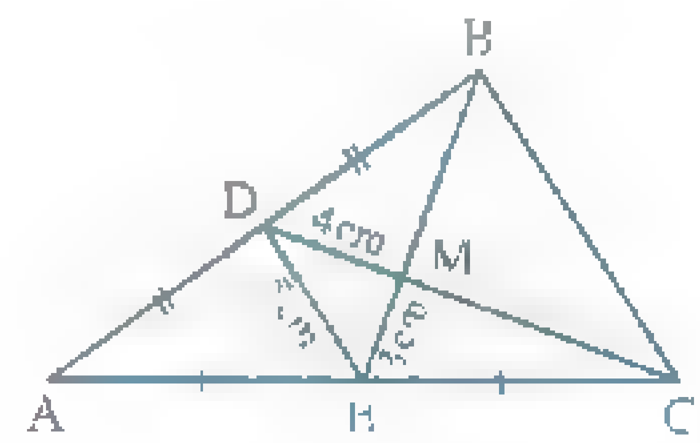
1 The longest side in the right-angled triangle is ....

2 The bisector of the vertex angle of an isosceles triangle .... the base and is perpendicular to it.

3 In the isosceles triangle ABC , if AB = AC , m ($\angle$ A) = $70°$, then AB <

4 In a triangle , if the lengths of two sides are 4 cm , 6 cm , then the length of the third side $\in$] .... , ... [

3 [a] In the opposite figure :



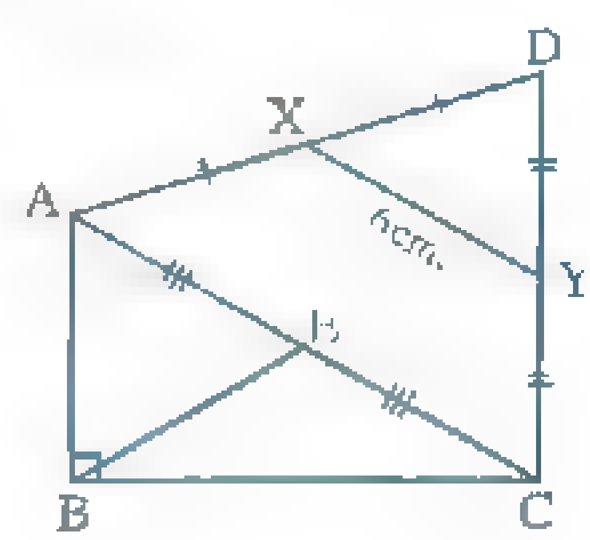
M is the intersection point of the medians of Δ ABC

, ME = 3 cm. , MD = 4 cm. , DE = 5 cm.

Find : The perimeter of Δ MBC

[b] Arrange ascendingly the measures of the angles of Δ ABC if AC = 12 cm. , BC = 5 cm. and AB = 13 cm.

4 [a] In the opposite figure :



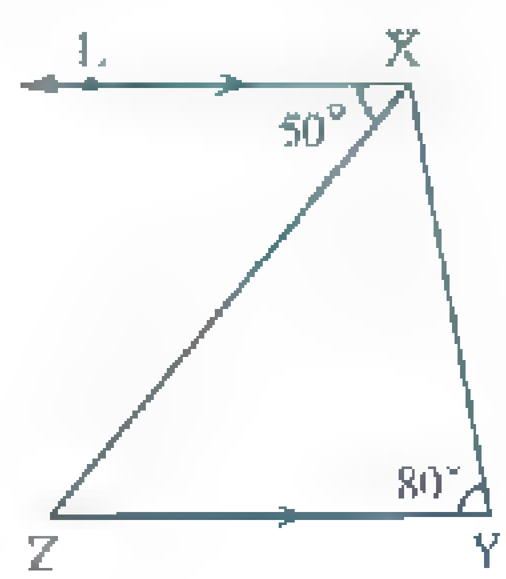
X is the midpoint of $\overline{AD}$, Y is the midpoint of $\overline{CD}$

, E is the midpoint of $\overline{AC}$

, m ($\angle$ ABC) = $90°$, XY = 6 cm.

Find : The length of $\overline{BE}$

[b] In the opposite figure :



$\overrightarrow{XL}$ // $\overrightarrow{YZ}$, m ($\angle$ Y) = $80°$

, m ($\angle$ LXZ) = $50°$

Prove that : XY = YZ

5 [a] In the opposite figure :

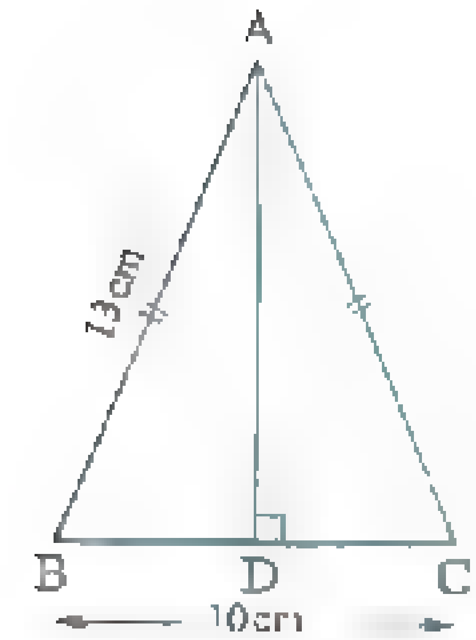
$AB = AC$, $\overline{AD} \perp \overline{BC}$

, $AB = 13$ cm.

, $BC = 10$ cm.

Find : 1 The length of $\overline{BD}$

2 The area of $\triangle ABC$



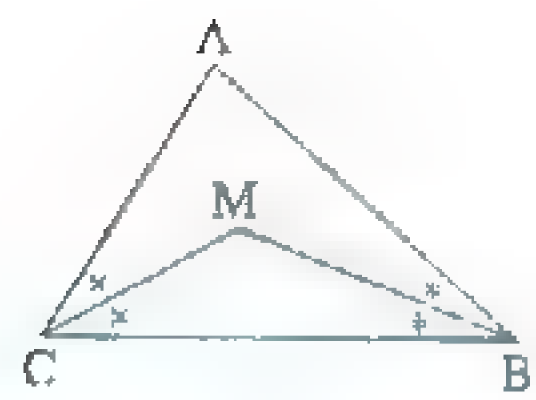
[b] In the opposite figure :

ABC is a triangle , $AB > AC$

, $\overrightarrow{BM}$ bisects $\angle ABC$

, $\overrightarrow{CM}$ bisects $\angle ACB$

Prove that : $MB > MC$



9 Suez Governorate — Math Inspection

Answer the following questions :

1 Complete :

1 ΔABC is an isosceles triangle , $AB = 3$ cm. , $BC = 7$ cm , then $AC =$ .... cm.

2 In ΔABC , if $AB = 3$ cm. , $BC = 5$ cm. , then $AC \in]$ .... , ... [

3 The measure of the exterior angle of the equilateral triangle is ....°

4 The bisector of the vertex angle of the isosceles triangle .... and ....

2 Choose the correct answer :

1 The triangle which has 3 axes of symmetry is .... triangle.

(a) an equilateral (b) an isosceles (c) a scalene (d) a right-angled

2 If the measure of the vertex angle of the isosceles triangle is 80° , then the measure of one base angle is ....

(a) 100° (b) 50° (c) 80° (d) 40°

3 If $A \in$ the axis of $\overline{BC}$, then $\overline{AB}$ .... $\overline{AC}$

(a) = (b) ≠ (c) // (d) $\perp$

4 The intersection point of the medians of any triangle divides each median in the ratio .... from the vertex

(a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 3 : 1

5 ABC is an isosceles triangle in which : AB = AC = 4 cm. , m (∠ A) = 60°
, then its perimeter is .... cm.

(a) 10 (b) 12 (c) 6 (d) 8

3 [a] In the triangle ABC , m (∠ B) = 40° , m (∠ C) = 80°

Arrange its side lengths ascendingly.

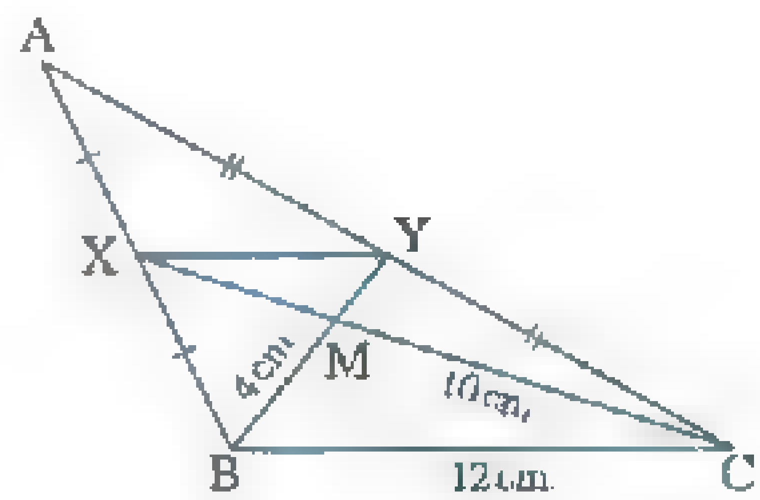
[b] **In the opposite figure :**

X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, $\overline{BY} \cap \overline{CX} = \{M\}$, MC = 10 cm.

, MB = 4 cm. , BC = 12 cm.

Find : The perimeter of Δ MXY



4 [a] **In the opposite figure :**

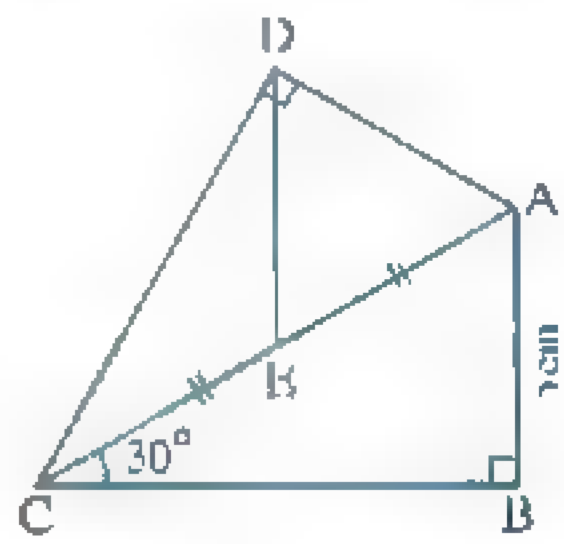
m (∠ B) = m (∠ ADC) = 90°

, m (∠ ACB) = 30° , AB = 5 cm.

, E is the midpoint of $\overline{AC}$

Find : 1 AC

2 DE

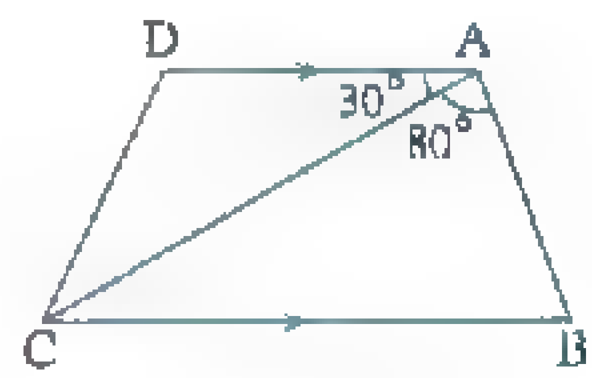


[b] **In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$, m (∠ CAD) = 30°

, m (∠ BAC) = 80°

Prove that : BC > AB

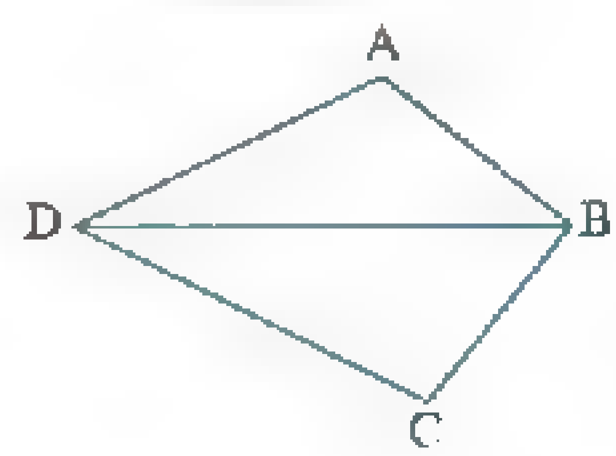


5 [a] **In the opposite figure :**

AD > AB and DC > BC

Prove that :

m (∠ ABC) > m (∠ ADC)



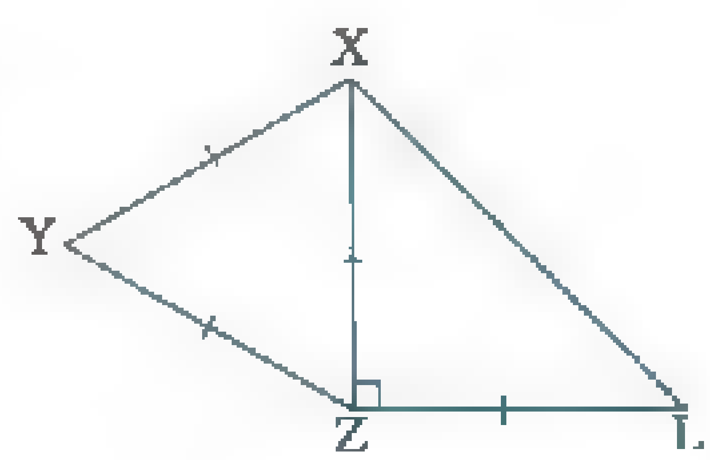
[b] **In the opposite figure :**

m (∠ XZL) = 90°

, LZ = ZX = XY = YZ

Find : 1 m (∠ YXZ)

2 m (∠ LXY)



10 El-Beheira Governorate

Maths Supervision

Answer the following questions :

1 Choose the correct answer :

1. The angle whose measure is more than 90° and less than 180° is ....
   (a) acute. (b) right. (c) obtuse. (d) straight.
2. The point of intersection of the medians of the triangle divides each of them by the ratio .... from the base.
   (a) 2 : 1 (b) 1 : 2 (c) 3 : 4 (d) 1 : 1
3. In ΔXYZ , $XY = XZ$, $m(\angle Y) = 40°$, then $m(\angle X) =$ ....
   (a) 40° (b) 55° (c) 70° (d) 100°
4. The measure of the exterior angle of an equilateral triangle is ....
   (a) 120° (b) 60° (c) 90° (d) 30°
5. In ΔABC , if $AB > AC$, then $m(\angle B)$ .... $m(\angle C)$
   (a) > (b) < (c) ≥ (d) =

2 Complete the following :

1. In ΔDEF , if $m(\angle E) = 120°$, then the longest side is ....
2. The isosceles triangle has .... axis of symmetry.
3. If $\overline{AB} \equiv \overline{CD}$ and $AB = 6$ cm. , then $AB + CD =$ .... cm.
4. The length of the side opposite to the angle whose measure is 30° in the right-angled triangle equals .... the length of the hypotenuse.

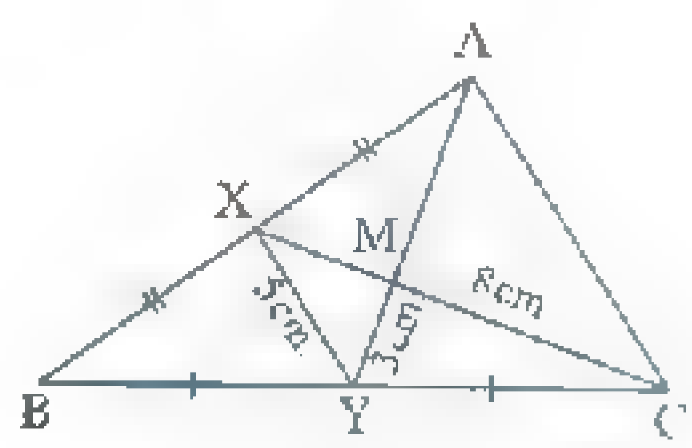
3 [a] In the opposite figure :

ABC is a triangle , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$, $\overline{XC} \cap \overline{AY} = \{M\}$

, $XY = 5$ cm. , $CM = 8$ cm. , $YM = 3$ cm.

Find : The perimeter of the triangle MAC



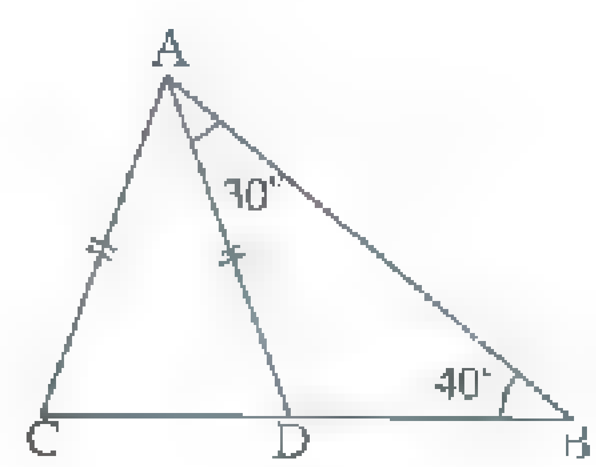
[b] In the opposite figure :

$AD = AC$, $D \in \overline{BC}$

, $m(\angle DAB) = 30°$

, $m(\angle ABD) = 40°$

Prove that : $AB = CB$



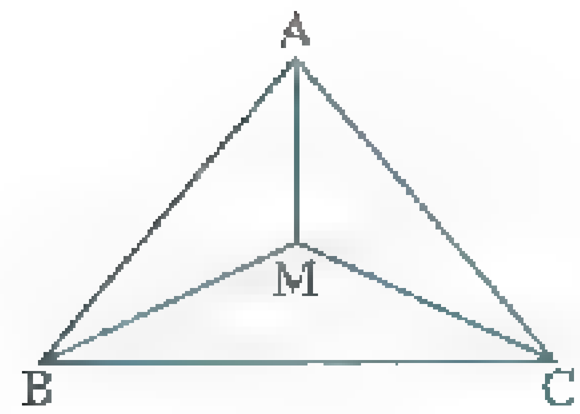
4 **[a] In the opposite figure :**

ABC is a triangle in which

M is a point inside it.

Prove that :

$MA + MB + MC > \frac{1}{2}$ the perimeter of the triangle ABC



[b] XYZ is a triangle in which : $m(\angle X) = 40°$, $m(\angle Y) = 80°$

Arrange the lengths of sides of ΔXYZ in an ascending order.

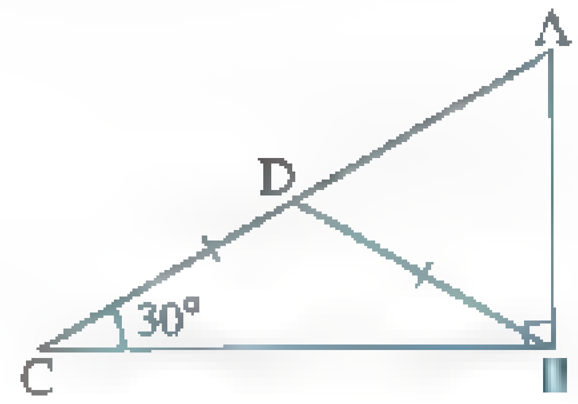
5 **[a] In the opposite figure :**

ABC is a right-angled triangle at B

, $m(\angle C) = 30°$, $D \in \overline{AC}$ where $DB = DC$

Prove that :

ΔABD is an equilateral triangle.



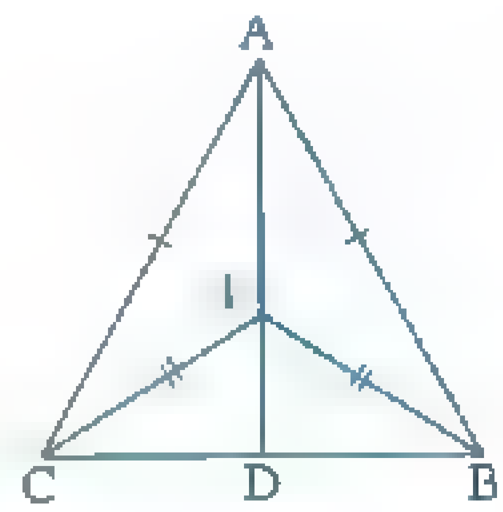
[b] In the opposite figure :

$AB = AC$ and $EB = EC$

, $\overrightarrow{AE} \cap \overline{BC} = \{D\}$

Prove that : [1] $\overleftrightarrow{AE}$ is the axis of $\overline{BC}$

[2] $BD = DC$



11 Beni Suef Governorate

Directorate of Official Language Schools
Education Administration

Answer the following questions :

1 **Choose the correct answer from those given :**

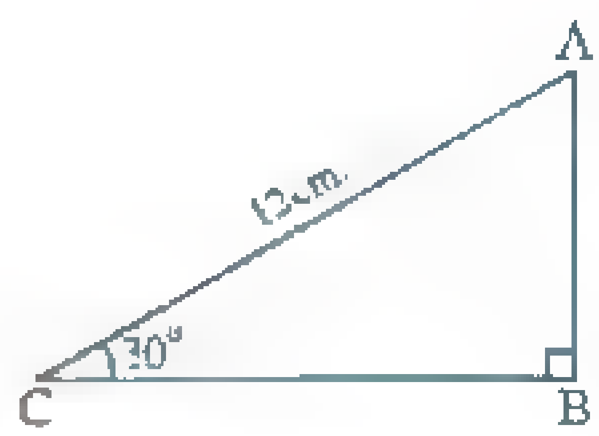
[1] **In the opposite figure :**

$m(\angle ABC) = 90°$, $m(\angle C) = 30°$ and $AC = 12$ cm.

, then the length of $\overline{AB} = \cdots\cdots\cdots$ cm.

(a) 3 (b) 4

(c) 6 (d) 12



[2] If the lengths of two sides in an isosceles triangle are 2 cm. and 5 cm. , then the length of the third side is

(a) 2 cm. (b) 3 cm. (c) 5 cm. (d) 7 cm.

[3] In ΔABC , if $m(\angle A) > m(\angle C)$, then ⋯

(a) $AB > BC$ (b) $AB < AC$ (c) $AB < BC$ (d) $AB = AC$

4 In ΔABC , if $m(\angle A) = 30°$, $m(\angle B) = 70°$, then $m(\angle C) = ...$

(a) 30° (b) 70° (c) 80° (d) 100°

5 If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm , then AC = ... cm.

(a) 10 (b) 28 (c) 100 (d) 14

2 Complete each of the following :

1 In ΔABC , if $m(\angle A) = 30°$ and $m(\angle B) = 90°$, then BC = ... AC

2 If two angles of a triangle are congruent , then the two sides opposite to these two angles are ...... and the triangle is ......

3 The number of axes of symmetry of the isosceles triangle equals ......

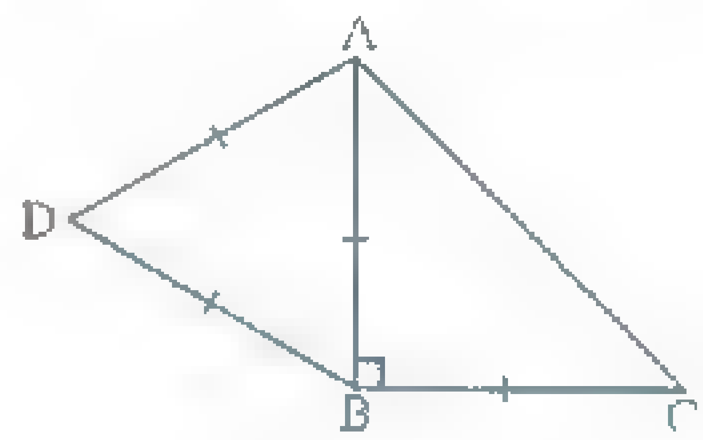
4 In ΔABC , if $m(\angle A) = 40°$ and $m(\angle B) = 60°$, then the shortest side in the triangle is ......

3 [a] In the opposite figure :

AB = BD = DA = BC

and $m(\angle ABC) = 90°$

Find : $m(\angle CAD)$



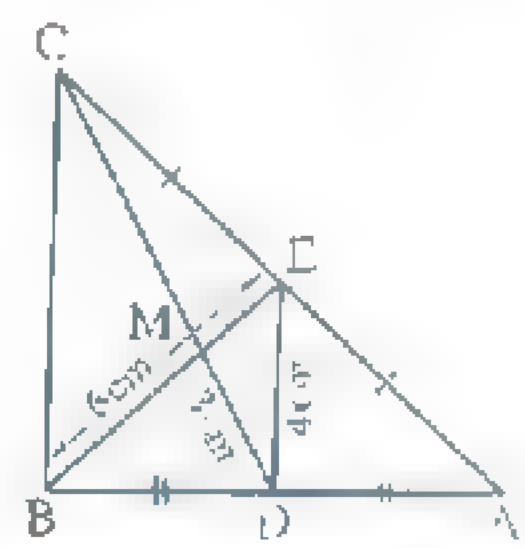
[b] In the opposite figure :

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, $\overline{BE} \cap \overline{DC} = \{M\}$, DE = 4 cm.

, MD = 3 cm. and BE = 6 cm.

Find : The perimeter of ΔBMC



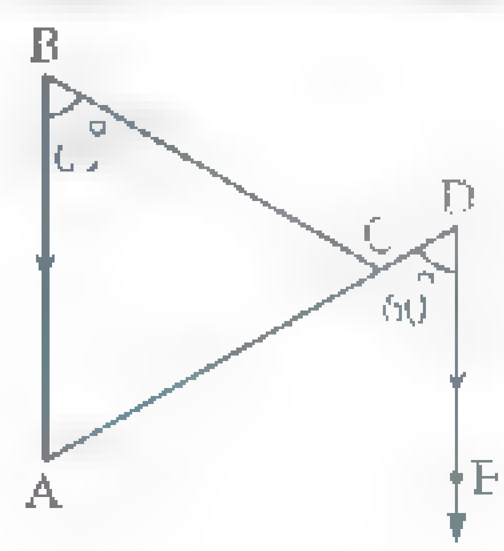
4 [a] In the opposite figure :

$\overrightarrow{DE} \parallel \overrightarrow{BA}$

, $C \in \overline{AD}$

and $m(\angle ABC) = m(\angle ADE) = 60°$

Prove that : ΔABC is an equilateral triangle.



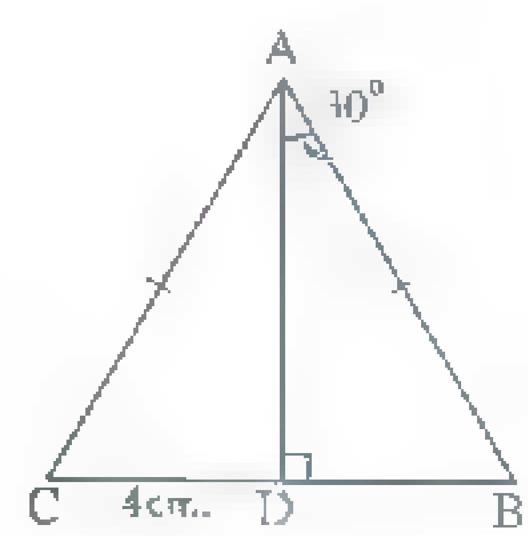
[b] In the opposite figure :

AB = AC , CD = 4 cm.

, $\overline{AD} \perp \overline{BC}$ and $m(\angle BAD) = 30°$

Find : 1 The length of $\overline{DB}$

2 $m(\angle BAC)$

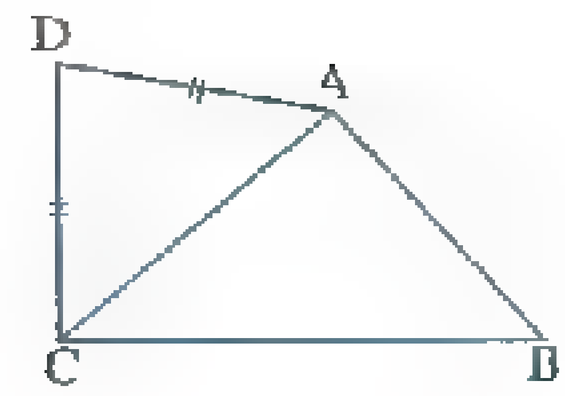


5 [a] **In the opposite figure :**

ABCD is a quadrilateral in which

BC > BA and DA = DC

Prove that : m (∠ BAD) > m (∠ BCD)

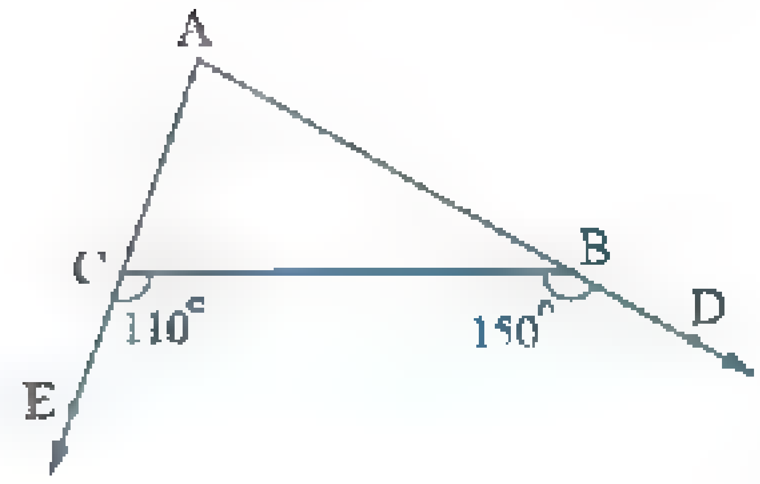


[b] **In the opposite figure :**

ABC is a triangle , D ∈ $\overrightarrow{AB}$, E ∈ $\overrightarrow{AC}$

, m (∠ DBC) = 150° , m (∠ ECB) = 110°

Prove that : CB > AB



12 Aswan Governorate

Edfo District
Mathematics Supervision

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1. XYZ is an isosceles triangle , m (∠ X) = 100° , then m (∠ Y) = ...

   (a) 100° (b) 80° (c) 60° (d) 40°

2. ABC is a right-angled triangle at B , AB = 6 cm. , m (∠ A) = 2 m (∠ C) , then AC = ... cm.

   (a) 6 (b) 3 (c) 12 (d) 24

3. If C ∈ the axis of symmetry of $\overline{AB}$, then AC – BC = ...

   (a) 0 (b) 1 (c) 2 (d) 3

4. If $\overline{AD}$ is a median in Δ ABC , M is the point of concurrence of the medians , then AD = ... DM

   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 3

5. The number of diagonals of the hexagon is ....

   (a) 3 (b) 6 (c) 9 (d) 12

2 **Complete the following :**

1. The bisector of the vertex angle of the isosceles triangle bisects the base and ....

2. A rectangle whose two dimensions are 8 cm. , 6 cm. , then the length of its diagonal is .... cm.

3. The longest side in the right-angled triangle is ....

4. In Δ ABC , if AB > BC , then m (∠ A) < ....

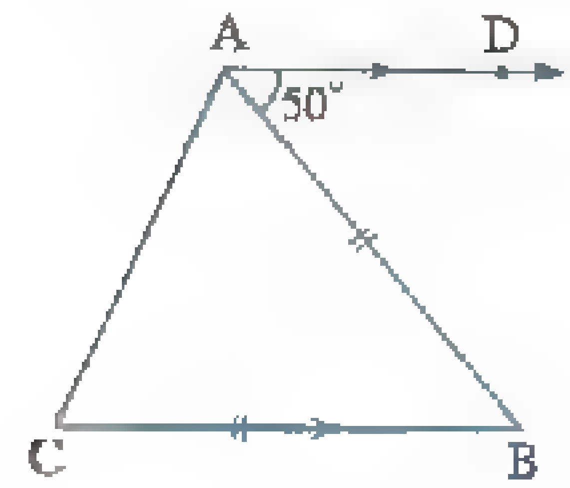
3 [a] **In the opposite figure :**

$\overrightarrow{AD} \parallel \overline{BC}$

, AB = BC

, m (∠ BAD) = 50°

Find by proof : m (∠ DAC)

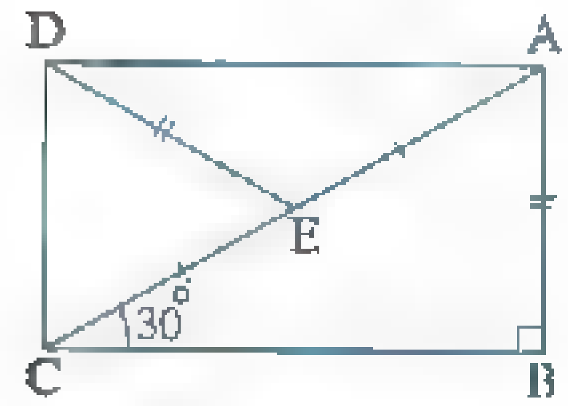


[b] **In the opposite figure :**

AB = DE , E is the midpoint of $\overline{AC}$

, m (∠ ABC) = 90° , m (∠ ACB) = 30°

Prove that : m (∠ ADC) = 90°



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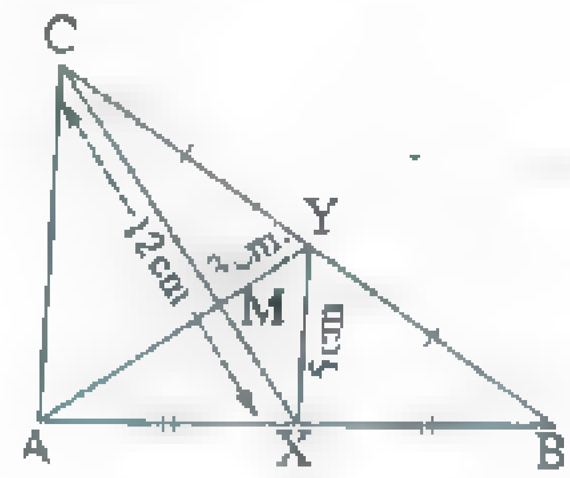
4 [a] **In the opposite figure :**

ABC is a triangle in which Y is the midpoint of $\overline{CB}$

, X is the midpoint of $\overline{AB}$, $\overline{AY} \cap \overline{XC} = \{M\}$

, YM = 3 cm. , YX = 5 cm. , CX = 12 cm.

Find : The perimeter of Δ AMC

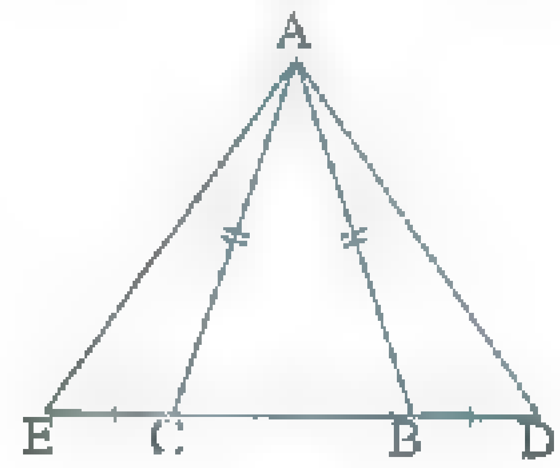


[b] **In the opposite figure :**

B ∈ $\overline{DE}$, C ∈ $\overline{DE}$

, AB = AC , BD = CE

Prove that : AD = AE



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5 [a] XYZ is a triangle , XY = 6 cm. , YZ = 7 cm. , XZ = 5 cm.

Arrange the measures of angles of Δ XYZ ascendingly.

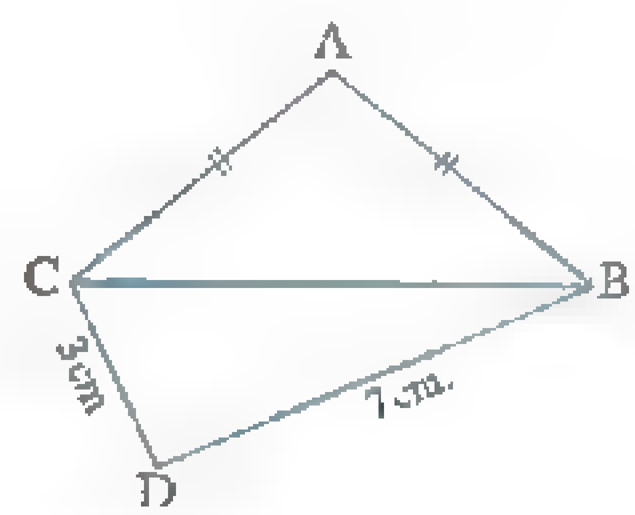
[b] **In the opposite figure :**

AB = AC

, BD = 7 cm.

, DC = 3 cm.

Prove that : m (∠ ACD) > m (∠ ABD)



El Moasser

By a group of supervisors

GUIDE ANSWERS

2nd PREP.
FIRST TERM

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Guide Answers

Of Algebra and Statistics Exercises

Answers of unit one

1

[1] 2 [2] 3 [3] 6 [4] 7

5 0.1 [6] $\frac{2}{3}$ [7] zero [8] 1

[9] 6 [10] 1 [11] zero [12] a

[13] $3a^7$ [14] 64 [15] 1000 [16] 64

[17] 25 [18] 61

2

[1] c [2] a [3] d [4] d [5] a

[6] d [7] c [8] c [9] d [10] b

3

[1] 125 [2] $-\frac{1}{64}$

[3] $\because \sqrt[3]{x} = -2$ $\therefore x = -8$

[4] $\because \sqrt[3]{x} = -1 + 3 = 2$ $\therefore x = 8$

[5] -2 6 4

[7] $\because x^3 = 32 - 5 = 27$ $\therefore x = 3$

[8] $\because x^3 = 54 \div 2 = 27$ $\therefore x = 3$

[9] $\because x^3 = -200 \div \frac{1}{5} = -1000$ $\therefore x = -10$

4

[1] $\because x^3 = -27$ $\therefore x = \sqrt[3]{-27} = -3$

$\therefore$ The S.S. $= \{-3\}$

[2] $\because 8x^3 = 8 - 7 = 1$ $\therefore x^3 = \frac{1}{8}$

$\therefore x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$ $\therefore$ The S.S. $= \{\frac{1}{2}\}$

[3] $\because 2x^3 - x^3 = 3 + 5$ $\therefore x^3 = 8$

$\therefore x = \sqrt[3]{8} = 2$ $\therefore$ The S.S. $= \{2\}$

[4] $\because x + 3 = \sqrt[3]{343} = 7$ $\therefore x = 7 - 3 = 4$

$\therefore$ The S.S. $= \{4\}$

[5] $\because (2x + 1)^3 = 20 + 7 = 27$ $\therefore 2x + 1 = \sqrt[3]{27} = 3$

$\therefore 2x = 3 - 1 = 2$ $\therefore x = 2 \div 2 = 1$

$\therefore$ The S.S. $= \{1\}$

[6] $\because (5x - 2)^3 = 18 - 10 = 8$ $\therefore 5x - 2 = \sqrt[3]{8} = 2$

$\therefore 5x = 2 + 2 = 4$ $\therefore x = \frac{4}{5}$

$\therefore$ The S.S. $= \{\frac{4}{5}\}$

5

[1] $\sqrt[3]{2^9 \times 3^6} = \sqrt[3]{(2^3 \times 3^2)^3} = -2^3 \times 3^2 = 8 \times 9 = 72$

[2] $\sqrt{\sqrt[3]{729}} = \sqrt{9} = 3$

[3] $\sqrt{27\sqrt[3]{27}} = \sqrt{27 \times 3} = \sqrt{81} = 9$

6

The edge length of the cube $= \sqrt[3]{27} = 3$ cm

[1] The area of one face $= 3^2 = 9 \text{ cm}^2$

[2] The total area $= 6 \times 3^2 = 54 \text{ cm}^2$

7

The length of the inner edge $= \sqrt[3]{1000} = 10$ cm.

8

The volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{1372}{81}\pi$

$\therefore r^3 = \frac{1372}{81} \times \frac{3}{4} = \frac{343}{27}$

$\therefore r = \sqrt[3]{\frac{343}{27}} = \frac{7}{3}$

$\therefore$ The diameter length of the sphere

$= 2 \times \frac{7}{3} = \frac{14}{3}$ length unit.

9

$\because$ The volume of the sphere $= \frac{4}{3}\pi r^3 = 113.04$

$\therefore \frac{4}{3} \times 3.14 \times r^3 = 113.04$

$r^3 = 27$ $\therefore r = \sqrt[3]{27} = 3$ cm

$\therefore$ The diameter length of the sphere $= 2 \times 3 = 6$ cm.

10

[1] $\because (x^2 + 6)^3 = 1000$ $\therefore x^2 + 6 = 10$

$\therefore x^2 = 4$ $\therefore x = \pm 2$

$\therefore$ The S.S. $= \{2, -2\}$

[2] $\because (x^3 - 14)^2 = 169$ $\therefore x^3 - 14 = \pm 13$

$\therefore x^3 = 14 \pm 13$ $\therefore x^3 = 27$ $\therefore x = 3$

or $x^3 = 1$ $\therefore x = 1$

$\therefore$ The S.S. $= \{3, 1\}$

[3] Cubing the two sides $\therefore (x - 1)^2 = 25$

$x - 1 = \pm 5$ $\therefore x = 6$ or $x = -4$

$\therefore$ The S.S. $= \{6, -4\}$

11

Cubing the two sides

$\therefore \sqrt{x} + 19 = 27$ $\qquad$ $\therefore \sqrt{x} = 8$

Squaring the two sides

$\therefore x = 64$ $\qquad$ $\therefore \sqrt[3]{x} = \sqrt[3]{64} = 4$

Answers of Exercise 2

1

The rational numbers are No.

1 , 2 , 3 , 4 , 5 , 8 , 9 ,

11 , 13 , 14 , 16 , 17 , 18 , 19

The remained numbers are irrational.

2 1) 3.32 $\qquad$ 2) 1.9 $\qquad$ 3) -2.1

3

1) $\because \sqrt{4} < \sqrt{5} < \sqrt{9}$ $\qquad$ $\therefore 2 < \sqrt{5} < 3$

$\therefore$ The two numbers are 2 and 3

2) $\because \sqrt{9} < \sqrt{12} < \sqrt{16}$ $\qquad$ $\therefore 3 < \sqrt{12} < 4$

$\therefore$ The two numbers are 3 and 4

3) $\because \sqrt[3]{8} < \sqrt[3]{10} < \sqrt[3]{27}$ $\qquad$ $\therefore 2 < \sqrt[3]{10} < 3$

$\therefore$ The two numbers are 2 and 3

4) $\because \sqrt[3]{-27} < \sqrt[3]{-20} < \sqrt[3]{-8}$ $\qquad$ $\therefore -3 < \sqrt[3]{-20} < -2$

$\therefore$ The two numbers are -2 and -3

4

1) $\because \sqrt{1} < \sqrt{2} < \sqrt{4}$ $\qquad$ $\therefore 1 < \sqrt{2} < 2$ $\qquad$ $\therefore x = 1$

2) $\because \sqrt{64} < \sqrt{80} < \sqrt{81}$ $\qquad$ $\therefore 8 < \sqrt{80} < 9$ $\qquad$ $\therefore x = 8$

3) $\because \sqrt[3]{1} < \sqrt[3]{5} < \sqrt[3]{8}$ $\qquad$ $\therefore 1 < \sqrt[3]{5} < 2$ $\qquad$ $\therefore x = 1$

4) $\because \sqrt[3]{27} < \sqrt[3]{50} < \sqrt[3]{64}$ $\qquad$ $\therefore 3 < \sqrt[3]{50} < 4$ $\qquad$ $\therefore x = 3$

5) $\because \sqrt[3]{-125} < \sqrt[3]{-100} < \sqrt[3]{-64}$

$\therefore -5 < \sqrt[3]{-100} < -4$ $\qquad$ $\therefore x = -5$

6) $\because \sqrt{25} < \sqrt{35} < \sqrt{36}$ $\qquad$ $\therefore 5 < \sqrt{35} < 6$ $\qquad$ $\therefore x = 5$

5

1) $\because \sqrt{16} < \sqrt{20} < \sqrt{25}$ $\qquad$ $\therefore 4 < \sqrt{20} < 5$

$\because (4.1)^2 = 16.81$, $(4.2)^2 = 17.64$, $(4.3)^2 = 18.49$,

$(4.4)^2 = 19.36$, $(4.5)^2 = 20.25$

$\therefore 4.4 < \sqrt{20} < 4.5$ $\qquad$ $\therefore \sqrt{20} \approx 4.4$ or 4.5

Using the calculator $\sqrt{20} \approx 4.47$

2) $\because \sqrt[3]{8} < \sqrt[3]{17} < \sqrt[3]{27}$ $\qquad$ $\therefore 2 < \sqrt[3]{17} < 3$

$\because (2.1)^3 = 9.261$, $(2.2)^3 = 10.648$, $(2.3)^3 = 12.167$

$(2.4)^3 = 13.824$, $(2.5)^3 = 15.625$, $(2.6)^3 = 17.576$

$\therefore 2.5 < \sqrt[3]{17} < 2.6$

$\sqrt[3]{17} \approx 2.5$ or 2.6

Using the calculator $\sqrt[3]{17} \approx 2.57$

3) $\because \sqrt{4} < \sqrt{5} < \sqrt{9}$ $\qquad$ $\therefore 2 < \sqrt{5} < 3$

$\because (2.1)^2 = 4.41$, $(2.2)^2 = 4.84$, $(2.3)^2 = 5.29$

$\therefore 2.2 < \sqrt{5} < 2.3$ $\qquad$ $\therefore 3.2 < \sqrt{5} + 1 < 3.3$

$\therefore \sqrt{5} + 1 \approx 3.2$ or 3.3

Using the calculator $\sqrt{5} + 1 \approx 3.24$

4) $\because \sqrt[3]{8} < \sqrt[3]{9} < \sqrt[3]{27}$ $\qquad$ $\therefore 2 < \sqrt[3]{9} < 3$

$\because (2.1)^3 = 9.261$ $\qquad$ $\therefore 2 < \sqrt[3]{9} < 2.1$

$\therefore 1 < \sqrt[3]{9} - 1 < 1.1$ $\qquad$ $\therefore \sqrt[3]{9} - 1 \approx 1$ or 1.1

Using the calculator $\sqrt[3]{9} - 1 \approx 1.08$

6

1) d $\quad$ 2) b $\quad$ 3) b $\quad$ 4) c $\quad$ 5) c $\quad$ 6) b

7) b $\quad$ 8) d $\quad$ 9) c $\quad$ 10) c $\quad$ 11) d

7

1) $x^2 = \frac{10}{5} = 2$ $\qquad$ $\therefore x = \pm\sqrt{2}$ $\qquad$ $\therefore x \notin \mathbb{Q}$

2) $x^2 = \frac{9}{4}$ $\qquad$ $\therefore x = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$ $\qquad$ $\therefore x \in \mathbb{Q}$

3) $x = \sqrt[3]{125}$ $\qquad$ $\therefore x = 5$ $\qquad$ $\therefore x \in \mathbb{Q}$

4) $x^3 = \frac{27}{3} = 9$ $\qquad$ $\therefore x = \sqrt[3]{9}$ $\qquad$ $\therefore x \notin \mathbb{Q}$

5) $x^2 = \frac{10}{0.1} = 100$ $\qquad$ $\therefore x = \pm\sqrt{100} = \pm 10$ $\qquad$ $\therefore x \in \mathbb{Q}$

6) $x^3 = \frac{-8}{0.001} = -8000$

$\therefore x = \sqrt[3]{-8000} = -20$ $\qquad$ $\therefore x \in \mathbb{Q}$

7) $x - 1 = \pm\sqrt{4} = \pm 2$ $\qquad$ $\therefore x = 2 + 1 = 3$

or $x = -2 + 1 = -1$ $\qquad$ $\therefore x \in \mathbb{Q}$

8) $x - 5 = \sqrt[3]{1} = 1$ $\qquad$ $\therefore x = 1 + 5 = 6$ $\qquad$ $\therefore x \in \mathbb{Q}$

[1] $x^2 = \frac{25}{2} \times \frac{5}{2} = \frac{125}{4}$ $\qquad \therefore x = \pm\sqrt{\frac{125}{4}}$

$\therefore$ The S.S. $= \{\sqrt{\frac{125}{4}} , -\sqrt{\frac{125}{4}}\}$

[2] $x^3 = -2 \times \frac{4}{5} = -\frac{8}{5}$ $\qquad \therefore x = \sqrt[3]{-\frac{8}{5}}$

$\therefore$ The S.S. $= \{\sqrt[3]{-\frac{8}{5}}\}$

[3] $125\,x^3 = 27$ $\qquad \therefore x^3 = \frac{27}{125}$

$\therefore x = \sqrt[3]{\frac{27}{125}} = \frac{3}{5}$

$\therefore$ The S.S. $= \varnothing$ because $\frac{3}{5} \notin \mathbb{Q}'$

[4] $\frac{1}{4}\,x^2 = 64$ $\qquad \therefore x^2 = 64 \times 4 = 256$

$\therefore x = \pm\sqrt{256} = \pm 16$

$\therefore$ The S.S. $= \varnothing$ because $16 \notin \mathbb{Q}'$, $-16 \notin \mathbb{Q}'$

[5] $\because (x^3 + 5)(x^2 - 3) = 0$

$\therefore x^3 + 5 = 0 \qquad \therefore x^3 = -5 \qquad \therefore x = -\sqrt[3]{5}$

or $x^2 - 3 = 0 \qquad \therefore x^2 = 3 \qquad \therefore x = \pm\sqrt{3}$

$\therefore$ The S.S. $= \{-\sqrt[3]{5} , \sqrt{3} , -\sqrt{3}\}$

[6] $\because (x + \sqrt{7})(x^3 - 6) = 0$

$\therefore x + \sqrt{7} = 0 \qquad \therefore x = -\sqrt{7}$

or $x^3 - 6 = 0 \qquad \therefore x^3 = 6$

$\therefore x = \sqrt[3]{6} \qquad \therefore$ The S.S. $= \{-\sqrt{7} , \sqrt[3]{6}\}$

[1] $\because (1.4)^2 = 1.96$, $(1.5)^2 = 2.25$, $(\sqrt{2})^2 = 2$

$\therefore \sqrt{2}$ is included between 1.4 , 1.5

[2] $\because (3.31)^2 \approx 10.96$, $(3.32)^2 \approx 11.02$, $(\sqrt{11})^2 = 11$

$\therefore \sqrt{11}$ is included between 3.31 , 3.32

[3] $\because (1.2)^3 = 1.728$, $(1.3)^3 = 2.197$, $(\sqrt[3]{2})^3 = 2$

$\therefore \sqrt[3]{2}$ is included between 1.2 , 1.3

[4] $\because (2.4)^3 = 13.824$, $(2.5)^3 = 15.625$

$(\sqrt[3]{15})^3 = 15$

$\therefore \sqrt[3]{15}$ is included between 2.4 , 2.5

[5] $\because (-2.6)^3 = -17.576$, $(-2.5)^3 = -15.625$

$(\sqrt[3]{-17})^3 = -17$

$\therefore \sqrt[3]{-17}$ is included between -2.6 , -2.5

[6] $\because 2.7 - 1 = 1.7$, $(1.7)^2 = 2.89$

$2.8 - 1 = 1.8$, $(1.8)^2 = 3.24$

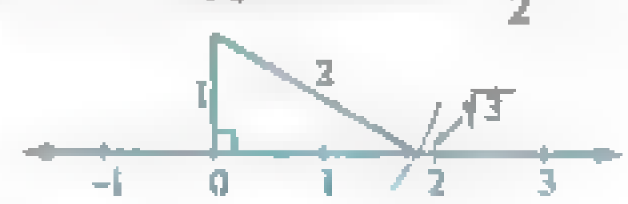
$\sqrt{3} + 1 - 1 = \sqrt{3}$, $(\sqrt{3})^2 = 3$

$\therefore \sqrt{3}$ is included between 1.7 , 1.8

$\therefore \sqrt{3} + 1$ is included between 2.7 , 2.8

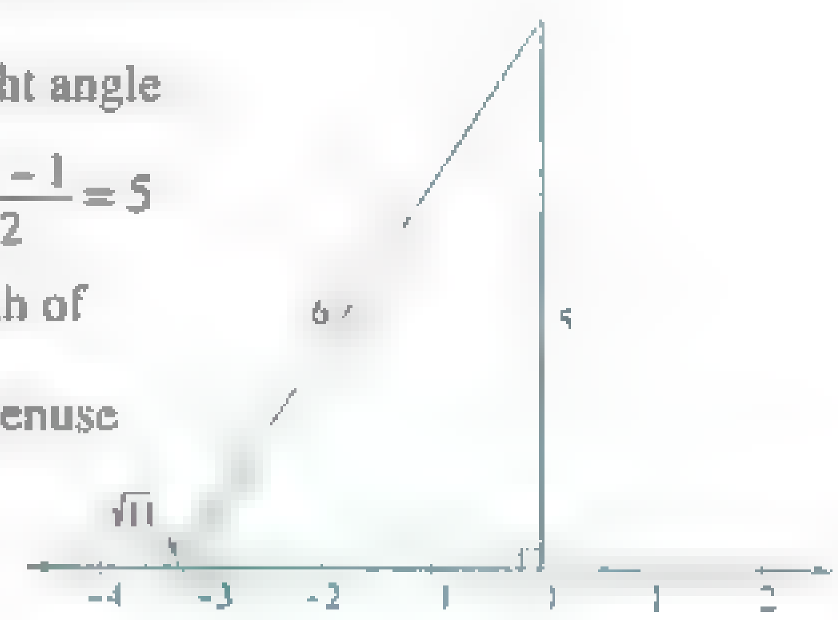
[1] The length of one side of the right angle $= \frac{3-1}{2} = 1$

The length of the hypotenuse $= \frac{3+1}{2} = 2$



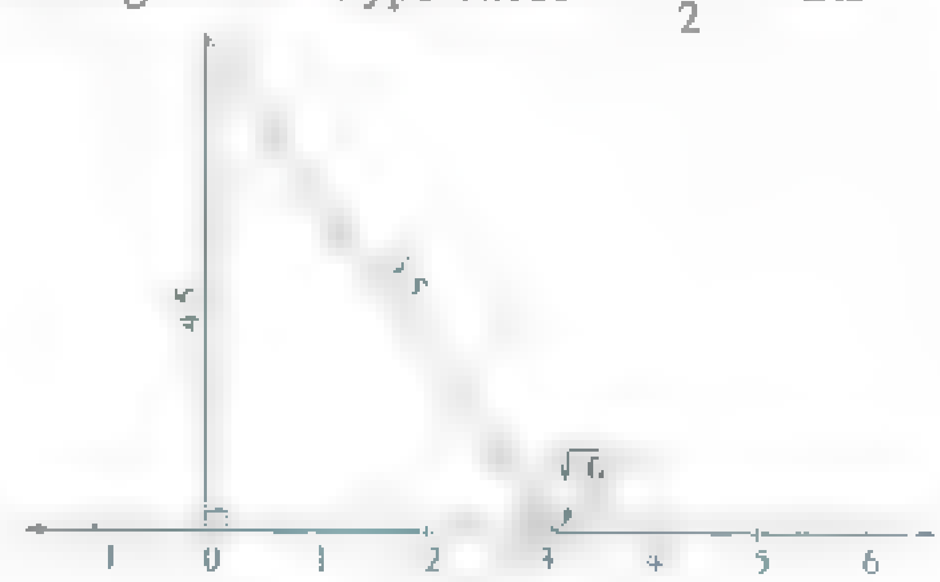
[2] The length of one side of the right angle equals $\frac{11-1}{2} = 5$

The length of the hypotenuse $= \frac{11+1}{2} = 6$



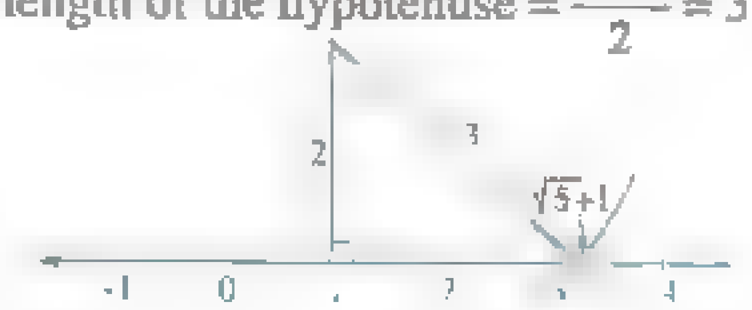
[3] The length of one side of the right angle $= \frac{10-1}{2} = 4.5$

The length of the hypotenuse $= \frac{10+1}{2} = 5.5$



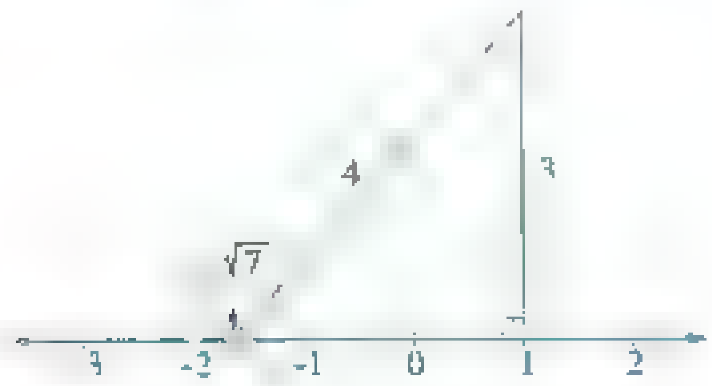
[4] The length of one side of the right angle $= \frac{5-1}{2} = 2$

The length of the hypotenuse $= \frac{5+1}{2} = 3$



5 The length of one side of the right angle $= \frac{7-1}{2} = 3$

The length of the hypotenuse $= \frac{7+1}{2} = 4$



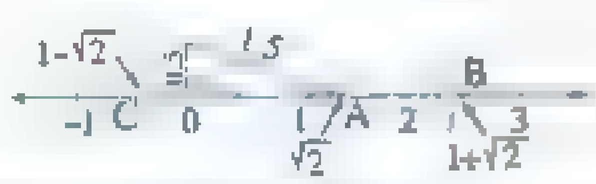
6 The length of one side of the right angle $= \frac{5-1}{2} = 2$

, the length of the hypotenuse $= \frac{5+1}{2} = 3$

11

The length of one side of the right angle $= \frac{2-1}{2} = 0.5$

The length of the hypotenuse $= \frac{2+1}{2} = 1.5$



12

The length of the side of the square $= \sqrt{10}$ cm

The square of the length of the diagonal

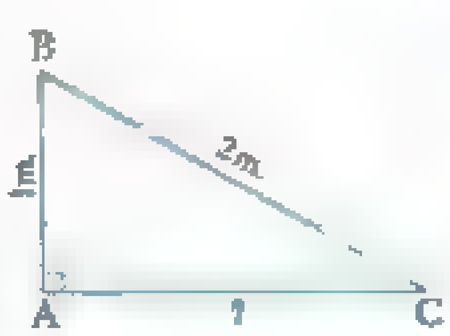
$= (\sqrt{10})^2 + (\sqrt{10})^2 = 10 + 10 = 20$

$\therefore$ The length of the diagonal $= \sqrt{20}$ cm.

$\because$ The length of the tree = 3 m.

$\therefore$ AB + BC = 3 m.

, $\because$ the length of the left part of the tree = 1 m.



$\therefore$ BC = 3 − 1 = 2 m.

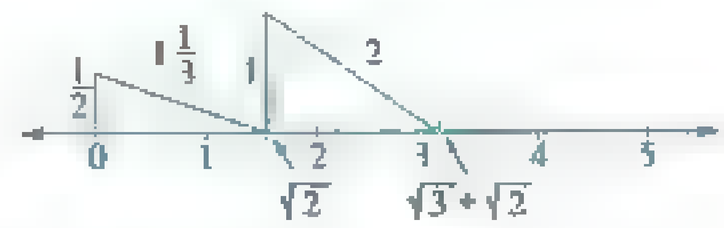
$\because$ In Δ ABC : m ($\angle$ A) = 90°

$\therefore (AC)^2 = (BC)^2 - (AB)^2 = 4 - 1 = 3$

$\therefore AC = \sqrt{3}$ m.

$\therefore$ The distance between the base of the tree and the point of touching of its top with the ground $= \sqrt{3}$ m.

We represent on the number line the point representing the number $\sqrt{3}+\sqrt{2}$ as shown in the figure :



We find that the point representing the number $\sqrt{3}+\sqrt{2}$ lies between the point representing the number 3 and the point representing the number 4

i.e. $\sqrt{3}+\sqrt{2}$ lies between 3 and 4

The number	Natural	Integer	Rational	Irrational	Real
-5	✗	✓	✓	✗	✓
$\sqrt{2}$	✗	✗	✗	✓	✓
$1\frac{1}{2}$	✗	✗	✓	✗	✓
$\sqrt[3]{9}$	✗	✗	✗	✓	✓
$\|-2\|$	✓	✓	✓	✗	✓
$-\sqrt{4}$	✗	✓	✓	✗	✓
$\frac{5}{2}$	✗	✗	✓	✗	✓
0.3	✗	✗	✓	✗	✓
$\sqrt{-1}$	✗	✗	✗	✗	✗

1 > 2 > 3 <
4 < 5 > 6 =
7 > 8 >

1 a 2 d 3 b 4 a
5 d 6 c 7 a 8 b
9 b 10 a 11 d 12 c
13 d

1 The ascending order is :

$-\sqrt{11}$, $-\sqrt{7}$, $-\sqrt{3}$, $\sqrt{5}$, $\sqrt{8}$ and $\sqrt{15}$

2 $\because 0.6 = \sqrt{0.36}$, $\sqrt[3]{-1} = -1 = -\sqrt{1}$

$\therefore$ The ascending order is :

$-\sqrt{45}$, $-\sqrt{1}$, $\sqrt{0.36}$, $\sqrt{20}$ and $\sqrt{27}$

i.e. $-\sqrt{45}$, $\sqrt[3]{-1}$, 0.6 , $\sqrt{20}$ and $\sqrt{27}$

1 $\because 8 = \sqrt{64}$

$\therefore$ The descending order is :

$\sqrt{70}$, $\sqrt{64}$, $\sqrt{62}$ and $-\sqrt{50}$

i.e. $\sqrt{70}$, 8 , $\sqrt{62}$ and $-\sqrt{50}$

2 $\because 9 = \sqrt{81}$

$\therefore$ The descending order is :

$\sqrt{101}$, $\sqrt{81}$, $\sqrt{6}$, $-\sqrt{7}$, $-\sqrt{10}$ and $-\sqrt{50}$

i.e. $\sqrt{101}$, 9 , $\sqrt{6}$, $-\sqrt{7}$, $-\sqrt{10}$ and $-\sqrt{50}$

$\because 2^2 = 4$ $\quad\therefore 4 > 3 > 2 > \frac{3}{2} > 0$

$\therefore 2 > \sqrt{3} > \sqrt{2} > \sqrt{\frac{3}{2}} > 0$

$\therefore$ The positive irrational numbers are

$\sqrt{3}$, $\sqrt{2}$ and $\sqrt{\frac{3}{2}}$

(There are other solutions)

The irrational numbers are

$-\sqrt{5}$, $-\sqrt{3}$ and $-\sqrt{2}$ (There are other solutions)

$\because (15)^2 = 225$, $(17)^2 = 289$

Then choosing 4 integers included

between 225 , 289

(except 256 because $\sqrt{256} = 16 \in \mathbb{Q}$)

$\therefore 225 < 235 < 245 < 255 < 265 < 289$

$\therefore 15 < \sqrt{235} < \sqrt{245} < \sqrt{255} < \sqrt{265} < 17$

$\therefore$ The four irrational numbers are

$\sqrt{235}$, $\sqrt{245}$, $\sqrt{255}$ and $\sqrt{265}$

(There are other solutions)

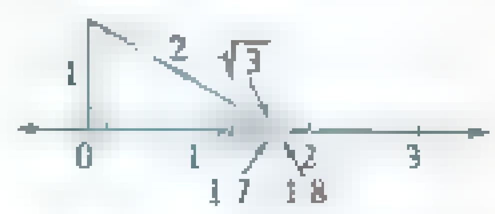
Using the calculator $\sqrt{3} \approx 1.73$

(to the nearest hundredth)

$\therefore 1.7 < \sqrt{3} < 1.8$ for representing $\sqrt{3}$

$\because$ The length of the hypotenuse $= \frac{3+1}{2} = 2$

, the length of one side of the right angle $= \frac{3-1}{2} = 1$



10

1 $x^2 = 6$ $\quad\therefore x = \pm\sqrt{6} \approx \pm 2.45$

2 $x^2 = 24 \times \frac{4}{3} = 32$ $\quad\therefore x = \pm\sqrt{32} \approx \pm 5.66$

3 $\frac{1}{2} x^2 = 5$ $\quad\therefore x^2 = 5 \times 2 = 10$

$\therefore x = \pm\sqrt{10}$ $\quad\therefore x \approx \pm 3.16$

4 $\frac{2}{x^3} = 16$ $\quad\therefore x^3 = \frac{1}{8}$ $\quad\therefore x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

5 $\because (x^2 - 9)(x^3 - 5) = 0$

$\therefore x^2 - 9 = 0$ $\quad\therefore x^2 = 9$

$\therefore x = \pm\sqrt{9} = \pm 3$

or $x^3 - 5 = 0$ $\quad\therefore x^3 = 5$

$\therefore x = \sqrt[3]{5} \approx 1.71$

6 $\because (2x^3 - 5)(x^2 + 1) = 0$

$\therefore 2x^3 - 5 = 0$ $\quad\therefore 2x^3 = 5$

$\therefore x^3 = \frac{5}{2}$ $\quad\therefore x = \sqrt[3]{\frac{5}{2}} \approx 1.36$

or $x^2 + 1 = 0$

$\therefore x^2 = -1$ (has no solution in $\mathbb{R}$)

The side length $= \sqrt{5}$ cm. , $\sqrt{5} \notin \mathbb{Q}$

12

The edge length $= \sqrt[3]{1.728} = \frac{6}{5}$ cm. , $\frac{6}{5} \in \mathbb{Q}$

13

$\because$ The total area of the cube $= 6\, l^2$

$\therefore 13.5 = 6\, l^2$ $\quad\therefore \frac{13.5}{6} = l^2$

$\therefore l = \sqrt{\frac{13.5}{6}} = 1.5$ cm. , $1.5 \in \mathbb{Q}$

The diagonal length $= \sqrt{6^2 + 6^2} = \sqrt{72}$ cm.

The diagonal length of the rectangle

$=\sqrt{(5)^2+(7)^2}=\sqrt{74}$ cm.

$\because$ The area of the square = the area of the rectangle $= 5\times 7 = 35$ cm.^{2}

$\therefore$ The side length of the square $=\sqrt{35}$ cm.

$\therefore$ The diagonal length of the square $=\sqrt{35+35}$

$=\sqrt{70}$ cm.

10

Cubing the two sides then squaring them we find that

$(\sqrt[3]{3})^3 = 3$, $3^2 = 9$, $(\sqrt{2})^3 = 2\sqrt{2}$, $(2\sqrt{2})^2 = 8$

$9 > 8$ $\qquad \therefore \sqrt[3]{3} > \sqrt{2}$

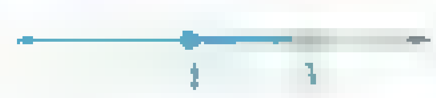
11

Let the other number $= x$ $\qquad \therefore x^2 + 2^2 = 7$

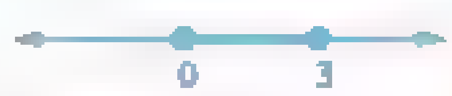
$\therefore x^2 = 7 - 4 = 3$ $\qquad \therefore x = \pm\sqrt{3}$

$\therefore$ The other number is $\sqrt{3}$ or $-\sqrt{3}$

2 $\{x : 1 \le x < 3 , x \in \mathbb{R}\}$

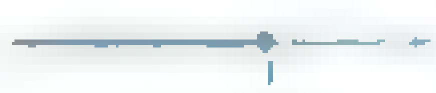


3 $]0 , 3]$



4 $]-2 , 3[$, $\{x : -2 < x < 3 , x \in \mathbb{R}\}$

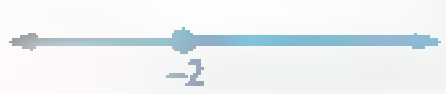
5 $\{x : x \le 1 , x \in \mathbb{R}\}$



6 $]0 , \infty[$, $\{x : x > 0 , x \in \mathbb{R}\}$

7 $]-\infty , 4[$

8 $\{x : x \ge -2 , x \in \mathbb{R}\}$



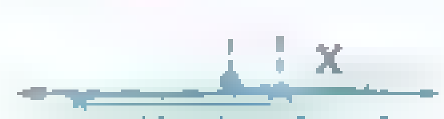
1 c $\quad$ 2 a $\quad$ 3 b $\quad$ 4 c $\quad$ 5 d

1 $\in$ $\quad$ 2 $\notin$ $\quad$ 3 $\in$ $\quad$ 4 $\in$ $\quad$ 5 $\in$

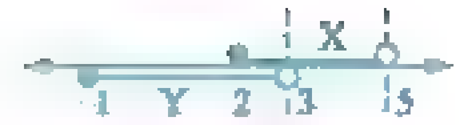
6 $\in$ $\quad$ 7 $\in$ $\quad$ 8 $\notin$ $\quad$ 9 $\notin$ $\quad$ 10 $\notin$

1 $[-1 , 5[$

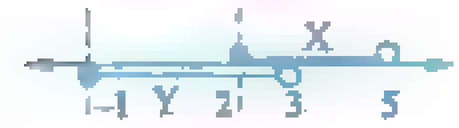
2 $[2 , 3[$



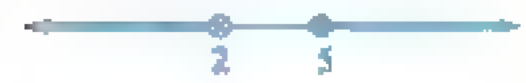
3 $[3 , 5[$



4 $[-1 , 2[$

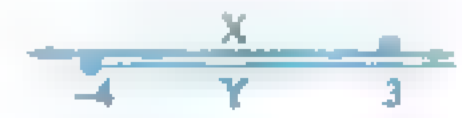


5 $]-\infty , 2] \cup [5 , \infty[$

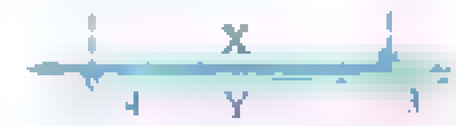


6 $]-\infty , -1[\cup [3 , \infty[$

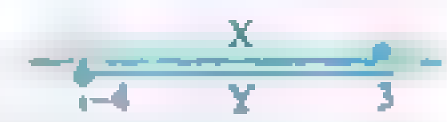
1 $\mathbb{R}$



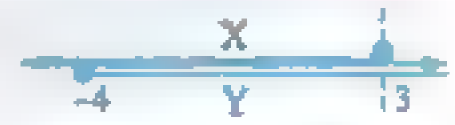
2 $[-4 , 3]$



3 $]-\infty , -4[$



4 $]3 , \infty[$



5 $]3 , \infty[$

6 $]-\infty , -4[$

Use the number line to get the following results :

1 $[-1 , \infty[$ $\quad$ 2 $[3 , 4]$ $\quad$ 3 $[-1 , 3[$

4 $[-1 , 4[- \{3\}$ $\quad$ 5 $\{3 , 4\}$

6 $]4 , \infty[$ $\quad$ 7 $]-\infty , -1[\cup]4 , \infty[$

8 $]-\infty , 3[$

Use the number line to get the following results

1 $[2 , 4]$ $\quad$ 2 $[-1 , 5]$ $\quad$ 3 $]0 , 1[$

4 $]-2 , 3]$ $\quad$ 5 $[3 , 6]$ $\quad$ 6 $[-1 , 2[$

7 $[-3 , 2] - \{0\}$ $\quad$ 8 $\varnothing$

9 $\varnothing$ $\quad$ 10 $[-2 , 1[\cup]2 , 4]$

11 $\varnothing$ $\quad$ 12 $\{-1 , 5\}$

Use the number line to get the following results :

1 $[-3 , \infty[$ $\quad$ 2 $[2 , 3[$ $\quad$ 3 $[-4 , 3]$

4 $\mathbb{R}$ $\quad$ 5 $]-\infty , -1[$ $\quad$ 6 $]-\infty , -3[$

7 $]0 , 2]$ $\quad$ 8 $\mathbb{R} - [3 , 4]$

1 $[3 , 5]$ $\quad$ 2 $[3 , 5]$ $\quad$ 3 $\{3 , 5\}$ $\quad$ 4 $\varnothing$

5 $]3 , 5[$ $\quad$ 6 $]3 , 5[$ $\quad$ 7 $\varnothing$ $\quad$ 8 $\{3 , 5\}$

9 $[3 , 5[$ $\quad$ 10 $[3 , 5[$ $\quad$ 11 $\{3 , 4\}$ $\quad$ 12 $]3 , 5]$

10

1 b | 2 d | 3 c
4 b | 5 b | 6 d

11

1 $[-3 , 3]$ 2 $\mathbb{R}$ 3 $]-\infty , -1[$
4 $]-\infty , -3[$ 5 $]-2 , 0]$ 6 $[0 , 2]$
7 $\{1 , 2\}$ 8 $\{0 , 1\}$
9 $\{-1 , 0 , 1 , 2\}$ 10 $]0 , 5]$
11 $[-3 , 0[$

12

1 d 2 c 3 c 4 c 5 b
6 d 7 c 8 c 9 c

13

$\because X \subset Y \quad \therefore X = X \cap Y = [4 , 7]$
$, Y = X \cup Y = [3 , 7] , Y - X = [3 , 4[$

1

1 $3\sqrt{3}$ 2 zero
3 $-\sqrt[3]{7}$ 4 $6\sqrt{5}$

2

1 $3\sqrt{5}$ 2 $3\sqrt{3} - 1$ 3 $8\sqrt{7} - 3\sqrt{2}$
4 $7\sqrt{2} - 2\sqrt[3]{2}$ 5 $\sqrt{2}$
6 $8 \times \frac{1}{2} + 2\sqrt[3]{3} - 4 - 5\sqrt[3]{3} = 4 + 2\sqrt[3]{3} - 4 - 5\sqrt[3]{3}$
$= -3\sqrt[3]{3}$

3

1 3 2 -30 3 $6\sqrt{2}$ 4 1 5 $15\sqrt{3}$
6 $2\sqrt{3} \times \frac{2\sqrt{7}}{7} \times \frac{5\sqrt{7}}{20\sqrt{3}} = 1$

4

1 $2\sqrt{2} + 2\sqrt{5}$ 2 $5\sqrt{2} + 2$
3 $7 + 2\sqrt{7}$ 4 $5\sqrt{3} + 3$
5 $-6\sqrt{5} + 10$ 6 $2 - 7 + 3\sqrt{7} = -5 + 3\sqrt{7}$
7 $-24 - 6\sqrt{3} + 6\sqrt{3} = -24$
8 $3\sqrt{5} - 5 - 2 - 2\sqrt{5} = \sqrt{5} - 7$

5

1 $(\sqrt{2})^2 - (1)^2 = 2 - 1 = 1$
2 $(4)^2 - (3\sqrt{2})^2 = 16 - 18 = -2$
3 $(\sqrt{5})^2 - 2 \times 1 \times \sqrt{5} + (-1)^2 = 5 - 2\sqrt{5} + 1$
$= 6 - 2\sqrt{5}$
4 $(2\sqrt{3})^2 + 2 \times 4 \times 2\sqrt{3} + (4)^2 = 12 + 16\sqrt{3} + 16$
$= 28 + 16\sqrt{3}$
5 $3 + \sqrt{3} - 2 = 1 + \sqrt{3}$
6 $(5)^2 - 2 \times 5 \times \sqrt{3} + (\sqrt{3})^2 - 28$
$= 25 - 10\sqrt{3} + 3 - 28 = -10\sqrt{3}$

6

1 $\frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$
2 $-\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{6\sqrt{3}}{3} = -2\sqrt{3}$
3 $\frac{2}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$
4 $\frac{6}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{6} = \sqrt{3}$
5 $\frac{\sqrt{2} + 3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2 + 3\sqrt{2}}{2}$
6 $\frac{\sqrt{5} - 15}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5 - 15\sqrt{5}}{10} = \frac{1 - 3\sqrt{5}}{2}$

7

1 b 2 c 3 a 4 d 5 c 6 b
7 d 8 b 9 c

8

1 1 , zero 2 $\sqrt{2} - 1$ 3 $5\sqrt{3}$
4 1 5 $2 , \sqrt{3}$ 6 $\frac{2}{3}$
7 $4\sqrt{3}$ 8 $3 + 2\sqrt{2}$ 9 $\pm\sqrt{5}$
10 $8\sqrt{2}$ 11 60 cm^2
12 The additive inverse 13 $\mathbb{R}$

9

1 $\sqrt{5} - 2 + \sqrt{5} + 2 = 2\sqrt{5}$
2 $\sqrt{5} - 2 - \sqrt{5} - 2 = -4$
3 $(\sqrt{5} - 2)(\sqrt{5} + 2) = 5 - 4 = 1$

[4] $x^2-y^2=(x-y)(x+y)=(-4)(2\sqrt{5})=-8\sqrt{5}$

[5] $x^2+2xy+y^2=(x+y)^2=(2\sqrt{5})^2=20$

[6] $x^2-2xy+y^2=(x-y)^2=(-4)^2=16$

[1] $x\approx 4+2=6$

and using the calculator $x\approx 5.9$ (accepted estimation)

, $y\approx 4-3=1$

and using the calculator $y\approx 1.08$ (accepted estimation)

[2] $x\times y\approx 6\times 1=6$

and using the calculator , the expression ≈ 6.3 (accepted estimation)

[3] $x+y\approx 6+1=7$

and using the calculator , the expression ≈ 6.9 (accepted estimation)

The perimeter $=2(6+\sqrt{5}+6-\sqrt{5})=2\times 12$
$=24$ cm.

The area $=(6+\sqrt{5})(6-\sqrt{5})=36-5=31$ cm^2.

$$(\sqrt{a}-1)\times\frac{\sqrt{a}+1}{4}=1$$

$\therefore \frac{a-1}{4}=1 \qquad \therefore a-1=4 \qquad \therefore a=5$

$x=\sqrt{2}$, $y=\frac{\sqrt{2}}{2}$, $z=\frac{\sqrt{2}}{4}$

$$\therefore x^2+2y^2+4z^2=(\sqrt{2})^2+2\times\left(\frac{\sqrt{2}}{2}\right)^2+4\left(\frac{\sqrt{2}}{4}\right)^2$$
$$=2+2\times\frac{2}{4}+4\times\frac{2}{16}=3\frac{1}{2}$$

$\frac{1}{2}(2y)=1-\sqrt{2} \quad \therefore y=1-\sqrt{2}$

$\therefore x=-1+\sqrt{2}$

$$\therefore xy-2\sqrt{2}=(-1+\sqrt{2})(1-\sqrt{2})-2\sqrt{2}$$
$$=-1+\sqrt{2}+\sqrt{2}-2-2\sqrt{2}=-3$$

Answers of Exercise 6

[1] $\sqrt{4\times 3}=2\sqrt{3}$ [2] $\sqrt{4\times 7}=2\sqrt{7}$

[3] $2\sqrt{36\times 2}=2\times 6\sqrt{2}=12\sqrt{2}$

[4] $\frac{2}{5}\sqrt{100\times 10}=\frac{2}{5}\times 10\sqrt{10}=4\sqrt{10}$

[5] $\sqrt{4\times\frac{1}{2}}=\sqrt{2}$ [6] $2\sqrt{\frac{2}{3}\times 9}=2\sqrt{6}$

[1] $5\sqrt{2}+2\sqrt{2}=7\sqrt{2}$

[2] $2\sqrt{5}-3\sqrt{5}=-\sqrt{5}$

[3] $3\sqrt{2}+2\sqrt{2}-3\sqrt{2}=2\sqrt{2}$

[4] $7\sqrt{2}-8\sqrt{2}-3\sqrt{2}+4\sqrt{2}=$ zero

[5] $2\times 3\sqrt{2}+5\sqrt{2}+\frac{1}{3}\times 9\sqrt{2}$
$=6\sqrt{2}+5\sqrt{2}+3\sqrt{2}=14\sqrt{2}$

[6] $7\sqrt{2}+5\sqrt{2}-\frac{1}{2}\times 10\sqrt{2}-\sqrt{2}$
$=7\sqrt{2}+5\sqrt{2}-5\sqrt{2}-\sqrt{2}=6\sqrt{2}$

[7] $3\sqrt{3}+5\times 3\sqrt{2}-10\sqrt{3}=15\sqrt{2}-7\sqrt{3}$

[1] $2\sqrt{5}+4\times 2\sqrt{5}-\sqrt{25\times\frac{1}{5}}=2\sqrt{5}+8\sqrt{5}-\sqrt{5}$
$=9\sqrt{5}$

[2] $4\sqrt{2}-6\sqrt{2}+3\sqrt{4\times\frac{1}{2}}=4\sqrt{2}-6\sqrt{2}+3\sqrt{2}$
$=\sqrt{2}$

[3] $2\sqrt{5}+2\sqrt{9\times\frac{1}{3}}-2\sqrt{3}-\sqrt{25\times\frac{1}{5}}$
$=2\sqrt{5}+2\sqrt{3}-2\sqrt{3}-\sqrt{5}=\sqrt{5}$

[4] $\sqrt{3}+\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}-\sqrt{12}=\sqrt{3}+\sqrt{3}-2\sqrt{3}=$ zero

[5] $3\sqrt{2}-\sqrt{\frac{12}{6}}=3\sqrt{2}-\sqrt{2}=2\sqrt{2}$

[6] $5+3\sqrt{2}-\frac{6}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=5+3\sqrt{2}-3\sqrt{2}=5$

[1] $10\sqrt{6}$ [2] $6\sqrt{36}=6\times 6=36$

[3] $2\sqrt{50}=2\times 5\sqrt{2}=10\sqrt{2}$

[4] $\sqrt{\frac{2}{7}\times\frac{7}{2}}=\sqrt{1}=1$ [5] $3\sqrt{\frac{15}{5}}=3\sqrt{3}$

[6] $12\times\sqrt{\frac{2}{3}\times 54}=12\sqrt{36}=12\times 6=72$

1 $\sqrt{18}-\sqrt{12}=3\sqrt{2}-2\sqrt{3}$

2 $20+5\sqrt{24}=20+5\times2\sqrt{6}=20+10\sqrt{6}$

3 $(3\sqrt{5})^2-(\sqrt{7})^2=45-7=38$

4 $(\sqrt{3})^2-2\times\sqrt{3}\times\sqrt{2}+(-\sqrt{2})^2=3-2\sqrt{6}+2$

$=5-2\sqrt{6}$

5 $(\sqrt{3})^2+2\times\sqrt{3}\times\sqrt{5}+(\sqrt{5})^2-2\sqrt{15}$

$=3+2\sqrt{15}+5-2\sqrt{15}=8$

6 $3\sqrt{2}-\frac{12}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}+2\sqrt{6}-3\sqrt{2}$

$=3\sqrt{2}-2\sqrt{6}+2\sqrt{6}-3\sqrt{2}=$ zero

1 $\frac{\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$

2 $\sqrt{\frac{5}{3}}=\frac{\sqrt{5}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{15}}{3}$

3 $\frac{5\sqrt{3}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{5\sqrt{15}}{5}=\sqrt{15}$

4 $\frac{4\sqrt{3}-\sqrt{2}}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{12-\sqrt{6}}{6}$

1 a	2 b	3 c	4 a	5 b
6 c	7 a	8 a	9 c	10 b

1 $\frac{1}{2}$	2 $\sqrt{2}$	3 $\sqrt{3}$	4 $\frac{3}{2}$
5 2	6 $\sqrt{125}$	7 $\pm\frac{2\sqrt{2}}{3}$	8 20 , zero

1 $x+y=3+\sqrt{5}+1-\sqrt{5}=4$

$x\times y=(3+\sqrt{5})(1-\sqrt{5})=3-2\sqrt{5}-5$

$=-2-2\sqrt{5}$

2 $x+y=\sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2}=2\sqrt{3}$

$x\times y=(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})=3-2=1$

3 $x+y=5-3\sqrt{2}+5-3\sqrt{2}=10-6\sqrt{2}$

$x\times y=(5-3\sqrt{2})(5-3\sqrt{2})$

$=25-30\sqrt{2}+18=43-30\sqrt{2}$

10

$\because x=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$, $y=\frac{\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$

$\therefore 6(x+y)=6\left(\frac{\sqrt{6}}{3}+\frac{\sqrt{6}}{2}\right)=6\times\frac{\sqrt{6}}{3}+6\times\frac{\sqrt{6}}{2}$

$=2\sqrt{6}+3\sqrt{6}=5\sqrt{6}$

11 We know that $(x+y)^2=x^2+2xy+y^2$

$\therefore x^2+2xy+y^2=(2\sqrt{5}+\sqrt{2}+2\sqrt{5}-\sqrt{2})^2$

$=(4\sqrt{5})^2=16\times5=80$

$a^{x+y}=a^x\times a^y=a^x\div a^{-y}=6\div\sqrt{3}=\frac{6}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{6\sqrt{3}}{3}=2\sqrt{3}$

1 $\frac{(\sqrt{5})^3\times(\sqrt{5})^5}{(\sqrt{2})^6\times(\sqrt{5})^6}=\frac{(\sqrt{5})^{3+5-6}}{(\sqrt{2})^6}=\frac{5}{8}$

2 $\frac{2\sqrt{2}\times(\sqrt{2})^{-3}\times(\sqrt{3})^{-3}}{(\sqrt{3})^{3}}=2\times(\sqrt{2})^{-2}$

$=\frac{2}{(\sqrt{2})^2}=\frac{2}{2}=1$

1 $\sqrt{5}-\sqrt{3}$ 2 $5+2\sqrt{7}$

3 $\because \sqrt{5}+\frac{2}{\sqrt{2}}=\sqrt{5}+\sqrt{2}$

$\therefore$ The conjugate number $=\sqrt{5}-\sqrt{2}$

1 $\frac{5}{\sqrt{7}-\sqrt{2}}\times\frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}=\frac{5(\sqrt{7}+\sqrt{2})}{7-2}=\sqrt{7}+\sqrt{2}$

2 $\frac{\sqrt{3}}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{\sqrt{3}(2+\sqrt{3})}{4-3}=2\sqrt{3}+3$

3 $\frac{\sqrt{7}+3}{\sqrt{7}-3}\times\frac{\sqrt{7}+3}{\sqrt{7}+3}=\frac{16+6\sqrt{7}}{7-9}=-8-3\sqrt{7}$

$\because x = \frac{2}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{2(\sqrt{7}+\sqrt{5})}{7-5}$

$= \sqrt{7} + \sqrt{5}$

$\because (x+y)^2 = (\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5})^2$

$= (2\sqrt{7})^2 = 28$

$x^2 y^2 = (x y)^2 = \left(\frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{4}{\sqrt{7}+\sqrt{3}}\right)^2$

$= \left(\frac{16}{7-3}\right)^2 = 4^2 = 16$

L.H.S. $= \frac{4}{x} + 2x$

$= \frac{4}{\sqrt{5}+\sqrt{3}} + 2(\sqrt{5}+\sqrt{3})$

$= \frac{4(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} + 2(\sqrt{5}+\sqrt{3})$

$= \frac{4(\sqrt{5}-\sqrt{3})}{2} + 2(\sqrt{5}+\sqrt{3})$

$= 2(\sqrt{5}-\sqrt{3}) + 2(\sqrt{5}+\sqrt{3})$

$= 2\sqrt{5} - 2\sqrt{3} + 2\sqrt{5} + 2\sqrt{3}$

$= 4\sqrt{5} =$ R.H.S.

$\because b = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3-2}$

$= \sqrt{3} - \sqrt{2}$

We know that : $(a-b)(a+b) = a^2 - b^2$

$\therefore (\sqrt{3}+\sqrt{2}-\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2})$

$= 2\sqrt{2} \times 2\sqrt{3} = 4\sqrt{6}$

$\because y = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$

$= \sqrt{5} + \sqrt{3}$

$\because x^2 + 2xy + y^2 = (x+y)^2$

$= (\sqrt{5}-\sqrt{3}+\sqrt{5}+\sqrt{3})^2 = (2\sqrt{5})^2 = 20$

$\because y = \frac{3}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}+\sqrt{2})}{5-2}$

$= \sqrt{5} + \sqrt{2}$

, $x = \sqrt{5} - \sqrt{2}$

$\therefore x$ and y are two conjugate numbers.

$\therefore x^2 - 2xy + y^2 = (x-y)^2 = (\sqrt{5}-\sqrt{2}-\sqrt{5}-\sqrt{2})^2$

$= (-2\sqrt{2})^2 = 8$

$\because x = 3 + \sqrt{5}$, $y = \frac{4}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{4(3-\sqrt{5})}{9-5}$

$= 3 - \sqrt{5}$

$\therefore x$ and y are two conjugate numbers

1 $xy = (3+\sqrt{5})(3-\sqrt{5}) = 9 - 5 = 4$

2 $x^2 + y^2 = (x+y)^2 - 2xy$

$= (3+\sqrt{5}+3-\sqrt{5})^2 - 2 \times 4 = 36 - 8 = 28$

$\because x = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$

$= \sqrt{5} + \sqrt{3}$

, $y = \frac{2}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{2(\sqrt{5}-\sqrt{3})}{5-3}$

$= \sqrt{5} - \sqrt{3}$

$\therefore x^2 - xy + y^2 = (x-y)^2 + xy$

$= (\sqrt{5}+\sqrt{3}-\sqrt{5}+\sqrt{3})^2 + (\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$

$= (2\sqrt{3})^2 + 2 = 14$

$\frac{x+y}{xy-1} = \frac{\sqrt{5}+\sqrt{2}+\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})-1}$

$= \frac{2\sqrt{5}}{5-2-1} = \frac{2\sqrt{5}}{2} = \sqrt{5}$

12

$\because a = \frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3}$

$= \sqrt{7} + \sqrt{3}$

$$, b = \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{4(\sqrt{7}-\sqrt{3})}{7-3} = \sqrt{7}-\sqrt{3}$$

$$\therefore \frac{a+b}{ab} = \frac{\sqrt{7}+\sqrt{3}+\sqrt{7}-\sqrt{3}}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})} = \frac{2\sqrt{7}}{7-3} = \frac{\sqrt{7}}{2}$$

13

$\because x = 2\sqrt{2}-\sqrt{3}$, $y = \frac{5}{2\sqrt{2}-\sqrt{3}}$

$$\therefore y = \frac{5}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}} = \frac{5(2\sqrt{2}+\sqrt{3})}{8-3} = 2\sqrt{2}+\sqrt{3}$$

$\therefore$ x and y are conjugate numbers.

$$\therefore \frac{x+y}{xy} = \frac{2\sqrt{2}-\sqrt{3}+2\sqrt{2}+\sqrt{3}}{(2\sqrt{2}-\sqrt{3})(2\sqrt{2}+\sqrt{3})} = \frac{4\sqrt{2}}{8-3} = \frac{4\sqrt{2}}{5}$$

14

$$x = \frac{5\sqrt{2}+3\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{10}+15}{5} = \sqrt{10}+3$$

$$, y = \frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{10}-6}{2} = \sqrt{10}-3$$

[1] $x^2+y^2 = (x+y)^2 - 2xy$

$$= (\sqrt{10}+3+\sqrt{10}-3)^2 - 2(\sqrt{10}+3)(\sqrt{10}-3)$$
$$= (2\sqrt{10})^2 - 2\times(10-9) = 40-2 = 38$$

[2] $xy = (\sqrt{10}+3)(\sqrt{10}-3) = 10-9 = 1$

$\therefore x^2+y^2 = 38\,xy$

15

$$\because x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$, y = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$\therefore x^2+y = (2-\sqrt{3})^2 + 4\sqrt{3} = 4-4\sqrt{3}+3+4\sqrt{3} = 7$$

16

$\because y = \sqrt{3}-\sqrt{2}$

$$, x = \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \sqrt{3}+\sqrt{2}$$

$$\therefore (x+y)^2 = (\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2})^2 = (2\sqrt{3})^2 = 12$$

17

$\because xy = 1$

$$\therefore y = \frac{1}{x} = \frac{1}{\sqrt{13}+\sqrt{6}} = \frac{1}{\sqrt{13}+\sqrt{6}} \times \frac{\sqrt{13}-\sqrt{6}}{\sqrt{13}-\sqrt{6}} = \frac{\sqrt{13}-\sqrt{6}}{7}$$

$$\therefore x^2 - 49y^2 = (x-7y)(x+7y)$$
$$= \left(\sqrt{13}+\sqrt{6}-7\left(\frac{\sqrt{13}-\sqrt{6}}{7}\right)\right)\left(\sqrt{13}+\sqrt{6}+7\left(\frac{\sqrt{13}-\sqrt{6}}{7}\right)\right)$$
$$= (\sqrt{13}+\sqrt{6}-\sqrt{13}+\sqrt{6})(\sqrt{13}+\sqrt{6}+\sqrt{13}-\sqrt{6})$$
$$= 2\sqrt{6}\times 2\sqrt{13} = 4\sqrt{78}$$

18

$$\because x = \frac{4(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3} = \sqrt{7}+\sqrt{3}$$

$, y = \sqrt{7}-\sqrt{3}$

$\therefore$ x and y are two conjugate numbers.

$$\therefore x^2y^2 = (xy)^2 = [(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})]^2 = (7-3)^2 = 4^2 = 16$$

19

$$\because y = \frac{2}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{2(\sqrt{7}-\sqrt{5})}{7-5} = \sqrt{7}-\sqrt{5}$$

$$\therefore \frac{x+y}{xy} = \frac{\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5}}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} = \frac{2\sqrt{7}}{7-5} = \sqrt{7}$$

$\because x = \dfrac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} \times \dfrac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \dfrac{11+2\sqrt{30}}{6-5}$

$= 11+2\sqrt{30}$

$, \dfrac{1}{x} = \dfrac{1}{11+2\sqrt{30}} \times \dfrac{11-2\sqrt{30}}{11-2\sqrt{30}} = \dfrac{11-2\sqrt{30}}{121-120}$

$= 11-2\sqrt{30}$

$\therefore x + \dfrac{1}{x} = 11+2\sqrt{30}+11-2\sqrt{30} = 22$

1 4   2 $3-\sqrt{2} , 7$ 3 $\sqrt{5}-\sqrt{3}$

4 $1-\sqrt{7}$ 5 $\sqrt{3}-\sqrt{2}$ 6 20

7 20   8 $\sqrt{5}+2$ 9 $(-1 , 2\sqrt{3})$

10 -1

1 $\dfrac{11}{2\sqrt{5}+3} = \dfrac{11(2\sqrt{5}-3)}{(2\sqrt{5}+3)(2\sqrt{5}-3)}$

$= \dfrac{11(2\sqrt{5}-3)}{20-9} = 2\sqrt{5}-3$

$\therefore a = 2 , b = -3$

2 $\dfrac{3}{2\sqrt{2}-\sqrt{5}} = \dfrac{3(2\sqrt{2}+\sqrt{5})}{(2\sqrt{2}-\sqrt{5})(2\sqrt{2}+\sqrt{5})}$

$= \dfrac{3(2\sqrt{2}+\sqrt{5})}{8-5} = 2\sqrt{2}+\sqrt{5}$

$\therefore a = 2 , b = 1$

3 $\dfrac{7}{\sqrt{8}+1} = \dfrac{7(\sqrt{8}-1)}{(\sqrt{8}+1)(\sqrt{8}-1)}$

$= \dfrac{7(\sqrt{8}-1)}{8-1} = \sqrt{8}-1$

$= 2\sqrt{2}-1 = a+b\sqrt{2}$

$\therefore a = -1 , b = 2$

$(x+y)^2 = x^2+2xy+y^2$

$= \left(\sqrt{4+\sqrt{7}}\right)^2 + 2\left(\sqrt{4+\sqrt{7}}\right)\left(\sqrt{4-\sqrt{7}}\right) + \left(\sqrt{4-\sqrt{7}}\right)^2$

$= 4+\sqrt{7}+2\sqrt{\left(4+\sqrt{7}\right)\left(4-\sqrt{7}\right)}+4-\sqrt{7}$

$= 8+2\sqrt{16-7} = 8+2\sqrt{9} = 8+6 = 14$

$x y^{-1} + y x^{-1} = \dfrac{x}{y} + \dfrac{y}{x} = \dfrac{x^2+y^2}{xy}$

$= \dfrac{(\sqrt{5}+1)^2+(\sqrt{5}-1)^2}{(\sqrt{5}+1)(\sqrt{5}-1)}$

$= \dfrac{6+2\sqrt{5}+6-2\sqrt{5}}{5-1} = \dfrac{12}{4} = 3$

$\because \dfrac{3x^2+3y^2}{xy} = \dfrac{3(x^2+y^2)}{xy}$

$= 3\left(\dfrac{x^2}{xy}+\dfrac{y^2}{xy}\right) = 3\left(\dfrac{x}{y}+\dfrac{y}{x}\right)$

$, \because \dfrac{x}{y} = \sqrt{3}-\sqrt{2}$

$\therefore \dfrac{y}{x} = \dfrac{1}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \sqrt{3}+\sqrt{2}$

$\therefore$ The expression $= 3\left(\sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2}\right)$

$= 3\left(2\sqrt{3}\right) = 6\sqrt{3}$

$\because \dfrac{x^8y^9-y}{(x+y)^5} = \dfrac{y(x^8y^8-1)}{(x+y)^5}$ (1)

$, x^8y^8 = (xy)^8 = [(\sqrt{7}+\sqrt{6})(\sqrt{7}-\sqrt{6})]^8$

$= 1^8 = 1$

, from (1) : $\therefore \dfrac{x^8y^9-y}{(x+y)^5} = \dfrac{(\sqrt{7}-\sqrt{6})(1-1)}{(\sqrt{7}+\sqrt{6}+\sqrt{7}-\sqrt{6})^5}$

$= \dfrac{\text{zero}}{(2\sqrt{7})^5} = \text{zero}$

8

1 $\sqrt[3]{8\times 2} = 2\sqrt[3]{2}$ 2 $\sqrt[3]{-27\times 2} = -3\sqrt[3]{2}$

3 $2\sqrt[3]{125\times 2} = 2\times 5\sqrt[3]{2} = 10\sqrt[3]{2}$

4 $\dfrac{2}{3}\sqrt[3]{-27\times 5} = \dfrac{2}{3} \times -3\sqrt[3]{5} = -2\sqrt[3]{5}$

5 $\sqrt[3]{\dfrac{1}{3}\times 27} = \sqrt[3]{9}$

6 $-2\sqrt[3]{\dfrac{2}{5}\times 125} = -2\sqrt[3]{50}$

1) $\sqrt[3]{2\times 32}=\sqrt[3]{64}=4$ 2) $\sqrt[3]{\frac{72}{9}}=\sqrt[3]{8}=2$

3) $\frac{4}{2}\sqrt[3]{\frac{-54}{-2}}=2\sqrt[3]{27}=2\times 3=6$

4) $\frac{1}{2}\times 6\sqrt[3]{10\times 100}=3\sqrt[3]{1000}=3\times 10=30$

5) $\sqrt[3]{\frac{2}{5}\times\frac{4}{25}}=\sqrt[3]{\frac{8}{125}}=\frac{2}{5}$

6) $\sqrt[3]{\frac{3}{4}\div\frac{2}{9}}=\sqrt[3]{\frac{3}{4}\times\frac{9}{2}}=\sqrt[3]{\frac{27}{8}}=\frac{3}{2}$

---

1) $2\sqrt[3]{2}-\sqrt[3]{2}=\sqrt[3]{2}$ 2) $5-2\sqrt[3]{3}$

3) $3\sqrt[3]{3}-2\sqrt[3]{3}=\sqrt[3]{3}$

4) $3\sqrt[3]{2}+2\sqrt[3]{2}-5\sqrt[3]{2}=\text{zero}$

5) $2\times 3\sqrt[3]{2}-5\sqrt[3]{2}+2\sqrt[3]{2}=3\sqrt[3]{2}$

6) $2\sqrt[3]{2}-\frac{1}{3}\times 3\sqrt[3]{2}-\sqrt[3]{2}=\text{zero}$

7) $2\sqrt[3]{2}+\sqrt[3]{250}=2\sqrt[3]{2}+5\sqrt[3]{2}=7\sqrt[3]{2}$

8) $2\sqrt[3]{3}-2\times 3\sqrt[3]{\frac{125}{9}}=2\sqrt[3]{3}-2\sqrt[3]{\frac{125}{9}\times 27}$

$=2\sqrt[3]{3}-2\times 5\sqrt[3]{3}=-8\sqrt[3]{3}$

---

1) The left hand side $=4\sqrt[3]{2}+2\sqrt[3]{2}-2\times 3\sqrt[3]{2}=\text{zero}$

= the right hand side.

2) The left hand side $=3\sqrt[3]{2}\times 2\sqrt[3]{2}\div(6\sqrt[3]{4})$

$=6\sqrt[3]{4}\div 6\sqrt[3]{4}=1=$ the right hand side.

---

1) $3\sqrt[3]{3}-2\sqrt[3]{3}-\sqrt[3]{27\times\frac{1}{9}}=3\sqrt[3]{3}-2\sqrt[3]{3}-\sqrt[3]{3}=\text{zero}$

2) $3\sqrt[3]{2}-4\sqrt[3]{\frac{1}{4}\times 8}+5\times 2\sqrt[3]{2}$

$=3\sqrt[3]{2}-4\sqrt[3]{2}+10\sqrt[3]{2}=9\sqrt[3]{2}$

3) $3\sqrt[3]{4}-2\sqrt[3]{4}-\sqrt[3]{\frac{4}{8}}=3\sqrt[3]{4}-2\sqrt[3]{4}-\frac{1}{2}\sqrt[3]{4}=\frac{1}{2}\sqrt[3]{4}$

4) $\sqrt[3]{3}-\sqrt[3]{24}+\sqrt[3]{27\times\frac{1}{9}}=\sqrt[3]{3}-2\sqrt[3]{3}+\sqrt[3]{3}=\text{zero}$

---

1) $\frac{7}{3}\times 3\sqrt{2}+3\sqrt[3]{2}-7\sqrt{2}+2\sqrt[3]{2}$

$=7\sqrt{2}+3\sqrt[3]{2}-7\sqrt{2}+2\sqrt[3]{2}=5\sqrt[3]{2}$

2) $3\sqrt{3}+\frac{1}{3}\times 3-3\sqrt{9\times\frac{1}{3}}-1$

$=3\sqrt{3}+1-3\sqrt{3}-1=\text{zero}$

3) $-2\sqrt[3]{2}+\frac{14}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}-2\sqrt{7}+3\sqrt[3]{2}$

$=-2\sqrt[3]{2}+2\sqrt{7}-2\sqrt{7}+3\sqrt[3]{2}=\sqrt[3]{2}$

4) $3\sqrt{2}+3\sqrt[3]{2}-\sqrt{\frac{216}{12}}-2\sqrt[3]{2}$

$=3\sqrt{2}+\sqrt[3]{2}-\sqrt{18}=3\sqrt{2}+\sqrt[3]{2}-3\sqrt{2}=\sqrt[3]{2}$

5) $5\sqrt{2}-\frac{1}{2}\times 10\sqrt{2}+\sqrt[3]{125}=5\sqrt{2}-5\sqrt{2}+5=5$

---

1) c 2) a 3) c 4) d 5) a

---

1) -2 2) 9 3) 250

4) $\frac{2}{3}\sqrt[3]{7}$ 5) $-\frac{1}{2}$ 6) 1

---

1) $(\sqrt[3]{5}+1-\sqrt[3]{5}+1)^5=2^5=32$

2) $(\sqrt[3]{5}+1+\sqrt[3]{5}-1)^3=(2\sqrt[3]{5})^3=8\times 5=40$

$\because x-y=3+\sqrt[3]{6}-3+\sqrt[3]{6}=2\sqrt[3]{6}$

, $x+y=3+\sqrt[3]{6}+3-\sqrt[3]{6}=6$

$\therefore\left(\frac{x-y}{x+y}\right)^3=\left(\frac{2\sqrt[3]{6}}{6}\right)^3=\left(\frac{\sqrt[3]{6}}{3}\right)^3=\frac{(\sqrt[3]{6})^3}{3^3}$

$=\frac{6}{27}=\frac{2}{9}$

L.H.S. $=x^2+y^2=(x+y)^2-2xy$

$=(\sqrt[3]{2}+1+\sqrt[3]{2}-1)^2-2(\sqrt[3]{2}+1)(\sqrt[3]{2}-1)$

$=(2\sqrt[3]{2})^2-2((\sqrt[3]{2})^2-1)$

$=4\sqrt[3]{4}-2\sqrt[3]{4}+2=2\sqrt[3]{4}+2=$ R.H.S.

12

$$\frac{2}{\sqrt[3]{2}} = \frac{2 \times \sqrt[3]{4}}{\sqrt[3]{2} \times \sqrt[3]{4}} = \frac{2\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{2\sqrt[3]{4}}{2} = \sqrt[3]{4}$$

Another solution : $\frac{2}{\sqrt[3]{2}} = \frac{\sqrt[3]{8}}{\sqrt[3]{2}} = \sqrt[3]{\frac{8}{2}} = \sqrt[3]{4}$

1

$\boxed{1}$ 125 $\boxed{2}$ 96 $\boxed{3}$ $4\ell^2$ $\boxed{4}$ $6\ell^2$ $\boxed{5}$ $8\ell^3$

2

$\because$ Area of one face $= \frac{36}{4} = 9$ cm^2.

$\therefore$ The edge length of the cube $= \sqrt{9} = 3$ cm.

$\boxed{1}$ Its total area $= 6\ell^2 = 6 \times 3^2 = 54$ cm^2.

$\boxed{2}$ Its volume $= \ell^3 = 3^3 = 27$ cm^3.

The edge length of the cube $= \frac{12}{4} = 3$ cm.

$\boxed{1}$ Its volume $= \ell^3 = 3^3 = 27$ cm^3.

$\boxed{2}$ Its lateral area $= 4\ell^2 = 4 \times 3^2 = 36$ cm^2.

The edge length of the cube $= \frac{60}{12} = 5$ cm.

$\boxed{1}$ Its volume $= \ell^3 = 5^3 = 125$ cm^3.

$\boxed{2}$ Its total area $= 6\ell^2 = 6 \times 5^2 = 150$ cm^2.

$\boxed{1}$ d $\boxed{2}$ c $\boxed{3}$ d $\boxed{4}$ a $\boxed{5}$ b

$\boxed{6}$ d $\boxed{7}$ b $\boxed{8}$ a

$\boxed{1}$ The volume of the cuboid $= X \times y \times z$

$= 9 \times 10 \times 5 = 450$ cm^3.

$\boxed{2}$ Its lateral area $= 2(X + y) \times z = 2(9 + 10) \times 5$

$= 190$ cm^2.

$\boxed{3}$ Its total area $= 2(Xy + yz + zX)$

$= 2(9 \times 10 + 10 \times 5 + 5 \times 9)$

$= 2(90 + 50 + 45) = 370$ cm^2.

The volume $= X \times y \times z = \sqrt{2} \times \sqrt{3} \times \sqrt{6} = 6$ cm^3.

$\because$ The lateral area = the perimeter of the base $\times$ height

$\therefore$ The height $= \frac{480}{4 \times 10} = 12$ cm.

The area of the base $= \frac{\text{volume}}{\text{height}} = \frac{720}{5} = 144$ cm^2

$\therefore$ The side length of the base $= \sqrt{144} = 12$ cm.

$\therefore$ The total area $= 2(Xy + yz + zX)$

$= 2 \times (12 \times 12 + 12 \times 5 + 12 \times 5)$

$= 528$ cm^2.

$\because$ The area of the face of the cube $= \frac{294}{6} = 49$ cm^2.

$\therefore$ The edge length $= \sqrt{49} = 7$ cm.

$\therefore$ The volume of the cube $= \ell^3 = 7 \times 7 \times 7 = 343$ cm^3.

, $\because$ the volume of the cuboid $= X \times y \times z$

$= 7\sqrt{2} \times 5\sqrt{2} \times 5$

$= 350$ cm^3.

$\therefore$ The volume of the cuboid is greater than the volume of the cube

The volume of the cuboid $= X \times y \times z = 17 \times 7 \times 4$

$= 476$ cm^3.

, the total area $= 2(X + y) \times z + Xy$

$= 2(17 + 7) \times 4 + 17 \times 7$

$= 192 + 119 = 311$ cm^2.

The circumference of the circle $= 2\pi r$

$= 2 \times \frac{22}{7} \times 10.5$

$= 66$ cm.

The area of the circle $= \pi r^2 = \frac{22}{7} \times (10.5)^2$

$= 346.5$ cm^2.

$\because$ The area of the circle $= \pi r^2$

$\therefore 154 = \frac{22}{7} r^2$ $\therefore r^2 = \frac{154 \times 7}{22} = 49$

$\therefore r = \sqrt{49} = 7$ cm.

The circumference $= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm.

The diameter length $= 2 \times 7 = 14$ cm.

$\because$ The area of the circle $= \pi r^2$

$\therefore 64\pi = \pi r^2$ $\qquad \therefore r^2 = 64$

$\therefore r = \sqrt{64} = 8$ cm.

The circumference of the circle $= 2\pi r$

$= 2 \times 3.14 \times 8 \approx 50$ cm.

$\because$ The area of the circle $= 2 \times 12.32 = 24.64$ cm^2.

$\therefore \pi r^2 = 24.64$ $\qquad \therefore r^2 = 24.64 \times \frac{7}{22} = 7.84$

$\therefore r = \sqrt{7.84} = 2.8$ cm.

$\therefore$ The perimeter of the figure $= \pi r + 2r$

$= \frac{22}{7} \times 2.8 + 2 \times 2.8 = 14.4$ cm.

The area of the shaded part = the area of the great circle − the area of the small circle $= \pi r_1^2 - \pi r_2^2$

$= \pi \times 25 - \pi \times 9 = 16\pi$ cm^2.

Let the radius length of the circle $= x$ cm.

$\therefore$ The side length of the square $= 2x$ cm.

$\therefore$ The area of the shaded part

$= \frac{\text{the area of the square} - \text{the area of the circle}}{2}$

$= \frac{4x^2 - \pi x^2}{2} = 10\frac{5}{7} = \frac{75}{7}$

$\therefore 4x^2 - \frac{22}{7}x^2 = \frac{75}{7} \times 2$

$\therefore \frac{6}{7}x^2 = \frac{150}{7}$ $\qquad \therefore x^2 = \frac{150}{7} \times \frac{7}{6} = 25$

$\therefore x = \sqrt{25} = 5$ cm.

$\because$ The perimeter of the shaded part

$= \frac{1}{2}$ the circumference of the circle

$+ \frac{1}{2}$ the perimeter of the square.

$= \frac{22}{7} \times 5 + 20 = 35\frac{5}{7}$ cm.

The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (14)^2 \times 20 = 12320$ cm^3.

The total area of the cylinder $= 2\pi rh + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 14 \times 20 + 2 \times \frac{22}{7} \times (14)^2 = 2992$ cm^2.

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 924 = \frac{22}{7} \times r^2 \times 6$

$\therefore r^2 = \frac{924 \times 7}{6 \times 22} = 49$ $\qquad \therefore r = 7$ cm.

$\therefore$ The lateral area $= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 6$

$= 264$ cm^2.

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 7536 = 3.14 \times r^2 \times 24$

$\therefore r^2 = \frac{7536}{3.14 \times 24} = 100$ $\qquad \therefore r = 10$ cm.

$\therefore$ The total area $= 2\pi rh + 2\pi r^2$

$= 2 \times 3.14 \times 10 \times 24 + 2 \times 3.14 \times (10)^2 = 2135.2$ cm^2

The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 10 = 1540$ cm^3.

The volume of the cube $= \ell^3 = (11)^3 = 1331$ cm^3.

$\therefore$ The volume of the cylinder is greater than the volume of the cube.

---

[1] $2\pi rh$, $\pi r^2 h$ $\qquad$ [2] 2 cm. $\qquad$ [3] 20 cm.

[4] r cm. $\qquad$ [5] r cm.

The circumference of the base $= 2\pi r$

$\therefore 44 = 2 \times \frac{22}{7} \times r$

$\therefore r = \frac{44}{2} \times \frac{7}{22} = 7$ cm.

$\therefore$ The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 25 = 3850$ cm^3.

The lateral area $= 2\pi rh$

$\therefore 52 = 2 \times \frac{22}{7} \times 4 \times h$

$\therefore h = \frac{52 \times 7}{2 \times 22 \times 4} = \frac{91}{44}$ cm.

$\therefore$ The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 4^2 \times \frac{91}{44} = 104$ cm^3.

∵ The volume of the cylinder $= \pi r^2 h$

$\therefore 36\pi = \pi \times r^2 \times 4$

$\therefore r^2 = \frac{36}{4} = 9$ $\quad \therefore r = 3$ cm.

∴ The edge length of the cube = 3 cm.

∴ The total area of the cube $= 6\,\ell^2 = 6 \times 3^2 = 54$ cm^2.

∵ The volume of the cylinder $= \pi r^2 h$

∵ $h = r$

∴ The volume of the cylinder $= \pi r^3$

$\therefore 72\pi = \pi r^3$ $\quad \therefore r^3 = 72$ $\quad \therefore r = 2\sqrt[3]{9}$

∴ The height of the cylinder $= 2\sqrt[3]{9}$ cm.

27

The circumference of the base of the cylinder = BC

$\therefore 2\pi r = 44$ $\quad \therefore r = \frac{44 \times 7}{2 \times 22} = 7$ cm

The height = AB = 10 cm.

The volume $= \pi r^2 h = \frac{22}{7} \times (7)^2 \times 10 = 1540$ cm^3.

The volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 = 38.808$ cm^3.

The surface area of the sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times (2.1)^2 = 55.44$ cm^2.

∵ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 4188 = \frac{4}{3} \times 3.141 \times r^3$

$\therefore r^3 = \frac{4188 \times 3}{4 \times 3.141} = 1000$ $\quad \therefore r = 10$ cm.

The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 562.5\pi = \frac{4}{3}\pi r^3$ $\quad \therefore r^3 = \frac{562.5 \times 3}{4} = 421.875$

$\therefore r = \sqrt[3]{421.875} = 7.5$ cm.

The surface area of the sphere

$= 4\pi r^2 = 4 \times \pi \times (7.5)^2 = 225\pi$ cm^2.

---

1 b — 2 a — 3 c — 4 c

5 b — 6 d — 7 b

The volume of the cylinder $= \pi r^2 h$

$= \pi \times (4)^2 \times 18 = 288\pi$ cm^3.

∵ The volume of the cylinder = the volume of the sphere.

∴ The volume of the sphere $= 288\pi$ cm^3.

$\therefore \frac{4}{3}\pi r^3 = 288\pi$

$\therefore r^3 = \frac{288 \times 3}{4} = 216$

∴ The radius length of the sphere = 6 cm.

∵ The volume of the cylinder $= \pi r^2 h$

$\therefore 7536 = 3.14 \times r^2 \times 24$

$\therefore r^2 = \frac{7536}{3.14 \times 24} = 100$

$\therefore r = \sqrt{100} = 10$ cm.

, ∵ the radius length of the sphere

= the radius length of the cylinder base

∴ The volume of the sphere $= \frac{4}{3} \times 3.14 \times (10)^3$

$= 4186\frac{2}{3}$ cm^3.

The volume of the cuboid $= 77 \times 24 \times 21 = 38808$ cm^3.

∵ The volume of the cuboid = the volume of the sphere

$\therefore 38808 = \frac{4}{3}\pi r^3$ $\quad \therefore r^3 = \frac{38808 \times 3 \times 7}{4 \times 22} = 9261$

$\therefore r = \sqrt[3]{9261} = 21$ cm.

The volume of the sphere $= \frac{4}{3}\pi (3)^3 = 36\pi$ cm^3.

∵ The volume of the cylinder = the volume of the sphere

∴ The volume of the cylinder $= 36\pi$ cm^3.

$\therefore \pi r^2 h = 36\pi$ $\quad \therefore 9\pi h = 36\pi$ $\quad \therefore h = 4$ cm.

∵ The sphere touches the six faces of the cube

∴ The edge length of the cube = 2 r

∵ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 36\pi = \frac{4}{3}\pi r^3$ $\quad \therefore r^3 = \frac{36 \times 3}{4} = 27$

$\therefore r = 3$ cm.

∴ The edge length of the cube $= 2 \times 3 = 6$ cm.

∴ The volume of the cube $= \ell^3 = 6^3 = 216$ cm^3

∵ The volume of the sphere

= the volume of 8 small spheres

$\therefore \frac{4}{3}\pi r_1^3 = 8 \times \frac{4}{3}\pi r_2^3$ $\qquad \therefore (16.8)^3 = 8 r_2^3$

$\therefore r_2^3 = \frac{(16.8)^3}{8}$ $\qquad \therefore r_2 = \frac{16.8}{2} = 8.4$ cm.

The volume of the sphere $= \frac{4}{3}\pi (15)^3 = 4500\pi$ cm³.

∵ The volume of the cylinder

$= \frac{4}{9}$ the volume of the sphere

$\therefore \pi r^2 h = \frac{4}{9} \times 4500\pi$ $\qquad \therefore r^2 \times 20 = 2000$

$\therefore r^2 = \frac{2000}{20} = 100$ $\qquad \therefore r = \sqrt{100} = 10$ cm.

∵ The sum of lengths of all edges = 52 cm.

, the sum of the 4 heights = 3 × 4 = 12 cm.

∴ The sum of the remained edges = 52 − 12 = 40 cm.

∵ The base is a square.

∴ The side length of the square $= \frac{40}{8} = 5$ cm.

∴ The volume = 5 × 5 × 3 = 75 cm³.

The volume of the metal = the outer volume

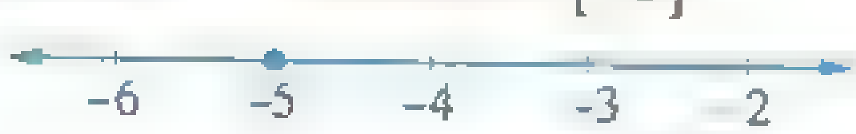
− the inner volume $= \frac{4}{3}\pi r_1^3 - \frac{4}{3}\pi r_2^3$

$= \frac{4}{3} \times \pi \left((3.5)^3 - (2.1)^3\right) = \frac{88}{21} \times 33.614 \simeq 140.859$ cm³.

∴ The mass of the metal = 140.859 × 20 ≃ 2817 gm.

…f Exer… 10

1 $\because x = -5$ $\qquad \therefore$ The S.S. $= \{-5\}$



2 $\because 5x = 1 - 6 = -5$ $\qquad \therefore x = \frac{-5}{5} = -1$

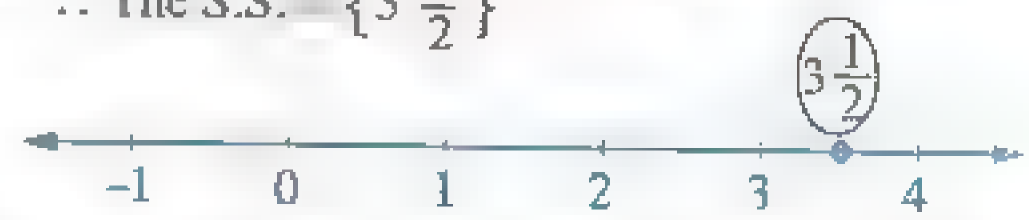
∴ The S.S. $= \{-1\}$

3 $\because 2x = 3 - 4 = -1$ $\qquad \therefore x = -\frac{1}{2}$

∴ The S.S. $= \{-\frac{1}{2}\}$

4 $\because 2x = 4 + 3 = 7$ $\qquad \therefore x = \frac{7}{2} = 3\frac{1}{2}$

∴ The S.S. $= \{3\frac{1}{2}\}$



5 $\because 4x - 1 = 2$ $\qquad \therefore 4x = 2 + 1 = 3$ $\qquad \therefore x = \frac{3}{4}$

∴ The S.S. $= \{\frac{3}{4}\}$

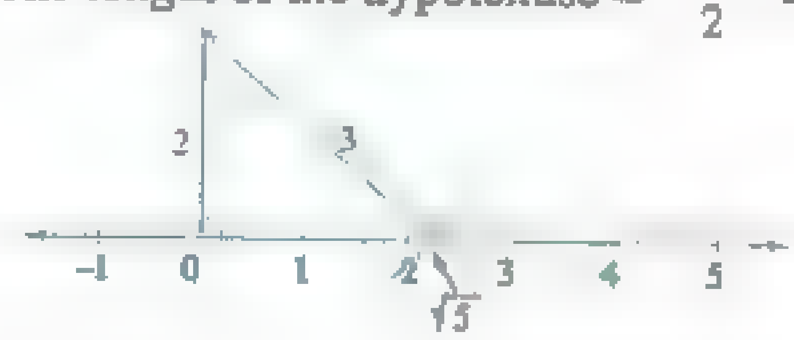
6 $\because \sqrt{5}\,x = 4 + 1 = 5$

$\therefore x = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

∴ The S.S. $= \{\sqrt{5}\}$

The length of one side of the right angle $= \frac{5-1}{2} = 2$

∴ The length of the hypotenuse $= \frac{5+1}{2} = 3$

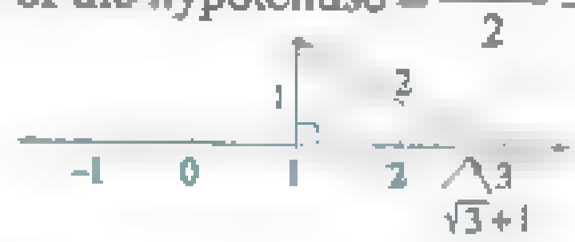


7 $\because x = \sqrt{3} + 1$ $\qquad \therefore$ The S.S. $= \{\sqrt{3} + 1\}$

The length of one side of the right angle

$= \frac{3-1}{2} = 1$

The length of the hypotenuse $= \frac{3+1}{2} = 2$



8 $\because 2 - \sqrt{6}\,x = 8$ $\qquad \therefore -\sqrt{6}\,x = 8 - 2 = 6$

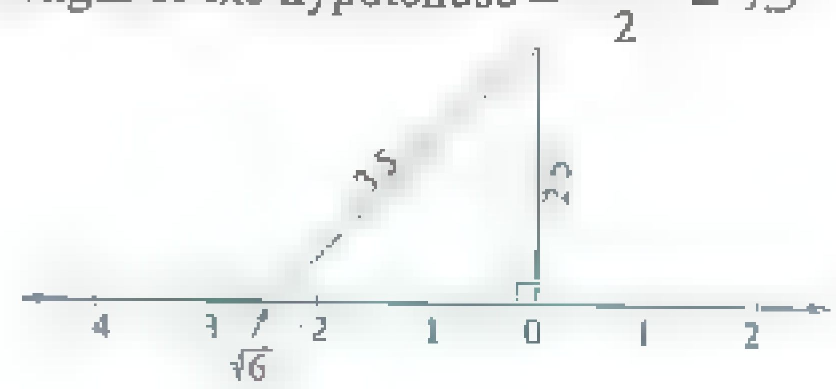
$\therefore x = -\frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{-6\sqrt{6}}{6} = -\sqrt{6}$

∴ The S.S $= \{-\sqrt{6}\}$

The length of one side of the right angle $= \frac{6-1}{2}$

$= 2.5$

The length of the hypotenuse $= \frac{6+1}{2} = 3.5$

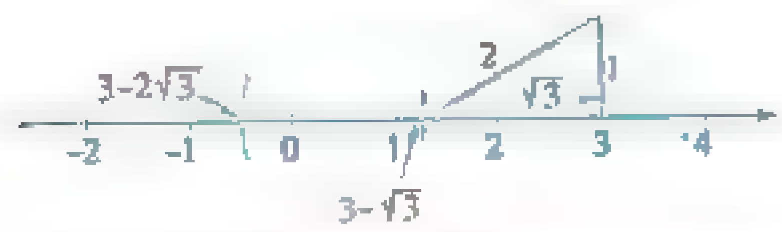


[5] $\because x = 3 - 2\sqrt{3}$ $\therefore$ The S.S. $= \{3 - 2\sqrt{3}\}$

The length of one side of the right angle $= \frac{3-1}{2}$

$= 1$

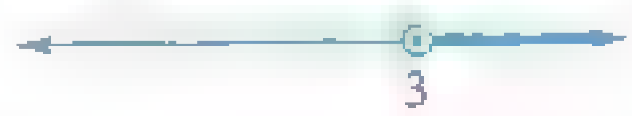
The length of the hypotenuse $= \frac{3+1}{2} = 2$



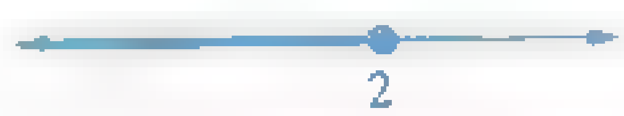
2

[1] a [2] c [3] c [4] d [5] a [6] d

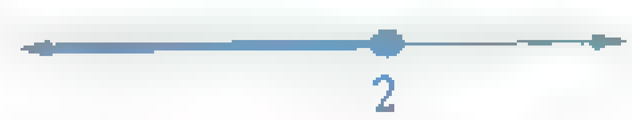
[1] $\because x > 3$ $\therefore$ The S.S. $=]3, \infty[$



[2] $\because x \le 2$ $\therefore$ The S.S. $=]-\infty, 2]$

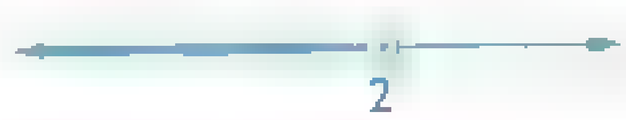


[3] $\because x \le 2$ $\therefore$ The S.S. $=]-\infty, 2]$



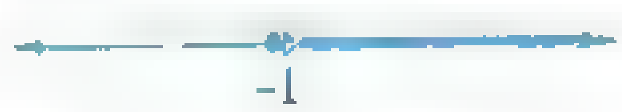
[4] $\because -x > -2$ $\therefore x < 2$

$\therefore$ The S.S. $=]-\infty, 2[$



[5] $\because 2x \ge -2$ $\therefore x \ge -1$

$\therefore$ The S.S. $= [-1, \infty[$

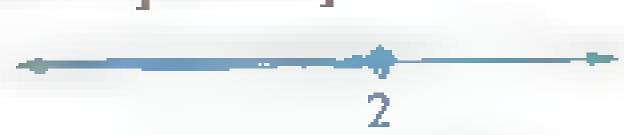


[6] $\because -5x < 5$ $\therefore x > -1$

$\therefore$ The S.S $=]-1, \infty[$

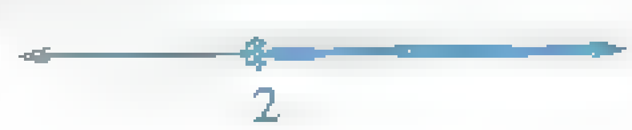
[7] $\because \frac{1}{2}x \le 1$ $\therefore x \le 2$

$\therefore$ The S.S $=]-\infty, 2]$

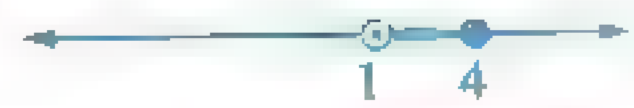


[8] $\because -2x \le 4$ $\therefore x \ge -2$

$\therefore$ The S.S $= [-2, \infty[$

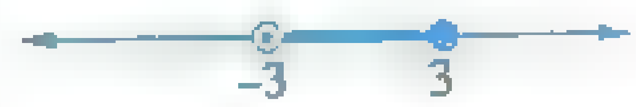


[1] $\because 1 < x \le 4$ $\therefore$ The S.S. $=]1, 4]$



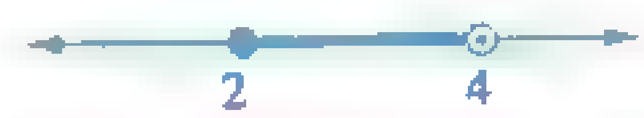
[2] $\because -8 < x < 6$ $\therefore$ The S.S $=]-8, 6[$

[3] $\because 3 \ge x > -3$ $\therefore$ The S.S. $=]-3, 3]$



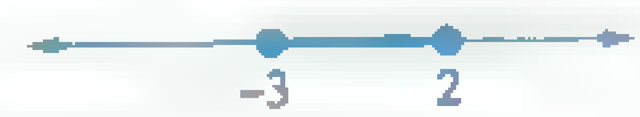
[4] $\because -4 < -x \le -2$ $\therefore 4 > x \ge 2$

$\therefore$ The S.S. $= [2, 4[$



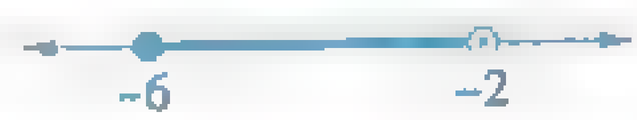
[5] $\because -2 \le x + 1 \le 3$ $\therefore -3 \le x \le 2$

$\therefore$ The S.S. $= [-3, 2]$



[6] $\because 2 < -x \le 6$ $\therefore -2 > x \ge -6$

$\therefore$ The S.S. $= [-6, -2[$



[7] $\because -9 \le 3x \le 3$ $\therefore -3 \le x \le 1$

$\therefore$ The S.S. $= [-3, 1]$

[8] $\because 3 < 2x - 1 < 5$ $\therefore 4 < 2x < 6$

$\therefore 2 < x < 3$ $\therefore$ The S.S. $=]2, 3[$

[9] $\because -1 < \frac{1}{2}x \le 2$ $\therefore -2 < x \le 4$

$\therefore$ The S.S. $=]-2, 4]$

[10] By multiplying by 3 $\therefore 0 \le -2x + 6 < 12$

$\therefore -6 \le -2x < 6$ $\therefore 3 \ge x > -3$

$\therefore$ The S.S. $=]-3, 3]$

Represent by yourself the S.S. on the number line :

[1] $\because 3x - 2x < 4$ $\therefore x < 4$

$\therefore$ The S.S. $=]-\infty, 4[$

[2] $\because 7x - 4x \ge 9$ $\therefore 3x \ge 9$ $\therefore x \ge 3$

$\therefore$ The S.S. $= [3, \infty[$

$\boxed{3}$ $\because 5x - 2x < 9 + 3 \quad \therefore 3x < 12 \quad \therefore x < 4$

$\therefore$ The S.S. $=]-\infty , 4[$

$\boxed{4}$ $\because 7x - 5x \geq -8 + 12 \quad \therefore 2x \geq 4 \quad \therefore x \geq 2$

$\therefore$ The S.S. $= [2 , \infty[$

$\boxed{5}$ $\because x + x \leq 3 + 1 \quad \therefore 2x \leq 4 \quad \therefore x \leq 2$

$\therefore$ The S.S. $=]-\infty , 2]$

$\boxed{6}$ $\because -x + 2x \geq -3 - 1 \quad \therefore x \geq -4$

$\therefore$ The S.S. $= [-4 , \infty[$

6

Represent by yourself the S.S. on the number line :

$\boxed{1}$ $\because x + 3 - x \geq 2x - x \geq x - 2 - x$

$\therefore 3 \geq x \geq -2 \quad \therefore$ The S.S. $= [-2 , 3]$

$\boxed{2}$ $\because -x + x < x + x < 4 - x + x \quad \therefore 0 < 2x < 4$

$\therefore 0 < x < 2 \quad \therefore$ The S.S. $=]0 , 2[$

$\boxed{3}$ $\because 4x - 4x \leq 5x + 2 - 4x < 4x + 3 - 4x$

$\therefore 0 \leq x + 2 < 3 \quad \therefore -2 \leq x < 1$

$\therefore$ The S.S. $= [-2 , 1[$

$\boxed{4}$ $\because x - 1 - x < 3x - 1 - x \leq x + 1 - x$

$\therefore -1 < 2x - 1 \leq 1 \quad \therefore 0 < 2x \leq 2$

$\therefore 0 < x \leq 1 \quad \therefore$ The S.S. $=]0 , 1]$

$\boxed{5}$ $\because 2 + 2x - 2x \leq 3x + 3 - 2x < 5 + 2x - 2x$

$\therefore 2 \leq x + 3 < 5 \quad \therefore -1 \leq x < 2$

$\therefore$ The S.S. $= [-1 , 2[$

$\boxed{6}$ By multiplying by 6

$\therefore 3x - 4 < 6x + 6 < 3x + 9$

$\therefore -4 < 3x + 6 < 9$

$\therefore -10 < 3x < 3 \quad \therefore -\frac{10}{3} < x < 1$

The S.S. $=]-\frac{10}{3} , 1[$

7

$\boxed{1}$ ≥ 3	$\boxed{2}$ < 3	$\boxed{3}$ < -3
$\boxed{4}$ $\geq -\frac{3}{2}$	$\boxed{5}$ $< 2\sqrt{2}$	$\boxed{6}$ $]2 , 4[$
$\boxed{7}$ $]-2 , 5]$	$\boxed{8}$ $]2 , \infty[$	$\boxed{9}$ 6

8

$\boxed{1}$ a $\quad \boxed{2}$ b $\quad \boxed{3}$ c $\quad \boxed{4}$ c $\quad \boxed{5}$ c

9

$\because$ The weight of one box $= 45$ kg.

Let the number of boxes be x

, $\because$ the maximum weight that the lift can carry is 2200 kg.

$\therefore$ The weight of boxes $\leq$ the maximum weight that the lift can carry.

$\therefore 45x \leq 2200 \quad \therefore x \leq 48\frac{8}{9}$

$\therefore$ The maximum number of boxes can the lift carry in one time is 48 boxes.

10

$\because -4 < -2x < 2 \quad \therefore 2 > x > -1 \quad \therefore$ The S.S $=]-1 , 2[$

$\because \sqrt{3} \approx 1.7 \quad \therefore \sqrt{3} \in]-1 , 2[$

11

$\because a + 3 \leq x \leq b + 3 \quad \therefore$ The S.S. $= [a + 3 , b + 3]$

$\therefore [4 , 7] = [a + 3 , b + 3]$

$\therefore a + 3 = 4 \quad \therefore a = 1$

$, b + 3 = 7 \quad \therefore b = 4$

12

$\because \frac{1}{5} \leq \frac{2x + 1}{5} \leq 1 \quad \therefore 1 \leq 2x + 1 \leq 5$

$\therefore 0 \leq 2x \leq 4 \quad \therefore 0 \leq x \leq 2$

$\therefore$ The S.S. $= [0 , 2] \quad \therefore m = 0 , m + n = 2 \quad \therefore n = 2$

13

$\because 5 \leq \frac{2x}{3} + 1 \leq 7 \quad \therefore 4 \leq \frac{2x}{3} \leq 6$

$\therefore 12 \leq 2x \leq 18 \quad \therefore 6 \leq x \leq 9$

$\therefore 4 \leq x - 2 \leq 7 \quad \therefore$ The smallest value of $x - 2$ is 4

14

Multiply both sides by $(\sqrt{3} - \sqrt{5})$

$\therefore x \leq (\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})$

"Note that the sign changed because $(\sqrt{3} - \sqrt{5})$ is a negative number because $\sqrt{3} < \sqrt{5}$"

$\therefore x \leq -2 \quad \therefore$ The S.S. $=]-\infty , -2]$

Answers of unit two

1 (5 , 14) , (2 , 5) , (0 , − 1) , (− 3 , − 10)

2 The ordered pair (− 1 , 3) satisfies the relation.

3 [1] (1 , − 3) , (2 , − 1) , (3 , 1) , (5 , 5)
[2] (0 , 5) , (2 , 6) , (4 , 7) , (6 , 8)
[3] (0 , 2) , (3 , 2) , (5 , 2) , (− 4 , 2)
[4] (2.5 , 7) , (2.5 , 3) , (2.5 , − 7) , (2.5 , 4)
There are other solutions.

4 [1]

x	0	1	2	3
y	1	5	9	13

[2]

x	− 4	− 3	− 2
y	− 5	0	5

[3]

a	1	4	3
b	− 3	0	− 1

[4]

a	2	5	− 1
b	− 1	0	− 2

5 [1] 7 [2] − 9 [3] zero [4] − 1

6 ∵ (3 , 6) satisfies the relation y = k x
∴ 6 = 3 k   k = 2

7 ∵ (3 , 1) satisfies the relation y − 3 x = a
∴ 1 − 3 × 3 = a   ∴ a = − 8

8 ∵ (− 3 , 2) satisfies the relation 3 x + b y = 1
∴ 3 × (− 3) + b × 2 = 1   ∴ 2 b = 9 + 1
∴ 2 b = 10   ∴ b = 5

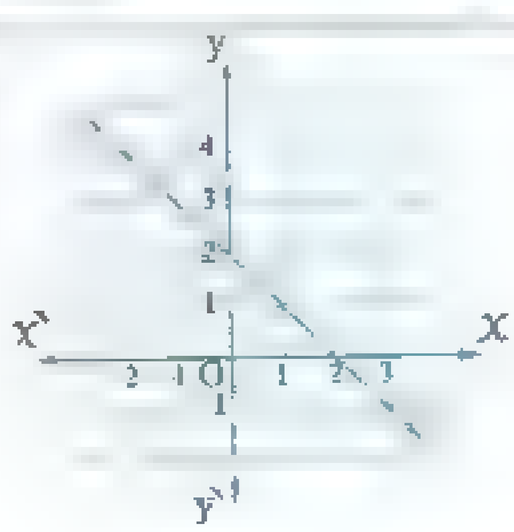
9 ∵ (3 , a) satisfies the relation y − 2 x = 4
∴ a − 2 × 3 = 4   ∴ a = 10

10 ∵ (k , 2 k) satisfies the relation x + y = 15
∴ k + 2 k = 15   ∴ 3 k = 15   ∴ k = 5

11 [1] x = 3   [2] y = zero

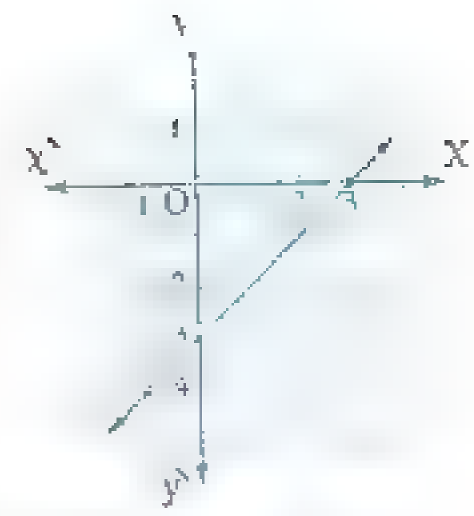
12 [1]

x	1	3	− 2
y	1	− 1	4



[2]

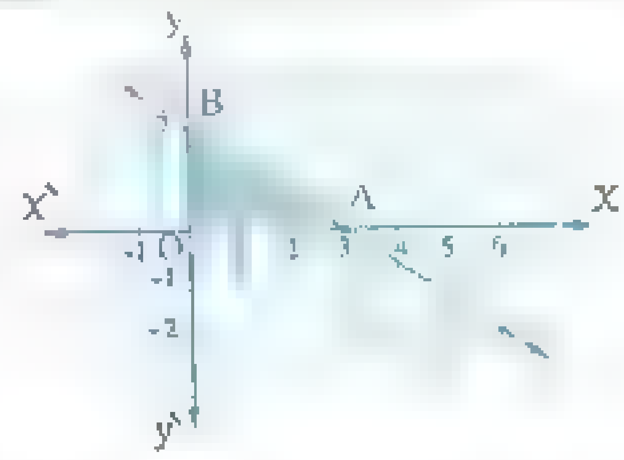
x	0	3	1
y	− 3	0	− 4



From [3] to [8] represent the relations graphically by yourself.

13 $y = -\frac{2}{3}x + 2$

x	0	3	6
y	2	0	− 2



From the graph :
The area of Δ OAB = $\frac{1}{2}$ × 3 × 2 = 3 square units.

14 ∵ The straight line intersects x-axis at (3 , b)
∴ b = 0
∴ (3 , 0) satisfies the relation 2 x − y = a
∴ 2 × 3 − 0 = a   ∴ a = 6

15 [1] b [2] c [3] b [4] b
[5] d [6] a [7] b [8] a
[9] d [10] c [11] c [12] c
[13] c [14] c

16 Let the first number be x and the second be y
∴ 2 x + y = 12   ∴ y = 12 − 2 x
∵ The two numbers are even natural numbers.
∴ x has the values 0 , 2 , 4 , 6 then we can register the different possibilities to the two numbers in the following table :

x	0	2	4	6
y	12	8	4	0

17 Let the length of the rectangle = x cm. and the width = y cm.   ∴ x > y
, ∵ the perimeter of the rectangle = 14 cm.
∴ 2 (x + y) = 14   ∴ x + y = 7
We can record the different possibilities of the length and the width of the rectangle in the opposite table :

x	6	5	4
y	1	2	3

Let the number of bills of L.E. 5 be X , then its value $= 5X$

and let the number of bills of L.E. 20 be y

, then its value $= 20y$

$\therefore 5X + 20y = 65$ where X and y are natural numbers.

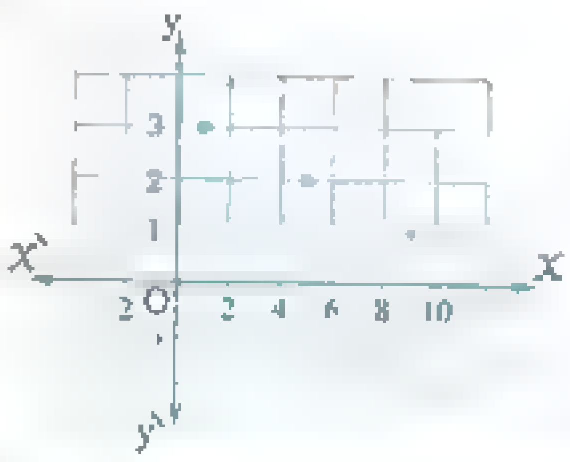
$\therefore X + 4y = 13$ $\qquad \therefore y = \frac{13 - X}{4}$

$\because X \leq 10$, $(13 - X)$ is divisible by 4

i.e. X has the values 9 , 5 and 1

, then we can write the different possibilities in the following table :

X	1	5	9
y	3	2	1



19 Let the store sold in this week X computer's table and y chairs.

$\therefore 100X + 50y = 500$

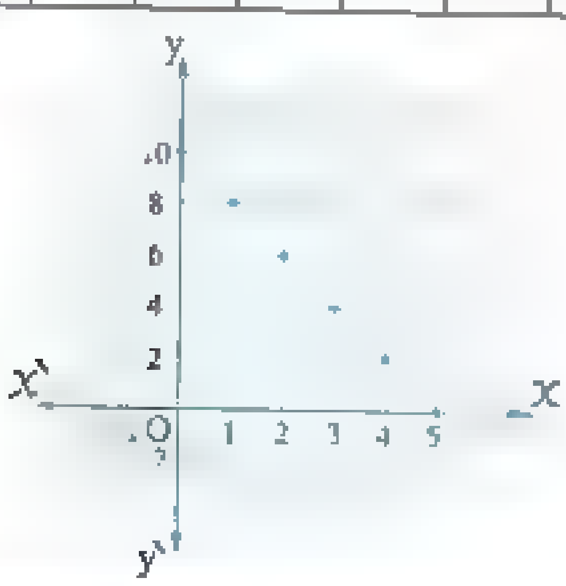
where X and y are two natural numbers.

$\therefore 2X + y = 10$ $\qquad \therefore y = 10 - 2X$

$\therefore X$ is not more than 5

$\therefore$ We can write the expectations which represent the number of computer's tables and the number of chairs in the following table :

X	0	1	2	3	4	5
y	10	8	6	4	2	0



20 Let the length of any of the two congruent sides in the triangle be X cm. and the length of the third side be y cm.

$\because$ the perimeter of the triangle $= 19$

$\therefore 2X + y = 19$ $\qquad \therefore y = 19 - 2X$

$\therefore X$ and y are positive integers then X is not more than 9 and from the inequality of the triangle then X has the values 5 , 6 , 7 , 8 and 9

, then we can write all the possibilities in the following table :

X	5	6	7	8	9
y	9	7	5	3	1

Figure (1) the slope is positive.

Figure (2) the slope is negative.

Figure (3) the slope is undefined.

Figure (4) the slope equals zero.

---

[1] negative [2] zero [3] undefined [4] positive

---

[1] zero [2] undefined [3] X-axis [4] $\overleftrightarrow{BC}$ or $\overleftrightarrow{AC}$

---

[1] The slope of $\overleftrightarrow{AB} = \frac{4-3}{3-1} = \frac{1}{2}$

[2] The slope of $\overleftrightarrow{AB} = \frac{0-2}{5-1} = -\frac{1}{2}$

[3] The slope of $\overleftrightarrow{AB} = \frac{5-2}{6-3} = 1$

[4] The slope of $\overleftrightarrow{AB} = \frac{-1+1}{4-2} = 0$

[5] The slope of $\overleftrightarrow{AB} = \frac{3-3}{2-1} = 0$

[6] The slope of $\overleftrightarrow{AB} = \frac{4-2}{5-5} = \frac{2}{\text{zero}}$ undefined.

[7] The slope of $\overleftrightarrow{AB} = \frac{2+1}{3-3} = \frac{3}{\text{zero}}$ undefined.

[8] The slope of $\overleftrightarrow{AB} = \frac{1+2}{4-3} = \frac{3}{1} = 3$

[9] The slope of $\overleftrightarrow{AB} = \frac{1-3}{2+1} = \frac{-2}{3}$

[10] The slope of $\overleftrightarrow{NK} = \frac{7+2}{-1-4} = \frac{-5}{-5} = 1$

[11] The slope of $\overleftrightarrow{EO} = \frac{0+1}{0+3} = \frac{1}{3}$

[12] The slope of $\overleftrightarrow{AB} = \frac{-1+9}{1+6}$ $\frac{8}{5}$

---

[1] Taking the two points (0 , 0) , (1 , 2) which lie on the straight line we find that :

the slope $= \frac{2-0}{1-0} = 2$

[2] Taking the two points (0 , − 1) , (− 2 , 3) which lie on the straight line we find that :

the slope $= \frac{3-(-1)}{-2-0} = \frac{4}{-2} = -2$

$\because m(\angle M) = 45^\circ$ $\because \Delta MNL$ is an isosceles triangle.

$\therefore ML = LN$

$\because$ The length of $\overline{ML} = 4$ units.

$\therefore$ The length of $\overline{LN} = 4$ units.

$N = (3 , 6)$

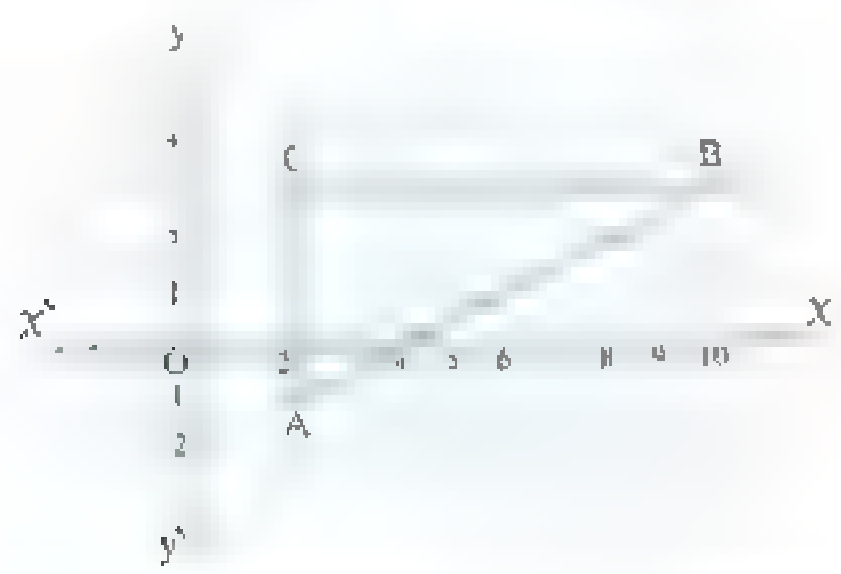
, the slope of $\overleftrightarrow{MN} = \frac{6-2}{3-7} = \frac{4}{-4} = -1$

---

the slope of $\overleftrightarrow{AB} = \frac{3+1}{10-2} = \frac{1}{2}$

the slope of $\overleftrightarrow{BC} = \frac{3-3}{2-10} = \text{zero}$

the slope of $\overleftrightarrow{AC} = \frac{3+1}{2-2} = \frac{4}{0}$ (undefined)



from the graph we find that ΔABC is right-angled.

$\because$ The slope of the straight line which passes through the two points $(1 , 3)$ and $(3 , k)$ equals 3

$\frac{k-3}{3-1} = 3$ $\therefore \frac{k-3}{2} = 3$

$k - 3 = 6$ $\therefore k = 9$

$\because$ The slope of the straight line which passes through the two points $(3 , c)$ and $(5 , -2)$ equals -3

$\frac{-2-c}{5-3} = -3$ $\therefore \frac{-2-c}{2} = -3$

$\therefore -2 - c = -6$ $\therefore c = 4$

$2 = \frac{2-4}{x-(-1)}$ $\therefore -2 = \frac{-2}{x+1}$

$x + 1 = 1$ $x = 0$

$\because \frac{-1-y}{3-(-2)} = -0.6$ $\therefore \frac{-1-y}{5} = -0.6$

$\therefore -1 - y = -3$ $\therefore y = 2$

$\because$ The straight line is parallel to x axis

$\therefore$ The slope = zero $\therefore \frac{k-4}{2-3} = \text{zero}$

$\therefore k - 4 = \text{zero}.$ $\therefore k = 4$

$\because$ The straight line is parallel to y-axis

$\therefore$ The slope is undefined

$\therefore x_2 - x_1 = \text{zero}$ $\therefore 6 - 2x = 0$

$\therefore -2x = -6$ $\therefore x = 3$

$\because$ The straight line is perpendicular to y-axis

$\therefore$ The straight line is parallel to x-axis

$\therefore$ The slope = zero $\therefore y_2 - y_1 = \text{zero}$

$\therefore 3y - 6 = 0$ $\therefore 3y = 6$ $\therefore y = 2$

$\because$ The slope of the straight line passing through the two points $(-5 , 11)$ and $(0 , 8) = \frac{8-11}{0-(-5)} = \frac{-3}{5}$ (1)

$\because$ The slope of the straight line passing through the two points $(0 , 8)$ and $(5 , 5) = \frac{5-8}{5-0} = \frac{-3}{5}$ (2)

from (1) and (2) we find that the three points are collinear

(lying on the straight line whose slope $= -\frac{3}{5}$)

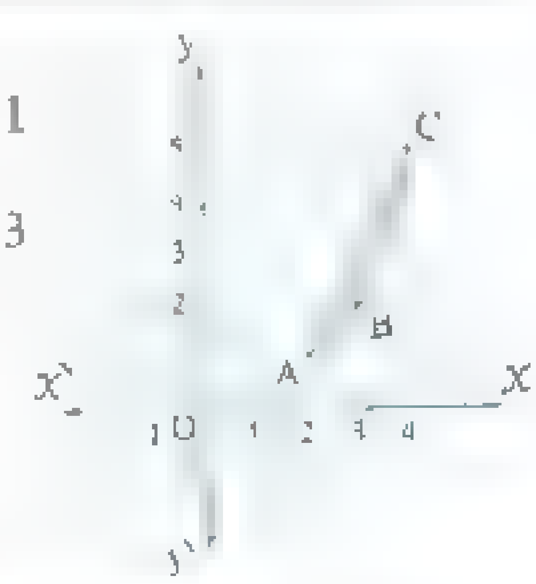
The slope of $\overleftrightarrow{AB} = \frac{2-1}{3-2} = 1$

, the slope of $\overleftrightarrow{BC} = \frac{5-2}{4-3} = 3$

and the slope of $\overleftrightarrow{AC}$

$= \frac{5-1}{4-2} = 2$

We observe that the three points are not collinear.



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[1] $\because$ The slope of $\overleftrightarrow{AB} = \frac{2-1}{2-1} = 1$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{3-2}{3-2} = 1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

$\therefore$ The points A , B and C are collinear.

[2] ∵ The slope of $\overleftrightarrow{AB} = \frac{7-(-3)}{-6-4} = \frac{10}{-10} = -1$

∵ The slope of $\overleftrightarrow{BC} = \frac{4-7}{5-(-6)} = \frac{-11}{11} = -1$

∵ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

∴ The points A , B and C are collinear.

[3] ∵ The slope of $\overleftrightarrow{AB} = \frac{-4-12}{2-(-2)} = \frac{-8}{4} = -2$

∵ The slope of $\overleftrightarrow{BC} = \frac{-4-4}{6-2} = \frac{-8}{4} = -2$

∴ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

∴ The points A , B and C are collinear

10

[1] ∵ The slope of $\overleftrightarrow{AB} = \frac{0-1}{3-2} = \frac{-1}{1} = -1$

∵ The slope of $\overleftrightarrow{BC} = \frac{-1-0}{5-3} = -\frac{1}{2}$

∴ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

∴ The points A , B and C are not collinear.

[2] ∵ The slope of $\overleftrightarrow{AB} = \frac{1-2}{3-(-1)} = -\frac{1}{4}$

∵ The slope of $\overleftrightarrow{BC} = \frac{2-1}{7-3} = \frac{1}{4}$

∴ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

∴ The points A , B and C are not collinear.

[3] ∵ The slope of $\overleftrightarrow{AB} = \frac{2-(-3)}{2-0} = \frac{5}{2}$

∵ The slope of $\overleftrightarrow{BC} = \frac{-3-2}{-3-2} = 1$

∴ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

∴ The points A , B and C are not collinear

∵ The slope of $\overleftrightarrow{AB} = \frac{5-3}{2+1} = \frac{2}{3}$

∵ The slope of $\overleftrightarrow{BC} = \frac{5-1}{2-8} = -\frac{2}{3}$

∴ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

∴ $C \notin \overleftrightarrow{AB}$

∵ The slope of the straight line which passes through the two points (4 , 1) and (– 2 , 7)

$= \frac{7-1}{-2-4} = \frac{6}{-6} = -1$

∵ The slope of the straight line which passes through the two points (– 2 , 7) and (3 , y)

$= \frac{y-7}{3-(-2)} = \frac{y-7}{5}$

∵ The three points are collinear.

∴ $\frac{y-7}{5} = -1$ ∴ $y - 7 = -5$

∴ $y = -5 + 7$ ∴ $y = 2$

∵ The slope of the straight line which passes through the two points (3 , – 1) and (x , 1) equals $\frac{2}{3}$

∴ $\frac{1-(-1)}{x-3} = \frac{2}{3}$ ∴ $\frac{2}{x-3} = \frac{2}{3}$

∴ $x - 3 = 3$ ∴ $x = 6$

∵ The slope of the straight line which passes through the two points (3 , – 1) and (9 , y) equals $\frac{2}{3}$

∴ $\frac{y-(-1)}{9-3} = \frac{2}{3}$ ∴ $\frac{y+1}{6} = \frac{2}{3}$ ∴ $3y + 3 = 12$

∴ $3y = 9$ ∴ $y = 3$

∵ The uniform velocity $= \frac{\text{the covered distance}}{\text{the taken time}}$

$= \frac{180}{3} = 60$ km./hr.

∴ The covered distance = the taken time × the uniform velocity = 60 × 5 = 300 km.

∵ The rate of consumption of fuel

$= \frac{\text{the amount of consumed fuel}}{\text{time}}$

$= \frac{2.47}{3} = \frac{247}{300}$ litre/hr.

∴ The consumed amount

= the rate of consumption × time

$= \frac{247}{300} \times 10 = 8\frac{7}{30}$ litre.

[1] At the moment of starting the motion , the body is at a distance of 2 metres from the fixed point.

At t = 6 , the body is at a distance of 8 metres.

Taking the two points (0 , 2) and (6 , 8) on the straight line which represents the relation between t and d

$\therefore$ The slope $= \frac{8 - 2}{6 - 0} = \frac{6}{6} = 1$

It represents the velocity of the body within a going trip

2 At the moment of starting the motion , the body is at t distance of 12 metres from the fixed point.

At t = 6 the body is at a distance of 2 metres. Taking the two points (0 , 12) and (6 , 2) on the straight line representing the relation between t and d

$\therefore$ The slope $= \frac{2 - 12}{6 - 0} = \frac{-10}{6} = -\frac{5}{3}$

It represents the velocity of the body within the returning back.

3 On starting the motion , the body is at a distance of 8 metres from the fixed point.

At t = 6 the body is at a distance of 8 metres.

$\because$ The straight line representing the relation is horizontal. $\therefore$ The slope = zero

It means that the body is rest.

Taking two points on the straight line representing the relation between t and d say (0 , 50) and (4 , 150)

$\therefore$ The uniform velocity = the slope of the straight line

$= \frac{150 - 50}{4 - 0} = \frac{100}{4} = 25$ km./hr.

1 Taking two points on the straight line representing the relation between t and d say (0 , 50) and (2 , 200)

$\therefore$ The unifrom velocity = the slope of the straight line $= \frac{200 - 50}{2 - 0} = 75$ km./hr.

2 from the graph :

The car is at a distance = 275 km. from the point 0 after passing 3 hours from the moment of beginning the motion.

1 The velocity within the first 3 hours = the slope of the striaght line $\overleftrightarrow{OB} = \frac{125}{3} = 41\frac{2}{3}$ km./hr.

The velocity within the next 2 hours = the slope of the straight line $\overleftrightarrow{BC} = \frac{125}{2} = 62\frac{1}{2}$ km./hr.

2 The average velocity within the all trip

$= \frac{\text{total distance}}{\text{total time}} = \frac{250}{5} = 50$ km./hr.

1 The velocity within the first 3 hours = the slope of the striaght line $= \frac{60 - 20}{3 - 0} = \frac{40}{3} = 13\frac{1}{3}$ km./hr.

2 The velocity within the next 4 hours = the slope of the straight line $= \frac{0 - 60}{7 - 3} = \frac{-60}{4} = -15$ km./hr.

The negative sign means that the bicycle returns back with velocity 15 km./hr.

The total distance = 40 + 60 = 100 km.

1 The slope of the straight line $\overleftrightarrow{AB}$

$= \frac{60 - 20}{4 - 0} = \frac{40}{4} = 10$

It means the increasing of the capital within the first 4 years with rate equals 10 thousands pounds/year.

The slope of $\overleftrightarrow{BC} = \frac{60 - 60}{6 - 4} =$ zero

It means constancy of the capital within the fifth and sixth years.

The slope of $\overleftrightarrow{CD} = \frac{50 - 60}{8 - 6} = \frac{-10}{2} = -5$

It means decreasing of the capital within the 7th and 8th years with rate = 5 thousands/year.

2 The starting capital of the company = 20 thousand pounds.

1 The slope of $\overleftrightarrow{AB} = \frac{125 - 50}{8 - 0} = \frac{75}{8} = 9\frac{3}{8}$

It means that the increase in height goes with respect to the increase in age.

The slope of $\overleftrightarrow{BC} = \frac{175 - 125}{18 - 8} = \frac{50}{10} = 5$

It means that the increase in height goes with respect to the increase in age but with a rate less than the rate within the first 8 years.

The slope of $\overleftrightarrow{CD} = \frac{175 - 175}{22 - 18} = 0$

It denotes the constancy in height inspite of the increase in age after 18 years.

2 $\because$ The height of the person at age 30 years = 175 cm. and the height of the person at age 8 years = 125 cm.

$\therefore$ The difference = 175 − 125 = 50 cm.

1 The greatest capacity of the tank = 70 litre.

2 The tank will be empty after 30 hours.

3 The remained fuel = 35 litre

4 Taking the two points (0 , 70) , (30 , 0) on the straight line representing the relation.

∴ The rate of consumption of the fuel =

The slope of the straight line $= \frac{70-0}{0-30} = -2\frac{1}{3}$ litre/hr

The negative sign means the rate of consumption.

i.e. The amount of fuel is consumed with rate $2\frac{1}{3}$ litre/hr.

---

1 100 pages.

2 Taking the two points (0 , 100) and (3 , 40) on the straight line representing the relation.

we find that the rate of decreasing the number of pages = the slope of the straight line

$= \frac{40-100}{3-0} = \frac{-60}{3} = -20$ pages/hr.

The negative sign expresses the decreasing in the number of remained pages with rate 20 page/h.

3 ∵ The remained pages decreases with rate 20 page/h.

There are 100 pages.

∴ The person finishes reading the book after $\frac{100}{20} = 5$ hours.

Let the covered distance be d km

The amount of the remained fuel in the tank be y litre.

At the beginning the covered distance = zero

The amount of remained fuel = 40 litre

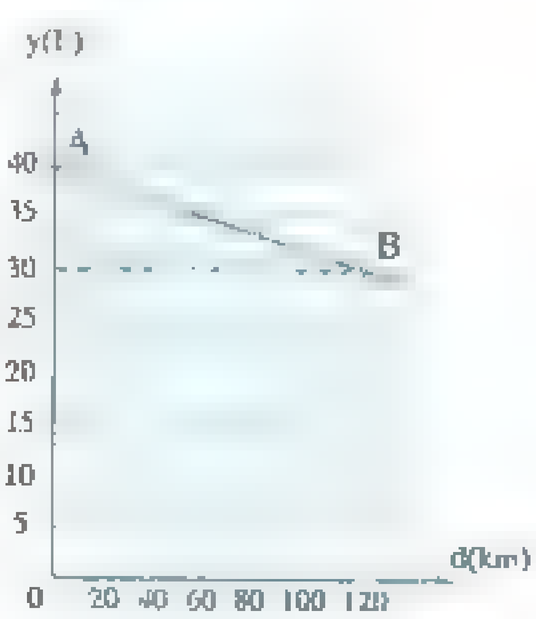
We express this by the point A (0 , 40)

∴ After covering distance.

The amount of remained fuel

$= \frac{3}{4} \times 40 = 30$ litre.

We expresses this by the point B (120 , 30)



∴ The rate of decreasing the amount of fuel

= the slope of $\overleftrightarrow{AB}$

$= \frac{30-40}{120-0} = \frac{-10}{120} = -\frac{1}{12}$

∴ The amount of fuel decreases with the rate one litre for every 12 km.

∴ The distance covered by the car when the tank becomes empty = 12 × 40 = 480 km.

13

1 100 km.

2 The train A took 2 hours

the train B took $2\frac{1}{2}$ hours.

3 The average speed $= \frac{\text{total distance}}{\text{total time}}$

with respect to the train A

The average speed $= \frac{100}{2} = 50$ km./hr.

with respect to the train B

the average speed $= \frac{100}{2.5} = 40$ km./hr.

4 It means that the train A was at rest from half past ten till half past eleven

---

1 The velocity of the bicycle = the slope of the straight line $= \frac{200-125}{8-2} = 12.5$ km./h.

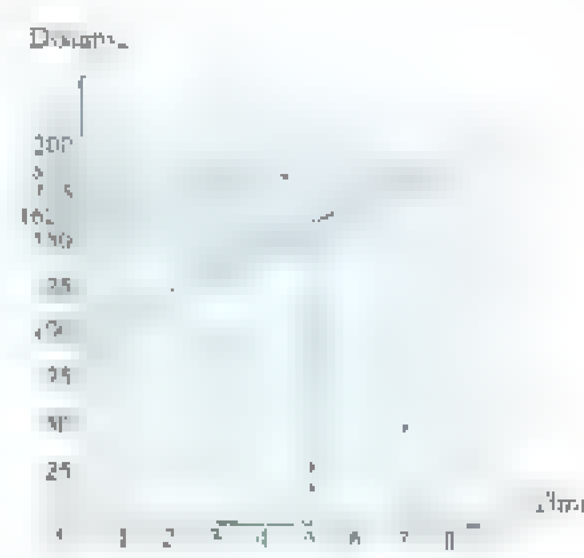
2 300 minutes = 5 hours ,

then the bicycle is at distance = 162.5 km.

3 7 hours.

4 From the graph.

the starting point is far from the fixed point = 100 km.



Answers of unit three

14

1

Sets	Tallies	Freq
30 –	////	4
40 –	~~////~~	5
50 –	~~////~~ //	7
60 –	~~////~~ ///	8
70	~~////~~ /	6
80 –	////	4
90 –	~~////~~ /	6
Total		40

Sets	Freq
30 –	4
40 –	5
50 –	7
60 –	8
70 –	6
80 –	4
90 –	6
Total	40

The set which has the highest frequency is 60 –

The sets which have the lowest frequency are 80 – , 30 –

2

1

Sets	Tallies	Freq.
20 –	///	3
24 –	//	2
28 –	~~////~~ /	6
32 –	~~////~~ //	7
36 –	~~////~~ ~~////~~ //	12
Total		30

Sets	Freq.
20 –	3
24 –	2
28 –	6
32 –	7
36 –	12
Total	30

2 12 students.

3 The least height = 165 cm.

The greatest height = 200 cm.

The range = 200 – 165 = 35 cm.

Sets	Tallies	Freq.
165 –	~~////~~ ///	8
170 –	~~////~~ ~~////~~	10
175 –	~~////~~ ~~////~~ ~~////~~	15
180 –	~~////~~ /	6
185 –	~~////~~ ~~////~~	10
190 –	////	4
195 –	/	1
200 –	/	1
Total		55

Sets	Freq.
165 –	8
170 –	10
175 –	15
180 –	6
185 –	10
190 –	4
195 –	1
200 –	1
Total	55

1 39 soldiers

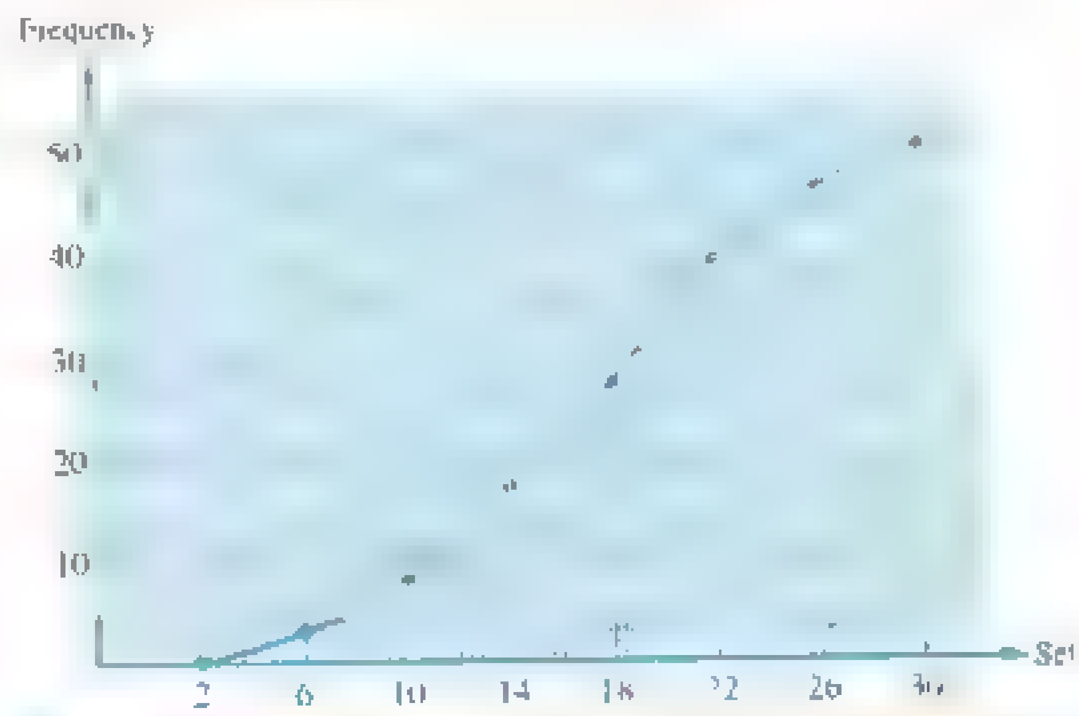
2 $\frac{22}{55} \times 100\% = 40\%$

15

First : Problems on the ascending cumulative frequency curve.

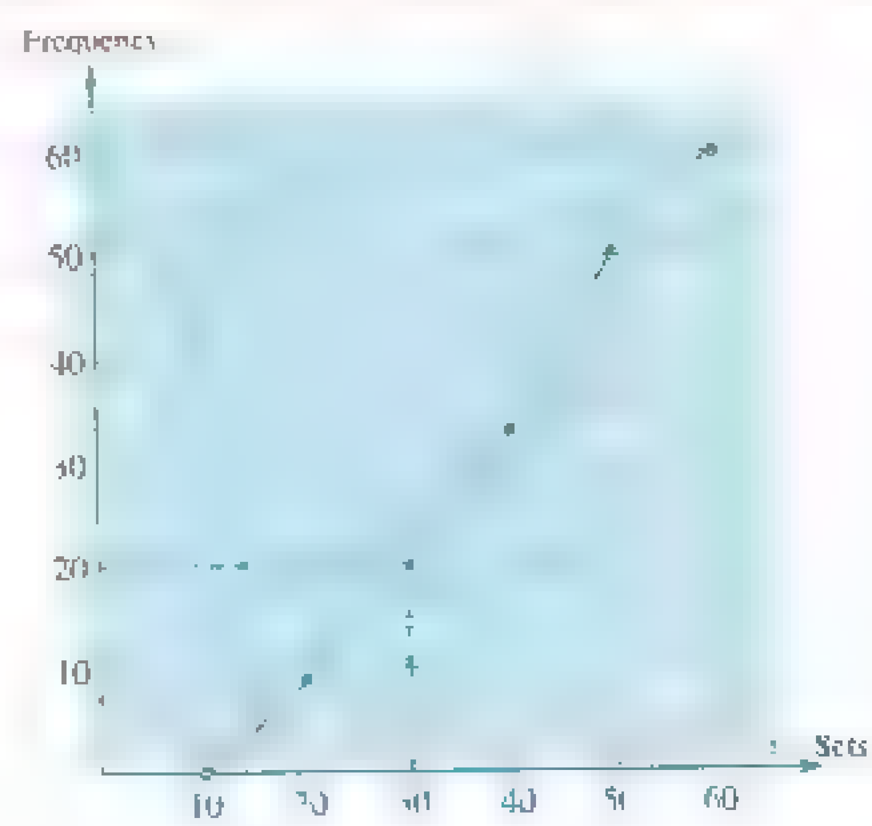
1

The upper boundaries of sets	Ascending cumulative frequency
less than 2	0
less than 6	3
less than 10	8
less than 14	17
less than 18	27
less than 22	39
less than 26	46
less than 30	50



2

The upper boundaries of sets	Ascending cumulative frequency
less than 10	0
less than 20	9
less than 30	20
less than 40	33
less than 50	50
less than 60	60

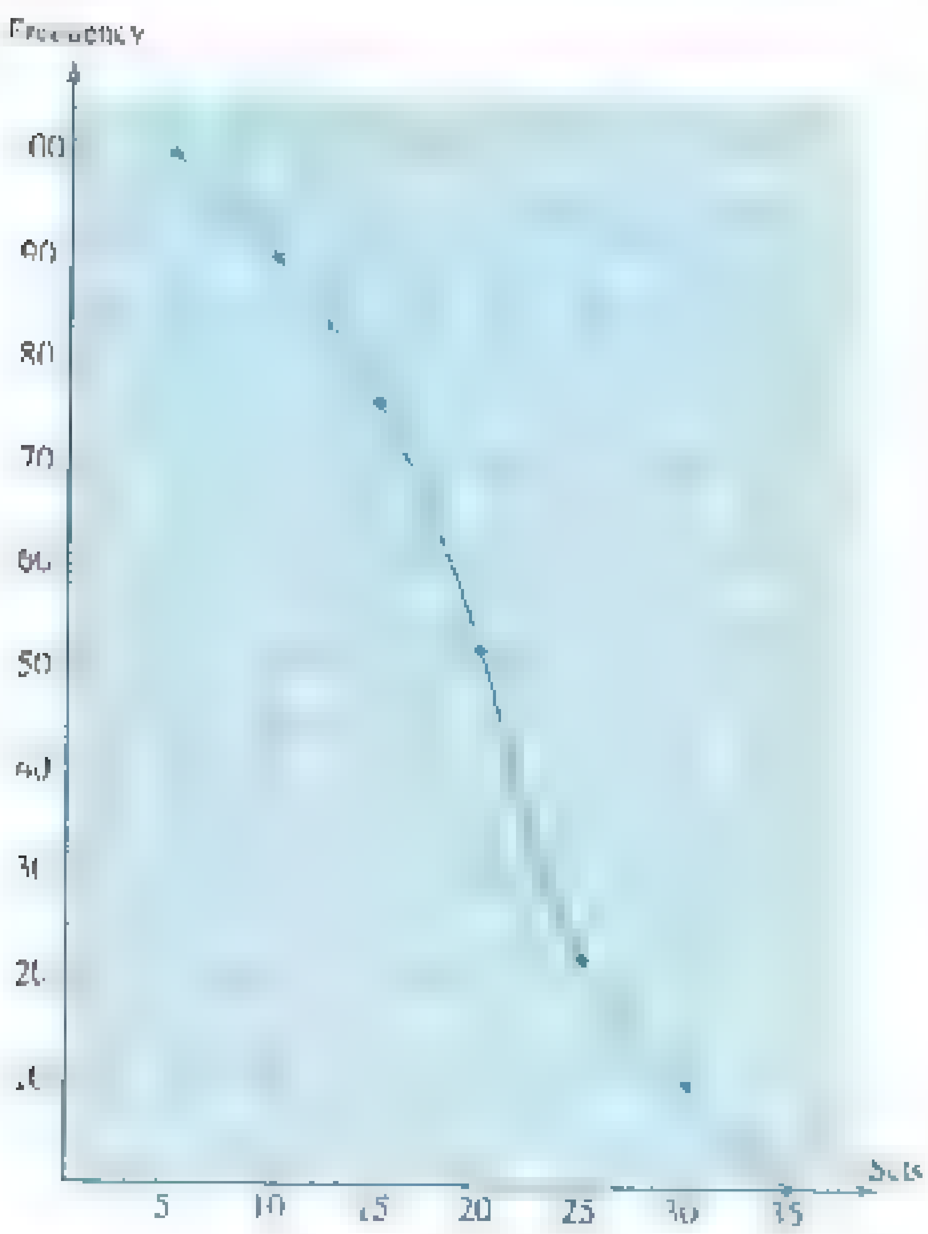


From the graph :

The number of failed pupils = 20 pupils.

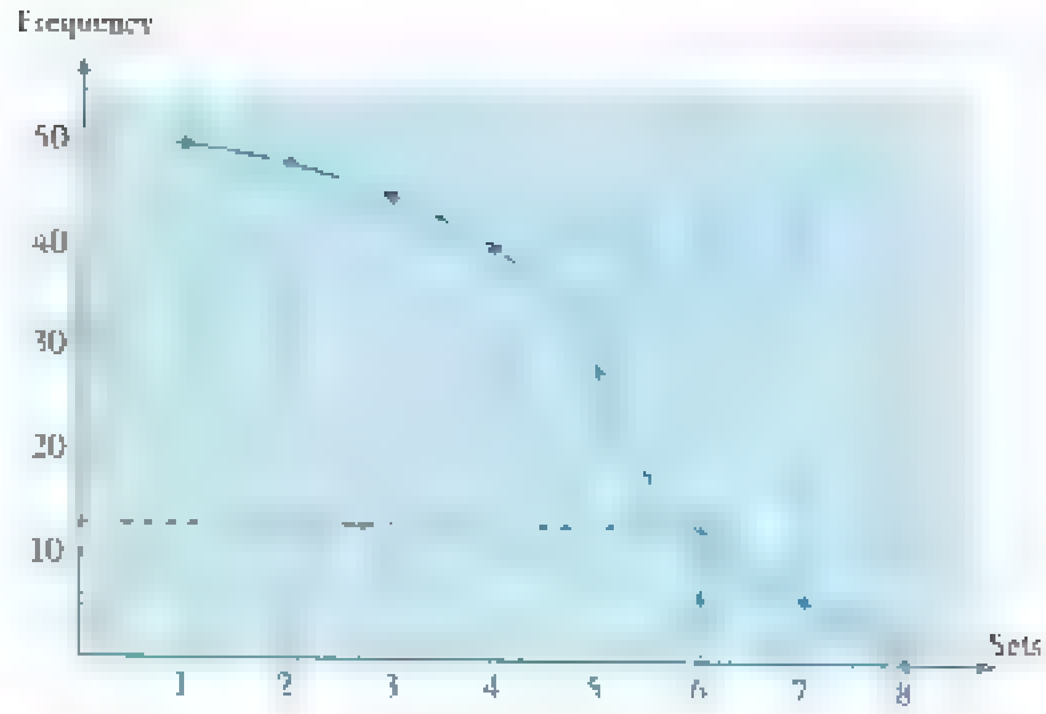
Second : Problems on the descending cumulative frequency curve.

The lower limits of sets	Descending cumulative frequency
5 and more	100
10 and more	90
15 and more	76
20 and more	52
25 and more	22
30 and more	10
35 and more	0



1

The lower boundaries of sets	Descending cumulative frequency
1 and more	50
2 and more	48
3 and more	45
4 and more	40
5 and more	28
6 and more	13
7 and more	6
8 and more	0

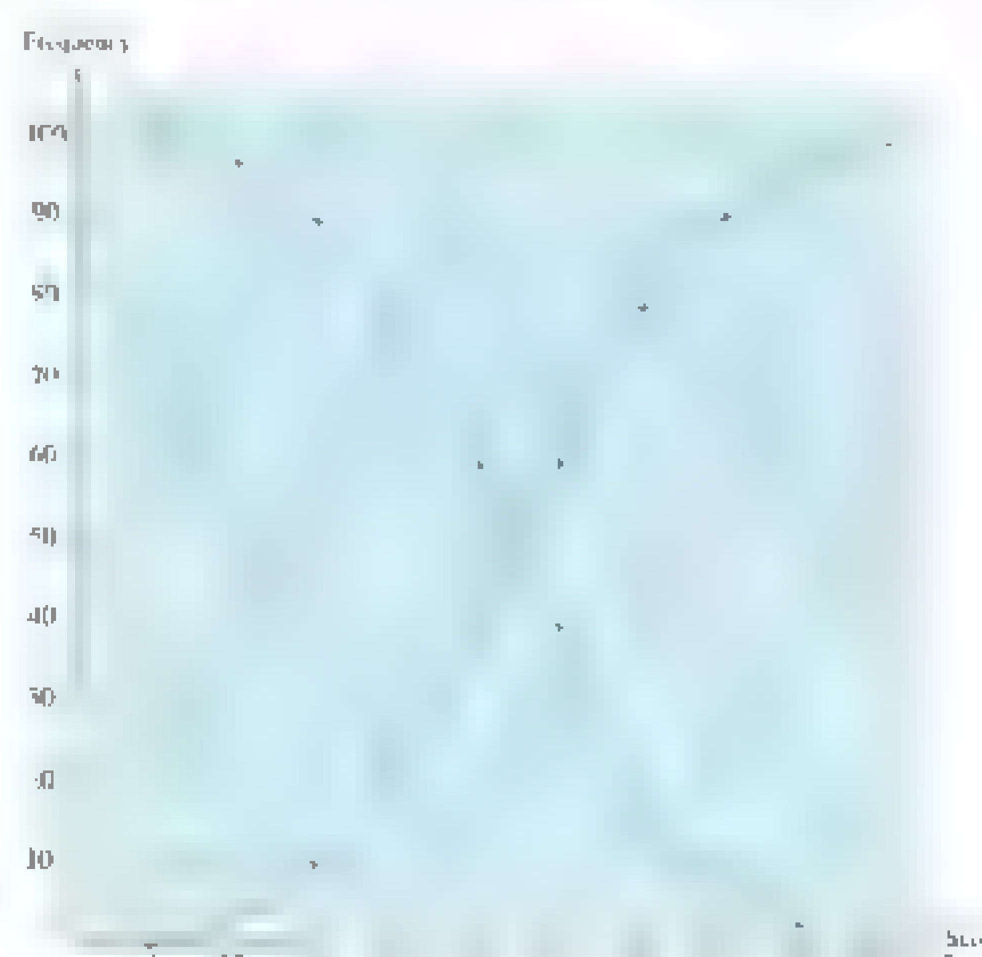


2 **From the graph :** The number of pupils who study 6 hours and more daily = 13 pupils.

3 The percentage of the number of pupils who study 6 hours and more daily = $\frac{13}{50} \times 100\ \% = 26\ \%$

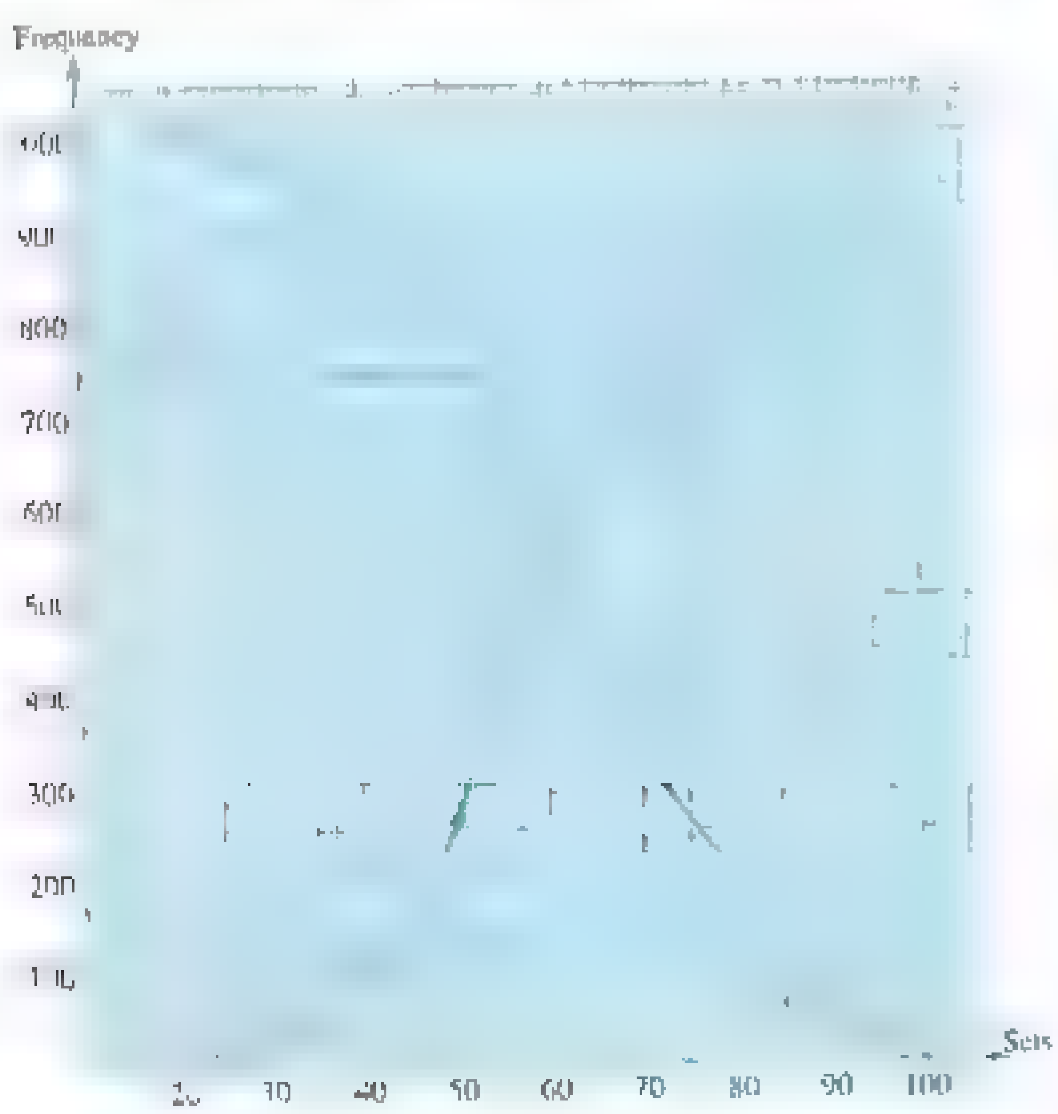
Third : Problems on the two curves together.

The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 8	0	8 and more	100
less than 12	4	12 and more	96
less than 16	11	16 and more	89
less than 20	23	20 and more	77
less than 24	41	24 and more	59
less than 28	61	28 and more	39
less than 32	80	32 and more	20
less than 36	91	36 and more	9
less than 40	97	40 and more	3
less than 44	100	44 and more	0



1

The upper boundaries of sets	Ascending cumulative frequency	The lower boundaries of sets	Descending cumulative frequency
less than 20	0	20 and more	1000
less than 30	30	30 and more	970
less than 40	100	40 and more	900
less than 50	260	50 and more	740
less than 60	520	60 and more	480
less than 70	670	70 and more	330
less than 80	800	80 and more	200
less than 90	910	90 and more	90
less than 100	1000	100 and more	0



2 740 students. 3 140 students.

1 $\frac{\text{The sum of values}}{\text{Number of values}}$

2 Its lower limit + its upper limit

3 10 4 11 5 14 6 3940

1 c 2 d 3 c 4 b 5 a

Sets	Centre of Sets "X"	Frequency "f"	Center of sets × frequency "$X \times f$"
5 –	10	6	60
15 –	20	8	160
25 –	30	4	120
35 –	40	2	80
Total		20	420

$\therefore$ The mean $= \frac{420}{20} = 21$

1

Sets	"X"	"f"	"$X \times f$"
10 –	15	1	15
20 –	25	2	50
30 –	35	4	140
40 –	45	2	90
50 –	55	1	55
Total		10	350

$\therefore$ The mean of marks of students $= \frac{350}{10} = 35$ marks.

2 The number of failed students = 3 students.

Sets	"X"	"f"	"$X \times f$"
15 –	20	2	40
25 –	30	3	90
35 –	40	5	200
45 –	50	8	400
55 –	60	6	360
65 –	70	4	280
75 –	80	2	160
Total		30	1530

$\therefore$ The mean $= \frac{1530}{30} = 51$

Sets	"X"	"f"	$X \times f$
140 –	142	12	1704
144 –	146	20	2920
148 –	150	38	5700
152 –	154	22	3388
156 –	158	17	2686
160 –	162	11	1782
Total		120	18180

$\therefore$ The mean $= \frac{18180}{120} = 151.5$ cm.

7

1 25 – , 10

2

Sets	"X"	"f"	"$X \times f$"
5 –	10	3	30
15 –	20	10	200
25 –	30	12	360
35 –	40	10	400
45 –	50	5	250
Total		40	1240

$\therefore$ The mean $= \frac{1240}{40} = 31$ marks.

3 The number of students whose marks are not less than 35 = 15 students

8 The missed number is 5

Sets	"X"	"f"	"$X \times f$"
6 –	8	2	16
10 –	12	3	36
14 –	16	5	80
18 –	20	8	160
22 –	24	6	144
26 –	28	4	112
30 –	32	2	64
Total		30	612

$\therefore$ The mean $= \frac{612}{30} = 20.4$ kg.

9

1 $3k + 4k = 50 - (7 + 10 + 8 + 4)$

$\therefore 7k = 21 \qquad \therefore k = \frac{21}{7} = 3$

2

Sets	"X"	"f"	"$X \times f$"
30 –	32.5	7	227.5
35 –	37.5	9	337.5
40 –	42.5	12	510
45 –	47.5	10	475
50 –	52.5	8	420
55	57.5	4	230
Total		50	2200

$\therefore$ The mean $= \frac{2200}{50} = 44$ kg.

10

1 $k - 2 = 50 - (4 + 5 + 8 + 7 + 5 + 1)$

$\therefore k - 2 = 20 \qquad \therefore k = 22$

2

Sets	"X"	"f"	"$X \times f$"
2 –	4	4	16
6 –	8	5	40
10 –	12	8	96
14 –	16	20	320
18 –	20	7	140
22 –	24	5	120
26 –	28	1	28
Total		50	760

$\therefore$ The mean $= \frac{760}{50} = 15.2$ days.

11 $\because$ The total of marks of the student in 5 months

$= 5 \times 23.8 = 119$ marks.

, let the required mark of the sixth month be X

$\therefore \frac{119 + X}{6} = 24 \qquad \therefore 119 + X = 144$

$\therefore X = 144 - 119 = 25$ marks.

$\therefore$ The mark of the student in the 6^{th} month is 25

12 $\because$ The total of marks of Magdi in 4 exams

$= 4 \times 16 = 64$ marks

, let the required mark be $X \qquad \frac{64 + X}{5} = 18$

$\therefore 64 + X = 90 \quad \therefore X = 90 - 64 \quad \therefore X = 26$ marks.

$\therefore$ The mark of Magdi in the 5^{th} exam should be 26 marks.

17

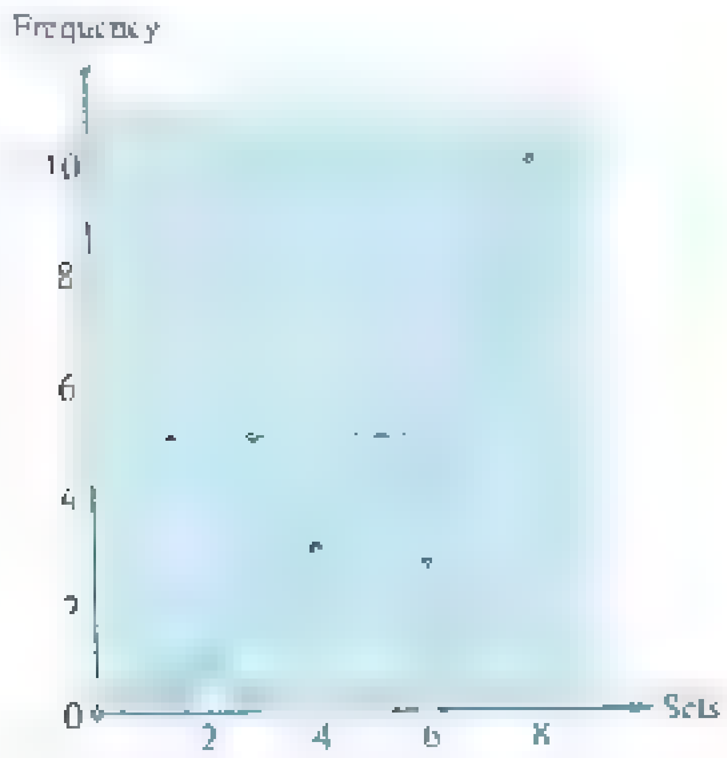
1

1 b 2 c 3 a 4 d

5 b 6 d 7 d

2

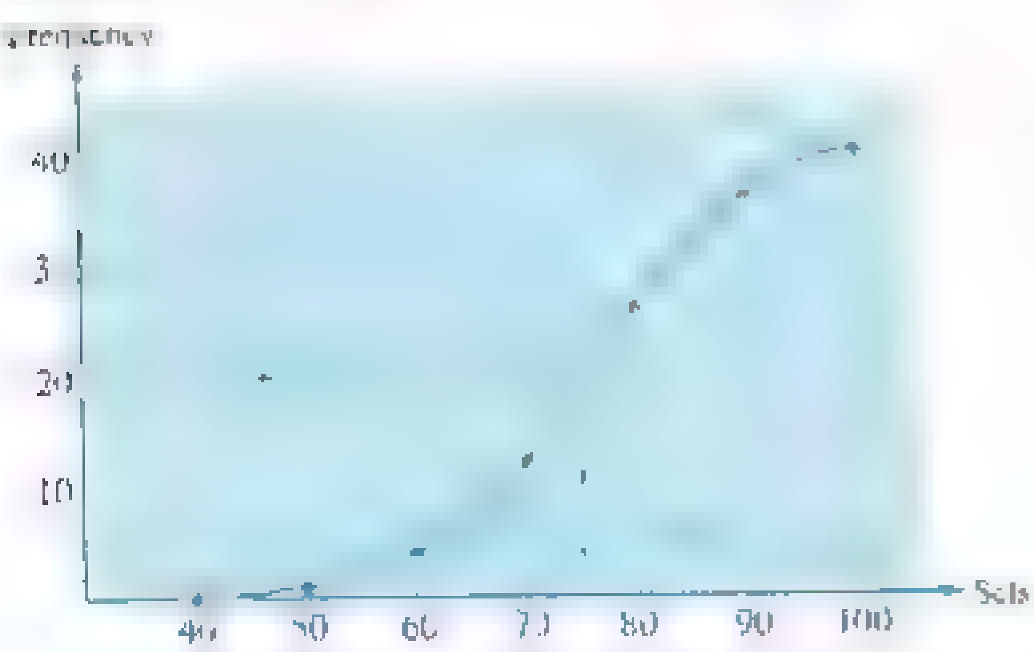
The upper limits of sets	Ascending cumulative frequency
less than 0	0
less than 2	1
less than 4	3
less than 6	5
less than 8	10



∵ The order of the median $= \frac{10}{2} = 5$

∴ The median = 6

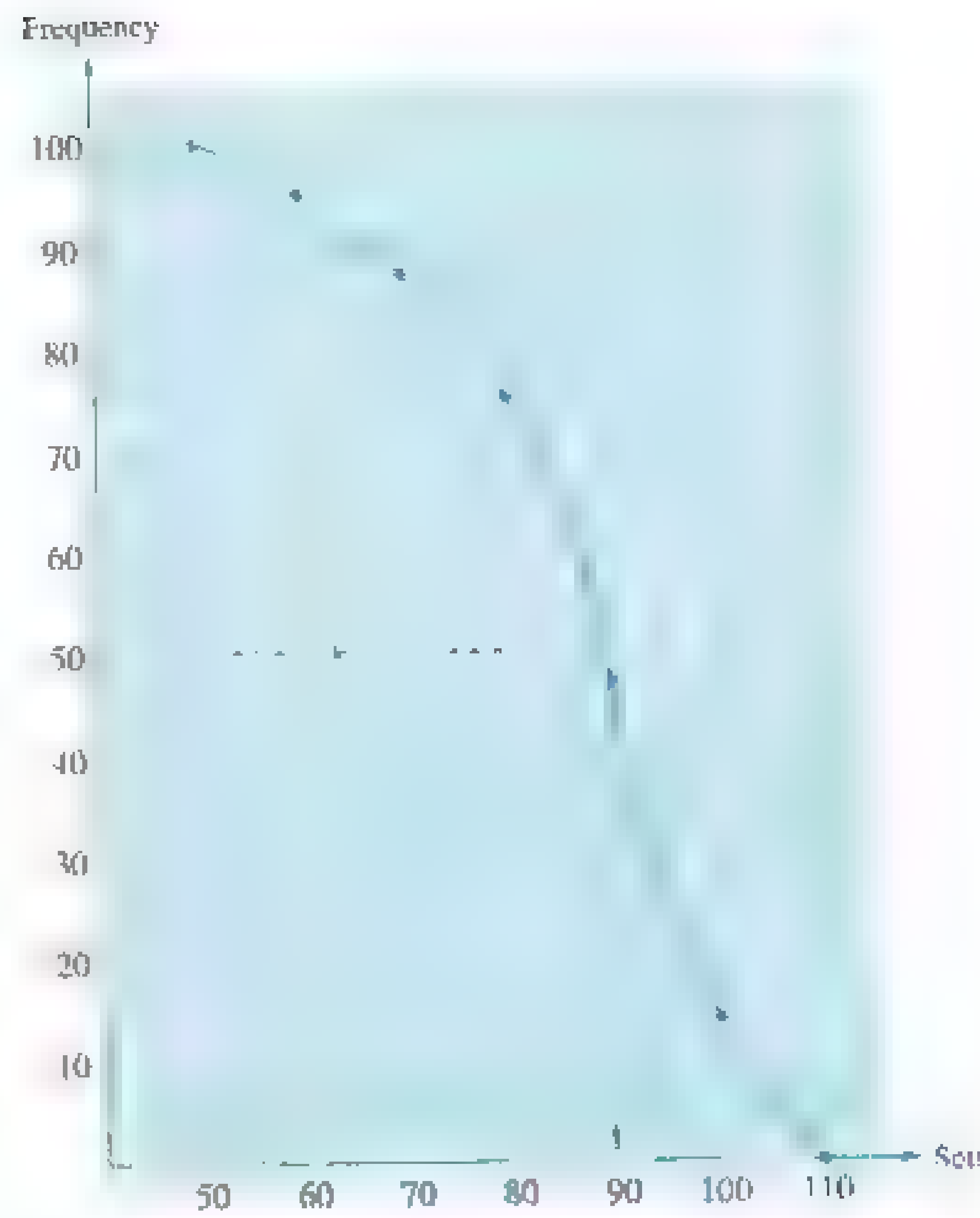
The upper boundaries of sets	Ascending cumulative frequency
less than 40	0
less than 50	1
less than 60	4
less than 70	12
less than 80	26
less than 90	36
less than 100	40



∵ The order of the median $= \frac{40}{2} = 20$

∴ The percentage of intelligence ≈ 75%

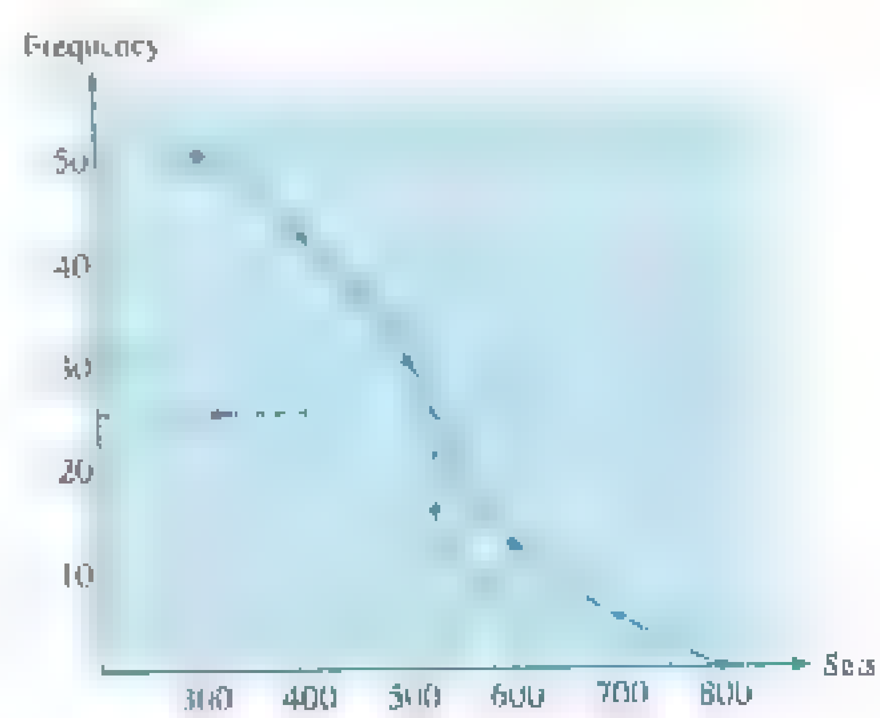
The lower boundaries of sets	Descending cumulative frequency
50 and more	100
60 and more	95
70 and more	87
80 and more	75
90 and more	47
100 and more	14
110 and more	0



∵ The order of the median $= \frac{100}{2} = 50$

∴ The median of working hours ≈ 89.5 hours.

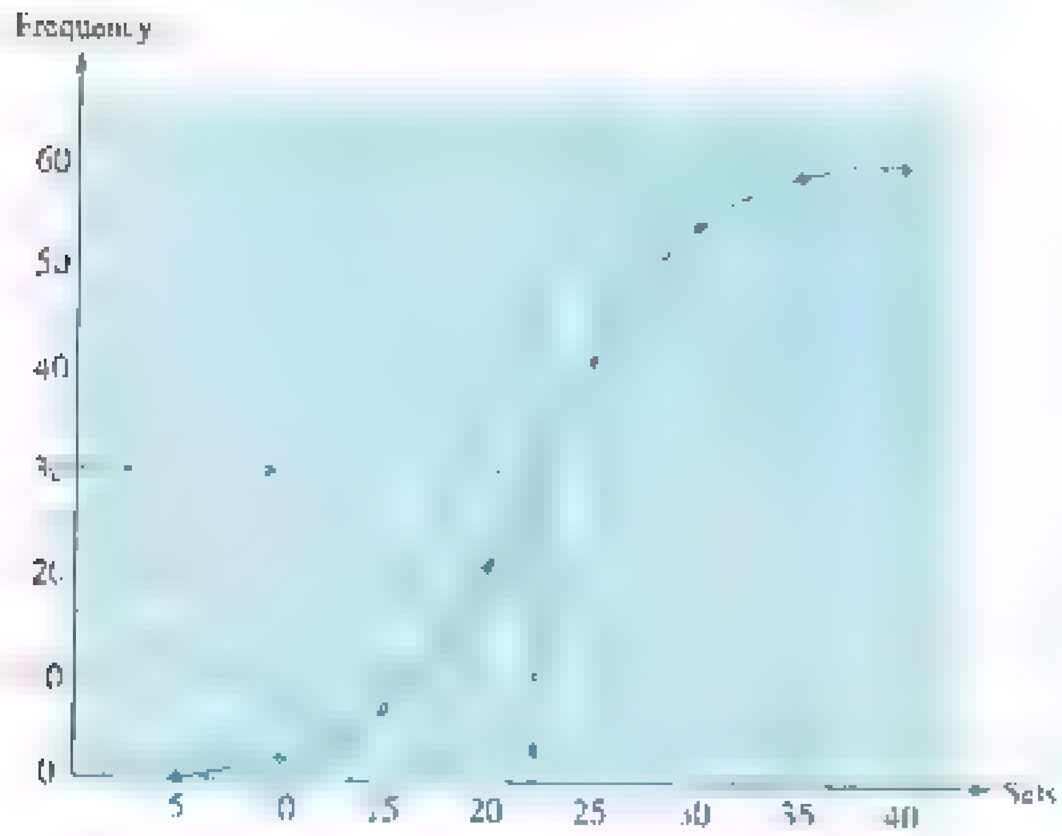
The lower boundaries of sets	Descending cumulative frequency
300 and more	50
400 and more	42
500 and more	30
600 and more	12
700 and more	5
800 and more	0



∵ The order of the median $= \frac{50}{2} = 25$

∴ The median wage = 520 pounds.

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 10	2
less than 15	7
less than 20	21
less than 25	41
less than 30	54
less than 35	59
less than 40	60

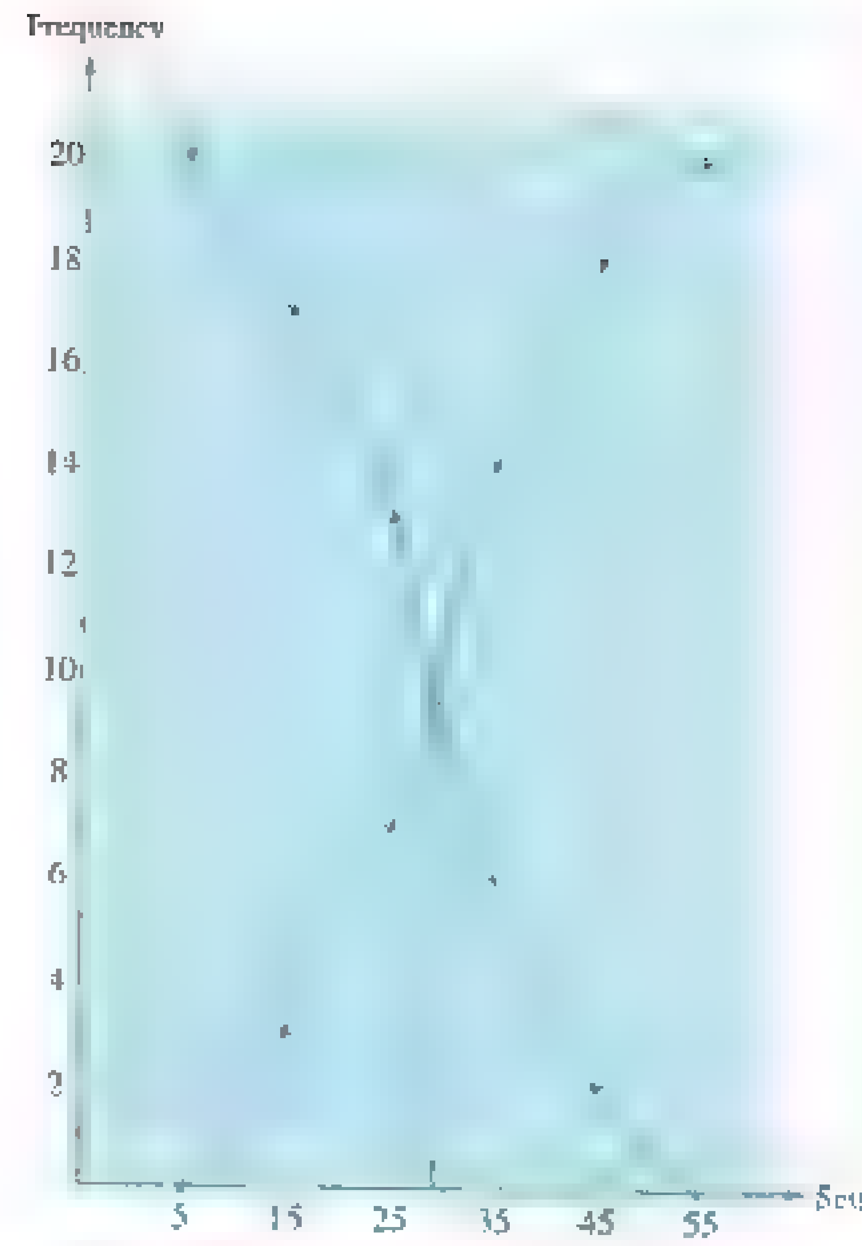


∵ The order of the median $= \frac{60}{2} = 30$

∴ The median mark ≈ 22 marks.

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 15	3
less than 25	7
less than 35	14
less than 45	18
less than 55	20

The lower limits of sets	Descending cumulative frequency
5 and more	20
15 and more	17
25 and more	13
35 and more	6
45 and more	2
55 and more	0



From the graph we find that the median weight ≈ 29 kg.

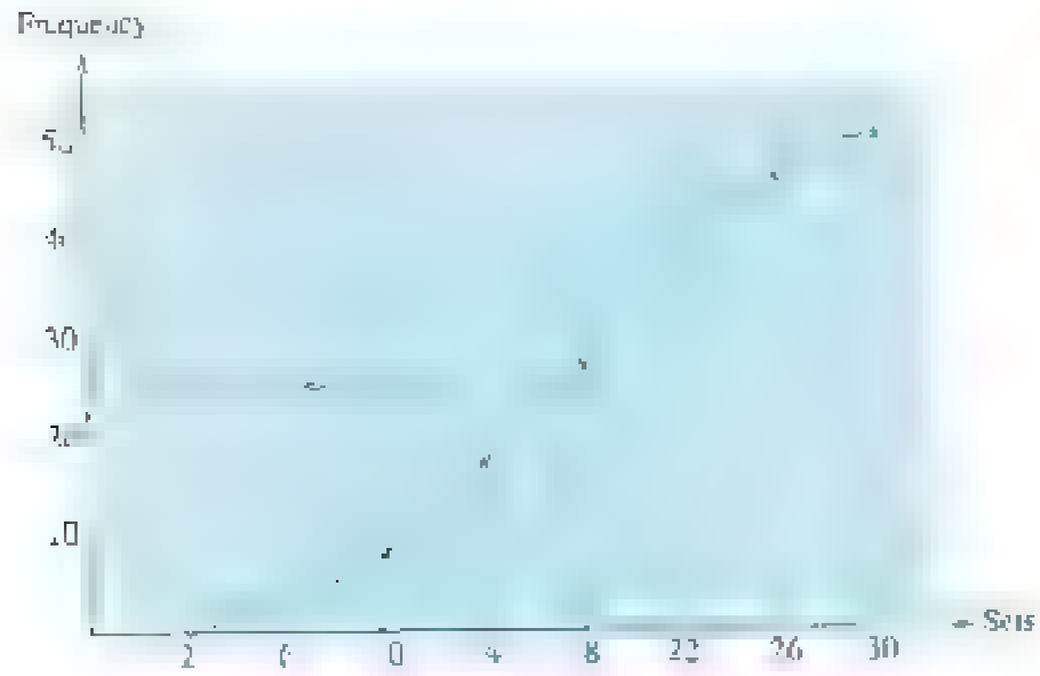
[1]

Sets	"X"	f"	"X × f"
2 –	4	3	12
6 –	8	5	40
10 –	12	9	108
14 –	16	10	160
18 –	20	12	240
22 –	24	7	168
26 –	28	4	112
Total		50	840

∴ The mean $= \frac{840}{50} = 16.8$

[2] We form the ascending cumulative frequency table

The upper limits of sets	Ascending cumulative frequency
less than 2	0
less than 6	3
less than 10	8
less than 14	17
less than 18	27
less than 22	39
less than 26	46
less than 30	50



$\because$ The order of the median $= \frac{50}{2} = 25$

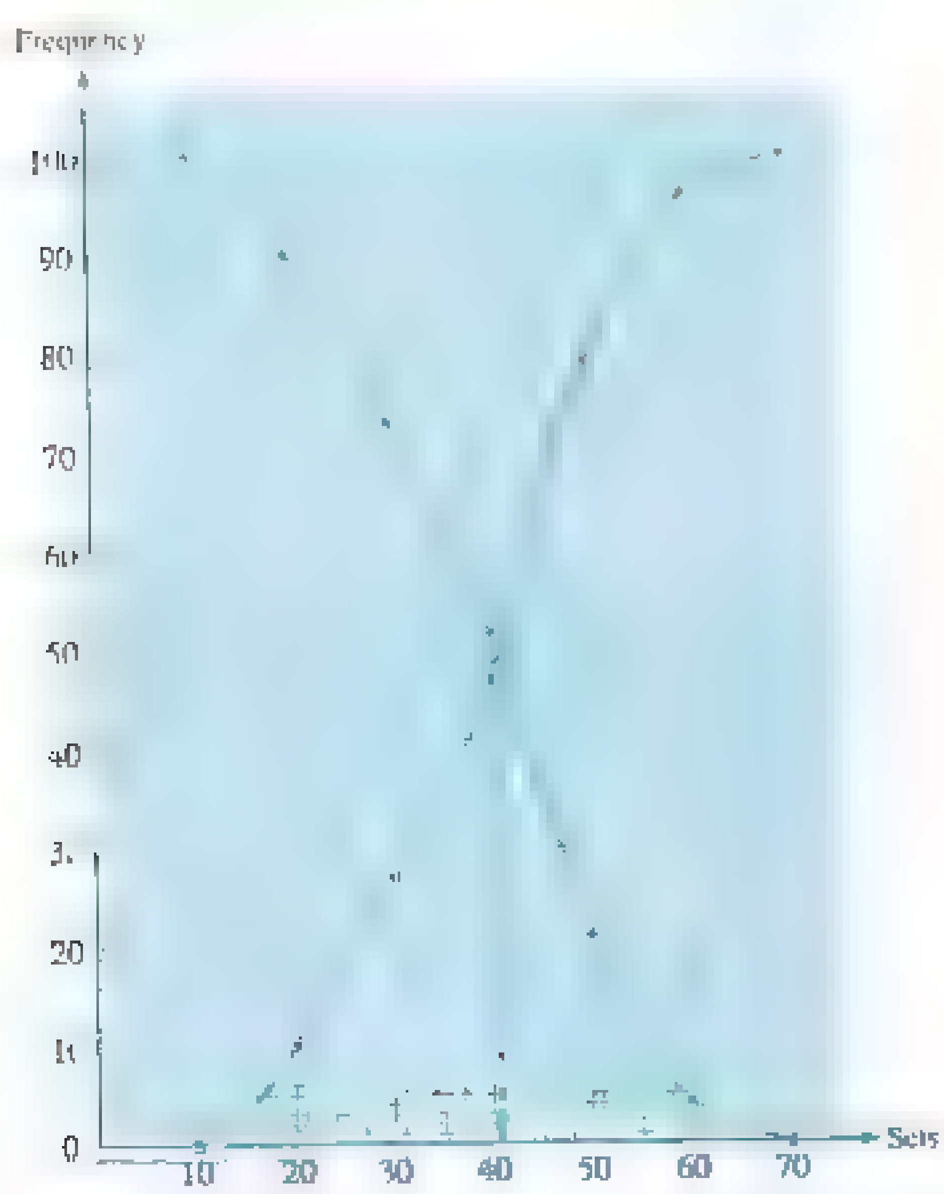
$\therefore$ The median $\simeq 17.6$

1 $X = 30$, $k + 2 = 100 - (10 + 17 + 20 + 32 + 4)$

$\therefore k + 2 = 17 \quad \therefore k = 15$

2

The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 10	0	10 and more	100
less than 20	10	20 and more	90
less than 30	27	30 and more	73
less than 40	47	40 and more	53
less than 50	79	50 and more	21
less than 60	96	60 and more	4
less than 70	100	70 and more	0

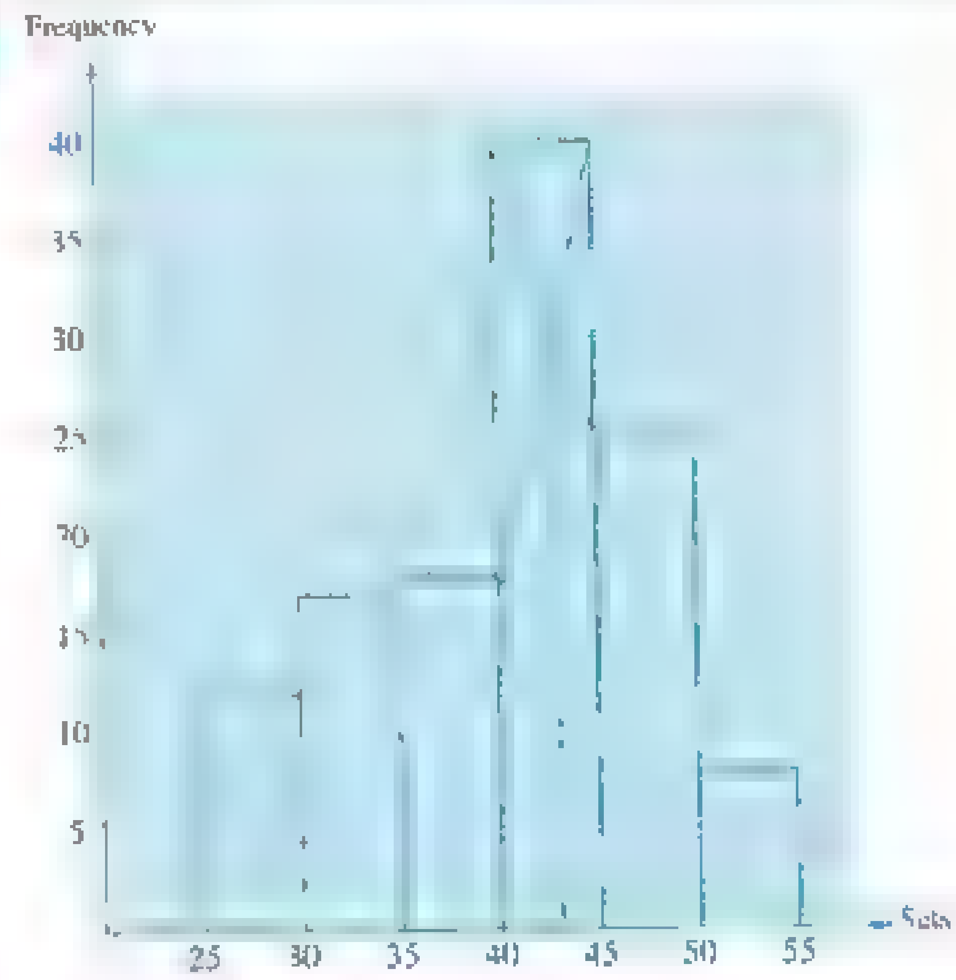


The median $\simeq 41$

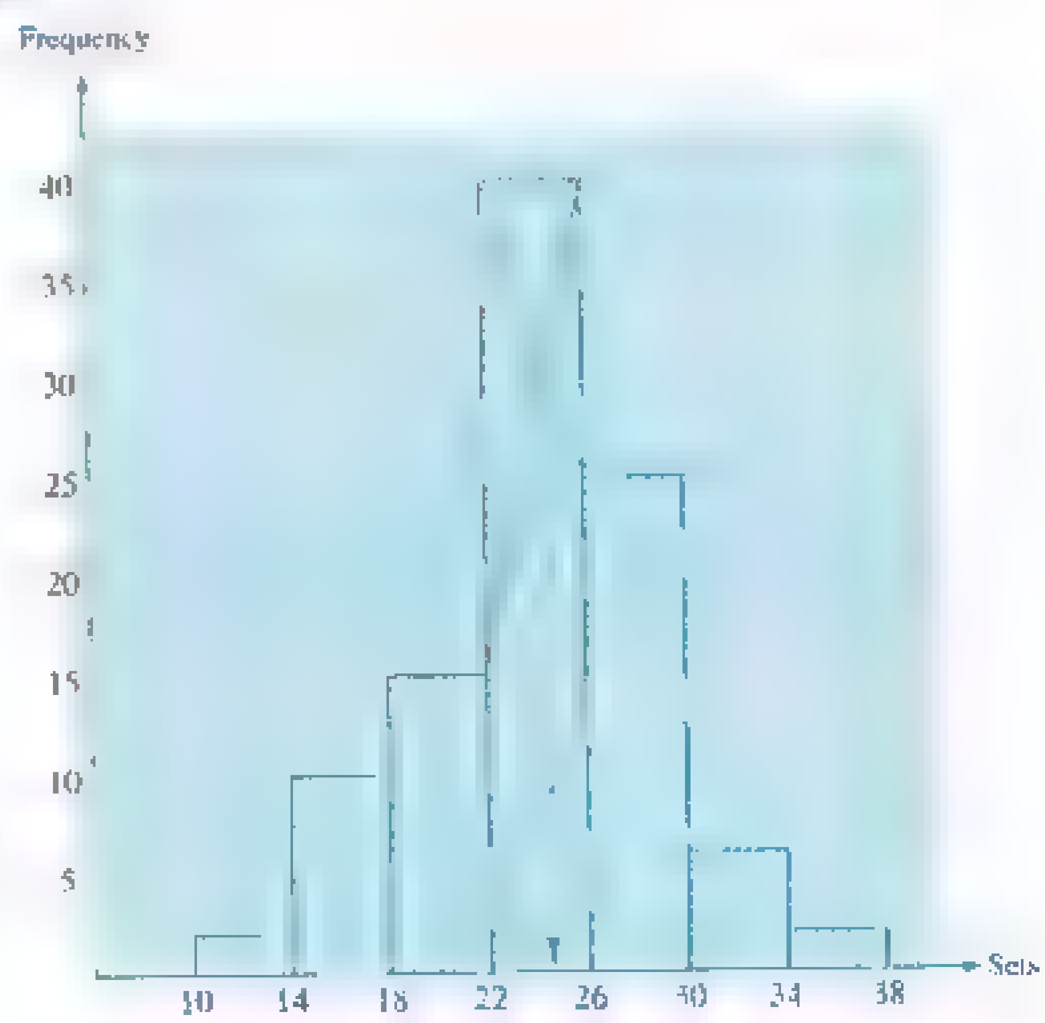
13

1 b   2 b   3 a   4 c

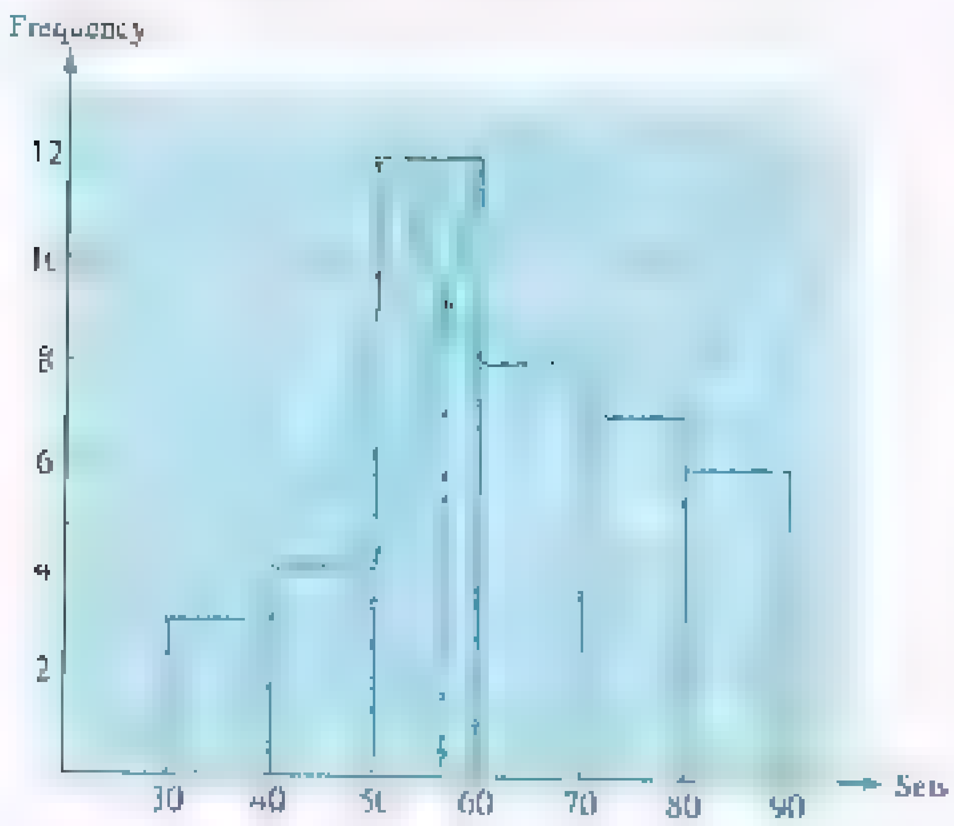
5 d   6 b   7 d



From the graph : The mode age = 43 years.



From the graph : The mode mark $\simeq$ 24.5 marks.



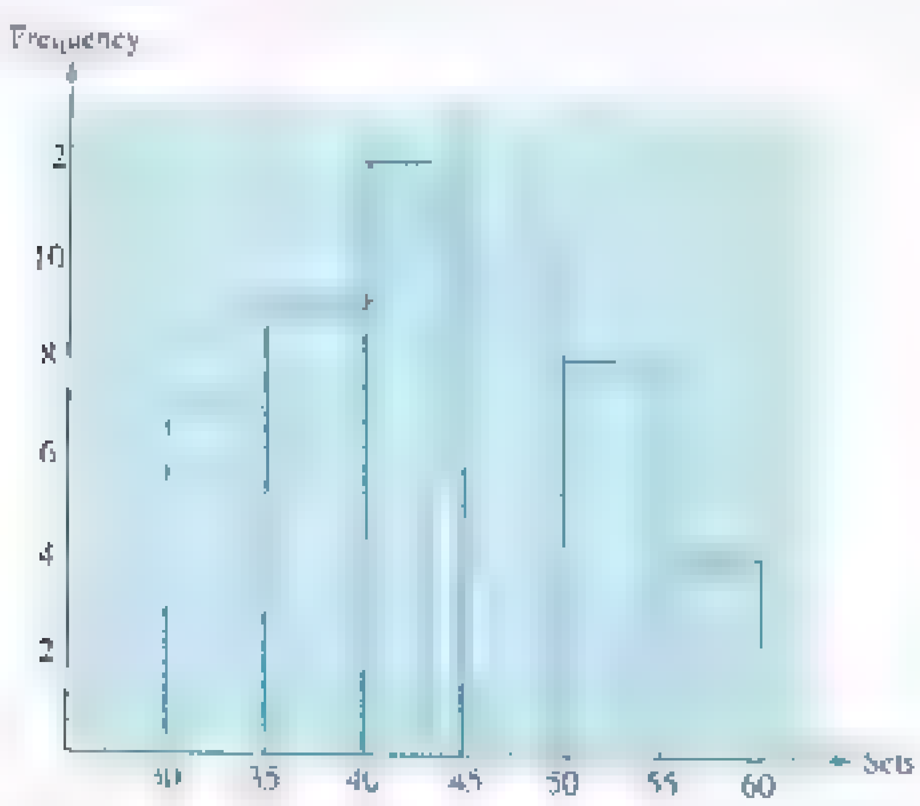
From the graph : The mode = 57 marks.

1. $\because k+4+3k+4k+3k+1+3k-1+k+1=50$

$\therefore 15k+5=50 \quad \therefore 15k=45 \quad \therefore k=3$

2

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
number of students	7	9	12	10	8	4	50



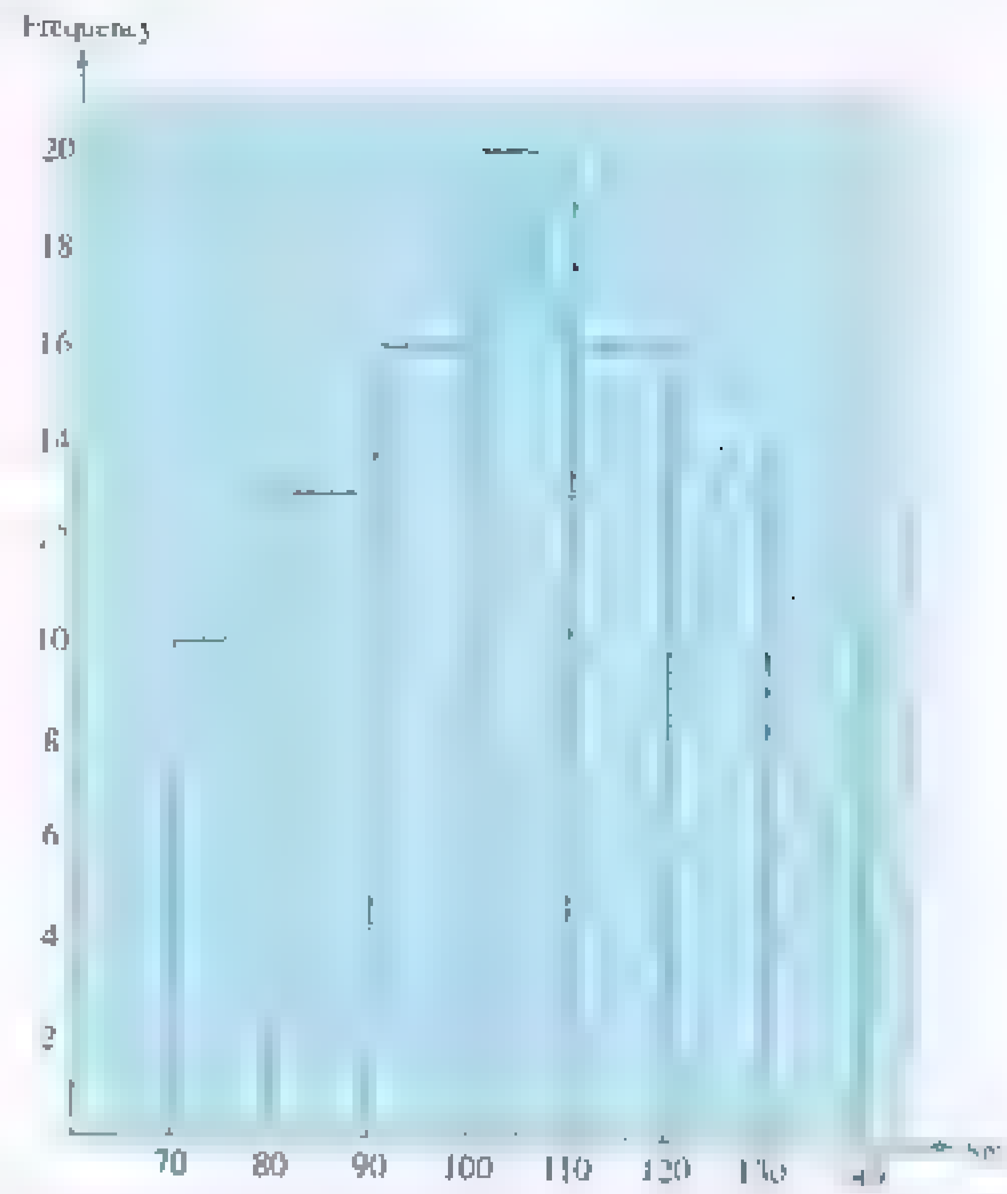
From the graph : The mode weight ≈ 43 kg.

1 $X = 110$

, $k-4=100-(10+13+20+16+14+11)$

$\therefore k-4=16 \quad \therefore k=20$

2 ,



From the graph : The mode wage = L.E. 105

1 $\because 3k+4k=50-(7+10+8+4)$

$\therefore 7k=21 \quad \therefore k=\frac{21}{7}=3$

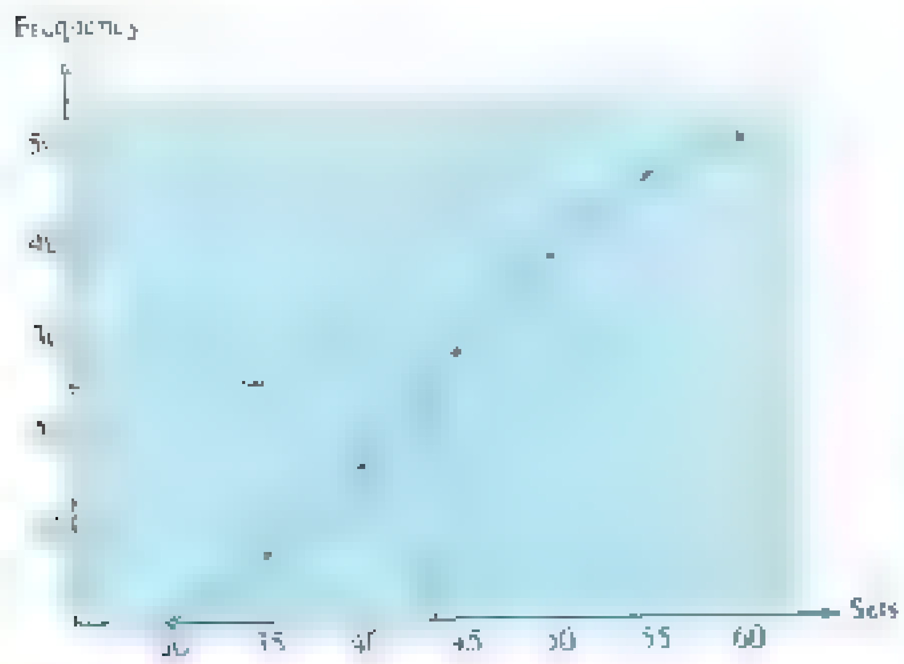
2

Sets	"X"	"f"	"$X \times f$"
30 –	32.5	7	227.5
35 –	37.5	9	337.5
40 –	42.5	12	510
45 –	47.5	10	475
50 –	52.5	8	420
55 –	57.5	4	230
Total		50	2200

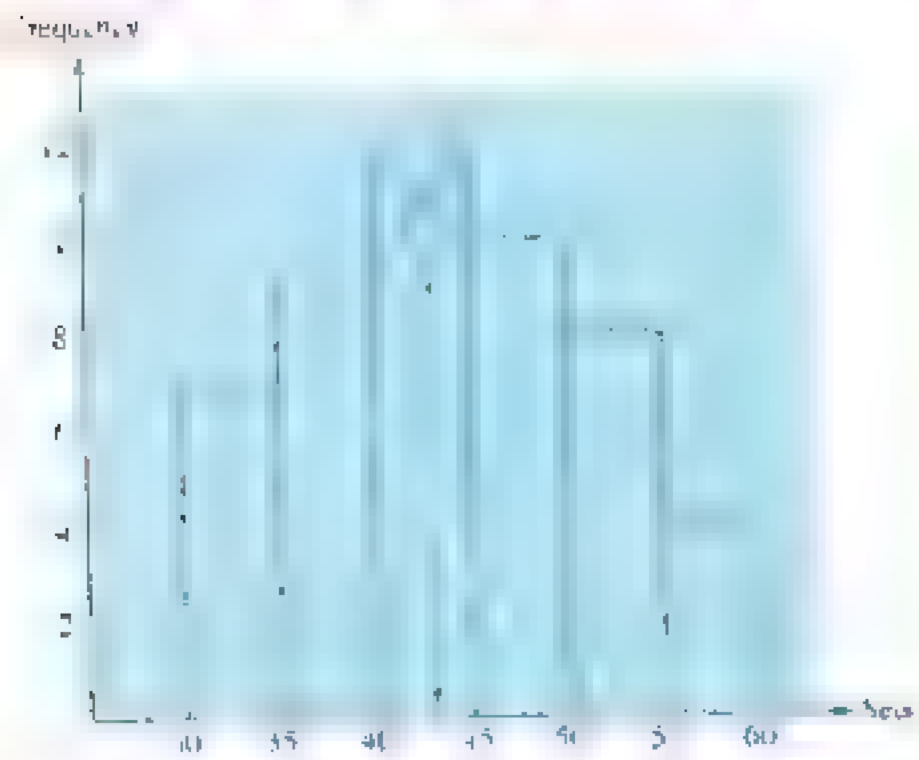
, The mean $=\frac{2200}{50}=44$ kg.

3

The upper limits of sets	Ascending cumulative frequency
less than 30	0
less than 35	7
less than 40	16
less than 45	28
less than 50	38
less than 55	46
less than 60	50



4



From the graph : The mode weight = 43 kg.

5 The order of the median = $\frac{50}{2} = 25$

∴ The median ≈ 43.5 kg.

Answers of accumulative basic skills

1 6	2 0	3 15	4 154	5 21
6 $\frac{2}{3}$	7 7500	8 4	9 12	10 9
11 27	12 6 , 8 , 2			

1 c	2 c	3 a	4 a	5 d
6 c	7 d	8 b	9 d	10 d
11 c	12 a			

Guide Answers

Of Geometry Exercises

Answers of unit four

Answers of Exercise

1

[1] a median [2] 3 [3] one point [4] 1 : 2

[5] 2 : 1 [6] 4 [7] 16

2

[1] d [2] a [3] d [4] d

[5] a [6] d [7] c

3

[1] 8 cm. , 5 cm. [2] 6 cm. , 4 cm. , $\frac{1}{3}$, $\frac{2}{3}$

[3] 6 cm. , 3 cm. , 4 cm. [4] 5 cm. , 12 cm. , 27 cm.

4

∵ $\overline{AD}$, $\overline{BE}$ are two medians in Δ ABC ,

$\overline{AD} \cap \overline{BE} = \{M\}$

∴ M is the point of concurrence of the medians of Δ ABC

∴ $MD = \frac{1}{3} AD = \frac{1}{3} \times 6 = 2$ cm. (1)

, $ME = \frac{1}{3} BE = \frac{1}{3} \times 9 = 3$ cm. (2)

∵ D is the midpoint of $\overline{BC}$, E is the midpoint of $\overline{AC}$ in Δ ABC

∴ $DE = \frac{1}{2} AB = \frac{1}{2} \times 9 = 4.5$ cm. (3)

From (1) , (2) and (3) :

∴ The perimeter of Δ MDE = 2 + 3 + 4.5 = 9.5 cm. (The req.)

5

∵ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

∴ BC = 2 DE ∴ BC = 8 cm.

∵ M is the intersection point of medians of Δ ABC

∴ MC = 2 DM ∴ MC = 6 cm.

, $BM = \frac{2}{3} BE$ ∴ BM = 4 cm.

∴ The perimeter of Δ BMC = 8 + 6 + 4 = 18 cm. (The req.)

6

∵ M is the intersection point of the medians of Δ ABC

∴ $XM = \frac{1}{2} MC = 4$ cm.

∴ The perimeter of Δ MXY = 4 + 5 + 3 = 12 cm. (First req.)

, AM = 2 MY = 6 cm.

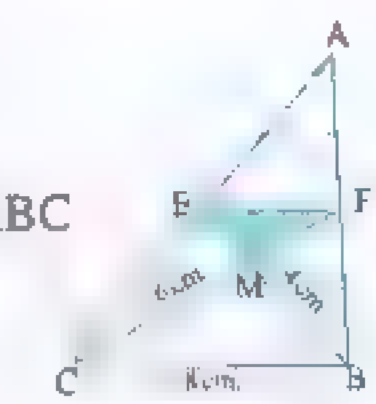
∵ X is the midpoint of AB ,

Y is the midpoint of $\overline{BC}$

∴ AC = 2 XY = 10 cm.

∴ The perimeter of Δ MAC = 6 + 8 + 10 = 24 cm. (Second req.)

7



∵ F is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

∴ $\overline{BE}$, $\overline{CF}$ are two medians in Δ ABC

∴ M is the intersection point of the medians of Δ ABC

∴ $ME = \frac{1}{2} MB = 2$ cm. (1)

, $MF = \frac{1}{2} MC = 3$ cm. (2)

∵ F is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

∴ $FE = \frac{1}{2} BC = 4$ cm. (3)

From (1) , (2) and (3) :

∴ The perimeter of Δ MFE = 2 + 3 + 4 = 9 cm. (The req.)

8

∵ M is the intersection point of medians of Δ ABC

∴ $MF = \frac{1}{2} AM$ (1)

, $MD = \frac{1}{2} MC$ (2)

∵ D is the midpoint of $\overline{AB}$,

F is the midpoint of $\overline{BC}$ in Δ ABC

∴ $DF = \frac{1}{2} AC$ (3)

By adding (1) , (2) and (3) :

∴ $MF + MD + DF = \frac{1}{2} AM + \frac{1}{2} MC + \frac{1}{2} AC$

∴ The perimeter of Δ MFD

$= \frac{1}{2} (AM + MC + AC)$

$= \frac{1}{2}$ the perimeter of Δ AMC

$= \frac{1}{2} \times 36 = 18$ cm (The req.)

9

∵ M is the point of concurrence of the medians of Δ ABC

∴ $\overline{CD}$ is a median in Δ ABC

$\therefore DM = \frac{1}{2} MC = 3$ cm.

, $\because \Delta AMD$ is a right angled triangle at M

$\therefore (AM)^2 = (AD)^2 - (DM)^2 = 25 - 9 = 16$

$\therefore AM = 4$ cm

$\therefore ME = \frac{1}{2} AM = 2$ cm. (The req.)

$\because$ ABCD is a parallelogram

$\therefore$ The two diagonals bisect each other.

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore \overline{DM}$ is a median in ΔADC

$\because DE = 2\ EM$

$\therefore$ E is the intersection point of the medians of ΔADC

$\because E \in \overline{FC}$

$\therefore \overline{CF}$ is a median in ΔACD $\therefore AF = FD$ (Q.E.D.)

$\because$ The two diagonals of the rectangle bisect each other.

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore \overline{BM}$ is a median in ΔABC

$\because$ E is the midpoint of $\overline{AB}$

$\therefore \overline{CE}$ is a median in ΔABC

$\because \overline{CE} \cap \overline{BM} = \{F\}$

$\therefore$ F is the intersection point of the medians of ΔABC

(First req.)

$\therefore BF = \frac{2}{3} BM$ $\therefore 4 = \frac{2}{3} BM$ $\therefore BM = 6$ cm.

$\because$ The two diagonals of the rectangle are equal in length and bisect each other.

$\therefore AM = BM = 6$ cm. (Second req.)

$\because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}$ is a median in ΔABC

$\because AM = \frac{2}{3} AD$

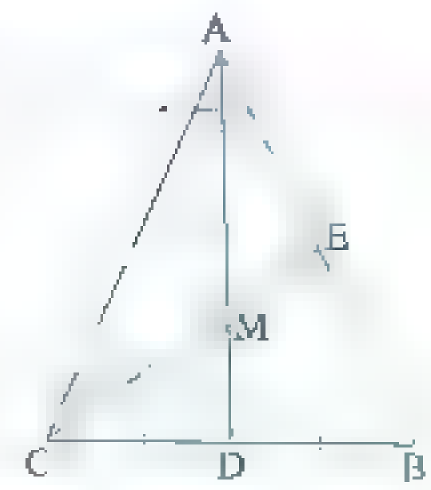
$\therefore$ M is the intersection point of the medians of ΔABC

$\therefore \overline{CF}$ is a median in ΔABC

$\therefore$ F is the midpoint of $\overline{AB}$ $\therefore BF = \frac{1}{2} AB$

$\because AC = AB$ $\therefore BF = \frac{1}{2} AC$ (Q.E.D.)

13



$\because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}$ is a median in ΔABC

$\because AM = 2\ MD$

$\therefore$ M is the intersection point of the medians of ΔABC

$\because M \in \overline{CE}$

$\therefore \overline{CE}$ is a median in ΔABC

$\therefore EM = \frac{1}{3} EC = \frac{1}{3} \times 12 = 4$ cm. (The req.)

14

$\because$ M is the point of concurrence of the medians of ΔABC

$\therefore \overline{CD}$ is a median in ΔABC

$\therefore$ D is the midpoint of $\overline{AB}$

In ΔAMB :

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BM}$

$\therefore \overline{MD}$, $\overline{AE}$ are two medians in ΔAMB

$\therefore$ N is the point of concurrence of the medians of ΔAMB

$\therefore MN = 2\ ND$ $\therefore x + 3 = 2(x - 1)$

$\therefore x + 3 = 2x - 2$ $\therefore 3 + 2 = 2x - x$

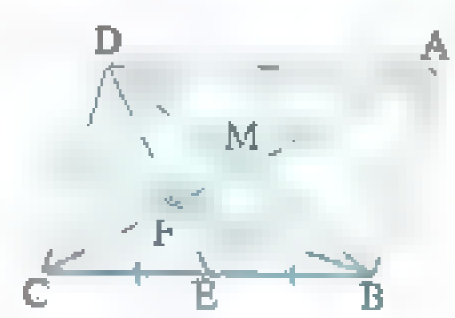
$\therefore x = 5$

$\therefore ND = 5 - 1 = 4$ cm. , $MN = 5 + 3 = 8$ cm.

$\therefore MD = ND + MN = 12$ cm.

, $\because \overline{CD}$ is a median in ΔABC

$\therefore MC = 2\ MD = 24$ cm. (The req.)



$\because$ ABCD is a parallelogram

$\therefore$ The two diagonals bisect each other.

$\therefore$ M is the midpoint of $\overline{BD}$

$\therefore \overline{CM}$ is a median in ΔDBC

$\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{DE}$ is a median in ΔDBC

$\therefore$ F is the intersection point of the medians of ΔDBC

$\therefore \overrightarrow{BF}$ bisects $\overline{CD}$ (Q.E.D. 1)

$\therefore CF = \frac{2}{3} CM$, $\because CM = \frac{1}{2} AC$

$\therefore CF = \frac{2}{3} \times \frac{1}{2} AC = \frac{1}{3} AC$ (Q.E.D. 2)

$\because \overline{AD}$ and $\overline{BE}$ are medians in ΔABC

M is the intersection point of the medians of ΔABC

$\because M \in \overline{CF}$ $\therefore \overline{CF}$ is a median in ΔABC

$\therefore$ F is the midpoint of $\overline{AB}$

In ΔABM :

F is the midpoint of $\overline{AB}$, N is the midpoint of $\overline{BM}$

$\therefore \overline{NF} \parallel \overline{AM}$ $\therefore \overline{NF} \parallel \overline{MD}$ (1)

In ΔBMC :

$\because$ D is the midpoint of $\overline{BC}$, N is the midpoint of $\overline{BM}$

$\overline{ND} \parallel \overline{CM}$ $\therefore \overline{ND} \parallel \overline{MF}$ (2)

From (1) and (2) :

The figure FNDM is a parallelogram. (Q.E.D.)

D is the midpoint of $\overline{BC}$

$\overline{AD}$ is a median in ΔABC

, AM = 2 MD , $M \in \overline{AD}$

M is the intersection point of the medians of ΔABC

, $\because M \in \overline{BE}$

$\overline{BE}$ is a median in ΔABC $\therefore$ BM = 2 ME

. BM = 4 cm. . BE = 2 + 4 = 6 cm.

, $\because \Delta BCE$ in which :

D is the midpoint of $\overline{BC}$, $\overline{DF} \parallel \overline{BE}$

F is the midpoint of $\overline{EC}$

$DF = \frac{1}{2} BE = 3$ cm. (The req.)

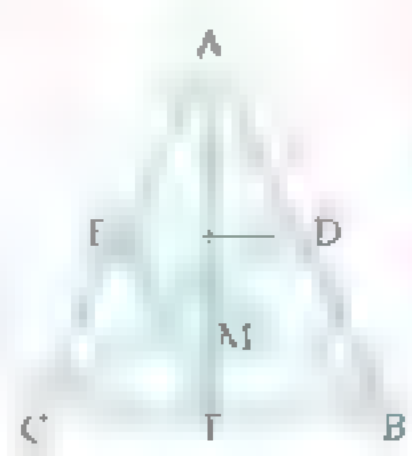
$\overline{CD}$ and $\overline{BE}$ are two medians in ΔABC

M is the intersection point of the medians of ΔABC

$\overline{AF}$ is a median in ΔABC

F is the midpoint of $\overline{BC}$

, E is the midpoint of $\overline{AC}$



$\overline{FE} \parallel \overline{AB}$, $FE = \frac{1}{2} AB$

$\overline{FE} \parallel \overline{BD}$, FE = BD

DBFE is a parallelogram (Q.E.D.)

1 3 | 2 half the length of the hypotenuse

3 right | 4 half the length of the hypotenuse

5 twice | 6 twice

1 3 2 10 3 8 4 18 , 9 , $\frac{1}{3}$, 3

5 5 , 5 , 15 6 9 , 8 , 10 , 27

1 b 2 b 3 c

4 a 5 b 6 a

In ΔADC

$\because m(\angle D) = 90°$, E is the midpoint of $\overline{AC}$

$\therefore DE = \frac{1}{2} AC$ (1)

In ΔABC :

$\because m(\angle B) = 90°$, $m(\angle ACB) = 30°$

$\therefore AB = \frac{1}{2} AC$ (2)

From (1) and (2) :

AB = DE (Q.E.D.)

In ΔLXZ :

$\because$ D is the midpoint of $\overline{LX}$, E is the midpoint of $\overline{LZ}$

$\therefore DE = \frac{1}{2} XZ$ (1)

From ΔXYZ :

$m(\angle Y) = 90°$, M is the midpoint of $\overline{XZ}$

$\therefore YM = \frac{1}{2} XZ$ (2)

From (1) and (2) : $\therefore$ DE = YM (Q.E.D.)

In ΔACD :

E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{CD}$

$\therefore EF = \frac{1}{2} AC$ $\therefore$ AC = 8 cm.

In ΔABC :

$\because m(\angle B) = 90°$, $m(\angle ACB) = 30°$

$\therefore AB = \frac{1}{2} AC = 4$ cm. (The req.)

In ΔABC :

$\because m(\angle BAC) = 90°$, D is the midpoint of $\overline{BC}$

$\therefore BC = 2 AD = 2 \times 3 = 6$ cm.

In Δ CBE : $\because m(\angle CBE) = 90°$, $m(\angle E) = 30°$

$\therefore EC = 2\,BC = 2 \times 6 = 12$ cm.

, $\because$ F is the midpoint of $\overline{CE}$

$\therefore BF = \frac{1}{2} EC = \frac{1}{2} \times 12 = 6$ cm. (The req.)

In Δ ABC :

$\because m(\angle B) = 90°$, $m(\angle ACB) = 60°$

$m(\angle CAB) = 30°$ $\therefore BC = \frac{1}{2} AC$

$\because DE = BC$ $\therefore DE = \frac{1}{2} AC$

$\therefore \overline{DE}$ is a median in Δ ACD

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

In Δ ABC :

$m(\angle B) = 90°$, $m(\angle ACB) = 30°$

$\therefore AB = \frac{1}{2} AC$

, $\because AB = DE = 5$ cm. $\therefore DE = \frac{1}{2} AC$

$\because \overline{DE}$ is a median in Δ ACD

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

In Δ DBC :

$\because$ E is the midpoint of $\overline{BC}$, $\overline{EF} \parallel \overline{BD}$

$\therefore EF = \frac{1}{2} BD$

, $\because AM = EF$ $\therefore AM = \frac{1}{2} BD$

$\therefore \overline{AM}$ is a median in Δ ABD

$\therefore m(\angle BAD) = 90°$ (Q.E.D.)

$\because \angle ADC$ is an exterior angle of Δ ABD

$\therefore m(\angle ADC) = 33° + 27° = 60°$

$\therefore$ In Δ ADC : $m(\angle DAC) = 180° - (60° + 90°) = 30°$

$\therefore DC = \frac{1}{2} AD$

$\therefore AD = 8$ cm. (The req.)

In Δ ADB :

$\because m(\angle ADB) = 90°$, $AE = EB$

$\therefore DE = \frac{1}{2} AB$

Similarly in Δ ACB :

$\because m(\angle ACB) = 90°$, $AE = EB$

$\therefore CE = \frac{1}{2} AB$ $\therefore DE = CE$

$\therefore \Delta$ CED is an isosceles triangle. (Q.E.D.)

In Δ LYE :

$\because m(\angle YLE) = 90°$, $m(\angle E) = 30°$

$\therefore LY = \frac{1}{2} YE = 5$ cm.

In Δ ZYX :

$\because m(\angle ZYX) = 90°$, L is the midpoint of $\overline{ZX}$

$\therefore YL = \frac{1}{2} ZX$ $\therefore ZX = 10$ cm. (The req.)

In Δ ABC :

$m(\angle ABC) = 90°$, $m(\angle C) = 30°$

$\therefore AC = 2\,AB = 14$ cm.

, $\because$ D is the midpoint of $\overline{AC}$

$\therefore BD = \frac{1}{2} AC = 7$ cm.

, in Δ DEC :

$\because m(\angle DEC) = 90°$, $m(\angle C) = 30°$

$\therefore DE = \frac{1}{2} DC$, $\because DC = \frac{1}{2} AC = 7$ cm.

$\therefore DE = 3.5$ cm. (The req.)

In Δ ABC :

$\because m(\angle ABC) = 90°$, $m(\angle C) = 30°$

$AB = \frac{1}{2} AC = 4$ cm

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2} AC = 4$ cm.

In Δ XBY :

$m(\angle XBY) = 90°$

, Z is the midpoint of XY

$\therefore BZ = \frac{1}{2} XY = 2$ cm. (The req.)

In Δ MED :

$\because m(\angle MED) = 90°$ $\therefore (MD)^2 = 3^2 + 4^2 = 25$

$\therefore MD = \sqrt{25} = 5$ cm.

, $\because$ M is the point of concurrence of the medians of Δ ABC

$\therefore AD = 3\,MD = 15$ cm.

, ∵ m (∠ BAC) = 90°

, $\overline{AD}$ is a median in Δ ABC

∴ BC = 2 AD = 30 cm. (The req.)

17

∵ ABCD is a parallelogram

m (∠ C) = m (∠ A) = 60°

∴ In Δ DEC :

m (∠ EDC) = 180° – (60° + 90°) = 30°

∴ CE = $\frac{1}{2}$ DC ∴ DC = 8 cm

∴ The perimeter of the parallelogram ABCD

= (12 + 8) × 2 = 40 cm. (The req.)

18

∵ m (∠ BAD) = 90° , m (∠ BAE) = 30°

∴ m (∠ DAF) = 60°

From Δ AFD :

∵ m (∠ AFD) = 90° , m (∠ DAF) = 60°

∴ m (∠ ADF) = 30° ∴ AD = 2 AF = 8 cm.

∴ The area of the square = 64 cm^2. (The req.)

19

In Δ BCE : ∵ m (∠ EBC) = 30°

∴ CE = $\frac{1}{2}$ BE (1)

, ∵ m (∠ EBC) = 30° ∴ m (∠ ABE) = 60°

∴ In Δ ABE :

∵ m (∠ EAB) = 30°

, ∵ m (∠ AEB) = 90°

∴ BE = $\frac{1}{2}$ AB (2)

From (1) and (2) :

∴ CE = $\frac{1}{2} \times \frac{1}{2}$ AB = $\frac{1}{4}$ AB (Q.E.D.)

20

In Δ ABC :

∵ m (∠ ABC) = 90° , m (∠ A) = 30°

∴ AC = 2 BC = 16 cm.

, m (∠ C) = 180° – (90° + 30°) = 60°

In Δ BCD :

m (∠ BDC) = 90° , m (∠ C) = 60°

∴ m (∠ CBD) = 180° – (90° + 60°) = 30°

∴ CD = $\frac{1}{2}$ BC = 4 cm.

∴ AD = AC – CD = 16 – 4 = 12 cm. (The req.)

21

In Δ ABC : ∵ m (∠ B) = 30° , m (∠ C) = 90°

∴ AC = $\frac{1}{2}$ AB

, ∵ E is the midpoint of $\overline{BC}$

, O is the midpoint of $\overline{AC}$

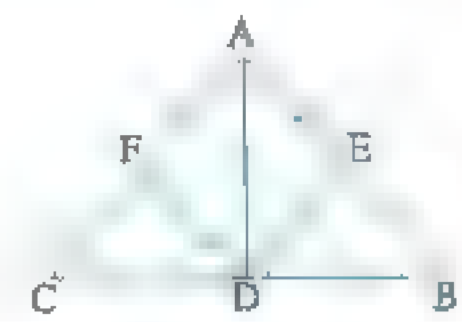
∴ EO = $\frac{1}{2}$ AB ∴ EO = AC

In Δ DEO : ∵ X is the midpoint of $\overline{DE}$

, Y is the midpoint of $\overline{DO}$

∴ XY = $\frac{1}{2}$ EO ∴ XY = $\frac{1}{2}$ AC (Q.E.D)

22



In Δ ADB

∵ m (∠ ADB) = 90°

, E is the midpoint of $\overline{AB}$

∴ DE = $\frac{1}{2}$ AB

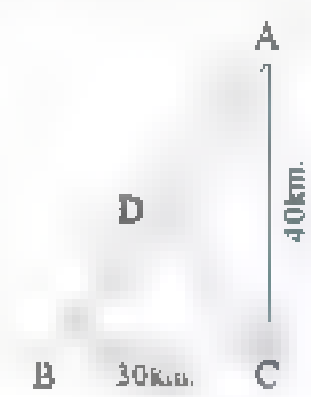
In Δ ADC : ∵ m (∠ ADC) = 90°

, F is the midpoint of $\overline{AC}$ ∴ DF = $\frac{1}{2}$ AC

∴ DE + DF = $\frac{1}{2}$ AB + $\frac{1}{2}$ AC but AB = AC (Given)

∴ DE + DF = $\frac{1}{2}$ AB + $\frac{1}{2}$ AB = AB (Q.E.D.)

23



Let the service station lie at the point D which is the midpoint of $\overline{AB}$

∴ The road length = the length of $\overline{CD}$

In Δ ACB :

∵ m (∠ ACB) = 90°

∴ $(AB)^2 = (AC)^2 + (BC)^2$ = 1600 + 900 = 2500

∴ AB = 50 km.

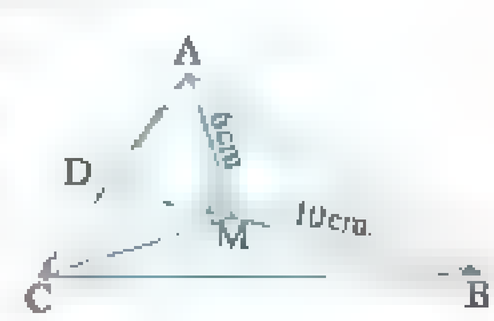
, ∵ D is the midpoint of $\overline{AB}$

∴ CD = $\frac{1}{2}$ AB = $\frac{1}{2}$ × 50 = 25 km.

∴ The length of the road 25 km. (The req.)

24

Constr : Draw $\overrightarrow{BM}$ to intersect $\overline{AC}$ at D



Proof : ∵ M is the point of concurrence of the medians of Δ ABC

∴ MD = $\frac{1}{2}$ BM = 5 cm.

In Δ AMC : ∵ m (∠ AMC) = 90°

, $\overline{MD}$ is a median

$\therefore MD = \frac{1}{2} AC$ $\therefore AC = 10$ cm. (First req.)

In Δ AMC : $\because$ m ($\angle$ AMC) = 90°

$\therefore (MC)^2 = (10)^2 - (6)^2 = 64$

$\therefore MC = \sqrt{64} = 8$ cm. (Second req.)

25

$\because \overline{DA} \parallel \overline{CB}$

, $\overleftrightarrow{DC}$ is a transversal.

$\therefore$ m ($\angle$ ADC) + m ($\angle$ DCB) = 180°

$\therefore \frac{1}{2}$ m ($\angle$ ADC) + $\frac{1}{2}$ m ($\angle$ DCB) = 90°

$\therefore$ m ($\angle$ XDC) + m ($\angle$ DCX) = 90°

but the sum of the measures of the interior angles of a triangle XDC = 180°

$\therefore$ m ($\angle$ DXC) = 90° , $\because$ DY = YC

$\therefore XY = \frac{1}{2}$ DC $\therefore$ XY = YC (Q.E.D.)

1 $x = 50°$ 2 $x = 56°$ 3 $y = 63°$

4 $\ell = 65°$, $z = 50°$ 5 $x = 54°$, $y = 117°$

6 $x = 69°$, $y = 111°$ 7 $x = 120°$

8 $x = 68°$, $y = 127°$

1 congruent 2 60° 3 F

4 50° 5 70° 6 20°

1 b 2 c 3 c 4 b 5 a

6 b 7 d 8 a 9 b 10 b

In Δ ABC :

$\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$= \frac{180° - 40°}{2} = 70°$ (First req.)

$\because$ m ($\angle$ ABC) = m ($\angle$ ACB) ,

$\angle$ ABD supplements $\angle$ ABC

, $\angle$ ACE supplements $\angle$ ACB

$\because$ The supplementaries of the congruent angles are congruent

$\therefore \angle ABD \equiv \angle ACE$ (Second req.)

From Δ ABC :

$\because$ AB = AC $\therefore$ m ($\angle$ B) = m ($\angle$ ACB) = 70°

$\therefore$ m ($\angle$ BAC) = 180° − (2 × 70°) = 40°

In Δ ACD :

$\because$ AC = CD $\therefore$ m ($\angle$ CAD) = m ($\angle$ D)

$\because \angle$ ACB is an exterior angle of Δ ACD

$\therefore$ m ($\angle$ ACB) = m ($\angle$ CAD) + m ($\angle$ D)

$\therefore$ m ($\angle$ CAD) = $\frac{70°}{2}$ = 35°

$\therefore$ m ($\angle$ BAD) = m ($\angle$ BAC) + m ($\angle$ CAD)

= 40° + 35° = 75° (The req.)

$\because \angle$ ACD is an exterior angle of Δ ABC

$\therefore$ m ($\angle$ ACD) = 30° + 40° = 70°

From Δ ACD : $\because$ AC = AD

$\therefore$ m ($\angle$ D) = m ($\angle$ ACD) = 70° (First req.)

$\therefore$ m ($\angle$ CAD) = 180° − (70° + 70°) = 40° (Second req.)

$\because \Delta$ ACD is an equilateral triangle

$\therefore$ m ($\angle$ CAD) = 60° (1)

From Δ ABC : $\because$ AB = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ BCA) = $\frac{180° - 40°}{2}$ = 70° (2)

From (1) and (2) :

$\therefore$ m ($\angle$ BAD) = 60° + 70° = 130° (The req.)

In Δ ABD : $\because$ AB = AD

$\therefore$ m ($\angle$ ADB) = m ($\angle$ ABD) = $\frac{180° - 120°}{2}$ = 30° (First req.)

$\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{DC}$ is a transversal to them

$\therefore$ m ($\angle$ C) + m ($\angle$ ADC) = 180°

$\therefore$ m ($\angle$ C) = 180° − (65° + 30°) = 85° (Second req.)

$\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore$ m ($\angle$ C) = m ($\angle$ DAC) = 30° (alternate angles)

In Δ ABC : $\because$ AC = BC

$\therefore$ m ($\angle$ CAB) = m ($\angle$ B) = $\frac{180° - 30°}{2}$ = 75° (The req.)

10

Δ DEC is an equilateral triangle.

$\therefore$ m ($\angle$ ECD) = 60° (1)

From Δ ABC :

AB = AC $\therefore$ m ($\angle$ B) = m ($\angle$ ACB)

, $\because$ m ($\angle$ B) + m ($\angle$ ACB) = 180° − 80° = 100°

$\therefore$ m ($\angle$ B) = m ($\angle$ ACB) = $\frac{100°}{2}$ = 50° (2)

From (1) and (2) :

m ($\angle$ BCD) = 50° + 60° = 110° (The req.)

11

In Δ ABC :

$\because$ BA = BC , m ($\angle$ B) = 80°

$\therefore$ m ($\angle$ BAC) = m ($\angle$ BCA) = $\frac{180° - 80°}{2}$ = 50°

$\therefore$ m ($\angle$ DAC) = 114° − 50° = 64°

In Δ ADC :

$\because$ DA = DC , m ($\angle$ DAC) = 64°

$\therefore$ m ($\angle$ ADC) = 180° − (64° × 2) = 52° (The req.)

12

From Δ ABC :

AB = AC

$\therefore$ m ($\angle$ B) = m ($\angle$ BCA)

$\because$ m ($\angle$ B) + m ($\angle$ BCA) = 180° − 48° = 132°

$\therefore$ m ($\angle$ B) = m ($\angle$ BCA) = $\frac{132°}{2}$ = 66° (First req.)

$\because$ $\overrightarrow{CD}$ bisects $\angle$ ACB

$\therefore$ m ($\angle$ BCD) = $\frac{66°}{2}$ = 33° (Second req.)

13

$\because$ Δ ABC is an equilateral triangle

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = 60°

$\therefore$ $\frac{1}{2}$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\angle$ ACB) = 30°

$\because$ $\overrightarrow{BD}$ bisects $\angle$ ABC , $\overrightarrow{CD}$ bisects $\angle$ ACB

m ($\angle$ DBC) = m ($\angle$ DCB) = 30°

From Δ DBC :

m ($\angle$ D) = 180° − (2 × 30°) = 120° (The req.)

14

$\because$ Δ ABC is an equilateral triangle.

$\therefore$ m ($\angle$ ABC) = 60° (1)

From Δ DBC :

$\because$ DB = DC , m ($\angle$ D) = 100°

$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB) = $\frac{180° - 100°}{2}$ = 40° (2)

From (1) and (2) :

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ABC) − m ($\angle$ DBC)

= 60° − 40° = 20° (The req.)

15

$\because$ Δ ABC is an equilateral triangle.

$\therefore$ m ($\angle$ ACB) = m ($\angle$ B) = m ($\angle$ BAC) = 60°

$\therefore$ m ($\angle$ ACD) = 120°

In Δ ACD :

$\because$ AC = CD $\therefore$ m ($\angle$ CAD) = m ($\angle$ D)

$\because$ m ($\angle$ CAD) + m ($\angle$ D) = 180° − 120° = 60°

$\therefore$ m ($\angle$ CAD) = $\frac{60°}{2}$ = 30°

$\therefore$ m ($\angle$ BAD) = 60° + 30° = 90°

$\therefore$ $\overline{BA} \perp \overline{AD}$ (Q.E.D.)

Another solution :

$\because$ C is the midpoint of $\overline{BD}$

$\therefore$ $\overline{AC}$ is a median in Δ ABD

, $\because$ AC = $\frac{1}{2}$ BD

$\therefore$ $\angle$ BAD is right

$\therefore$ $\overline{BA} \perp \overline{AD}$ (Q.E.D.)

16

From Δ ABC :

$\because$ AB = AC $\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\Delta\Delta$ ABD , ACE in them :

$$\begin{cases} AB = AC \\ m(\angle B) = m(\angle C) \\ BD = EC \end{cases}$$

$\therefore$ Δ ABD $\equiv$ Δ ACE then we deduce that AD = AE

$\therefore$ Δ ADE is an isosceles triangle. (Q.E.D. 1)

$\therefore$ m ($\angle$ ADE) = m ($\angle$ AED)

$\therefore$ $\angle$ ADE $\equiv$ $\angle$ AED (Q.E.D. 2)

17

$\Delta\Delta$ ADE , BCE in them :

$$\begin{cases} AD = CB \\ AE = EB \\ m(\angle A) = m(\angle B) \end{cases}$$

$\because \Delta ADE \equiv \Delta BCE$, then we deduce that $DE = CE$

In ΔDEC :

$\because DE = CE$ $\therefore m(\angle EDC) = m(\angle ECD)$

$\because m(\angle DEC) = 40^\circ$

$\therefore m(\angle EDC) + m(\angle ECD) = 180^\circ - 40^\circ = 140^\circ$

$\therefore m(\angle EDC) = \frac{140^\circ}{2} = 70^\circ$ (The req.)

[illegible]

$\because \angle LZX$ is an exterior angle of ΔXYZ

$\therefore m(\angle X) + m(\angle Y) = 130^\circ$

$\because ZX = ZY$ $\therefore m(\angle X) = m(\angle Y)$

$\therefore m(\angle Y) = \frac{130^\circ}{2} = 65^\circ$

$\because \overleftrightarrow{LM} \parallel \overleftrightarrow{XY}$, $\overleftrightarrow{LY}$ is a transversal to them

$\therefore m(\angle MLY) = m(\angle Y) = 65^\circ$ (The req.)

19

$\because \overleftrightarrow{AE} \parallel \overleftrightarrow{BC}$ and $\overleftrightarrow{BD}$ is a transversal to them.

$\therefore m(\angle B) = m(\angle DAE)$ (corresponding angles)

, $\because \overleftrightarrow{AE} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle C) = m(\angle EAC)$ (alternate angles)

but $m(\angle B) = m(\angle C)$ because $AB = AC$

$\therefore m(\angle DAE) = m(\angle EAC)$

i.e. $\overrightarrow{AE}$ bisects $\angle DAC$ (Q.E.D.)

20

$\because B \in \overline{AD}$

$\therefore m(\angle ABC) + m(\angle CBE) + m(\angle EBD) = 180^\circ$ (1)

$\because$ The sum of measures of the angles of the triangle $= 180^\circ$

$\therefore m(\angle ABC) + m(\angle A) + m(\angle C) = 180^\circ$ (2)

From (1) and (2) :

$\therefore m(\angle CBE) + m(\angle EBD) = m(\angle A) + m(\angle C)$

, $\because m(\angle CBE) = m(\angle EBD)$ (Given)

, $m(\angle A) = m(\angle C)$ (because $BA = BC$)

$\therefore m(\angle CBE) = m(\angle C)$ and they are alternate angles.

$\therefore \overleftrightarrow{BE} \parallel \overleftrightarrow{AC}$ (Q.E.D.)

[illegible]

In ΔDEC :

$\because DE = DC$

$\therefore m(\angle DEC) = m(\angle C) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$

$\because \overleftrightarrow{AD} \parallel \overleftrightarrow{EC}$, $\overleftrightarrow{DE}$ is a transversal to them

$\therefore m(\angle ADE) = m(\angle DEC) = 70^\circ$ (alternate angles)

$\because AD = AE$

$\therefore m(\angle AED) = m(\angle ADE) = 70^\circ$ (First req.)

In ΔAED :

$\because m(\angle EAD) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$

$\because m(\angle BAD) = m(\angle C) = 70^\circ$

(from properties of the parallelogram)

$\therefore m(\angle BAE) = 70^\circ - 40^\circ = 30^\circ$ (Second req.)

22

From ΔABC :

$\because AB = AC$ $\therefore m(\angle B) = m(\angle C)$

$\therefore 2x + 13 = 3x - 17$ $\therefore x = 30^\circ$

$\therefore m(\angle B) = m(\angle C) = 2 \times 30 + 13 = 73^\circ$

$\therefore m(\angle A) = 180^\circ - (73^\circ + 73^\circ) = 34^\circ$ (The req.)

23

$\boxed{1}$ $x = 60^\circ$, $y = 121^\circ$ $\boxed{2}$ $x = 45^\circ$, $y = 105^\circ$

$\boxed{3}$ $x = 44^\circ$

$\boxed{4}$ $x = 75^\circ$, $y = 15^\circ$ $\boxed{5}$ $x = 25^\circ$, $y = 92^\circ$

$\boxed{6}$ $y = 110^\circ$, $\ell = 40^\circ$, $z = 70^\circ$

$\boxed{7}$ $x = 30^\circ$, $y = 40^\circ$ $\boxed{8}$ $x = 70^\circ$, $y = 50^\circ$

$\boxed{9}$ $x = 120^\circ$

[illegible]

$\boxed{1}$ 3 cm. $\boxed{2}$ 5° $\boxed{3}$ 5°

$\boxed{4}$ 66.5° $\boxed{5}$ 7° $\boxed{6}$ 22°

[illegible]

In $\Delta\Delta$ EBD and CBD :

$\left\{ \begin{array}{l} \overline{BD} \text{ is a common side} \\ m(\angle EBD) = m(\angle CBD) \\ m(\angle EDB) = m(\angle CDB) \end{array} \right.$

$\therefore \Delta EBD \equiv \Delta CBD$, then we deduce that :

$BE = BC$, $m(\angle BED) = m(\angle C)$

, $\because BA = BC$ $\therefore BA = BE$

$\therefore m(\angle A) = m(\angle BEA)$

, $\because m(\angle BEA) + m(\angle BED) = 180^\circ$

$\therefore m(\angle A) + m(\angle C) = 180^\circ$ (Q.E.D.)

$\Delta\Delta$ XYM , MZL in them :

$\begin{cases} XY = MZ \\ YM = LZ \\ m(\angle Y) = m(\angle Z) = 90^\circ \end{cases}$

$\therefore \Delta$ XYM $\equiv \Delta$ MZL , then we deduce that :

XM = ML , m ($\angle$ XMY) = m ($\angle$ MLZ)

$\because \angle$ MLZ complements $\angle$ LMZ

$\therefore \angle$ XMY complements $\angle$ LMZ

$\therefore$ m ($\angle$ XML) = 90°

$\therefore$ From Δ XLM :

MX = ML , m ($\angle$ XML) = 90°

$\therefore$ m ($\angle$ MXL) = m ($\angle$ MLX) = $\frac{180^\circ - 90^\circ}{2}$ = 45°

(The req.)

From Δ BDC :

$\because$ BD = CD

$\therefore$ m ($\angle$ DBC) = m ($\angle$ BCD) (1)

$\because \angle$ ADB is an exterior angle of Δ CBD

$\therefore$ m ($\angle$ ADB) = m ($\angle$ DBC) + m ($\angle$ BCD)

from (1) : m ($\angle$ ADB) = 2 m ($\angle$ BCD) (2)

In Δ ABD :

$\because$ AB = AD

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ADB)

from (2) :

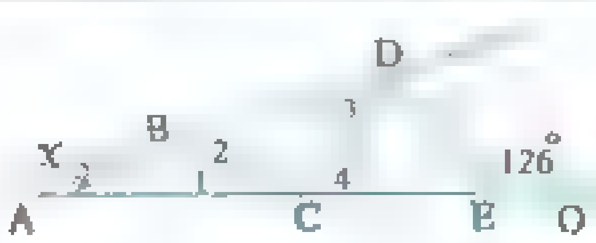
$\therefore$ m ($\angle$ ABD) = m ($\angle$ ADB) = 2 m ($\angle$ BCD)

$\because \angle$ BAE is an exterior angle of Δ ABD

$\therefore$ m ($\angle$ BAE) = m ($\angle$ ABD) + m ($\angle$ ADB)

= 2 m ($\angle$ BCD) + 2 m ($\angle$ BCD)

= 4 m ($\angle$ BCD) (Q.E.D.)



In Δ ABC :

$\because$ BC = BA

$\therefore$ m ($\angle$ A) = m ($\angle$ 1) = x

, $\because \angle$ 2 is an exterior angle of Δ ABC

$\therefore$ m ($\angle$ 2) = m ($\angle$ A) + m ($\angle$ 1) = $x + x = 2x$

In Δ DBC : $\because$ CB = CD

$\therefore$ m ($\angle$ 3) = m ($\angle$ 2) = $2x$

, $\because \angle$ 4 is an exterior angle of Δ ACD

$\therefore$ m ($\angle$ 4) = m ($\angle$ A) + m ($\angle$ 3) = $x + 2x = 3x$ (1)

, $\because$ m ($\angle$ DEC) = 180° − 126° = 54°

In Δ CDE : $\because$ DC = DE

$\therefore$ m ($\angle$ 4) = m ($\angle$ DEC) = 54° (2)

From (1) and (2) : $\therefore 3x = 54^\circ$

$\therefore x = \frac{54^\circ}{3} = 18^\circ$ (The req.)

4

1 AB = AC 2 YX = YZ 3 XY = XZ

4 AB = AC = BC 5 ML = MN 6 BA = BC

7 ZX = ZY 8 CB = CA 9 AC = AB

1 congruent , isosceles 2 equilateral

3 isosceles 4 isosceles

5 equilateral 6 6 7 60°

$\because$ B $\in \overline{DC}$ $\therefore$ m ($\angle$ ABC) = 180° − 125° = 55°

In Δ ABC : m ($\angle$ C) = 180° − (55° + 70°) = 55°

$\therefore$ m ($\angle$ ABC) = m ($\angle$ C)

$\therefore$ AB = AC

$\therefore \Delta$ ABC is an isosceles triangle. (Q.E.D.)

$\because$ Y $\in \overline{ZL}$ $\therefore$ m ($\angle$ XYZ) = 180° − 120° = 60°

, $\because$ XY = XZ

$\therefore \Delta$ XYZ is an equilateral triangle. (Q.E.D.)

$\because$ B $\in \overrightarrow{AD}$ $\therefore$ m ($\angle$ ABC) = 180° − 120° = 60°

Similarly : m ($\angle$ ACB) = 60°

$\therefore$ m ($\angle$ A) = 180° − (60° + 60°) = 60°

$\therefore$ m ($\angle$ A) = m ($\angle$ ABC) = m ($\angle$ ACB)

$\therefore \Delta$ ABC is an equilateral triangle. (Q.E.D.)

$\because \overline{AD} \,/\!/\, \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them.

$\therefore$ m ($\angle$ DBC) = m ($\angle$ ADB) = 40° (alternate angles)

In Δ DBC : m ($\angle$ C) = 180° − (100° + 40°) = 40°

$\therefore$ m ($\angle$ DBC) = m ($\angle$ C) $\therefore$ DB = DC

$\therefore \Delta$ DBC is an isosceles triangle. (Q.E.D.)

$\because \overleftrightarrow{XY} \,//\, \overline{AC}$, $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore$ m ($\angle$ A) = m ($\angle$ ABX) = 62° (alternate angles)

$\therefore$ m ($\angle$ ABC) = 180° − (62° + 56°) = 62°

$\therefore$ m ($\angle$ ABC) = m ($\angle$ A) $\therefore$ CA = CB (Q.E.D.)

$\because$ AB = AC $\therefore$ m ($\angle$ B) = m ($\angle$ C) (1)

, $\because \overleftrightarrow{XY} \,//\, \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (corresponding angles) (2)

Similarly m ($\angle$ AYX) = m ($\angle$ C) (3)

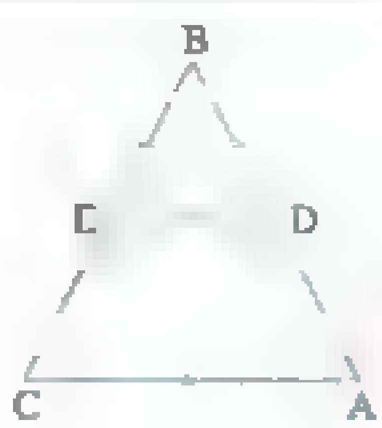
From (1) , (2) and (3) :

$\therefore$ m ($\angle$ AXY) = m ($\angle$ AYX) $\therefore$ AX = AY

$\therefore \Delta$ AXY is an isosceles triangle. (Q.E.D. 1)

$\because$ AB = AC , AX = AY subtracting

$\therefore$ XB = YC (Q.E.D. 2)



From Δ EBD : $\because$ DB = EB

$\therefore$ m ($\angle$ BDE) = m ($\angle$ BED) (1)

$\because \overline{DE} \,//\, \overline{AC}$, $\overleftrightarrow{AD}$ is a transversal to them

$\therefore$ m ($\angle$ A) = m ($\angle$ BDE) (corresponding angles) (2)

Similarly m ($\angle$ C) = m ($\angle$ BED) (3)

From (1) , (2) and (3) ; $\therefore$ m ($\angle$ A) = m ($\angle$ C)

$\therefore$ AB = BC (Q.E.D.)

$\because$ MB = MC $\therefore$ m ($\angle$ B) = m ($\angle$ C) (1)

$\because \overline{AD} \,//\, \overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore$ m ($\angle$ A) = m ($\angle$ C) (alternate angles) (2)

similarly m ($\angle$ D) = m ($\angle$ B) (3)

from (1) , (2) and (3) $\therefore$ m ($\angle$ A) = m ($\angle$ D)

$\therefore$ MA = MD (Q.E.D.)

$\because$ AB = AC $\therefore$ m ($\angle$ B) = m ($\angle$ C) (1)

$\because$ AB // $\overline{DE}$, $\overline{BE}$ is a transversal to them.

$\therefore$ m ($\angle$ B) = m ($\angle$ DEF) (corresponding angles) (2)

similarly m ($\angle$ C) = m ($\angle$ DFE) (3)

from (1) , (2) and (3)

$\therefore$ m ($\angle$ DEF) = m ($\angle$ DFE)

$\therefore$ DE = DF (Q.E.D. 1)

In $\Delta\Delta$ ABC , DEF :

$\because$ m ($\angle$ B) = m ($\angle$ DEF) , m ($\angle$ C) = m ($\angle$ DFE)

$\therefore$ m ($\angle$ BAC) = m ($\angle$ EDF) (Q.E.D. 2)

$\because \overline{ED} \,//\, \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them.

$\therefore$ m ($\angle$ EDB) = m ($\angle$ DBC) (alternate angles)

but m ($\angle$ EBD) = m ($\angle$ DBC)

$\therefore$ m ($\angle$ EDB) = m ($\angle$ EBD) $\therefore$ EB = ED

$\therefore \Delta$ EBD is an isosceles triangle. (Q.E.D.)

$\because \overleftrightarrow{AE} \,//\, \overline{BC}$ and $\overleftrightarrow{DB}$ is a transversal to them.

$\therefore$ m ($\angle$ DAE) = m ($\angle$ B) (corresponding angles)

$\because \overleftrightarrow{AE} \,//\, \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ EAC) = m ($\angle$ C) (alternate angles)

but m ($\angle$ DAE) = m ($\angle$ EAC)

$\therefore$ m ($\angle$ B) = m ($\angle$ C) $\therefore$ AB = AC (Q.E.D.)

$\because$ m ($\angle$ ABC) = m ($\angle$ ACB) $\therefore$ AB = AC

$\therefore \Delta\Delta$ ADB , AEC in them :

$\left\{\begin{array}{l} AB = AC \\ DB = EC \\ m(\angle D) = m(\angle E) = 90^\circ \end{array}\right.$

$\therefore \Delta ADB \equiv \Delta AEC$

$\therefore$ m ($\angle$ DAB) = m ($\angle$ CAE) (Q.E.D.)

In Δ YZX :

$\because$ YZ = YX $\therefore$ m ($\angle$ Z) = $\frac{180^\circ - 50^\circ}{2} = 65^\circ$

$\because$ m ($\angle$ ZMX) = 50° + 15° = 65°

$\therefore$ In Δ MZX : m ($\angle$ Z) = m ($\angle$ ZMX) $\therefore$ MX = ZX

$\therefore \Delta$ MZX is an isosceles triangle. (Q.E.D.)

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC) = $\frac{180^\circ - 70^\circ}{2} = 55^\circ$

$\because$ m ($\angle$ MCA) = 25° $\therefore$ m ($\angle$ MCB) = 55° − 25° = 30°

$\therefore$ m ($\angle$ MBC) = m ($\angle$ MCB) $\therefore$ MB = MC

$\therefore \Delta$ MBC is an isosceles triangle. (Q.E.D.)

17

$\angle$ ADC is an exterior angle of Δ ADB

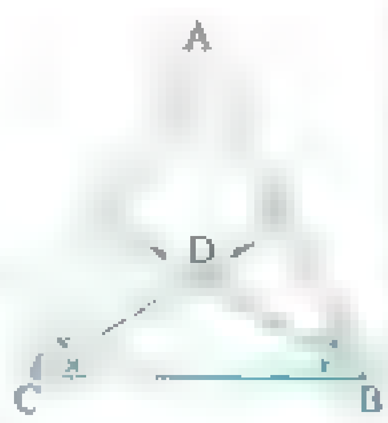
$\therefore$ m ($\angle$ ADC) = 40° + 30° = 70°

$\because$ AD = AC $\therefore$ m ($\angle$ C) = m ($\angle$ ADC) = 70°

$\therefore$ In Δ ABC : m ($\angle$ BAC) = 180° – (40° + 70°) = 70°

$\therefore$ m ($\angle$ BAC) = m ($\angle$ C) $\therefore$ AB = BC (Q.E.D.)

18



AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$\therefore$ $\frac{1}{2}$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\angle$ ACB)

$\because$ m ($\angle$ DBC) = $\frac{1}{2}$ m ($\angle$ ABC)

m ($\angle$ DCB) = $\frac{1}{2}$ m ($\angle$ ACB)

m ($\angle$ DBC) = m ($\angle$ DCB) $\therefore$ DB = DC

$\therefore$ Δ DBC is an isosceles triangle. (Q.E.D.)

19

$\because$ Δ ABC is an equilateral triangle

$\therefore$ m ($\angle$ ACB) = 60°

$\because$ $\angle$ ACB is an exterior angle of Δ DCF

$\therefore$ m ($\angle$ D) = 60° – 30° = 30°

$\therefore$ m ($\angle$ D) = m ($\angle$ F) $\therefore$ CD = CF

$\therefore$ Δ DCF is an isosceles triangle. (Q.E.D.)

20

$\because$ DA = DC $\therefore$ m ($\angle$ C) = m ($\angle$ DAC) = 30°

$\because$ $\angle$ ADB is an exterior angle of Δ ADC

$\therefore$ m ($\angle$ ADB) = 30° + 30° = 60°

$\because$ DA = DB

$\therefore$ Δ ABD is an equilateral triangle. (Q.E.D. 1)

$\because$ m ($\angle$ BAD) = 60° , m ($\angle$ DAC) = 30°

$\therefore$ m ($\angle$ BAC) = 90°

$\therefore$ Δ ABC is a right-angled triangle. (Q.E.D. 2)

21

$\because$ ED // $\overleftrightarrow{AC}$, $\overleftrightarrow{EC}$ is a transversal to them.

$\therefore$ m ($\angle$ DEC) = m ($\angle$ ACE) (alternate angles)

$\because$ m ($\angle$ DEC) = m ($\angle$ AEC)

$\therefore$ m ($\angle$ ACE) = m ($\angle$ AEC)

$\therefore$ AE = AC (1)

, $\because$ $\overleftrightarrow{DE}$ // $\overleftrightarrow{AC}$, $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore$ m ($\angle$ A) = m ($\angle$ BED) = 60°

(corresponding angles) (2)

from (1) and (2) :

$\therefore$ Δ AEC is an equilateral triangle. (Q.E.D.)

22

In Δ ABC : m ($\angle$ ACB) = 180° – (60° + 90°) = 30°

In Δ ECD : m ($\angle$ ECD) = 180° – (30° + 90°) = 60°

$\because$ C $\in$ $\overline{BD}$

$\therefore$ m ($\angle$ ACE) = 180° – (30° + 60°) = 90°

In Δ ACE : m ($\angle$ CAE) = 180° – (90° + 45°) = 45°

$\therefore$ m ($\angle$ CAE) = m ($\angle$ CEA) = 45° $\therefore$ CA = CE

In Δ ECD : $\because$ m ($\angle$ D) = 90° , m ($\angle$ CED) = 30°

$\therefore$ CE = 2 CD = 6 cm. but AC = CE

$\therefore$ AC = 6 cm. (The req.)

23

In Δ ADE : $\because$ $\angle$ ADE $\equiv$ $\angle$ AED $\therefore$ AD = AE

D $\in$ $\overline{BC}$, E $\in$ $\overline{BC}$

$\because$ $\angle$ ADB supplements $\angle$ ADE ,

$\angle$ AEC supplements $\angle$ AED

but m ($\angle$ ADE) = m ($\angle$ AED)

$\therefore$ m ($\angle$ ADB) = m ($\angle$ AEC)

(supplementaries of the congruent angles are congruent)

$\therefore$ $\Delta\Delta$ ADB , AEC in them :

$$\left\{\begin{array}{l} m(\angle ADB) = m(\angle AEC) \\ AD = AE \\ BD = CE \end{array}\right.$$

$\therefore$ Δ ADB $\equiv$ Δ AEC

We deduce that AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

24

In Δ BMC : $\because$ m ($\angle$ MBC) = m ($\angle$ MCB)

$\therefore$ MB = MC

$\because$ m ($\angle$ ABM) = m ($\angle$ MCD)

(complementaries of equal angles in measure are equal in measure)

$\therefore \Delta\Delta$ ABM , DCM in them :

$\left\{\begin{array}{l} AB = DC \text{ (two sides in a square)} \\ BM = CM \text{ (proved)} \\ m(\angle ABM) = m(\angle DCM) \text{ (proved)} \end{array}\right.$

$\therefore \Delta ABM \equiv \Delta DCM$ we deduce that AM = DM

$\therefore \Delta$ AMD is an isosceles triangle. (Q.E.D.)

In $\Delta\Delta$ ABF , AME : m ($\angle$ B) = m ($\angle$ AME) = 90°

m ($\angle$ BAF) = m ($\angle$ MAE) ($\overrightarrow{AE}$ bisects $\angle$ BAC)

$\therefore$ m ($\angle$ AFB) = m ($\angle$ E) (1)

, $\because \overleftrightarrow{AD} \,/\!/\, \overleftrightarrow{BF}$, $\overleftrightarrow{AF}$ is a transversal to them.

$\therefore$ m ($\angle$ DAE) = m ($\angle$ AFB) (alternate angles) (2)

from (1) and (2) : $\therefore$ m ($\angle$ E) = m ($\angle$ DAE)

$\therefore$ DA = DE (Q.E.D.)

$\because$ m ($\angle$ B) = m ($\angle$ C) $\therefore$ AB = AC

$\therefore 2x - 1 = x + 3 \quad \therefore 2x - x = 3 + 1 \quad \therefore x = 4$

$\therefore$ AB = AC = 2 × 4 – 1 = 7 cm. , BC = 9 – 4 = 5 cm.

$\therefore$ The perimeter of Δ ABC = 7 + 7 + 5 = 19 cm. (The req.)

---

1 $\because 3x + x + 50° + 30° = 180°$

$\therefore 4x + 80° = 180°$

$\therefore 4x = 180° - 80° = 100° \quad \therefore x = \frac{100°}{4} = 25°$

$\therefore$ m ($\angle$ A) = 3 × 25° = 75° ,

m ($\angle$ B) = 25° + 50° = 75°

$\therefore$ m ($\angle$ A) = m ($\angle$ B) $\therefore$ CB = CA

2 $\because 2z + 3z - 10° + z + 40° = 180°$

$\therefore 6z + 30° = 180°$

$\therefore 6z = 180° - 30° = 150° \quad \therefore z = \frac{150°}{6} = 25°$

$\therefore$ m ($\angle$ B) = 3 × 25° – 10° = 65°

, m ($\angle$ C) = 25° + 40° = 65°

$\therefore$ m ($\angle$ B) = m ($\angle$ C) $\therefore$ AB = AC

3 $\because \angle$ DBC is an exterior angle of Δ ABC

$\therefore 3x = x - 20° + x + 70°$

$\therefore 3x = 2x + 50° \qquad x = 50°$

$\therefore$ m ($\angle$ A) = 50° – 20° = 30° ,

m ($\angle$ C) = 50° + 70° = 120°

$\therefore$ m ($\angle$ ABC) = 180° – (30° + 120°) = 30°

$\therefore$ m ($\angle$ A) = m ($\angle$ ABC) $\therefore$ CB = CA

---

1 c 2 b

---

1 An axis of symmetry. 2 3 3 1 4 zero

5 Bisects it and it is perpendicular to the base.

6 Bisects the base and is perpendicular to it

7 Bisects each of the base and the vertex angle.

8 The straight line perpendicular to it at its middle.

9 at equal distances 10 3 11 1 12 3

---

1 35° 2 70° 3 55° 4 2 5 $\overline{AD}$

---

1 a 2 c 3 b 4 b 5 a

6 a 7 b 8 a

$\because$ BA = BC , $\overline{BD} \perp \overline{AC}$

$\therefore \overrightarrow{BD}$ bisects each of $\angle$ ABC , $\overline{AC}$

$\therefore$ AC = 2 AD = 40 cm.

, m ($\angle$ DBC) = $\frac{1}{2}$ m ($\angle$ ABC) = 45° (1) (First req.)

$\because \Delta$ABC in which m ($\angle$B) = 90° , BA = BC

$\therefore$ m ($\angle$ C) = 45° (2)

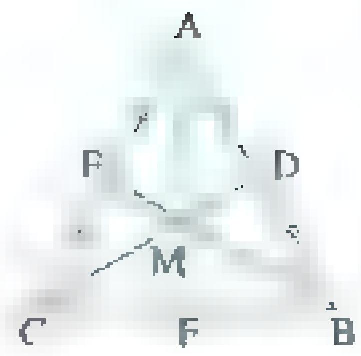
From (1) and (2) : $\therefore$ DB = DC

$\therefore \Delta$DBC is an isosceles triangle. (Second req.)

In ΔABC : $AB = AC$,

M is the point of

intersection of its medians



$\therefore \overline{AF}$ is a median of ΔABC

$\therefore \overrightarrow{AM} \perp \overline{BC}$ (Q.E.D. 1)

, $\overrightarrow{AM}$ bisects $\angle BAC$ (Q.E.D. 2)

In ΔABC : $\because AB = AC$, $\overline{AD} \perp \overline{BC}$

$\therefore BC = 2 \times 5 = 10$ cm. (First req.)

In the right-angled triangle ADB at D

$AD = \sqrt{(13)^2 - (5)^2} = 12$ cm.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 10 \times 12 = 60$ cm^2.

(Second req.)

$\because AB = AC$, $\overline{AD} \perp \overline{BC}$ $\therefore BD = \frac{1}{2} BC = 5$ cm.

, $m(\angle BAC) = 2 \times 30^\circ = 60^\circ$

$\therefore \Delta ABC$ is an equilateral triangle.

$\therefore AB = 10$ cm.

$\because$ In ΔADB which is right-angled at D

$\therefore AD = \sqrt{(10)^2 - (5)^2} = 5\sqrt{3}$ cm. (First req.)

The number of axes of symmetry of $\Delta ABC = 3$

(Second req.)

The area of $\Delta ABC = \frac{1}{2} \times 10 \times 5\sqrt{3} = 25\sqrt{3}$ cm^2.

(Third req.)

In ΔABC : $\because AB = AC$, $\overrightarrow{AE}$ bisects $\angle BAC$

$\therefore BE = \frac{1}{2} BC$ (Q.E.D.1)

, $\overline{AE} \perp \overline{BC}$

$\therefore \overleftrightarrow{AE}$ is the axis of symmetry of $\overline{BC}$, $D \in \overleftrightarrow{AE}$

$\therefore BD = CD$ (Q.E.D.2)

$\because C \in \overline{BD}$, $m(\angle ACD) = 130^\circ$

$\therefore m(\angle ACB) = 180^\circ - 130^\circ = 50^\circ$

From ΔABC : $m(\angle B) = 180^\circ - (80^\circ + 50^\circ) = 50^\circ$

$\therefore m(\angle B) = m(\angle ACB)$

$\therefore \Delta ABC$ is an isosceles triangle.

, $\overrightarrow{AE}$ bisects $\angle BAC$

$\therefore \overrightarrow{AE} \perp \overline{BC}$, E is the midpoint of $\overline{BC}$ (Q.E.D.)

$\because m(\angle ABX) = m(\angle ACY)$

$\therefore m(\angle ABC) = m(\angle ACB)$

(The supplementaries of congruent angles are congruent)

$\therefore AB = AC$

$\because \overline{AD}$ is a median of ΔABC which is isosceles.

$\therefore \overline{AD} \perp \overline{BC}$ (Q.E.D.)

$\because \overline{AD} \mathbin{/\!/} \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them

$\therefore m(\angle ADB) = m(\angle DBC)$ (alternate angles)

but $m(\angle ABD) = m(\angle DBC)$

$\therefore m(\angle ADB) = m(\angle ABD)$

$\therefore$ In ΔABD : $AB = AD$ (Q.E.D.1)

, $\because \overrightarrow{AE}$ bisects $\angle BAD$ $\therefore \overrightarrow{AE} \perp \overline{BD}$ (Q.E.D.2)

, $BE = ED$ (Q.E.D.3)

In ΔACD :

$\because$ E is the midpoint of $\overline{AD}$

, $\overline{CE} \perp \overline{AD}$ $\therefore DC = AC$

$\therefore \Delta ACD$ is an isosceles triangle.

, $\because \angle ADC$ is an exterior angle of ΔADB

$\therefore m(\angle ADC) = 20^\circ + 30^\circ = 50^\circ$

From ΔCDE :

$m(\angle DCE) = 180^\circ - (90^\circ + 50^\circ) = 40^\circ$

, $\because \overrightarrow{CE}$ bisects $\angle ACD$

$\therefore m(\angle ACE) = m(\angle DCE) = 40^\circ$ (The req.)

In ΔADC :

$\because$ E is the midpoint of $\overline{DC}$

, $\overline{AE} \perp \overline{DC}$ $\therefore AD = AC$

$\therefore \Delta ADC$ is an isosceles triangle.

$\therefore m(\angle ADC) = m(\angle C) = 70°$

, $\because \angle ADC$ is an exterior angle of ΔABD

$\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$

, $\because BD = AC$, $AD = AC$

$\therefore m(\angle B) = m(\angle BAD) = \frac{70°}{2} = 35°$ (The req.)

14

In ΔXYL : $\because XL = XY$, M is the midpoint of $\overline{LY}$

$\therefore \overleftrightarrow{XM}$ is the axis of $\overline{LY}$

similarly in ΔZYL , $\overleftrightarrow{ZM}$ is the axis of $\overline{LY}$

$\therefore$ X , M and Z are on the same straight line. (Q.E.D.)

15

$\because AB = AC$ $\therefore A \in$ the axis of $\overline{BC}$ (1)

, $m(\angle ABC) = m(\angle ACB)$,

$\because m(\angle ABD) = m(\angle ACD)$

by subtracting :

$\therefore m(\angle DBC) = m(\angle DCB)$

$\therefore DB = DC$ $\therefore D \in$ the axis of $\overline{BC}$ (2)

From (1) and (2) :

$\therefore \overleftrightarrow{AD}$ is the axis of $\overline{BC}$ (Q.E.D.)

16

$\because \overline{AD}$ bisects the base of ΔABC which is an isosceles triangle

$\therefore \overline{AD} \perp \overline{BC}$ $\therefore m(\angle ADB) = 90°$

$\because \overleftrightarrow{XY} // \overline{BC}$, $\overleftrightarrow{AD}$ is a transversal to them.

$\therefore m(\angle YAD) = m(\angle ADB) = 90°$ (alternate angles)

$\therefore \overline{AD} \perp \overleftrightarrow{XY}$ (Q.E.D.)

17

$\because AB = AC$, $EB = EC$ $\therefore \overleftrightarrow{AE}$ is the axis of $\overline{BC}$

$\therefore BD = DC$ (First req.)

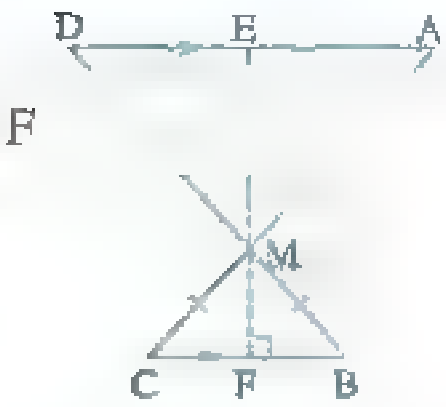
$\therefore DC = 3$ cm

In ΔADC which is right angled at D

$\therefore AD = \sqrt{(10)^2 - (3)^2} = \sqrt{100 - 9} = \sqrt{91}$ cm.

(Second req.)

18 **Constr. :**



Draw $\overrightarrow{MF} \perp \overline{BC}$ to meet $\overline{BC}$ at F and $\overrightarrow{AD}$ at E

Proof : $\because \overleftrightarrow{AD} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle A) = m(\angle C)$ similarly $m(\angle B) = m(\angle D)$

$\because MB = MC$ $\therefore m(\angle B) = m(\angle C)$

$\therefore m(\angle A) = m(\angle D)$ $\therefore AM = DM$

$\therefore \Delta AMD$ is an isosceles triangle. (Q.E.D.1)

In ΔMBC : $\because MB = MC$, $\overrightarrow{MF} \perp \overline{BC}$

$\therefore \overleftrightarrow{MF}$ is the axis of symmetry of ΔMBC

$\because \overleftrightarrow{AD} // \overline{BC}$, $\overleftrightarrow{FE}$ is a transversal to them.

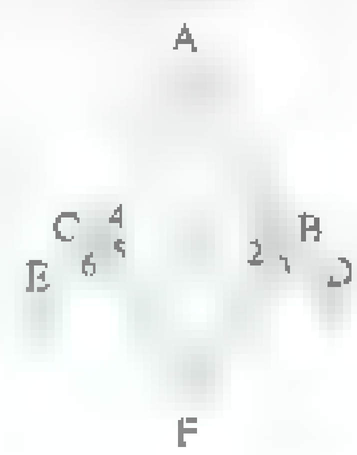
$\therefore m(\angle AEM) = m(\angle BFM) = 90°$

$\therefore \overline{ME} \perp \overline{AD}$

, $\because MA = MD$ $\therefore \overleftrightarrow{ME}$ is the axis of ΔAMD

$\therefore \overleftrightarrow{EF}$ is the axis of symmetry of each of ΔAMD , ΔBMC (Q.E.D.2)

19



$\because AB = AC$

$\therefore m(\angle 1) = m(\angle 4)$ (1)

$\because m(\angle DBC) = 180° - m(\angle 1)$ (2)

, $m(\angle BCE) = 180° - m(\angle 4)$ (3)

From (1) , (2) and (3) :

$\therefore m(\angle DBC) = m(\angle BCE)$

$\therefore \frac{1}{2} m(\angle DBC) = \frac{1}{2} m(\angle BCE)$

$\therefore m(\angle 2) = m(\angle 5)$ $\therefore FB = FC$

$\therefore \Delta BFC$ is an isosceles triangle. (Q.E.D.1)

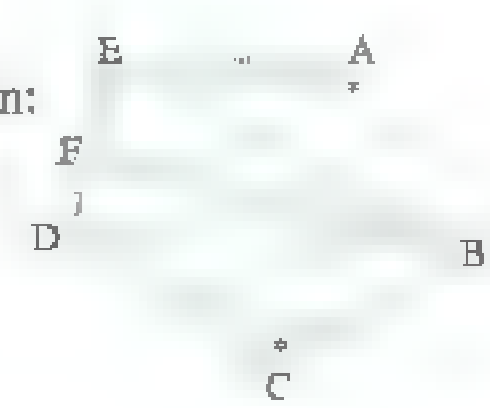
$\because AB = AC$, $FB = FC$

$\therefore \overleftrightarrow{AF}$ is the axis of symmetry of $\overline{BC}$ (Q.E.D.2)

20

Constr. : Draw $\overline{BD}$, $\overline{BE}$

Proof: $\Delta\Delta ABE$, CBD in them:



$$\begin{cases} m(\angle A) = m(\angle C) \\ AB \quad CB , \\ AE = CD \end{cases}$$

$\therefore \Delta ABE \equiv \Delta CBD$

, then we deduce that: BE = BD

, $\because \overline{BF}$ is a median of ΔBED which is isosceles

$\therefore \overline{BF} \perp \overline{DE}$ (Q.E.D.)

[1] c [2] b [3] c [4] b [5] a [6] b

In ΔABD :

E is the midpoint of $\overline{AB}$

, $\overline{DE} \perp \overline{AB}$ $\therefore DA = DB$

$\therefore m(\angle A) = m(\angle ABD)$ (1)

, in ΔDBC :

O is the midpoint of $\overline{BC}$

, $\overline{DO} \perp \overline{BC}$ $\therefore DB = DC$

$m(\angle DBC) = m(\angle C)$ (2)

, $\because m(\angle ABD) + m(\angle DBC) = 130°$ (3)

From (1) , (2) and (3) :

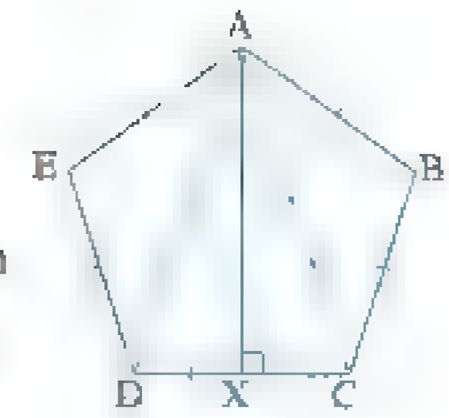
$\therefore m(\angle A) + m(\angle C) = 130°$

From the quadrilateral ABCD

$\therefore m(\angle ADC) = 360° - (130° + 130°) = 100°$ (The req.)

Constr. :

Draw $\overline{AD}$, $\overline{AC}$



Proof :

$\because$ ABCDE is a regular pentagon

$\therefore$ The measure of each interior angle = 108°

, AB = BC = CD = DE = EA

In ΔABC : $\because$ AB = BC , $m(\angle ABC) = 108°$

$\therefore m(\angle BAC) = \frac{180° - 108°}{2} = 36°$

$\therefore \Delta ABC$, ΔAED in them :

$$\begin{cases} AB = AE \\ BC = ED \\ m(\angle ABC) = m(\angle AED) = 108° \end{cases}$$

$\therefore \Delta ABC \equiv \Delta AED$

Then we deduce that : $m(\angle EAD) = m(\angle BAC) = 36°$

, AC = AD $\therefore \Delta ADC$ is an isosceles triangle.

, $\because \overline{AX} \perp \overline{CD}$

$\therefore m(\angle CAX) = m(\angle DAX)$

$= \frac{108° - (36° + 36°)}{2} = 18°$ (The req.)

Answers of unit five

1

1. $>$
2. $>$
3. $<$
4. $<$
5. $<$
6. $>$
7. $>$, $<$

2

1. $m(\angle 1) < m(\angle 3)$
2. $m(\angle 4) < m(\angle 2)$
3. $m(\angle 3) < m(\angle 5)$
4. $m(\angle 6) < m(\angle 2)$
5. $m(\angle 1) < m(\angle 3) < m(\angle 5)$
6. $m(\angle 3) < m(\angle 5) < m(\angle 7)$
7. $m(\angle 1) < m(\angle 3) < m(\angle 5) < m(\angle 7)$

3

$\because \overline{AD} \perp \overline{BC}$ from its midpoint.

$\therefore \overline{AD}$ is the axis of symmetry of $\overline{BC}$

$\therefore AB = AC$

$\because E \in \overline{AB}$ $\therefore AB > AE$

$\therefore AC > AE$ (Q.E.D.)

4

$\because \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore m(\angle BCD) = m(\angle ABC)$ (alternate angles)

$\therefore m(\angle BCD) + m(\angle ACB) > m(\angle ABC)$

$\therefore m(\angle ACD) > m(\angle ABC)$ (1) (Q.E.D.1)

$\because E \in \overrightarrow{CD}$ $\therefore \angle ADE$ is an exterior angle of ΔACD

$\therefore m(\angle ADE) > m(\angle ACD)$ (2)

From (1) and (2)

$\therefore m(\angle ADE) > m(\angle ABC)$ (Q.E.D.2)

5

$\because E \in \overrightarrow{CB}$ $\therefore \angle ABE$ is an exterior angle of ΔABC

$\therefore m(\angle ABE) > m(\angle A)$ (1)

$\Delta\Delta ABM$, CDM in them :

$$\begin{cases} AM = MC \\ MB = MD \\ m(\angle AMB) = m(\angle DMC) \quad \text{(V.O.A)} \end{cases}$$

$\therefore \Delta ABM \equiv \Delta CDM$, then we deduce that :

$m(\angle A) = m(\angle ACD)$ and from (1) :

$\therefore m(\angle ABE) > m(\angle ACD)$ (Q.E.D.)

6

$\because$ The figure is a parallelogram.

$\therefore AD = BC$, $AB = CD$

$\because DX < BY$ $\therefore AX > CY$

$\therefore AX + AB > CY + CD$ (Q.E.D.)

7

$\because D \in \overline{AB}$ $\therefore \angle ADC$ is an exterior angle of ΔDBC

$\therefore m(\angle ADC) > m(\angle B)$

But $m(\angle ADC) = m(\angle ACD)$

because ΔADC in which $AD = AC$

$\therefore m(\angle ACD) > m(\angle B)$

$\therefore m(\angle ACD) + m(\angle DCB) > m(\angle B)$

$\therefore m(\angle ACB) > m(\angle B)$ (Q.E.D.)

8

In ΔAXY : $\because m(\angle AXY) = m(\angle AYX)$

$\therefore AX = AY$ (1)

$\because AC > AB$ $\therefore AY + YC > AX + XB$ (2)

From (1) and (2) :

$\therefore YC > XB$ (Q.E.D.)

9

$\because \angle ADC$ is an exterior angle of ΔDBC

$\therefore m(\angle ADC) > m(\angle B)$

But $m(\angle B) = m(\angle ACB)$

(because $AB = AC$ in ΔABC)

$\therefore m(\angle ADC) > m(\angle ACB)$ (Q.E.D.)

10

$\because m(\angle ACB) > m(\angle ABC)$

$\therefore$ The supplement

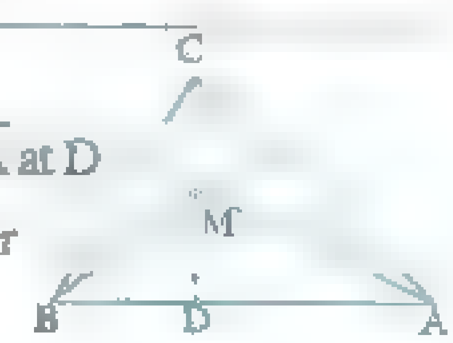
of $\angle ABC >$ the supplement of $\angle ACB$

$\therefore m(\angle ABD) > m(\angle ACE)$

$\therefore \frac{1}{2} m(\angle ABD) > \frac{1}{2} m(\angle ACE)$

i.e. $m(\angle ABX) > m(\angle ACY)$ (Q.E.D.)

11



Const : Draw $\overrightarrow{CM}$ to intersect $\overline{BA}$ at D

Proof : $\because \angle AMD$ is an exterior angle of ΔAMC

$\therefore m(\angle AMD) > m(\angle ACM)$ (1)

$\because \angle BMD$ is an exterior angle of ΔCMB

$\therefore m(\angle BMD) > m(\angle BCM)$ (2)

Adding (1) and (2)

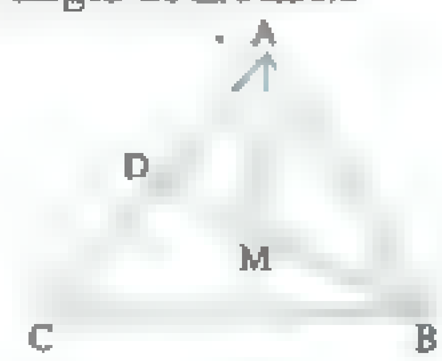
$\therefore$ m (∠ AMD) + m (∠ BMD) > m (∠ ACM) + m (∠ BCM)

$\therefore$ m (∠ AMB) > m (∠ C) (Q.E.D.)

Another solution :

Const : Draw $\overline{BM}$ to intersect $\overline{AC}$ at D

Proof : $\because$ ∠ AMB is an exterior angle of Δ ADM



$\therefore$ m (∠ AMB) > m (∠ ADM)

, $\because$ ∠ ADM is an exterior angle of Δ BCD

$\therefore$ m (∠ ADM) > m (∠ C)

$\therefore$ m (∠ AMB) > m (∠ C) (Q.E.D.)

$\because$ m (∠ B) > m (∠ C)

$\therefore$ m (∠B) + $\frac{1}{2}$m (∠ BAC) > m (∠ C) + $\frac{1}{2}$m (∠ BAC)

$\therefore$ m (∠ B) + m (∠ BAD) > m (∠C) + m (∠ CAD)

but m (∠ B) + m (∠ BAD) = m (∠ CDA)

(an exterior angle of ΔABD),

m (∠ C) + m (∠ CAD) = m (∠ BDA)

(an exterior angle of ΔACD)

$\therefore$ m (∠ ADC) > m (∠ ADB)

$\because$ Their sum = 180° $\therefore$ m (∠ ADC) > $\frac{180^\circ}{2}$

i.e. m (∠ ADC) > 90°

i.e. ∠ ADC is an obtuse angle. (Q.E.D.)

$\because$ AC = AD $\therefore$ m (∠ D) = m (∠ ACD)

$\because$ m (∠ ACB) > m (∠ ABC)

$\therefore$ m (∠ ACB) + m (∠ ACD) > m (∠ ABC) + m (∠ D)

$\therefore$ m (∠ BCD) > m (∠ B) + m (∠ D)

but the sum of measures of the interior angles of Δ BCD = 180°

$\therefore$ m (∠ BCD) > $\frac{180^\circ}{2}$ i.e. m (∠ BCD) > 90°

i.e. ∠ BCD is an obtuse angle. (Q.E.D.)

Answers of Exercise 7

1 The angle of the greater measure

2 ∠ A   3 m (∠ D)

4 m (∠ A) < m (∠ B) < m (∠ C)

1 > , > , <   2 < , < , >   3 > , > , >

3

1 $\because$ $\overline{BC}$ is the longest side

$\therefore$ ∠ A is the greatest angle in measure

$\because$ $\overline{AC}$ is the shortest side

$\therefore$ ∠ B is the smallest angle in measure

$\therefore$ The ascending order of measures of the angles is : m (∠ B) , m (∠ C) and m (∠ A)

2 $\because$ $\overline{BC}$ is the longest side.

$\therefore$ ∠ A is the greatest angle in measure.

$\because$ $\overline{AB}$ is the shortest side.

$\therefore$ ∠ C is the smallest angle in measure.

$\therefore$ The ascending order of the measures of the angles is : m (∠ C) , m (∠ B) and m (∠ A)

In ΔABC : $\because$ AC > AB

$\therefore$ m (∠ ABC) > m (∠ ACB) (1)

In ΔBDC : $\because$ DB = DC

$\therefore$ m (∠ DBC) = m (∠ DCB) (2)

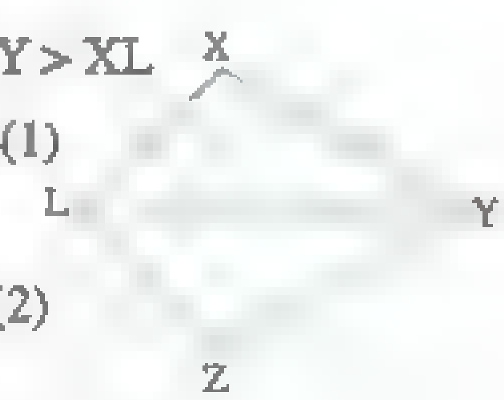
Adding (1) and (2) :

$\therefore$ m (∠ ABC) + m (∠ DBC) > m (∠ ACB) + m (∠ DCB)

$\therefore$ m (∠ ABD) > m (∠ ACD) (Q.E.D.)

Construction : Draw $\overline{YL}$

Proof : In ΔXYL $\because$ XY > XL



$\therefore$ m (∠ XLY) > m (∠ XYL) (1)

In ΔZYL : $\because$ YZ > ZL

$\therefore$ m (∠ ZLY) > m (∠ ZYL) (2)

Adding (1) and (2) :

$\therefore$ m (∠ XLY) + m (∠ ZLY) > m (∠ XYL) + m (∠ ZYL)

$\therefore$ m (∠ XLZ) > m (∠ XYZ) (Q.E.D.)

Construction : Draw $\overline{AC}$

Proof : In ΔABC

$\because$ BC > AB

$\therefore$ m (∠ BAC) > m (∠ ACB) (1)

In ΔDAC : $\because$ DA = DC

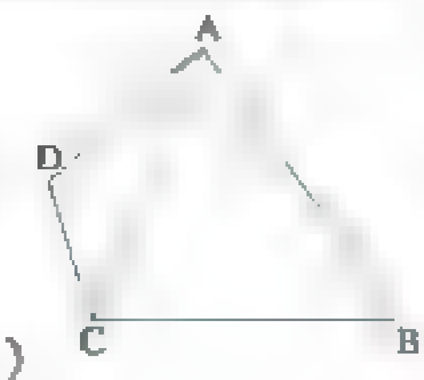
$\therefore$ m (∠ DAC) = m (∠ DCA) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ BAC) + m ($\angle$ DAC) > m ($\angle$ ACB) + m ($\angle$ DCA)

$\therefore$ m ($\angle$ BAD) > m ($\angle$ BCD) (Q.E.D.)

7



Construction : Draw $\overline{AC}$

Proof : In ΔABC

$\because$ AB > BC

$\therefore$ m ($\angle$ ACB) > m ($\angle$ BAC) (1)

In ΔADC : $\because$ AD > DC

$\therefore$ m ($\angle$ ACD) > m ($\angle$ CAD) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ BCD) > m ($\angle$ BAD) (Q.E.D.)

In ΔMBC : $\because$ MC > MB

$\therefore$ m ($\angle$ MBC) > m ($\angle$ MCB)

$\because$ m ($\angle$ MBC) = $\frac{1}{2}$ m ($\angle$ ABC)

, m ($\angle$ MCB) = $\frac{1}{2}$ m ($\angle$ ACB)

$\therefore$ $\frac{1}{2}$ m ($\angle$ ABC) > $\frac{1}{2}$ m ($\angle$ ACB)

$\therefore$ m ($\angle$ ABC) > m ($\angle$ ACB) (Q.E.D.)

In ΔDBC : $\because$ DB > DC

$\therefore$ m ($\angle$ DCB) > m ($\angle$ DBC)

In ΔABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC)

$\therefore$ m ($\angle$ ACB) − m ($\angle$ DCB) < m ($\angle$ ABC) − m ($\angle$ DBC)

$\therefore$ m ($\angle$ ACD) < m ($\angle$ ABD)

i.e. m ($\angle$ ABD) > m ($\angle$ ACD) (Q.E.D.)

In ΔABC : $\because$ AB > AC $\therefore$ m ($\angle$ C) > m ($\angle$ B) (1)

$\because$ $\overleftrightarrow{XY}$ // $\overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ AYX) = m ($\angle$ C) (corresponding angles) (2)

Similarly : $\because$ $\overleftrightarrow{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal.

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (3)

From (1) , (2) and (3) :

$\therefore$ m ($\angle$ AYX) > m ($\angle$ AXY) (Q.E.D.)

11

$\because$ AB > AC $\therefore$ m ($\angle$ C) > m ($\angle$ B)

But m ($\angle$ C) = m ($\angle$ AED) (corresponding angles)

, m ($\angle$ B) = m ($\angle$ ADE) (corresponding angles)

$\therefore$ m ($\angle$ AED) > m ($\angle$ ADE)

In ΔADE :

$\because$ m ($\angle$ A) = 90°

$\therefore$ m ($\angle$ AED) + m ($\angle$ ADE) = 90°

, $\because$ m ($\angle$ AED) > m ($\angle$ ADE)

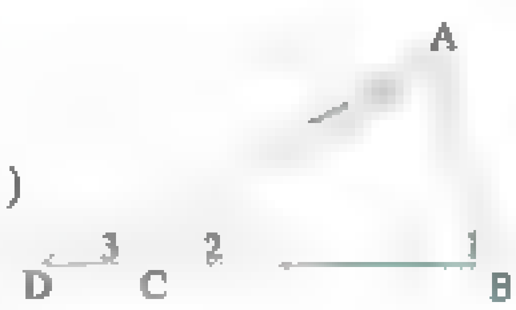
$\therefore$ m ($\angle$ AED) > $\frac{90°}{2}$

$\therefore$ m ($\angle$ AED) > 45° (Q.E.D.)

$\because$ AB > AC , BD = CE Subtracting $\therefore$ AD > AE

$\therefore$ In ΔADE : $\because$ AD > AE

$\therefore$ m ($\angle$ AED) > m ($\angle$ ADE) (Q.E.D.)



In ΔABC : $\because$ AC > AB

$\because$ m ($\angle$ 1) > m ($\angle$ 2) (1)

But $\angle$ 2 is an exterior angle of ΔACD

$\because$ m ($\angle$ 2) > m ($\angle$ 3) (2)

From (1) and (2) : $\therefore$ m ($\angle$ 1) > m ($\angle$ 3)

$\therefore$ m ($\angle$ ABD) > m ($\angle$ D) (Q.E.D.)

$\because$ ΔABC is an equilateral triangle.

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = 60°

$\because$ m ($\angle$ EBC) < m ($\angle$ ECB) Subtracting

$\therefore$ m ($\angle$ ABC) − m ($\angle$ EBC) > m ($\angle$ ACB) − m ($\angle$ ECB)

$\therefore$ m ($\angle$ ABE) > m ($\angle$ ACE) (1) (Q.E.D.1)

$\because$ m ($\angle$ A) = m ($\angle$ B)

$\therefore$ m ($\angle$ A) = m ($\angle$ ABE) + m ($\angle$ EBC)

$\therefore$ m ($\angle$ A) > m ($\angle$ ABE) and from (1) :

$\therefore$ m ($\angle$ A) > m ($\angle$ ABE) > m ($\angle$ ACE) (Q.E.D.2)

In ΔXBC : $\because$ XC > XB

$\therefore$ m ($\angle$ XBC) > m ($\angle$ XCB)

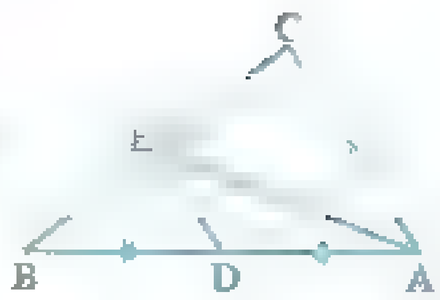
$\because$ ABCD is a rectangle.

i.e. m ($\angle$ ABC) = m ($\angle$ DCB) = 90°

$\therefore$ 90° − m ($\angle$ XBC) < 90° − m ($\angle$ XCB)

$\therefore$ m ($\angle$ ABX) < m ($\angle$ XCD) (Q.E.D.)

16



In ΔABC : $\because$ D is the midpoint of AB , $\overline{DE} \,//\, \overline{AC}$

$DE = \frac{1}{2} AC$

, $\because AD = \frac{1}{2} AB$, $\because AB > AC$

$\therefore \frac{1}{2} AB > \frac{1}{2} AC$

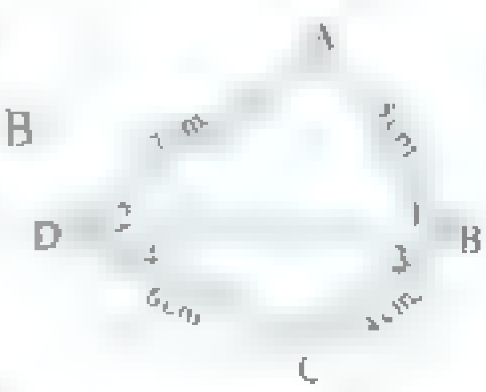
$\therefore AD > DE$ $\therefore m(\angle AED) > m(\angle DAE)$

But $m(\angle AED) = m(\angle CAE)$ (alternate angles)

$m(\angle CAE) > m(\angle DAE)$ (Q.E.D.)

17

1 **Construction :** Draw $\overline{BD}$



Proof : In ΔABD : $\because AD > AB$

$\therefore m(\angle 1) > m(\angle 2)$ (1)

In ΔCBD : $\because CD > CB$

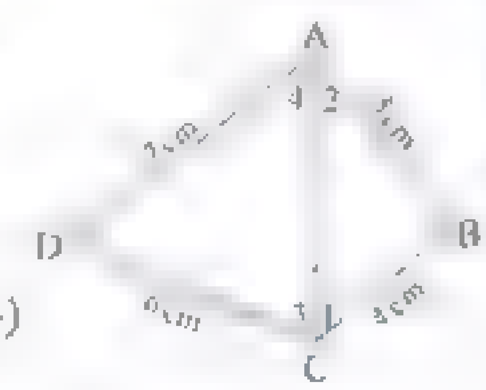
$\therefore m(\angle 3) > m(\angle 4)$ (2)

Adding (1) and (2) :

$\therefore m(\angle 1) + m(\angle 3) > m(\angle 2) + m(\angle 4)$

$\therefore m(\angle ABC) > m(\angle ADC)$ (Q.E.D.1)

2 **Construction :** Draw $\overline{AC}$



Proof : In ΔABC :

$BA > BC$

$\therefore m(\angle 1) > m(\angle 2)$ (3)

In ΔADC : $\because AD > DC$

$\therefore m(\angle 3) > m(\angle 4)$ (4)

Adding (3) and (4) :

$\therefore m(\angle 1) + m(\angle 3) > m(\angle 2) + m(\angle 4)$

$m(\angle BCD) > m(\angle BAD)$ (Q.E.D.2)

3 $\because$ The sum of measure of the interior angles of the quadrilateral = 360°
and from the two preceding requirements.

$m(\angle B) + m(\angle C) > \frac{360°}{2}$

$\therefore m(\angle B) + m(\angle C) > 180°$ (Q.E.D.3)

18

$\overline{AE}$ is a median in ΔABD , $m(\angle A) = 90°$

$\therefore AE = \frac{1}{2} BD$

, $\because$ E is the midpoint of $\overline{BD}$, $\overline{EX} \,//\, \overline{AC}$

$\therefore EX = \frac{1}{2} DC$, $\because AE > EX$

$\therefore \frac{1}{2} BD > \frac{1}{2} DC$ $\therefore BD > DC$

$\therefore m(\angle C) > m(\angle DBC)$ (Q.E.D.)

19

In ΔABM : $\because AM > BM \therefore m(\angle ABM) > m(\angle A)$ (1)

$\because AM = CM \therefore$ In ΔCBM : $MC > MB$

$\therefore m(\angle MBC) > m(\angle C)$ (2)

Adding (1) and (2) :

$\therefore m(\angle ABM) + m(\angle MBC) > m(\angle A) + m(\angle C)$

$\therefore m(\angle ABC) > m(\angle A) + m(\angle C)$

$\therefore \angle ABC$ is an obtuse angle. (Q.E.D)

20

In ΔABD : $\because m(\angle B) = 90° - m(\angle BAD)$ (1)

From ΔACD: $m(\angle C) = 90° - m(\angle CAD)$ (2)

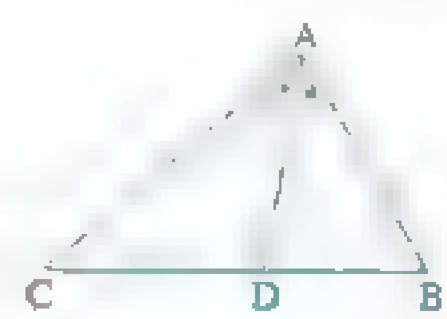
From ΔABC : $AC > AB \therefore m(\angle B) > m(\angle C)$ (3)

From (1) , (2) and (3) :

$\therefore 90° - m(\angle BAD) > 90° - m(\angle CAD)$

$\therefore m(\angle BAD) < m(\angle CAD)$ (Q.E.D.)

21



In ΔABC : $\because AC > AB$

$\therefore m(\angle B) > m(\angle C)$

$m(\angle BAD) = m(\angle DAC)$

($\overline{AD}$ bisects $\angle A$)

$\therefore m(\angle B) + m(\angle BAD) > m(\angle C) + m(\angle DAC)$

$\angle ADC$ is an exterior angle of ΔABD

$\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$

$\because \angle ADB$ is an exterior angle of ΔADC

$\therefore m(\angle ADB) = m(\angle C) + m(\angle DAC)$

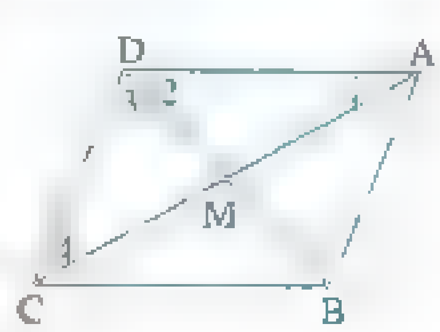
$\therefore m(\angle ADC) > m(\angle ADB)$

$\because m(\angle ADC) + m(\angle ADB) = 180°$

$\therefore m(\angle ADC) > \frac{180°}{2}$ *i.e.* $m(\angle ADC) > 90°$

i.e. $\angle ADC$ is an obtuse angle. (Q.E.D.)

22



Let $\overline{CA} \cap \overline{DB} = \{M\}$

$\because$ The two diagonals in the parallelogram bisects each other

, $\because AC > BD$ $\therefore MA > MD$,
$MC > MD$,

From ΔAMD : $\because AM > MD$

$\therefore m(\angle 2) > m(\angle 1)$ (1)

From ΔDMC : $\because MC > MD$

$\therefore m(\angle 3) > m(\angle 4)$ (2)

Adding (1) and (2) :

$\therefore m(\angle 2) + m(\angle 3) > m(\angle 1) + m(\angle 4)$

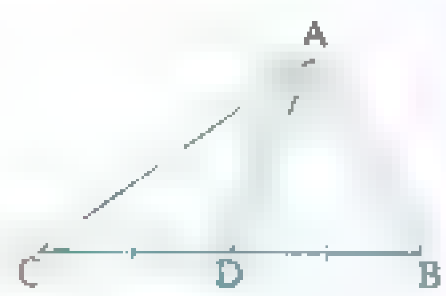
$\therefore m(\angle D) > m(\angle 1) + m(\angle 4)$

$\therefore$ In ΔADC :

$\because m(\angle D) > m(\angle CAD) + m(\angle ACD)$

$\therefore \angle D$ is an obtuse angle. (Q.E.D.)

23



$\because$ The perimeter of ΔACD

$= CD + DA + AC$

The perimeter of ΔABD

$= BD + DA + AB$

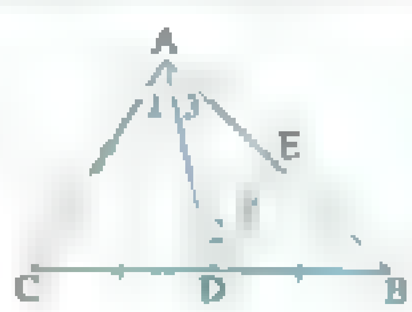
$\because$ The perimeter of $\Delta ACD >$ the perimeter of ΔABD

$\therefore CD + DA + AC > BD + DA + AB$

But $CD = BD$

$\therefore AC > AB \therefore m(\angle B) > m(\angle C)$ (Q.E.D.)

24



Construction : Draw $\overline{DE} \parallel \overline{AC}$ to intersect $\overline{AB}$ at E

Proof : In ΔABC : $\because \overline{DE} \parallel \overline{AC}$,

D is the midpoint of $\overline{BC}$

$\therefore$ E is the midpoint of $\overline{BA}$

$\therefore AE = \frac{1}{2} AB$, $DE = \frac{1}{2} AC$

, $\because AB > AC$ $\therefore AE > DE$

$\therefore m(\angle 2) > m(\angle 3)$ (1)

$\because \overline{DE} \parallel \overline{AC}$, $\overleftrightarrow{AD}$ is a transversal to them

$\therefore m(\angle 1) = m(\angle 2)$ (Alternate angles)

From (1) : $\therefore m(\angle 1) > m(\angle 3)$

$\therefore m(\angle BAD) < m(\angle CAD)$ (Q.E.D.)

1 A side greater in length than that opposite to the other angle , greater in measure than the measure of the angle opposite to the other side.

2 The shortest side.

3 The hypotenuse.

4 The length of the line segment drawn from the given point perpendicular to the given straight line.

5 $\overline{AB}$ 6 $\overline{AC}$ 7 $\overline{BC}$

1 c 2 a 3 d 4 a

1 > , > , < 2 > , > , >

3 > , > , > , > 4 > , < , < , >

$YZ < XY < XZ$

$AC > AB > BC$

$\because \overleftrightarrow{AE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle C) = m(\angle EAC) = 30°$ (alternate angles) (1)

, $\because \overleftrightarrow{AE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal.

$\therefore m(\angle B) = m(\angle DAE) = 70°$

(corresponding angles) (2)

From (1) and (2) : $\therefore m(\angle B) > m(\angle C)$

$\therefore AC > AB$ (Q.E.D.)

$\because C \in \overrightarrow{AE}$ $\therefore m(\angle ACB) = 180° - 120° = 60°$

$\because B \in \overrightarrow{CD}$ $\therefore m(\angle ABC) = 180° - 110° = 70°$

$\therefore m(\angle A) = 180° - (60° + 70°) = 50°$

$\because m(\angle ACB) > m(\angle A)$ $\therefore AB > BC$ (Q.E.D.)

In ΔABC : $\because AB = AC$

$\therefore m(\angle ACB) = m(\angle B) = 65°$

$\therefore m(\angle DCB) = 65° + 20° = 85°$

In ΔDBC : $\therefore m(\angle D) = 180° - (65° + 85°) = 30°$

$\therefore$ In ΔDAC : $m(\angle D) > m(\angle ACD)$

$\therefore AC > AD$ but $AB = AC$

$\therefore AB > AD$ (Q.E.D.)

In ΔDBC : $\because DB = DC$

$\therefore m(\angle B) = m(\angle DCB) = \frac{180° - 100°}{2} = 40°$

$\because \overrightarrow{CD}$ bisects $\angle ACB$ $\therefore m(\angle ACD) = 40°$

$\because D \in \overline{AB}$

$\therefore m(\angle ADC) = 180° - 100° = 80°$

$\therefore$ In ΔADC : $m(\angle A) = 180° - (40° + 80°) = 60°$

$\therefore m(\angle ADC) > m(\angle A)$

$\therefore AC > DC$ but $DC = DB$

$\therefore AC > DB$ (Q.E.D.)

10

$\because \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle ACB) = m(\angle DAC) = 30°$ (alternate angles)

In ΔABC $\because m(\angle BAC) > m(\angle ACB)$

$\therefore BC > AB$ (Q.E.D.)

In ΔACM : $\because m(\angle C) = 90°$ $\therefore AM > CM$ (1)

In ΔBDM : $\because m(\angle D) = 90°$ $\therefore BM > DM$ (2)

Adding (1) and (2) : $\therefore AM + MB > CM + MD$

$\therefore AB > CD$ (Q.E.D.)

12

In ΔABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\because m(\angle ABM) < m(\angle ACM)$

$\therefore m(\angle ABC) - m(\angle ABM) > m(\angle ACB) - m(\angle ACM)$

$\therefore m(\angle MBC) > m(\angle MCB)$

From ΔMBC : $\therefore MC > MB$ (Q.E.D.)

$\because \overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{DB}$ is a transversal.

$\therefore m(\angle ADE) = m(\angle B)$ (corresponding angles)

, $\because \angle B$ is an obtuse angle

$\therefore \angle ADE$ is an obtuse angle

$\therefore$ In ΔADE :

$m(\angle ADE) > m(\angle AED)$ (obtuse and acute angles)

$\therefore AE > AD$ (Q.E.D.)

14

In ΔABC : $\because AB > AC$ $\therefore m(\angle C) > m(\angle B)$ (1)

$\because \overleftrightarrow{DE} \parallel BC$ and $\overleftrightarrow{DC}$ is a transversal.

$\therefore m(\angle D) = m(\angle C)$ (alternate angles) (2)

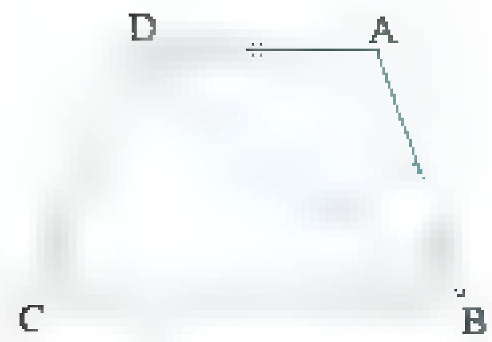
$\therefore \overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{BE}$ is a transversal.

$\therefore m(\angle E) = m(\angle B)$ (alternate angles) (3)

From (1) , (2) and (3) :

$\therefore m(\angle D) > m(\angle E)$ and from ΔADE

$\therefore AE > AD$ (Q.E.D.)



Const. : Draw BD

Proof : In ΔADB

$\because AD = AB$

$\therefore m(\angle ADB) = m(\angle ABD)$

, $\because m(\angle ADC) > m(\angle ABC)$

$\therefore m(\angle ADC) - m(\angle ADB) > m(\angle ABC) - m(\angle ABD)$

$\therefore m(\angle BDC) > m(\angle DBC)$

$\therefore$ In ΔBDC : $BC > CD$ (Q.E.D.)

In ΔABC : $\because AB > AC$

$\therefore m(\angle ABC) < m(\angle ACB)$

, $\because B \in \overline{AD}$, $C \in \overline{AE}$

$\therefore 180° - m(\angle ABC) > 180° - m(\angle ACB)$

$\therefore m(\angle CBD) > m(\angle BCE)$

$\because \overrightarrow{BF}$ bisects $\angle DBC$, $\overrightarrow{CF}$ bisects $\angle BCE$

$\therefore m(\angle FBC) > m(\angle BCF)$ (Q.E.D.1)

$\therefore CF > BF$ (Q.E.D 2)

17

In ΔABD : $\because BD = AD$

$\therefore m(\angle BAD) = m(\angle B)$

$\therefore m(\angle BAD) + m(\angle DAC) > m(\angle B)$

$\therefore m(\angle BAC) > m(\angle B)$ $\therefore BC > AC$ (Q.E.D.)

In ΔDBC : $\because m(\angle B) > m(\angle DCB)$

$\therefore DC > DB$ but $DB = AD$ $\therefore DC > AD$

$\therefore$ In ΔADC : $m(\angle A) > m(\angle ACD)$ (Q.E.D.1)

$\because m(\angle BDC) = 180° - (70° + 50°) = 60°$

$\because \angle BCD$ is an exterior angle of ΔADC

$\therefore m(\angle BDC) = m(\angle A) + m(\angle ACD) = 60°$

$\because m(\angle A) > m(\angle ACD)$ $\therefore m(\angle ACD) < 30°$

$\therefore m(\angle ACD) + m(\angle DCB) < 30° + 50°$

$\therefore m(\angle ACB) < 80°$

$\therefore \angle ACB$ is an acute angle. (Q.E.D.2)

In ΔAFB : $\because FA = FB$

$\therefore m(\angle FBA) = m(\angle FAB) = 50°$ (1)

$\because \angle AFD$ is an exterior angle of ΔAFB

$\therefore m(\angle AFD) = 50° + 50° = 100°$ (2)

∴ In Δ AFD :

∵ FA = FD   ∴ m (∠ FDA) = $\frac{180° - 100°}{2}$ = 40°

From (1) and (2) : ∴ In Δ ABD

m (∠ ABD) > m (∠ ADB)

∴ AD > AB   (Q.E.D.1)

In Δ ABD : ∵ $\overline{AF}$ is a median , AF = $\frac{1}{2}$ BD

∴ m (∠ DAB) = 90°

∴ $\overline{BC}$ is a hypotenuse of Δ BAC

∴ BC > AC   (Q.E.D.2)

20

∵ ∠ ADB is an exterior angle of Δ ADC

∴ m (∠ ADB) > m (∠ C)

, ∵ m (∠ C) = m (∠ B) , (AB = AC in Δ ABC)

∴ m (∠ ADB) > m (∠ B)

And from Δ ABD : AB > AD   (Q.E.D.)

21

ΔΔ ABD , AED in them

m (∠ B) = m (∠ AED) , m (∠ BAD) = m (∠ DAE)

∴ m (∠ ADB) = m (∠ ADE)

∴ ΔΔ ABD , AED

In them $\begin{cases} \text{m (∠ BAD) = m (∠ EAD)} \\ \text{m (∠ ADB) = m (∠ ADE)} \\ \overline{AD} \text{ is a common side} \end{cases}$

∴ Δ ABD ≡ Δ AED , then we deduce that :

BD = DE   (Q.E.D.1)

∴ In Δ DEC

∵ m (∠ DEC) = 90°   ∴ DC > DE

∵ DE = DB   ∴ DC > DB   (Q.E.D.2)

∵ m (∠ ADC) = 180° – 110° = 70°

∴ Δ ACD in which m (∠ ADC) > m (∠ C)

∴ AC > AD   (1)

∵ Δ ADB is an obtuse-angled at D

∴ AB > AD   (2)

By adding (1) and (2) : ∴ AB + AC > 2 AD (Q.E.D.)

In Δ ABC : ∵ m (∠ B) = 90°

∴ The hypotenuse $\overline{AC}$ is the longest side.

∴ AB < AC , BC < AC

By adding : ∴ AB + BC < 2 AC   (Q.E.D.)

24

∵ $\overline{AD}$ // $\overline{CE}$, $\overleftrightarrow{AC}$ is a transversal.

∴ m (∠ DAC) = m (∠ ACE)

∴ m (∠ BCE) > m (∠ DAC)

∴ m (∠ BCE) > m (∠ BAD) , ($\overrightarrow{AD}$ bisects ∠ BAC)   (1)

∵ $\overline{AD}$ // $\overline{CE}$ and $\overleftrightarrow{BE}$ is a transversal.

∴ m (∠ BAD) = m (∠ E) (corresponding angles)   (2)

From (1) and (2) :

∴ m (∠ BCE) > m (∠ E) and from Δ BCE

∴ BE > BC   (Q.E.D.)

∵ Δ XYM is right-angled at Y

∴ ∠ XMY is an acute angle.

∴ ∠ XMZ is an obtuse angle.

∵ Δ XMZ is an obtuse-angled at M

∴ XZ > XM   (Q.E.D.)

In Δ ABC : ∵ AB > BC

∴ m (∠ C) > m (∠ A)   (1)

, ∵ m (∠ ABC) = 90°

∴ ∠ A complements ∠ C   (2)

, in Δ ABD : ∵ m (∠ ADB) = 90°

∴ ∠ A complements ∠ ABD   (3)

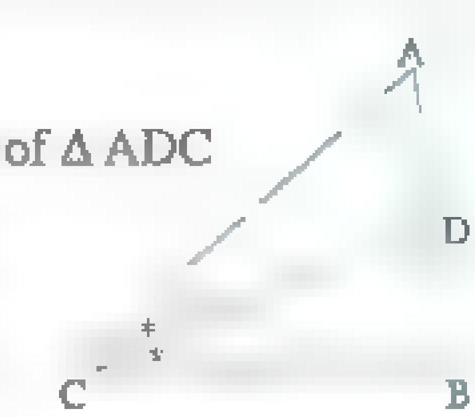
From (2) and (3) :

∴ m (∠ C) = m (∠ ABD)

, from (1) :

∴ m (∠ ABD) > m (∠ A)

∴ In Δ ABD : AD > BD   (Q.E.D.)



∵ ∠ BDC is an exterior angle of Δ ADC

∴ m (∠ BDC) > m (∠ ACD)

∵ m (∠ BCD) = m (∠ ACD)

∴ m (∠ BDC) > m (∠ BCD)

In Δ DBC : ∴ BC > BD   (Q.E.D.)

28

In ΔABC $\because$ $m(\angle B) = 90°$

$\therefore AC > BC$

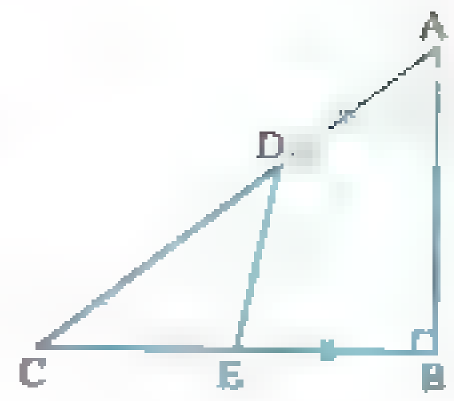
$\because AD = BE$

$\therefore AC - AD > BC - BE$

$\therefore DC > EC$

In ΔDEC : $\because DC > EC$

$\therefore m(\angle CED) > m(\angle CDE)$ (Q.E.D.)



29

$\because m(\angle A) + m(\angle B) + m(\angle C) = 180°$

$5X + 2° + 6X - 10° + X + 20° = 180°$

$\therefore 12X + 12° = 180°$ $\quad 12X = 180° - 12° = 168°$

$\therefore X = \frac{168°}{12°} = 14°$ $\quad \therefore m(\angle A) = 5 \times 14° + 2° = 72°$

$, m(\angle B) = 6 \times 14° - 10° = 74°$

$m(\angle C) = 14° + 20° = 34°$

$\therefore AB < BC < AC$ (The req.)

30

$\because \angle 1$ is an exterior angle of ΔXZC

$\therefore m(\angle 1) > m(\angle 2)$

But $m(\angle 3) = m(\angle 2)$, ($AB = AC$ in ΔABC)

$\therefore m(\angle 1) > m(\angle 3)$

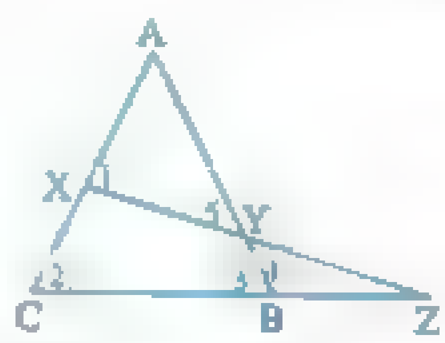
But $\angle 3$ is an exterior angle of ΔYZB

$\therefore m(\angle 3) > m(\angle 4)$ $\quad \therefore m(\angle 1) > m(\angle 4)$

$, \because m(\angle 4) = m(\angle 5)$ (V.O.A.)

$\therefore m(\angle 1) > m(\angle 5)$

and from ΔAYX $\therefore AY > AX$ (Q.E.D.)



1

1 $\because 3 + 4 < 9$ $\quad \therefore$ lengths are not suitable

2 $\because 5 + 7 > 8$ $\quad \therefore$ lengths are suitable

3 $\because 4 + 6 = 10$ $\quad \therefore$ lengths are not suitable

4 $\because 6 + 8 > 13$ $\quad \therefore$ lengths are suitable

5 $\because 3 + 4 > 5$ $\quad \therefore$ lengths are suitable

6 $\because 9 + 9 < 19$ $\quad \therefore$ lengths are not suitable

2

Let the length of the third side be ℓ

1 $\because 9 - 6 < \ell < 9 + 6$ $\quad \therefore 3 < \ell < 15$

$\therefore \ell \in]3 , 15[$

2 $\because 3 - 3 < \ell < 3 + 3$ $\quad \therefore 0 < \ell < 6$

$\therefore \ell \in]0 , 6[$

3 $\because 3.2 - 2.9 < \ell < 3.2 + 2.9$

$\therefore 0.3 < \ell < 6.1$ $\quad \therefore \ell \in]0.3 , 6.1[$

4 $\because 7.3 - 5.7 < \ell < 7.3 + 5.7$

$\therefore 1.6 < \ell < 13$ $\quad \therefore \ell \in]1.6 , 13[$

3

1 b	2 b	3 c	4 d	5 a
6 b	7 d	8 b	9 a	10 a

4

In ΔXLY : $XL + LY > XY$ (The triangle inequality)

But $XL = LZ$ $\quad \therefore LZ + LY > XY$

$\therefore YZ > XY$ (Q.E.D.)

5

In ΔABC :

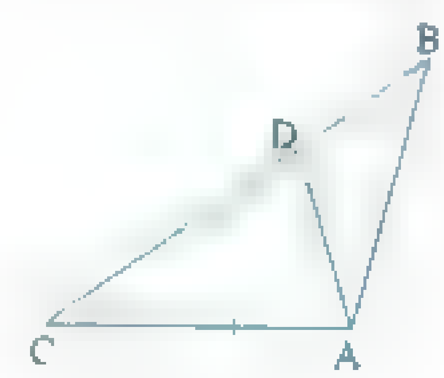
$\because CA + AB > BC$

(triangle inequality)

$\therefore CA + AB > BD + DC$

But $CA = DC$

$\therefore AB > BD$ (Q.E.D.)



6

From ΔABD :

$AD + DB > AB$

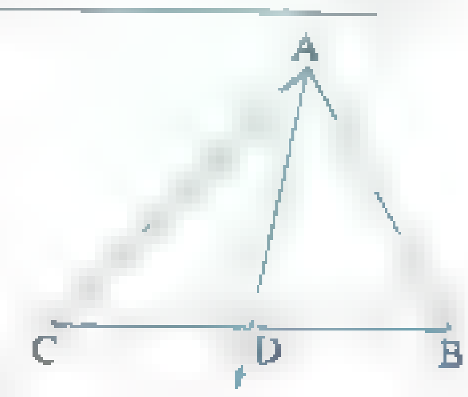
(triangle inequality) (1)

From ΔADC : $AD + DC > AC$

(Triangle inequality) (2)

Adding (1) and (2) :

$\therefore BD + DC + 2AD > AB + AC$ (Q.E.D.)



7

From ΔABM : $MA + MB > AB$

(Triangle inequality) (1)

From ΔBMC : $MB + MC > BC$

(Triangle inequality) (2)

From ΔAMC : $MA + MC > AC$ (Triangle inequality) (3)

Adding (1) , (2) and (3) :

$\therefore$ 2 MA + 2 MB + 2 MC > AB + BC + AC

$\therefore$ MA + MB + MC > $\frac{1}{2}$ the perimeter of Δ ABC

(Q.E.D.)

8

From Δ AEZ : AE + AZ > EZ (Triangle inequality) (1)

From Δ EBF :

EB + BF > EF (Triangle inequality) (2)

From Δ ZFC : ZC + CF > ZF (Triangle inequality) (3)

Adding (1) , (2) and (3) :

$\therefore$ AB + AC + BC > EZ + EF + ZF

$\therefore$ The perimeter of Δ ABC > the perimeter of Δ EFZ

(Q.E.D.)

9

In Δ DAC : DA + DC > AC (1)

In Δ DBC : DB + DC > BC (2)

In Δ DBA : DB + DA > AB (3)

Adding (1) , (2) and (3) :

$\because$ 2 (DA + DB + DC) > AC + BC + AB

$\therefore$ AC + BC + AB < 2 (DA + DB + DC)

$\therefore$ The perimeter of Δ ABC < 2 (DA + DB + DC)

(Q.E.D.)

10

7 − 3 < AC < 7 + 3 $\therefore$ 4 < AC < 10

$\because$ AC $\in$]4 , 10[$\therefore$ AB < AC

$\therefore$ m ($\angle$ C) < m ($\angle$ B) (The req.)

11

Assuming that ABC is a triangle

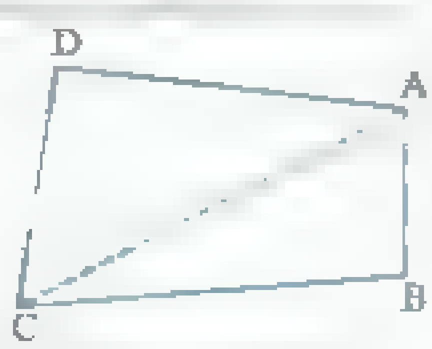
AB < AC + BC adding AB to both sides

$\therefore$ 2 AB < AC + BC + AB

AB < $\frac{1}{2}$ the perimeter of Δ ABC

$\therefore$ The length of any side in the triangle is less than the half of the perimeter of the triangle (Q.E.D.)

12



Construction : Draw $\overline{AC}$

Proof : From Δ ABC

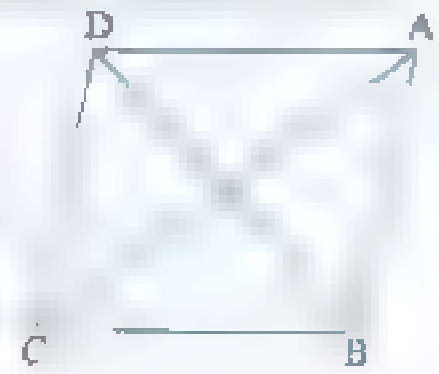
AB + BC > AC (Triangle inequality) (1)

From Δ ADC :

AC + CD > AD (Triangle inequality) (2)

From (1) and (2) : $\therefore$ AB + BC + CD > AD (Q.E.D.)

13



Let ABCD be a quadrilateral

In Δ ABC : AB + BC > AC (1)

In Δ BCD : BC + CD > BD (2)

In Δ ACD : AD + CD > AC (3)

In Δ ABD : AB + AD > BD (4)

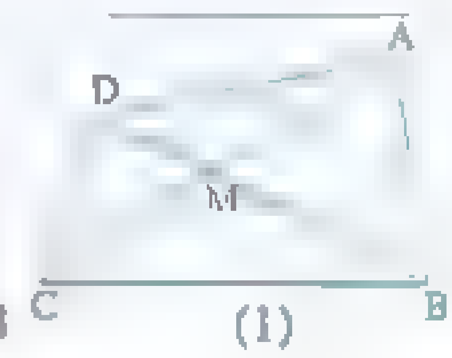
Adding (1) , (2) , (3) , and (4) :

$\because$ 2 AB + 2 BC + 2 CD + 2 AD > 2 AC + 2 BD

$\therefore$ AB + BC + CD + AD > AC + BD

$\therefore$ The sum of lengths of the two diagonals in any convex quadrilateral is less than the perimeter of the quadrilateral. (Q.E.D.)

14



Let ABCD be a quadrilateral ,

$\overline{AC} \cap \overline{BD}$ = {M}

From Δ ABM : AB < MA + MB (1)

From Δ BMC : BC < MB + MC (2)

From Δ CMD : CD < MC + MD (3)

From Δ AMD : AD < MA + MD (4)

Adding (1) , (2) , (3) and (4)

$\therefore$ AB + BC + CD + AD

< 2 MA + 2 MC + 2 MB + 2 MD

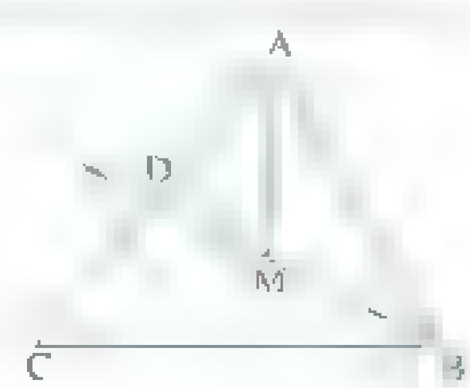
$\therefore$ AB + BC + CD + DA

< 2 (MA + MC) + 2 (MB + MD)

$\therefore$ AB + BC + CD + DA < 2 (AC + BD)

$\therefore$ The perimeter of the quadrilateral ABCD < twice the sum of lengths of the two diagonals. (Q.E.D.)

15



Construction :

Draw $\overrightarrow{BM}$ to cut $\overline{AC}$ at D

Proof :

In Δ BDC :

BC + DC > BD (Triangle inequality)

$\therefore$ BC + DC > BM + MD (1)

$\because$ In Δ AMD : AD + MD > AM (Triangle inequality)

$\therefore$ AD > AM − MD (2)

Adding (1) , (2) :

$\therefore$ BC + AD + DC > BM + MD + AM - MD

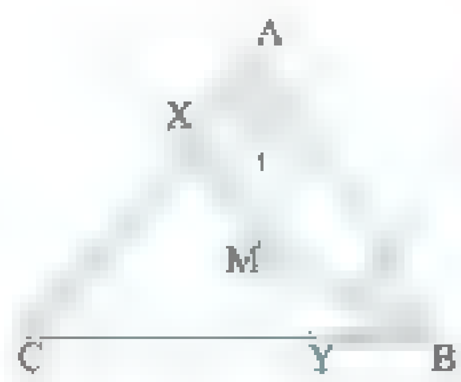
BC + AC > BM + AM

$\therefore$ AM + MB < BC + AC (Q.E.D.)

Another solution :

Construction :

Draw $\overline{XY}$ Passing through the point M where $X \in \overline{AC}$, $Y \in \overline{BC}$



Proof : In Δ CXY

$\because$ CY + CX > XM + MY adding BY and AX to both sides.

$\therefore$ CY + BY + CX + AX > XM + AX + MY + BY

$\therefore$ BC + AC > XM + AX + MY + BY

$\because$ XM + AX > AM , MY + BY > MB

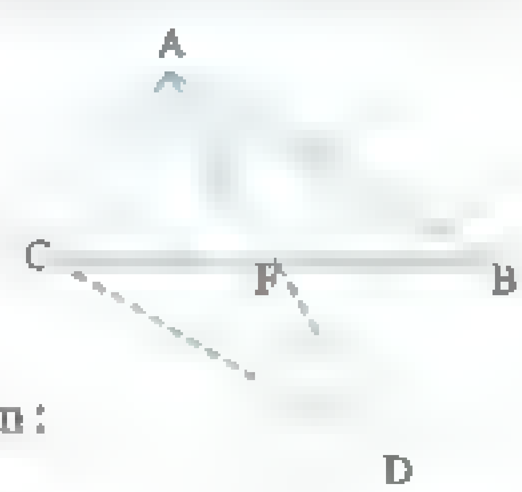
$\therefore$ BC + AC > AM + MB

$\therefore$ AM + MB < AC + BC (Q.E.D.)

16

Construction :

Extend $\overline{AF}$ as its length to D then draw $\overline{CD}$



Proof : $\Delta\Delta$ AFB , DFC in them :

$\begin{cases} AF = DF \text{ const.} \\ BF = FC \text{ (given)} \\ m(\angle AFB) = m(\angle DFC) \quad \text{(V.O.A.)} \end{cases}$

$\therefore$ The two triangles are congruent then we deduce that AB = DC but in Δ ACD we find that :

AC + CD > AD (triangle inequality)

$\therefore$ AC + AB > AD

$\because$ AD = 2 AF

AC + AB > 2 AF (1) (Q.E.D.1)

From Δ ABC : $\because$ AB + AC > BC

i.e. AB + AC > 2 BF (2)

Adding (1) and (2) : 2 AB + 2 AC > 2 AF + 2 BF

Dividing by 2 : $\therefore$ AB + AC > AF + BF (Q.E.D.2)

Answers of accumulative basic skills

1

1 $2\sqrt{10}$	2 2 : 3	3 5	4 150°
5 18	6 54	7 $\frac{1}{2}$ P − y	8 $5\sqrt{3}$
9 108°	10 60	11 9	12 19

2

1 (b)	2 (c)	3 (b)	4 (d)
5 (a)	6 (c)	7 (a)	8 (c)
9 (d)	10 (a)	11 (c)	12 (b)
13 (d)	14 (c)		

Guide Answers

Of The Notebook (Algebra and Statistics)

[illegible]

Accumulative test 1

1 [1] c [2] c [3] c [4] d

2 [1] -8 [2] zero [3] 5 [4] 2

3 [1] $\{-1\}$ [2] $\left\{\frac{1}{2}\right\}$

Accumulative test 2

1 [1] c [2] a [3] c [4] b

2 [1] 4 [2] 2 [3] 25 [4] zero

3 [a] Prove by yourself.

[b] Prove by yourself.

Accumulative test 3

1 [1] c [2] c [3] a

[4] c [5] d

2 [1] $\varnothing$ [2] $\varnothing$ [3] 4

[4] $\varnothing$ [5] $\{2\}$

Accumulative test 4

1 [1] c [2] b [3] c [4] c

2 [1] $\{3, 5\}$ [2] $\mathbb{N} - \{1\}$ [3] 4 [4] [illegible]

3 [1] $X = \{x : x \in \mathbb{Z}, 2 \le x \le 5\}$

[2] 

[3] $X - Y =]3, 5]$ $\because \sqrt{29} \simeq 5.39$

$\therefore \sqrt{29} \notin X - Y$

4 

$X \cup Y = [-1, \infty[$, $X \cap Y = [-1, 3[$

, $Y - X =]4, \infty[$

Accumulative test 5

1 [1] c [2] c [3] c [4] b

2 [1] $]-1, 5[$ [2] 96 [3] $5\sqrt{3}$ [4] 24

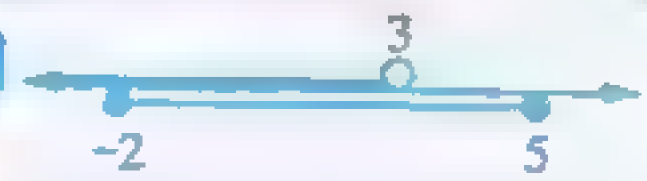
3 $4 + 2\sqrt{3}$

4 [1] 22 [2] 56

Accumulative test 6

1 [1] b [2] d [3] c [4] b

2 [1] $[2, 3]$ [2] $\varnothing$ [3] $\sqrt{5} - 3$ [4] $-\sqrt{2}$

3 

[1] $A \cap B = [-2, 3[$ [2] $B - A = [3, 5]$

4 [1] $2\sqrt{6}$ [2] zero

Accumulative test 7

1 [1] c [2] b [3] c [4] b

2 [1] zero [2] $\sqrt{3}$ [3] $\{0\}$ [4] 2

3 [a] 7

[b] $10 - 4\sqrt{5}$

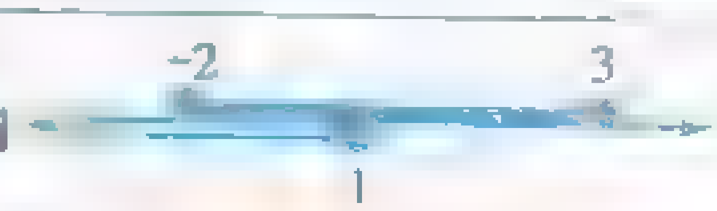
4 Prove by yourself , $\frac{4}{5}$

Accumulative test 8

1 [1] a [2] b [3] b [4] d

2 [1] $]-\infty, 0[$ [2] $3\sqrt{2}$ [3] $3\sqrt[3]{2}$ [4] 6

3 [1] $6\sqrt[3]{2}$ [2] -1

4 [a] 

[1] $X \cap Y = [-2, 1]$

[2] $X - Y =]1, 3]$

[b] 1

Accumulative test 9

1 [1] c [2] c [3] a [4] d

2 [1] 4 [2] 3 cm [3] $\{0, 1\}$ [4] $5\sqrt{5}$

3 [a] 4 cm.
[b] Prove by yourself , 4

4 [a] 36π cm^2
[b] $7\sqrt{5} - 4\sqrt[3]{2}$

Accumulative test 10

1 [1] b [2] b [3] d [4] c

2 [1] $[-2, \infty[$ [2] $]1, \infty[$ [3] $\{-1\}$ [4] 15

3 [a] 15 cm.
[b] $[-2, 3[$, represent by yourself.

4 [a] 

[1] $X \cup Y = [-1, 6]$
[2] $X \cap Y = [2, 4[$

[b] $[-1, 2[$, represent by yourself.

Accumulative test 11

1 [1] b [2] c [3] c [4] a

2 [1] 6 [2] 12 [3] y [4] 3

3 [a] Find by yourself
[b] $3\sqrt{3}$

4 [a] $[3, \infty[$, represent by yourself.
[b] Represent by yourself , 2 square units.

Accumulative test 12

1 [1] a [2] c [3] d [4] d

2 [1] 36 [2] zero [3] 5 [4] y

3 [a] Represent by yourself , -1
[b] $[0, 2]$, represent by yourself.

4 [a] Prove by yourself
[b] zero.

Accumulative test 13

1 [1] a [2] c

2 [1] zero [2] $(2, 3)$ [3] -3

3 [1] $\frac{75}{8}$, 5 , zero
[2] 50 cm

4 [a] 440 cm^2 , 1540 cm^3
[b] Represent by yourself , $\left(\frac{-1}{2}, 0\right)$, $(0, 1)$

5 [1] $]0, 3]$ [2] $[-2, \infty[$
[3] $]-\infty, 0]$

Accumulative test 14

1 [1] c [2] b

2 [1] 4 [2] 2

3 Form by yourself

4 [a] $]1, 5[$
[b] Find and represent by yourself.

5 [a] zero
[b] $y = \sqrt{5} - 2$, prove by yourself.

Accumulative test 15

1 [1] b [2] b

2 [1] 1 [2] 5

3 [1] 20 workers.
[2] Graph by yourself

4 [1] $\{\sqrt{2}\}$
[2] $[-1, 5[$, represent by yourself

5 [a] $\frac{2}{3}$, the point $C \notin \overleftrightarrow{AB}$

[b] 

1 $X \cap Y = [2 , 3[$

2 $X \cup Y =]-2 , \infty[$

3 $X - Y = [3 , \infty[$

Accumulative test 16

1 1 a 2 b 3 c 4 b

2 1 zero 2 20 3 $\sqrt{2} - 1$ 4 $2\sqrt{5}$

3 1 $X = 60$, $k = 14$

2 The arithmetic mean = 50.6

4 [a] 14 cm.

[b] $X = \sqrt{7} + \sqrt{3}$, $y = \sqrt{7} - \sqrt{3}$, $X^2 y^2 = 16$

Accumulative test 17

1 1 a 2 b 3 c 4 c

2 1 5 2 6 3 7 4 24

3 [a] Graph by yourself.

[b] 1 $k = 15$

2 The median 52 approximately.

4 [a] $[-2 , 1]$

[b] 

1 $X \cap Y = [5 , 7]$

2 $X \cup Y =]3 , \infty[$

3 $X - Y =]3 , 5[$

Accumulative test 18

1 1 c 2 d 3 c 4 d

2 1 6 2 3 3 3 4 3

3 [a] $\{\sqrt{7}\}$

[b] 7

4 [a] 4

[b] 1 $X = 110$, $k = 20$

2 The mode of wages ≃ L.E. 105

Answers of October tests on Algebra and Statistics

Answers of Test 1

1 1 d 2 c 3 a

2 1 $\mathbb{R}^*$ or $\mathbb{R} - \{0\}$

2 $\{\sqrt{5}, -\sqrt{3}\}$ 3 $]1, 4[$

3 $\because (\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = 2$, $(1.4)^2 = 1.96$

, $(1.5)^2 = 2.25$

$\therefore 1.96 < 2 < 2.25$ $\therefore \sqrt{1.96} < \sqrt{2} < \sqrt{2.25}$

$\therefore 1.4 < \sqrt{2} < 1.5$

$\therefore \sqrt{2}$ lies between 1.4 and 1.5

4 27 litres $\times$ 1000 = 27000 cm^3.

$\because$ Volume of the cube $= \ell^3$

$\therefore \ell^3 = 27000$ $\therefore \ell = \sqrt[3]{27000}$

$\therefore \ell = 30$ cm.

Answers of Test 2

1 1 d 2 c 3 c

2 1 3 2 $\mathbb{R}$ 3 $\varnothing$

3 $\because 2 + x^3 = 1$ $\therefore x^3 = 1 - 2 = -1$

$\therefore x = \sqrt[3]{-1} = -1$ $\therefore$ The S.S. $= \{-1\}$

4 $X \cup Y = [-1, \infty[$

$Y - X =]4, \infty[$

Answers of November tests on Algebra and Statistics

Answers of Test 1

1 1 c 2 b 3 c

2 1 96 2 $3\sqrt{2}$ 3 $\pm \frac{2\sqrt{2}}{3}$

3 $2\sqrt{9 \times 2} + \sqrt{25 \times 2} + \frac{1}{3} \times \sqrt{81 \times 2}$

$= 2 \times 3\sqrt{2} + 5\sqrt{2} + \frac{1}{3} \times 9\sqrt{2}$

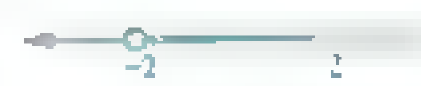
$= 6\sqrt{2} + 5\sqrt{2} + 3\sqrt{2} = 14\sqrt{2}$

4 $\because -1 < 3x + 5 < 11$

$\therefore -6 < 3x < 6$

$\therefore -2 < x < 2$

$\therefore$ The S.S. $=]-2, 2[$



Answers of Test 2

1 1 c 2 d 3 b

2 1 1 , zero 2 $\sqrt{125}$ 3 $2\sqrt[3]{2}$

3 $\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 924 = \frac{22}{7} \times r^2 \times 6$

$\therefore r^2 = \frac{924 \times 7}{22 \times 6} = 49$ $\therefore r = 7$ cm.

$\therefore$ The lateral area $= 2\pi r h = 2 \times \frac{22}{7} \times 7 \times 6$

$= 264$ cm^2.

4 $\because b = \frac{1}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \frac{\sqrt{3} - \sqrt{2}}{3 - 2}$

$= \sqrt{3} - \sqrt{2}$

$\therefore a^2 - b^2 = (\sqrt{3} + \sqrt{2})^2 - (\sqrt{3} - \sqrt{2})^2$

$= 3 + 2\sqrt{6} + 2 - 3 + 2\sqrt{6} - 2 = 4\sqrt{6}$

Answers of important questions

First: Answers of multiple choice questions

1 (a) 2 (c) 3 (a) 4 (d) 5 (b)
6 (c) 7 (c) 8 (a) 9 (b) 10 (a)
11 (c) 12 (b) 13 (b) 14 (c) 15 (c)
16 (d) 17 (c) 18 (c) 19 (b) 20 (d)
21 (a) 22 (a) 23 (c) 24 (b) 25 (d)
26 (c)

Second: Answers of complete questions

1 $\mathbb{R}$ 2 $\varnothing$ 3 12 4 $\{0\}$
5 $]3 , 4]$ 6 $\sqrt{3}-\sqrt{2}$ 7 $\sqrt{5}+2$
8 $]0 , \infty[$ 9 12 10 3 , 4 11 $\sqrt{50}$
12 64 cm³ 13 $\frac{4}{3}$ 14 3 15 12 cm.
16 20π 17 2 18 2 19 $(-1 , 2\sqrt{3})$
20 5 21 $]-2 , 3]$ 22 $\{-1 , 0 , 1\}$
23 $[-3 , 0[$ 24 2

Third: Answers of essay questions

1


1 $X \cup Y = [-3 , 5]$

2 $X \cap Y = [-1 , 2[$

3 $X - Y = [-3 , -1[$

2


• $B - A = [3 , 5]$ • $A \cap B = [-2 , 3[$

• $A \cup B =]-\infty , 5]$ • $\acute{A} = [3 , \infty[$

3 $\because y = \frac{2}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{2(\sqrt{3}-1)}{3-1} = \sqrt{3}-1$

$\therefore \frac{xy}{x-y} = \frac{(\sqrt{3}+1)(\sqrt{3}-1)}{\sqrt{3}+1-\sqrt{3}+1} = \frac{3-1}{2} = 1$

4 $\because x = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$

, $y = \sqrt{5}+2$ $\quad\therefore x , y$ are conjugate.

$\therefore x^2 y^2 = (xy)^2 = [(\sqrt{5}-2)(\sqrt{5}+2)]^2$

$= [5-4]^2 = 1$

5 $\because y = \frac{2}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{2(\sqrt{7}+\sqrt{5})}{7-5}$

$= \sqrt{7}+\sqrt{5}$

$\therefore x^2 + 2xy + y^2 = (x+y)^2$

$= (\sqrt{7}-\sqrt{5}+\sqrt{7}+\sqrt{5})^2$

$= (2\sqrt{7})^2 = 28$

6 $\because y = \frac{2}{x}$

$= \frac{2}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{2(\sqrt{7}-\sqrt{5})}{7-5}$

$= \sqrt{7}-\sqrt{5}$

, $x = \sqrt{7}+\sqrt{5}$

$\therefore x , y$ are two conjugate numbers

$\therefore x^2 + xy + y^2 = (x+y)^2 - xy$

$= (\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5})^2$

$- (\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})$

$= (2\sqrt{7})^2 - 2 = 26$

7 $(x+y)^2 = x^2 + 2xy + y^2 = \left(\sqrt{4+\sqrt{7}}\right)^2$

$+ 2\left(\sqrt{4+\sqrt{7}}\right)\left(\sqrt{4-\sqrt{7}}\right)$

$+ \left(\sqrt{4-\sqrt{7}}\right)^2$

$= 4 + \sqrt{7} + 2\sqrt{(4+\sqrt{7})(4-\sqrt{7})}$

$+ 4 - \sqrt{7}$

$= 8 + 2\sqrt{16-7} = 8 + 2\sqrt{9} = 8 + 6 = 14$

8 $\because x = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{11+2\sqrt{30}}{6-5}$

$= 11 + 2\sqrt{30}$

, $\frac{1}{x} = \frac{1}{11+2\sqrt{30}} \times \frac{11-2\sqrt{30}}{11-2\sqrt{30}}$

$= \frac{11-2\sqrt{30}}{121-120} = 11 - 2\sqrt{30}$

$\therefore x + \frac{1}{x} = 11 + 2\sqrt{30} + 11 - 2\sqrt{30} = 22$

9 $\because 5x - 3 < 2x + 9$ $\therefore 5x - 2x < 9 + 3$

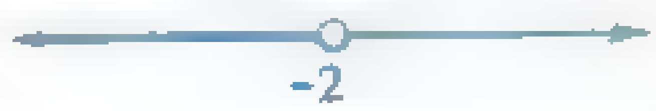
$\therefore 3x < 12$ $\therefore x < 4$

The S.S. $=]-\infty , 4[$

10 $\because 5 - 3x > 11$ $\therefore -3x > 6$

$\therefore x < -2$

$\therefore$ The S.S. $=]-\infty , -2[$



11 $\because 1 < 2x + 3 \le 9$ $\therefore -2 < 2x \le 6$

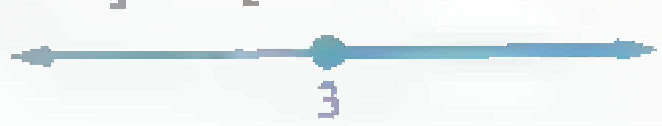
$\therefore -1 < x \le 3$

$\therefore$ The S.S. $=]-1 , 3]$

12 $\because \frac{x}{3} + 2 > 3$ $\therefore \frac{x}{3} > 1$

$\therefore x > 3$

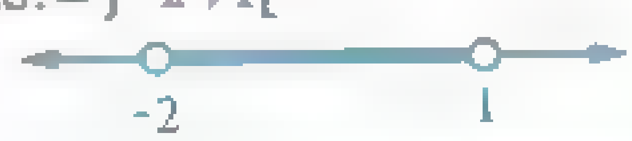
$\therefore$ The S.S. $=]3 , \infty[$



13 $\because 4x + 3 > 5x + 2 > 4x$ $\therefore 3 > x + 2 > 0$

$\therefore 1 > x > -2$

$\therefore$ The S.S. $=]-2 , 1[$



14 The expression $= \sqrt[3]{2 \times 4} + 2\sqrt[3]{2} - 2\sqrt[3]{2} - 2$

$= 2 - 2 = \text{zero}$

15 The expression $= \sqrt{25 \times 3} - 2\sqrt{9 \times 3} + \sqrt{9 \times \frac{1}{3}}$

$= 5\sqrt{3} - 6\sqrt{3} + \sqrt{3} = \text{zero}$

16 The expression $= \sqrt[3]{27 \times 3} + \sqrt[3]{8 \times 3} - \sqrt[3]{27 \times \frac{1}{9}}$

$= 3\sqrt[3]{3} + 2\sqrt[3]{3} - \sqrt[3]{3} = 4\sqrt[3]{3}$

17 The expression $= \sqrt{4 \times 3} + \sqrt[3]{27 \times 2} - 2\sqrt{3}$

$- \sqrt[3]{8 \times 2}$

$= 2\sqrt{3} + 3\sqrt[3]{2} - 2\sqrt{3} - 2\sqrt[3]{2}$

$= \sqrt[3]{2}$

18 The expression $= \sqrt{25 \times 2} + \sqrt[3]{27 \times 2} - 5\sqrt{4 \times \frac{1}{2}}$

$- \sqrt[3]{8 \times 2}$

$= 5\sqrt{2} + 3\sqrt[3]{2} - 5\sqrt{2} - 2\sqrt[3]{2}$

$= \sqrt[3]{2}$

19 $\because (x - 2)^3 = 125$ $\therefore x - 2 = 5$

$\therefore x = 7$

$\therefore$ The S.S. $= \{7\}$

20 [1] $\because$ The volume of the sphere $= \frac{4}{3} \pi r^3$

$\therefore 36\pi = \frac{4}{3} \times \pi \times r^3$ $\therefore r^3 = \frac{36 \times 3}{4} = 27$

$\therefore r = 3$ cm.

[2] The area of the sphere $= 4\pi r^2$

$= 4 \times 3^2 \times \pi = 36\pi$ cm^2

21 $\because$ The volume of the cylinder $= \pi r^2 h$

, $\because h = r$

$\therefore$ The volume of the cylinder $= \pi h^3$

$\therefore 27\pi = \pi h^3$ $\therefore h^3 = 27$ $\therefore h = 3$ cm.

22 The volume of the cylinder

$= \pi r^2 h = \pi (4\sqrt{2})^2 \times 9 = 288\pi$ cm^3

23 The volume of the cylinder

$= \pi r^2 h = \frac{22}{7} \times 7^2 \times 10 = 1540$ cm^3

, The lateral area $= 2\pi r h$

$= 2 \times \frac{22}{7} \times 7 \times 10 = 440$ cm^2

24 $\because$ The volume of the sphere

$= \frac{4}{3} \pi r^3 = \frac{4}{3} \times 3^3 \times \pi = 36\pi$ cm^3

, $\because$ The volume of the cylinder

= The volume of the sphere

$\therefore$ The volume of the cylinder $= 36\pi$ cm^3

$\therefore \pi r^2 h = 36\pi$ $\therefore 9\pi h = 36\pi$

$\therefore h = \frac{36}{9} = 4$ cm.

25 $\because$ The volume of the cylinder

$= \pi r^2 h = 3^2 \times 4 \times \pi = 36\pi$ cm^3

, ∵ the volume of the sphere

= The volume of the cylinder

The volume of the sphere = 36π cm^3

$\frac{4}{3}\pi r^3 = 36\pi$

∴ $r^3 = \frac{36 \times 3}{4} = 27$ ∴ $r = 3$ cm.

∵ The lateral area = $2\pi r h$

∴ $440 = 2 \times \frac{22}{7} \times 10 \times r$

$r = \frac{440 \times 7}{2 \times 22 \times 10} = 7$ cm.

The volume of the cylinder

$= \pi r^2 h = \frac{22}{7} \times 7^2 \times 10 = 1540$ cm^3.

First Answers of multiple choice questions

1 (b) 2 (c) 3 (b) 4 (a) 5 (c)

6 (b) 7 (c) 8 (c) 9 (a) 10 (b)

Second Answers of complete questions

1 1 2 undefined 3 zero

4 4 5 x-axis , zero 6 -11

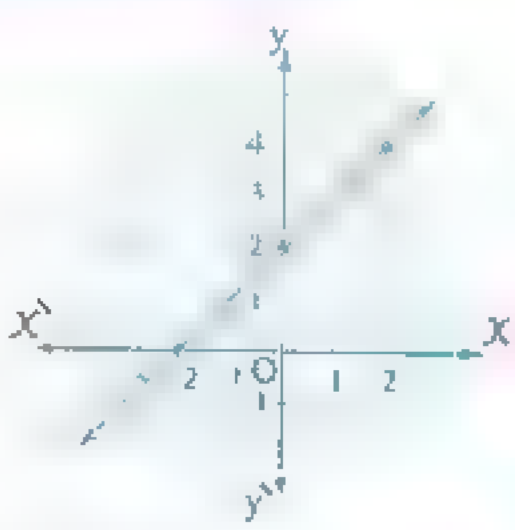
7 $\overleftrightarrow{BC}$ or $\overleftrightarrow{AC}$ 8 -1 , zero 9 3

10 $(0 , 8)$

Third Answers of essay questions

1 $y = x + 2$

x	-2	0	2
y	0	2	4



2 $y = 3 - x$

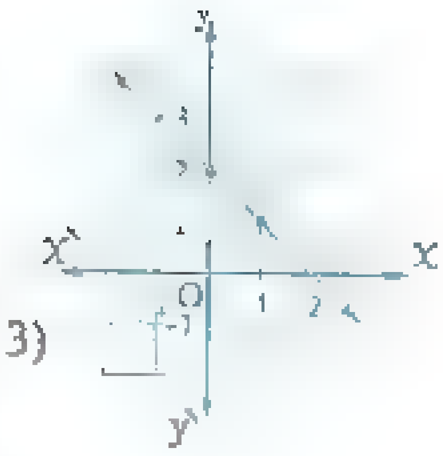
x	1	0	3
y	4	3	0

From the graph :

The point of intersection with x axis is : $(3 , 0)$

3 $y = 2 - x$

x	-1	0	1
y	3	2	1



Taking the two points $(-1 , 3)$ and $(0 , 2)$ which lie on the straight line we find that :

the slope $= \frac{2-3}{0+1} = -1$

4 At $x = 0$ ∴ $2 \times 0 + 3y = 12$

∴ $3y = 12$ ∴ $y = 4$

∴ The straight line intersects y-axis at $(0 , 4)$

At $y = 0$ ∴ $2x + 3 \times 0 = 12$

∴ $2x = 12$ ∴ $x = 6$

∴ The straight line intersects x-axis at $(6 , 0)$

5 ∵ The slope of $\overleftrightarrow{AB} = \frac{4-3}{2+1} = \frac{1}{3}$

, the slope of $\overleftrightarrow{BC} = \frac{5-4}{5-2} = \frac{1}{3}$

∴ The slope of $\overleftrightarrow{AB}$ = The slope of $\overleftrightarrow{BC}$

and the point B is a common point

∴ The points A , B and C are collinear.

6 ∵ The slope of $\overleftrightarrow{AB} = \frac{3+1}{-1-2} = \frac{-4}{3}$

, the slope of $\overleftrightarrow{BC} = \frac{3-3}{2+1}$ = zero

∴ The slope of $\overleftrightarrow{AB} \neq$ The slope of $\overleftrightarrow{BC}$

∴ The points A , B and C are not collinear

7 ∵ $\frac{k-17}{6-4} = 4$ ∴ $\frac{k-17}{2} = 4$

∴ $k - 17 = 8$ ∴ $k = 25$

8 ∵ $(k , 2k)$ satisfies the relation : $3x + y = 30$

∴ $3k + 2k = 30$ ∴ $5k = 30$ ∴ $k = 6$

9 The slope of $\overleftrightarrow{AB} = \frac{5-3}{2+1} = \frac{2}{3}$

, ∵ the slope of $\overleftrightarrow{BC} = \frac{1-5}{8-2} = \frac{-4}{6} = \frac{-2}{3}$

∴ The slope of $\overleftrightarrow{AB} \neq$ The slope of $\overleftrightarrow{BC}$

∴ $C \notin \overleftrightarrow{AB}$

10 ∵ The slope of $\overleftrightarrow{AB} = \frac{5-3}{3-3} = \frac{2}{\text{zero}}$

∴ The slope of $\overleftrightarrow{AB}$ is undefined

∴ $\overleftrightarrow{AB}$ // y-axis

First Answers of multiple choice questions

1 (b) 2 (d) 3 (c) 4 (a) 5 (d)
6 (c) 7 (a) 8 (b) 9 (c) 10 (c)
11 (c) 12 (c) 13 (b) 14 (a) 15 (c)

Second Answers of complete questions

1 5 2 3 3 7 4 the third
5 central tendency 6 4 7 40 8 3
9 10 10 median

Third Answers of essay questions

1

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

∴ The mean $= \frac{540}{20} = 27$

2 [1] n = 60

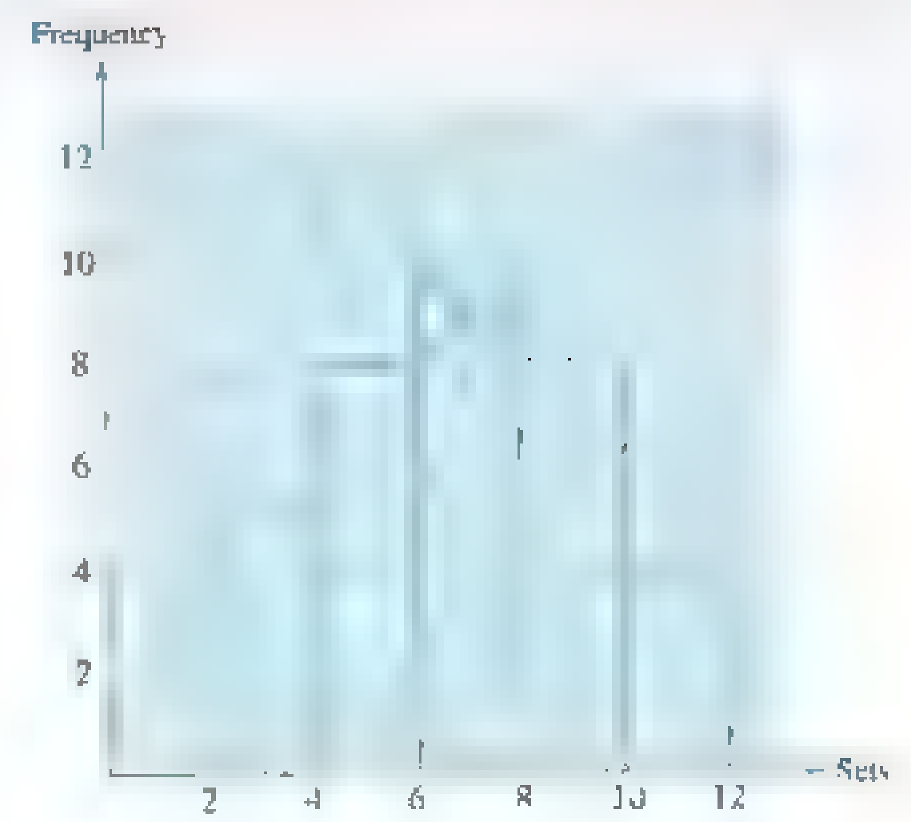
k = 100 – (10 + 22 + 25 + 20 + 8) = 15

[2]

Sets	x	f	$x \times f$
20 –	25	10	250
30 –	35	15	525
40 –	45	22	990
50 –	55	25	1375
60 –	65	20	1300
70 –	75	8	600
Total		100	5040

∴ The mean $= \frac{5040}{100} = 50.4$

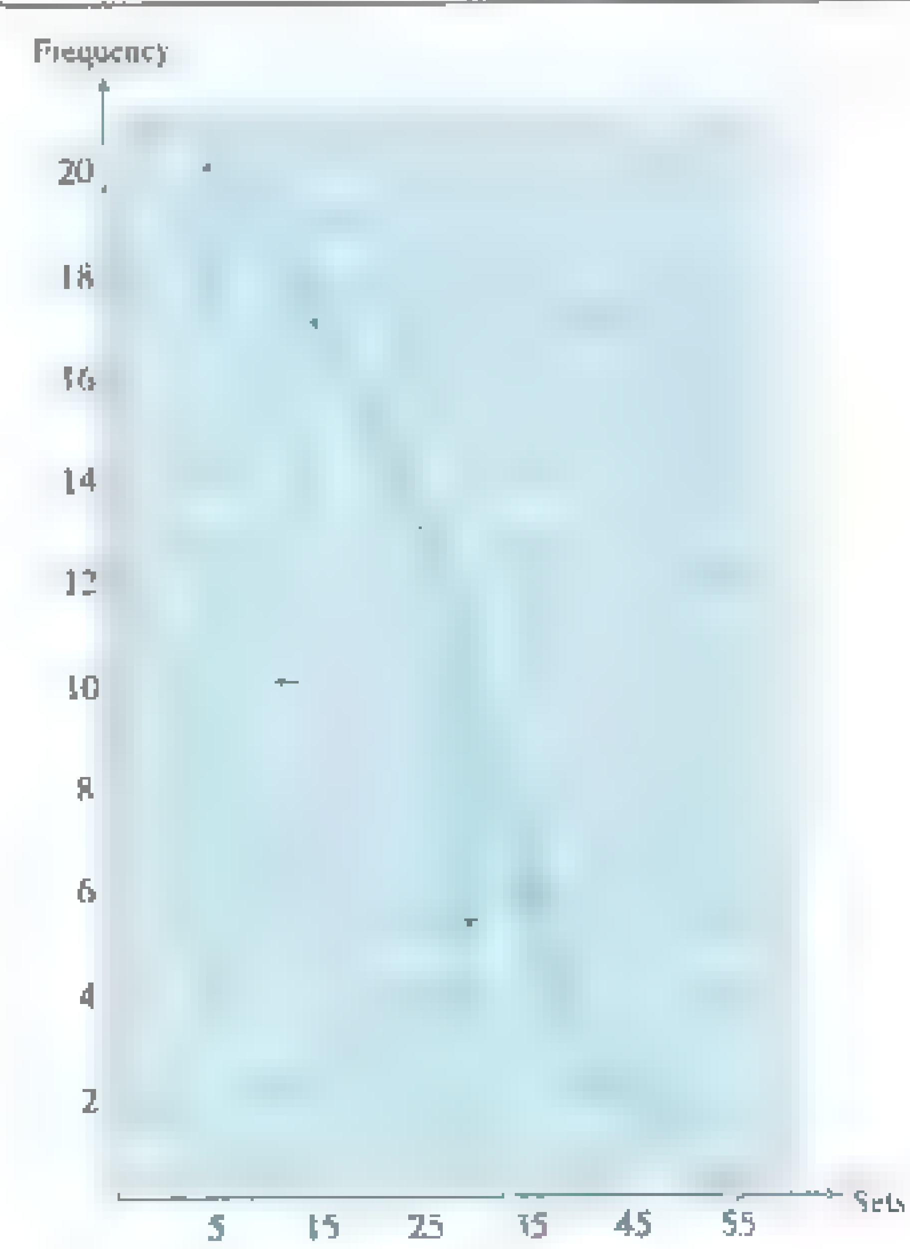
3



From the graph : The mode mark = 7 marks

4

The lower boundaries of sets	Descending cumulative frequency
5 and more	20
15 and more	17
25 and more	13
35 and more	6
45 and more	2
55 and more	0



∵ The order of the median $= \frac{20}{2} = 10$

∴ The median ≃ 29

Answers of the school book model examinations of Algebra and Statistics

1

1

1 $\{1\}$ 2 20 3 $[-2, 2]$

4 24 5 $\sqrt{3}-\sqrt{2}$

2

1 d 2 c 3 c

4 c 5 a 6 b

3

[a] $\sqrt{2 \times 9}+\sqrt[3]{2 \times 27}-3\sqrt{2}-\frac{1}{2}\sqrt[3]{2 \times 8}$

$=3\sqrt{2}+3\sqrt[3]{2}-3\sqrt{2}-\frac{1}{2} \times 2\sqrt[3]{2}$

$=2\sqrt[3]{2}$

[b] $\because X=\frac{3}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{3\left(\sqrt{5}+\sqrt{2}\right)}{5-2}$

$=\sqrt{5}+\sqrt{2}$

X and y are two conjugate numbers

4

[a] $\because$ The area of the square $=\frac{1}{2}$ d^2

$\therefore \frac{1}{2} d^2=1089$

$\therefore d^2=2178$

$\therefore d=\sqrt{2178}=33\sqrt{2}$ cm.

[b] $\because 6 \times \frac{3X+1}{6}<6(X+1)<6 \times \frac{X+4}{2}$

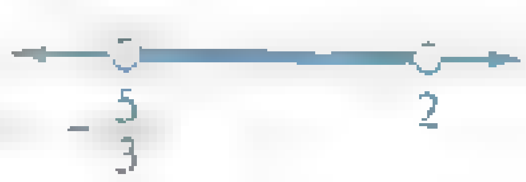
$3X+1<6X+6<3X+12$

$\therefore 3X-3X+1<6X-3X+6<3X-3X+12$

$\therefore 1<3X+6<12$ $\therefore 1-6<3X<12-6$

$\therefore -5<3X<6$ $\therefore -\frac{5}{3}<X<2$

$\therefore$ The S.S $=\left]-\frac{5}{3}, 2\right[$



5

[a] The volume of the cylinder $=\pi r^2 h$

$=\left(4\sqrt{2}\right)^2 \times 9 \times \pi$

$=288\pi$ cm^3

$\because$ the volume of the cylinder

= the volume of the sphere

$\therefore$ The volume of the sphere $=288\pi$ cm^3

$\therefore \frac{4}{3}\pi r^3=288\pi$

$\therefore r^3=288 \times \frac{3}{4}=216$

$\therefore r=\sqrt[3]{216}=6$ cm.

[b]

Sets	X	f	$X \times f$
5 –	10	7	70
15 –	20	10	200
25 –	30	12	360
35 –	40	13	520
45 –	50	8	400
Total		50	1550

$\therefore$ The mean $=\frac{1550}{50}=31$

2

1

1 $\sqrt{3}+\sqrt{5}$ 2 6 3 $3+\sqrt{10}$

4 3 5 $]3, 4]$

2

1 b 2 a 3 b

4 c 5 c 6 d

3

[a] $\frac{\sqrt{3}\left(\sqrt{5}+\sqrt{3}\right)+\sqrt{5}\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}$

$=\frac{\sqrt{15}+3+5-\sqrt{15}}{5-3}=\frac{8}{2}=4$

[b] The left hand side

$=\sqrt[3]{2 \times 64}+\sqrt[3]{2 \times 8}-2\sqrt[3]{2 \times 27}$

$=4\sqrt[3]{2}+2\sqrt[3]{2}-2 \times 3\sqrt[3]{2}=6\sqrt[3]{2}-6\sqrt[3]{2}=0$

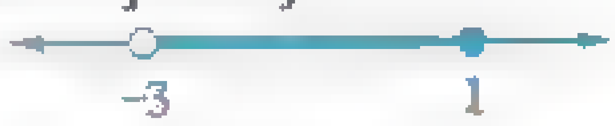
= the right hand side

[a] $\because -2-7 < 3X+7-7 \le 10-7$

$\therefore -9 < 3X \le 3$

$\therefore -3 < X \le 1$

$\therefore$ The S.S. $=]-3 , 1]$



[b] $X^4 - 2X^2 + 1 = (X^2-1)^2$

$= \left(\left(\sqrt{2}+\sqrt{3}\right)^2 - 1\right)^2 = \left(2+2\sqrt{6}+3-1\right)^2$

$= \left(4+2\sqrt{6}\right)^2$

$= 16 + 16\sqrt{6} + 24$

$= 40 + 16\sqrt{6}$

[a] 20

[b]

Sets	X	f	$X \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

1 $\sqrt{3}-\sqrt{2}$ 2 $3\sqrt{6}$ 3 3

4 5 5 $\varnothing$

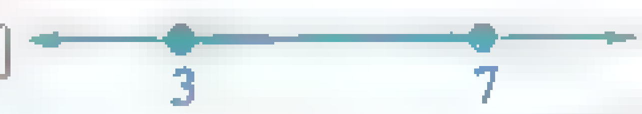
2

1 a 2 b 3 a 4 a 5 a

1 $\{5 , -5\}$ 2 $[0 , 2]$ 3 7

4 irrational

5



1 ✓ 2 ✓ 3 ✓ 4 ✗ 5 ✓

[a] The centre $= \frac{8+4}{2} = 6$

[b]

Sets	The centre of the set « X »	Frequency « f »	$X \times f$
5 –	10	7	$10 \times 7 = 70$
15 –	20	10	$20 \times 10 = 200$
25 –	30	12	$30 \times 12 = 360$
35 –	40	13	$40 \times 13 = 520$
45 –	50	8	$50 \times 8 = 400$
Total		50	1550

$\therefore$ The arithmetic mean $= \frac{\sum (X \times f)}{\sum (f)}$

$= \frac{1550}{50} = 31$

[illegible] on Algebra and Statistics

1 Cairo

1 1 (d)   2 (a)   3 (a)   4 (d)   5 (c)

2 1 $3\sqrt{2}$ 2 $\varnothing$ 3 3   4 $-\frac{6}{5}$

3

[a] $\sqrt{9\times 2}+\sqrt{25\times 2}-\sqrt{9\times 6}$

$=3\sqrt{2}+5\sqrt{2}-3\sqrt{6}$

$=8\sqrt{2}-3\sqrt{6}$

[b] $\because x=\frac{4}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}=\frac{4\left(3+\sqrt{5}\right)}{9-5}=3+\sqrt{5}$

$, y=3-\sqrt{5}$

$\therefore$ x and y are two conjugate numbers.

$, (x+y)^2=\left(3+\sqrt{5}+3-\sqrt{5}\right)^2=(6)^2=36$

4

[a] 

1 $X\cup Y=[-1 , 7]$

2 $X\cap Y=[2 , 4]$

[b] $\because$ (3 m , 2 m) satisfies the relation

$\therefore 2\text{ m}=2\times 3\text{ m}-8$ $\therefore 2\text{ m}=6\text{ m}-8$

$\therefore 2\text{ m}-6\text{ m}=-8$ $\therefore -4\text{ m}=-8$

$\therefore \text{m}=2$

5

[a] 1 $\because 8x^3-20=7$ $\therefore 8x^3=27$

$\therefore x^3=\frac{27}{8}$ $\therefore x=\frac{3}{2}$

$\therefore$ The S.S. $=\left\{\frac{3}{2}\right\}$

2 $\because 3x+7\le 10$

$\therefore 3x\le 3$ $\therefore x\le 1$

$\therefore$ The S.S. $=]-\infty , 1]$

[b]

Sets	x	f	$x\times f$
0 –	2	2	4
4 –	6	10	60
8 –	10	8	80
12 –	14	7	98
16 –	18	3	54
Total		30	296

$\therefore$ The arithmetic mean $=\frac{296}{30}=9\ \frac{13}{15}$

2 [illegible]

1 1 (a)   2 (b)   3 (a)   4 (c)   5 (a)

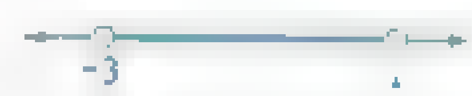
2 1 $4\sqrt{3}$ 2 -5 3 5   4 $\mathbb{R}$

3

[a] $\because -2<3x+7<10$

$\therefore -9<3x<3$ $\therefore -3<x<1$

$\therefore$ The S.S. $=]-3 , 1[$



[b] $\because y=\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}=\sqrt{5}+\sqrt{3}$

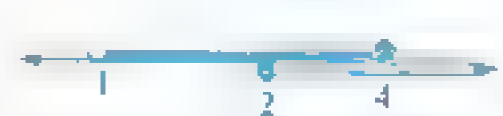
$, x=\sqrt{5}-\sqrt{3}$

$\therefore$ x and y are two conjugate numbers.

$, x^2+y^2=\left(\sqrt{5}-\sqrt{3}\right)^2+\left(\sqrt{5}+\sqrt{3}\right)^2$

$=5-2\sqrt{15}+3+5+2\sqrt{15}+3=16$

4

[a] 

1 $X\cap Y=[2 , 4]$ 2 $X\cup Y=]-1 , \infty[$

3 $X-Y=]-1 , 2[$

[b] $\because$ The volume of the sphere $=\frac{4}{3}\pi r^3$

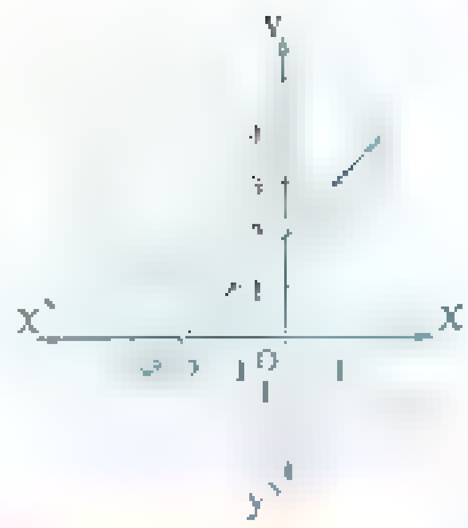
$\therefore 36\pi=\frac{4}{3}\pi r^3$ $\therefore r^3=27$

$\therefore r=3$ cm.

$\therefore$ The area $=4\pi r^2=4\pi\times 9=36\pi\text{ cm}^2$

[a] $y - x = 2$

x	1	0	1
y	1	2	3



[b]

Sets	x	f	$x \times f$
5	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

1 (c) 2 (c) 3 (a)

4 (a) 5 (b)

1 1 2 8 3 $\varnothing$ 4 2 , 3

[a] $\sqrt[3]{64 \times 2} + \sqrt[3]{8 \times 2} - 2\sqrt[3]{27 \times 2}$

$= 4\sqrt[3]{2} + 2\sqrt[3]{2} - 6\sqrt[3]{2} = 0$

[b] $x\,y = (\sqrt{5} - 3)(\sqrt{5} + 3) = 5 - 9 = -4$

[c] $\because$ The mode $= 6$

$\therefore 2x = 6$ $\therefore x = 3$

[a]

$]-2 , 2] \cap [1 , 5] = [1 , 2]$

[b] The volume $= 2 \times 2\sqrt{5} \times 3\sqrt{5} = 60$ cm^3.

[c] $\because$ 4 , 5 , 11 , 13 , 20

The median $= 11$

[a] $\because -3 \leq 2x + 1 < 7$

$\therefore -4 \leq 2x < 6$ $\therefore -2 \leq x < 3$

$\therefore$ The S.S. $= [-2 , 3[$

[b] $\because x\,y = 1$ $\therefore y = \frac{1}{x}$

$\therefore y = \frac{1}{\sqrt{5}+2} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5} - 2$

$\because x = \sqrt{5} + 2$

$\therefore x$ and y are two conjugate numbers.

[c]

Sets	x	f	$x \times f$
5 –	10	20	200
15 –	20	30	600
25 –	30	15	450
35 –	40	25	1000
45 –	50	10	500
Total		100	2750

$\therefore$ The mean $= \frac{2750}{100} = 27.5$

1 (c) 2 (b) 3 (c)

4 (d) 5 (b)

1 64 2 $288\,\pi$ 3 undefined 4 3

[a] $\sqrt[3]{8 \times 3} + \sqrt{4 \times 3} - 2\sqrt[3]{3} - 2\sqrt{3}$

$= 2\sqrt[3]{3} + 2\sqrt{3} - 2\sqrt[3]{3} - 2\sqrt{3} = 0$

[b] $\because y = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3} - \sqrt{2}$

$\therefore x^2 - y^2 = (\sqrt{3}+\sqrt{2})^2 - (\sqrt{3}-\sqrt{2})^2$

$= 3 + 2\sqrt{6} + 2 - 3 + 2\sqrt{6} - 2 = 4\sqrt{6}$

[a] The volume $= 3 \times 4 \times 5 = 60$ cm^3

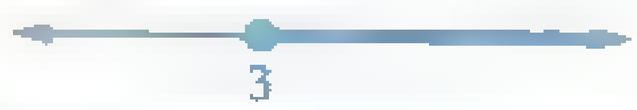
, the total area $= 2\,(3 \times 4 + 4 \times 5 + 5 \times 3)$

$= 2\,(12 + 20 + 15) = 94$ cm^2.

[b] $\because 3X - 1 \geq 8$

$\therefore 3X \geq 9$ $\therefore X \geq 3$

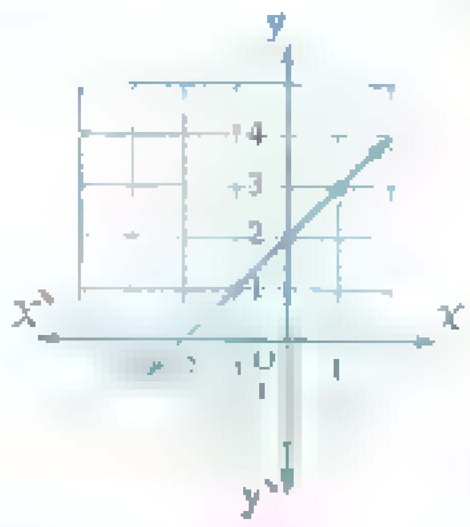
The S.S. $= [3, \infty[$



4

[a] $y = X + 2$

X	-1	0	1
y	1	2	3



[b]

Sets	X	f	$X \times f$
4 –	6	2	12
8 –	10	4	40
12 –	14	8	112
16 –	18	6	108
20 –	22	4	88
Total		24	360

$\therefore$ The mean $= \frac{360}{24} = 15$

5 Alexandria

1 [1] (a) [2] (c) [3] (c) [4] (b) [5] (c)

2 [1] zero [2] 11 [3] 3 [4] undefined

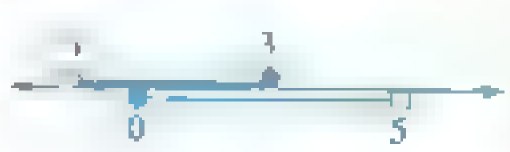
3

[a] $\sqrt[3]{27 \times 2} + 2\sqrt[3]{8 \times \frac{1}{4}} - \sqrt[3]{2}$

$= 3\sqrt[3]{2} + 2\sqrt[3]{2} - \sqrt[3]{2} = 4\sqrt[3]{2}$

[b] [1] $A \cap B = [0, 3]$

[2] $B - A =]3, 5[$



4

[a] $\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 288\pi = \frac{4}{3}\pi r^3$

$\therefore r^3 = 216$ $\therefore r = 6$ cm.

$\therefore$ The area $= 4\pi r^2 = 4\pi \times 36 = 144\pi$ cm^2

[b] $\because -2 < 3X + 7 \leq 10$

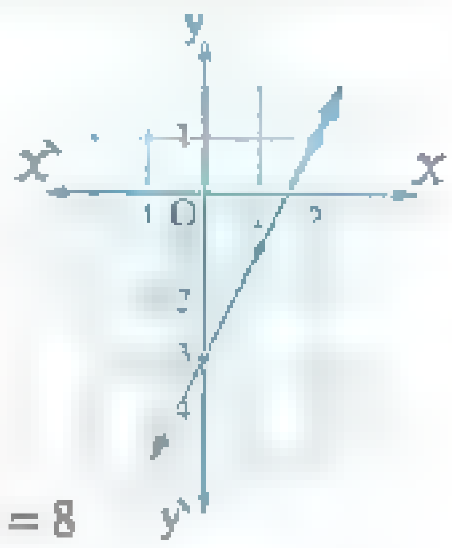
$\therefore -9 < 3X \leq 3$ $\therefore -3 < X \leq 1$

$\therefore$ The S.S. $=]-3, 1]$

5

[a] $y = 2X - 3$

X	0	1	2
y	-3	-1	1



[b] [1] $k = 40 - (7 + 9 + 12 + 4) = 8$

[2]

Sets	X	f	$X \times f$
5 –	10	7	70
15 –	20	9	180
25 –	30	12	360
35 –	40	8	320
45 –	50	4	200
Total		40	1130

$\therefore$ The arithmetic mean $= \frac{1130}{40} = 28.25$

6 El-Kalyoubia

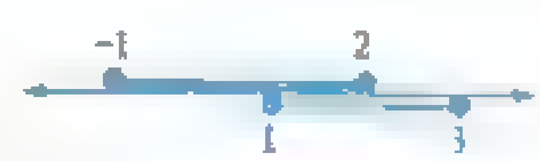
1 [1] (b) [2] (b) [3] (b) [4] (d) [5] (d)

2 [1] $]1, 5[$ [2] 2 [3] 3^5 [4] $(4, 0)$

3

[a] [1] $X \cap Y = [1, 2]$

[2] $X - Y = [-1, 1[$



[b] $\sqrt{25 \times 3} - 2\sqrt{9 \times 3} - 2\sqrt{9 \times \frac{1}{3}}$

$= 5\sqrt{3} - 6\sqrt{3} - 2\sqrt{3} = -3\sqrt{3}$

4

[a] $\because$ The slope $= \frac{1 - k - 3}{3 - 2k} = 1$

$\therefore 1 - k - 3 = 3 - 2k$ $\therefore -k + 2k = 3 + 3 - 1$

$\therefore k = 5$

[b] $\because 4X \le 5X + 2 \le 4X + 3$

$\therefore 0 \le X + 2 < 3$ $\therefore -2 \le X \le 1$

$\therefore$ The S.S. $= [-2, 1]$

[a] The volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 27 = 36\pi \text{ cm}^3$

, the area $= 4\pi r^2 = 4\pi \times 9 = 36\pi \text{ cm}^2$

[b]

Sets	X	f	$X \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

7 [illegible]

1 1 (d) 2 (b) 3 (b) 4 (c) 5 (a)

2 1 $\sqrt{5} - \sqrt{2}$ 2 3 3 $]-\infty, -3[$ 4 $]3, 7]$

[a] $\because$ The slope of $\overline{AB} = \frac{4+1}{-3-2} = -1$

, the slope of $\overline{BC} = \frac{5-4}{-4+3} = -1$

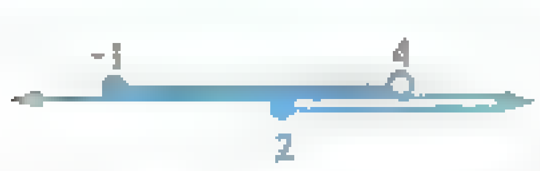
$\therefore$ The slope of $\overline{AB}$ = the slope of $\overline{BC}$ and the point B is a common point.

$\therefore$ The points A , B and C are collinear.

[b] The volume of the cylinder $= \pi r^2 \times h$

$\pi r^2 \times 4 = 64\pi$ $\therefore r^2 = \frac{64}{4} = 16$

$r = 4$ cm

[a] 

1 $X \cap Y = [2, 4[$ 2 $X \cup Y = [-1, \infty[$

3 $X \quad Y = [-1, 2[$

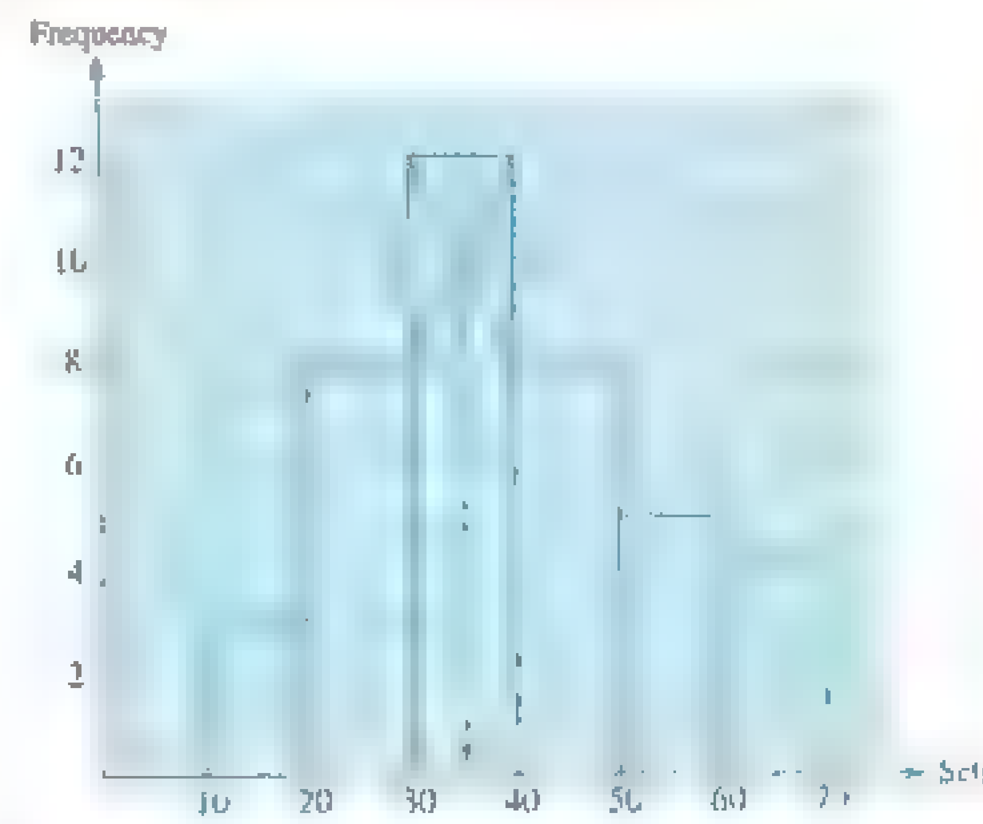
[b] $\because -2 < 5X + 3 < 13$

$\therefore -5 < 5X < 10$ $\therefore -1 < X < 2$

$\therefore$ The S.S. $=]-1, 2[$

[a] $3\sqrt[3]{8 \times 2} + \sqrt{25 \times 2} - 2\sqrt[3]{27 \times 2} - 2\sqrt{4 \times 2}$

$= 6\sqrt[3]{2} + 5\sqrt{2} - 6\sqrt[3]{2} - 4\sqrt{2} = \sqrt{2}$

[b] 

From the graph : The mode $= 35$

El-Gharbia

1 1 (d) 2 (b) 3 (c) 4 (c) 5 (b)

2 1 9 2 24 3 -3 4 2

[a] $\sqrt{25 \times 5} + 2\sqrt{16 \times 5} - \sqrt{4 \times 5} - \sqrt{9 \times 5}$

$= 5\sqrt{5} + 8\sqrt{5} - 2\sqrt{5} - 3\sqrt{5} = 8\sqrt{5}$

[b] 1 $\because (X - 3)^3 = 2^3$ $\therefore X - 3 = 2$

$\therefore X = 5$ $\therefore$ The S.S. $= \{5\}$

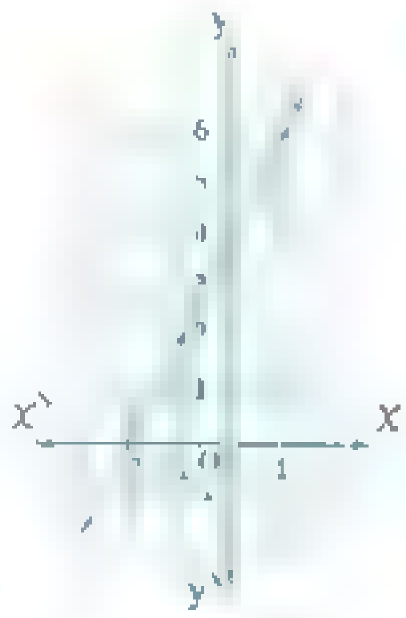
2 $\because 8 + 3X < 14$

$\therefore 3X < 6$ $\therefore X < 2$

$\therefore$ The S.S. $=]-\infty, 2[$

[a] $y - 2x = 4$

x	-2	-1	1
y	0	2	6



Taking the two points

$(-2 , 0) , (1 , 6)$

$\therefore$ The slope $= \frac{6-0}{1+2} = \frac{6}{3} = 2$

[b] $\because x = \frac{3}{\sqrt{6}-\sqrt{3}} \times \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}}$

$= \frac{3\left(\sqrt{6}+\sqrt{3}\right)}{6-3} = \sqrt{6}+\sqrt{3}$

$\therefore \frac{xy}{x-y} = \frac{\left(\sqrt{6}+\sqrt{3}\right)\left(\sqrt{6}-\sqrt{3}\right)}{\left(\sqrt{6}+\sqrt{3}\right)-\left(\sqrt{6}-\sqrt{3}\right)}$

$= \frac{6-3}{2\sqrt{3}} = \frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

[a] 1 $X \cap Y =]1 , 5]$

2 $Y - X =]5 , 7]$

[b] 1 $\because k - 1 = 20 - (4 + 6 + 3 + 2) = 5$

$\therefore k = 6$

2

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		**20**	**540**

$\therefore$ The arithmetic mean $= \frac{540}{20} = 27$

[illegible]

1 1 (c) 2 (d) 3 (b)

4 (a) 5 (c)

2 1 6 2 0 , 25

3 5 4 $\overleftrightarrow{AB}$ or $\overleftrightarrow{AC}$

[a] The slope of $\overleftrightarrow{AB} = \frac{5-3}{2+1} = \frac{2}{3}$

[b] $\sqrt[3]{64 \times 2} + \sqrt[3]{8 \times 2} - 2\sqrt[3]{27 \times 2}$

$= 4\sqrt[3]{2} + 2\sqrt[3]{2} - 6\sqrt[3]{2} = 0$

[a] $\because (x^2 + 4)(x^2 - 9) = 0$

$\therefore x^2 + 4 = 0$

$\therefore x^2 = -4$ (has no solution in $\mathbb{R}$)

or $x^2 - 9 = 0$ $\therefore x^2 = 9$

$\therefore x = \pm 3$ $\therefore$ The S.S. $= \{-3 , 3\}$

[b] The volume of the cylinder $= \pi r^2 h$

$\therefore \frac{22}{7} \times 7^2 \times 10 = 1540 \text{ cm}^3$

5

[a] $\because y = \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{\sqrt{3}+\sqrt{2}}{3-2} = \sqrt{3}+\sqrt{2}$

$\therefore x + y = \sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2} = 2\sqrt{3}$

[b]

Sets	x	f	$x \times f$
5 –	10	2	20
15 –	20	1	20
25 –	30	3	90
35 –	40	3	120
45 –	50	1	50
Total		**10**	**300**

$\therefore$ The arithmetic mean $= \frac{300}{10} = 30$

10 Kafr El-Sheikh

1 1 (d) 2 (a) 3 (d)

4 (d) 5 (b)

2 1 $\frac{3}{2}$ 2 $3\sqrt{2}$ 3 4 4 8

3

[a] $\because -1 < 3x - 7 \le 5$ $\therefore 6 < 3x \le 12$

$\therefore 2 < x \le 4$ $\therefore$ The S.S. $=]2 , 4]$

[b] $y = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3} = \sqrt{5}+\sqrt{3}$

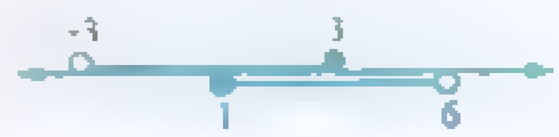
, $x = \sqrt{5}-\sqrt{3}$

$\therefore$ x and y are two conjugate numbers.

, $\frac{x+y}{xy} = \frac{\sqrt{5}-\sqrt{3}+\sqrt{5}+\sqrt{3}}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)} = \frac{2\sqrt{5}}{5-3} = \sqrt{5}$

4

[a] 1 $A \cap B = [1, 3]$



2 $A \cup B =]-3, 6[$

[b] $\sqrt{25 \times 2} - \sqrt{9 \times 2} - 2\sqrt{2} = 5\sqrt{2} - 3\sqrt{2} - 2\sqrt{2} = 0$

5

[a] (0 , 5) , (1 , 4) , (2 , 3)

[b]

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The arithmetic mean $= \frac{540}{20} = 27$ marks

11 El-Kalyoubia

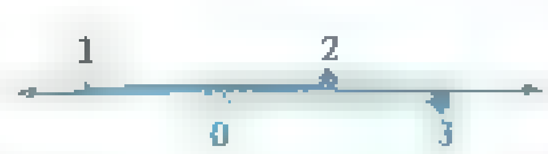
1 1 (b) 2 (a) 3 (b)

4 (a) 5 (d)

2 1 5 mn^2 2 7 3 $\frac{1}{4}$ 4 36

3

[a] 1 $A \cup B =]-1, 3]$



2 $A \cap B =]0, 2]$

[b] $\because x = \frac{5}{\sqrt{7}-\sqrt{2}} \times \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}$

$= \frac{5\left(\sqrt{7}+\sqrt{2}\right)}{7-2} = \sqrt{7}+\sqrt{2}$

, $y = \sqrt{7}-\sqrt{2}$

$\therefore$ x and y are two conjugate numbers

, $(x+y)^2 = \left(\sqrt{7}+\sqrt{2}+\sqrt{7}-\sqrt{2}\right)^2$

$= \left(2\sqrt{7}\right)^2 = 28$

4

[a] 1 $\because$ The lateral area of the cube $= 4\,\ell^2$

$\therefore$ $4\,\ell^2 = 64$ $\therefore$ $\ell^2 = 16$

$\therefore$ The edge length of the cube (ℓ) = 4 cm

2 The total area of the cube

$= 6\,\ell^2 = 6 \times 4^2 = 96$ cm².

3 The volume of the cube $= \ell^3 = 4^3 = 64$ cm³.

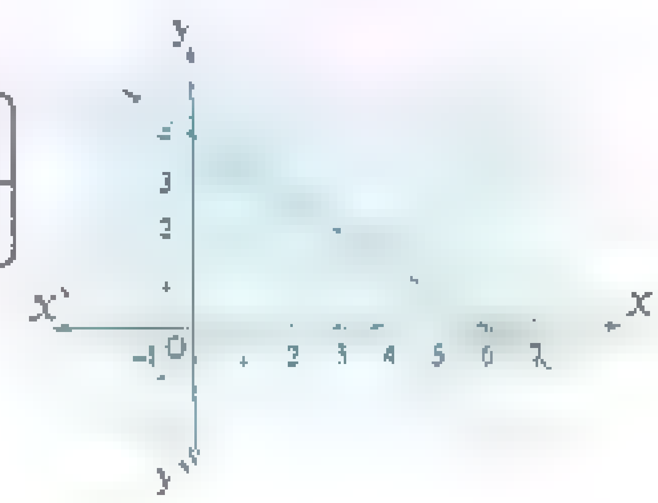
[b] $2\sqrt{9 \times 3} + 3\sqrt{4 \times 3} - 4\sqrt{3} + \frac{1}{3}\sqrt{9 \times 3}$

$= 6\sqrt{3} + 6\sqrt{3} - 4\sqrt{3} + \sqrt{3} = 9\sqrt{3}$

5

[a] $2x + 3y = 12$

x	0	3	6
y	4	2	0



[b] A = 12

, B = 40 – (5 + 10 + 14 + 6) = 5

Sets	x	f	$x \times f$
0 –	2	5	10
4 –	6	10	60
8 –	10	14	140
12 –	14	6	84
16 –	18	5	90
Total		40	384

$\therefore$ The arithmetic mean $= \frac{384}{40} = 9.6$

12 Sohag

1 1 (b) 2 (c) 3 (a)

4 (c) 5 (d) 6 (b)

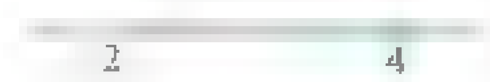
2 1 3 2 12 3 $6\sqrt{6}$

4 $\sqrt{5}+2$ 5 2 6 $\{-3, 3\}$

[a] $\because -2 < 3x + 4 \le 16$

$\therefore \quad 6 < 3x \le 12 \qquad \therefore -2 < x \le 4$

$\therefore$ The S.S. $=]-2 , 4]$



[b] $\sqrt{25 \times 3} - 2\sqrt{9 \times 3} + \sqrt{9 \times \frac{1}{3}}$

$= 5\sqrt{3} - 6\sqrt{3} + \sqrt{3} = 0$

[a] $\because r = h$

The volume of the cylinder $= \pi r^2 h$

$= \pi r^3 = 27\pi$

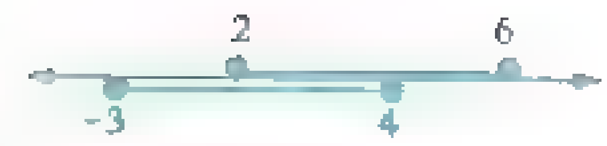
$\therefore r^3 = 27 \qquad \therefore r = 3$ cm.

[b] $\because y = \frac{2}{\sqrt{7} - \sqrt{5}} \times \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} + \sqrt{5}}$

$= \frac{2\left(\sqrt{7} + \sqrt{5}\right)}{7 - 5} = \sqrt{7} + \sqrt{5}$

$\therefore (x + y)^2 = \left(\sqrt{7} - \sqrt{5} + \sqrt{7} + \sqrt{5}\right)^2 = \left(2\sqrt{7}\right)^2 = 28$

[a]



1 $X \cup Y = [-3 , 6]$

2 $X - Y =]4 , 6]$

[b]

Sets	x	f	$x \times f$
5 –	10	7	70
15 –	20	10	200
25 –	30	12	360
35 –	40	13	520
45 –	50	8	400
Total		**50**	**1550**

$\therefore$ The arithmetic mean $= \frac{1550}{50} = 31$

Guide Answers

Of The Notebook (Geometry)

Answers of the accumulative tests on Geometry

Accumulative test 1

1 [1] c [2] b [3] a [4] c

2 [1] 2 : 1 [2] 4 [3] 3 [4] one point

3 6 cm.

4 15 cm.

Accumulative test 2

1 [1] a [2] a [3] c [4] c

2 [1] $\frac{1}{2}$ the length of the hypotenuse [2] 4 [3] 4 [4] 2.5 cm.

3 Prove by yourself.

4 [1] 12 cm. [2] 4 cm. [3] 26 cm.

Accumulative test 3

1 [1] c [2] a [3] a [4] b

2 [1] congruent [2] 4 [3] 50° [4] 120°

3 24 cm

4 70°

Accumulative test 4

1 [1] a [2] a [3] d [4] a

2 [1] equilateral [2] 1 : 2 [3] isosceles [4] 3

3 Prove by yourself.

4 Prove by yourself.

Accumulative test 5

1 [1] d [2] b [3] b [4] a

2 [1] the angle at this vertex is right [2] 30° [3] 12° [4] the vertex angle

3 Prove by yourself

4 6 cm.

Accumulative test 6

1 [1] b [2] a [3] d [4] a

2 [1] 50° [2] > [3] bisects the base and is perpendicular to it [4] one axis of symmetry

3 Prove by yourself.

4 [1] 25° [2] 2 cm.

Accumulative test 7

1 [1] c [2] c [3] c [4] b

2 [1] the angle of the greater measure [2] ≡ [3] $\overline{BC}$ [4] 105°

3 Prove by yourself.

4 Prove by yourself.

Accumulative test 8

1 [1] b [2] a [3] b [4] b

2 [1] The straight line perpendicular to it from its middle [2] ∠ B [3] > [4] a side greater in length than that opposite to the other angle

3 [a] AB < BC < AC

4 Prove by yourself.

Accumulative test 9

1 [1] b [2] c [3] a [4] b

2 [1] The perpendicular [2] < [3]]5 , 13[[4] obtuse

3 AC > AB > BC

4 Prove by yourself.

Answers of October tests on Geometry

Answers of Test 1

1 (1) d (2) a (3) d

2 (1) twice (2) 120° (3) 1

3 In ΔABC :

$\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$ (given)

$\therefore BC = 2DE = 2 \times 4 = 8$ cm

, $\because \overline{BE}$ and $\overline{CD}$ are two medians

, $\overline{BE} \cap \overline{CD} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore MC = 2DM = 2 \times 3 = 6$ cm

, $BM = \frac{2}{3} BE = \frac{2}{3} \times 6 = 4$ cm.

$\therefore$ The perimeter of $\Delta BMC = 8 + 6 + 4 = 18$ cm.

(The req.)

4 In ΔABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$

In ΔBCD : $\because BD = CD$

$\therefore m(\angle DBC) = m(\angle DCB) = \frac{180^\circ - 120^\circ}{2} = 30^\circ$

$\therefore m(\angle ABD) = 70^\circ - 30^\circ = 40^\circ$ (The req.)

Answers of Test 2

1 (1) b (2) a (3) c

2 (1) one point (2) the angle at this vertex is right (3) 50°

3 $\because \Delta ACD$ is an equilateral triangle

$\therefore m(\angle CAD) = 60^\circ$

In ΔABC : $\because AB = BC$

$\therefore m(\angle BAC) = m(\angle BCA) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$

$\therefore m(\angle BAD) = 60^\circ + 70^\circ = 130^\circ$ (The req.)

4 In ΔABC :

$\because m(\angle BAC) = 90^\circ$, $\overline{AD}$ is a median

$\therefore BC = 2AD = 2 \times 3 = 6$ cm.

In ΔCBE :

$\because m(\angle CBE) = 90^\circ$, $m(\angle E) = 30^\circ$

$\therefore EC = 2BC = 2 \times 6 = 12$ cm.

, $\because \overline{BF}$ is a median

$\therefore BF = \frac{1}{2} EC = \frac{1}{2} \times 12 = 6$ cm. (The req.)

Answers of November tests on Geometry

Answers of Test 1

1 (1) b (2) b (3) a

2 (1) bisects each of the base and the vertex angle (2) 60° (3) $\frac{5}{4}$

3 $\because B \in \overline{DC}$

$\therefore m(\angle ABC) = 180^\circ - 125^\circ = 55^\circ$

In ΔABC :

$m(\angle C) = 180^\circ - (55^\circ + 70^\circ) = 55^\circ$

$\therefore m(\angle ABC) = m(\angle C)$

$\therefore AB = AC$

$\therefore \Delta ABC$ is an isosceles triangle. (Q.E.D.)

4 $\because m(\angle BCE) = m(\angle BDF)$ (1)

$\therefore m(\angle BCL) = m(\angle BDL)$

(The supplementaries of equal angles are equal)

In ΔBCD : $\therefore BC = BD$

$\therefore \Delta BCD$ is an isosceles triangle

, $\because \overline{BL}$ is a median in ΔBCD

$\therefore \overline{BL} \perp \overline{CD}$ (Q.E.D.)

Answers of Test 2

1 1 b 2 a 3 b

2 1 bisects the base and is perpendicular to it.

2 isosceles

3 zero.

3 In Δ ADE : $\because$ AD = AE

m ($\angle$ ADE) = m ($\angle$ AED) (1)

, $\because$ $\overline{DE}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

m ($\angle$ ADE) = m ($\angle$ B) (corresponding angles) (2)

, $\because$ $\overline{DE}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

m ($\angle$ AED) = m ($\angle$ C) (corresponding angles) (3)

From (1) , (2) and (3) : $\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

4 In Δ ADC : $\because$ DA = DC

$\therefore$ m ($\angle$ C) = m ($\angle$ DAC) = 30°

$\because$ $\angle$ ADB is an exterior angle of Δ ADC

$\therefore$ m ($\angle$ ADB) = 30° + 30° = 60°

$\because$ DA = DB

$\therefore$ Δ ABD is an equilateral triangle. (Q.E.D.)

Answers of important questions on Geometry

First Answers of multiple choice questions

1 (c) 2 (c) 3 (c) 4 (c) 5 (a)
6 (d) 7 (c) 8 (b) 9 (b) 10 (a)
11 (d) 12 (b) 13 (a) 14 (c) 15 (d)
16 (b) 17 (b) 18 (d)

Second Answers of complete questions

1 one point 2 congruent

3 perpendicular , bisects it

4 perpendicular to the base , bisects the vertex angle

5 the base , the vertex angle

6 its axis of symmetry

7 equilateral 8 isosceles 9 zero

10 right 11 36°

Third Answers of essay questions

1 In Δ ABC :

$\because$ m ($\angle$ ABC) = 90°

, m ($\angle$ C) = 30°

$\therefore$ AC = 2 AB = 2 × 5 = 10 cm. (First req.)

, $\because$ D is the midpoint of $\overline{AC}$

$\therefore$ $\overline{BD}$ is a median from the vertex of the right angle

$\therefore$ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm. (Second req.)

2 $\because$ M is the intersection point of the medians of Δ ABC

$\therefore$ $\overline{BE}$ and $\overline{CD}$ are two medians of Δ ABC

$\therefore$ MD = $\frac{1}{3}$ DC = $\frac{1}{3}$ × 15 = 5 cm.

, ME = $\frac{1}{2}$ MB = $\frac{1}{2}$ × 6 = 3 cm.

, $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ DE = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 14 = 7 cm.

$\therefore$ The perimeter of Δ MDE = 5 + 3 + 7 = 15 cm.

(The req.)

3 $\because$ F and N are the midpoints of $\overline{AB}$ and $\overline{AC}$

$\therefore$ $\overline{BN}$ and $\overline{CF}$ are two medians of Δ ABC

, $\because$ $\overline{BN} \cap \overline{CF} = \{M\}$

$\therefore$ M is the point of concurrence of the medians of Δ ABC

$\therefore$ MF = $\frac{1}{3}$ CF = $\frac{1}{3}$ × 9 = 3 cm.

, MN = $\frac{1}{2}$ BM = $\frac{1}{2}$ × 4 = 2 cm.

, $\because$ F is the midpoint of $\overline{AB}$

$\therefore$ AF = $\frac{1}{2}$ AB = $\frac{1}{2}$ × 6 = 3 cm.

, $\because$ N is the midpoint of $\overline{AC}$

$\therefore$ AN = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm.

$\therefore$ The perimeter of the figure AFMN

= 3 + 2 + 3 + 5 = 13 cm. (The req.)

4 $\because$ Δ EBD is an equilateral triangle.

$\therefore$ m ($\angle$ EBD) = 60°

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ C) = $\frac{180° - 50°}{2}$ = 65°

$\therefore$ m ($\angle$ DBC) = 60° + 65° = 125° (The req.)

5 $\because$ $\overrightarrow{AD}$ // $\overrightarrow{CB}$, $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore$ m ($\angle$ B) = m ($\angle$ BAD) = 30° (alternate angles)

In Δ ABC : $\because$ AB = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ C) = $\frac{180° - 30°}{2}$ = 75°

(The req.)

6 $\because$ $\overrightarrow{AE}$ // $\overline{BC}$, $\overline{BD}$ is a transversal to them

$\therefore$ m ($\angle$ B) = m ($\angle$ DAE) (corresponding angles)

, $\because$ $\overrightarrow{AE}$ // $\overline{BC}$, $\overline{AC}$ is a transversal to them

$\therefore$ m ($\angle$ C) = m ($\angle$ CAE) (alternate angles)

In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

because : AB = AC

$\therefore$ m ($\angle$ DAE) = m ($\angle$ CAE)

$\therefore$ $\overrightarrow{AE}$ bisects $\angle$ DAC (Q.E.D.)

7 In Δ ABC which is right-angled at B

$\because$ m ($\angle$ ACB) = 30° $\therefore$ AB = $\frac{1}{2}$ AC

, $\because$ AB = DE $\therefore$ DE = $\frac{1}{2}$ AC (1)

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{DE}$ is a median in Δ ADC (2)

From (1) and (2) :

$\therefore$ m ($\angle$ ADC) = 90° (Q.E.D.)

In Δ ABC :

X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2} AC = \frac{1}{2} \times 20 = 10$ cm.

In Δ XBY which is right-angled at B

$\because$ D is the midpoint of $\overline{XY}$

$\therefore \overline{BD}$ is a median

$\therefore BD = \frac{1}{2} XY = \frac{1}{2} \times 10 = 5$ cm. (The req.)

In Δ ABD which is right-angled at D

$\because$ F is the midpoint of $\overline{AB}$

$\therefore \overline{DF}$ is a median

$\therefore DF = \frac{1}{2} AB = \frac{1}{2} \times 16 = 8$ cm.

Similarly ; $DE = \frac{1}{2} AC = \frac{1}{2} \times 18 = 9$ cm.

, $\because$ F is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore FE = \frac{1}{2} BC = \frac{1}{2} \times 20 = 10$ cm.

$\therefore$ The perimeter of Δ DEF = 8 + 9 + 10

= 27 cm. (The req.)

10 $\because \overleftrightarrow{ML} // \overleftrightarrow{YZ}$, $\overleftrightarrow{YL}$ is a transversal to them

$\therefore$ m ($\angle$ LYZ) = m ($\angle$ MLY) (alternate angles)

, $\because$ m ($\angle$ LYZ) = m ($\angle$ LYM)

$\therefore$ m ($\angle$ MLY) = m ($\angle$ LYM)

$\therefore$ MY = ML

$\therefore \Delta$ LMY is an isosceles triangle. (Q.E.D.)

In Δ MXY : $\because$ MX = MY

$\therefore$ m ($\angle$ X) = m ($\angle$ Y) (1)

, $\because \overleftrightarrow{XY} // \overleftrightarrow{ZL}$, $\overleftrightarrow{XL}$ is a transversal to them

$\therefore$ m ($\angle$ L) = m ($\angle$ X) (alternate angles) (2)

, $\because \overleftrightarrow{XY} // \overleftrightarrow{ZL}$, $\overleftrightarrow{YZ}$ is a transversal to them

$\therefore$ m ($\angle$ Z) = m ($\angle$ Y) (alternate angles) (3)

From (1) , (2) and (3) :

$\therefore$ m ($\angle$ L) = m ($\angle$ Z)

$\therefore$ ML = MZ

$\therefore \Delta$ MLZ is an isosceles triangle. (Q.E.D.)

12 In Δ ACD : $\because$ X is the midpoint of $\overline{AD}$

, Y is the midpoint of $\overline{CD}$

$\therefore XY = \frac{1}{2} AC$ (1)

, In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore AB = \frac{1}{2} AC$ (2)

From (1) and (2) : $\therefore$ AB = XY (Q.E.D.)

13 In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ In $\Delta\Delta$ ABD and ACE :

$$\begin{cases} m(\angle B) = m(\angle C) \\ m(\angle BAD) = m(\angle CAE) \\ AB = AC \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACE$, then we deduce that :

AD = AE (Q.E.D.)

In $\Delta\Delta$ ABD and ACE :

$$\begin{cases} m(\angle D) = m(\angle E) = 90° \\ m(\angle ABD) = m(\angle ACE) \\ BD = CE \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACE$, then we deduce that :

AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) (Q.E.D.)

15 In Δ ABC : $\because$ AB = AC

, $\overrightarrow{AD}$ bisects $\angle$ A

$\therefore \overline{AD} \perp \overline{BC}$ (First req.)

, D is the midpoint of $\overline{BC}$

$\therefore CD = \frac{1}{2} BC = \frac{1}{2} \times 8 = 4$ cm.

, $\because$ m ($\angle$ BAD) = 180° – (65° + 90°) = 25°

$\therefore$ m ($\angle$ DAC) = m ($\angle$ BAD) = 25° (Second req.)

16 In Δ ADE : $\because$ AD = AE

$\therefore$ m ($\angle$ ADE) = m ($\angle$ AED) (1)

, $\because \overleftrightarrow{DE} // \overleftrightarrow{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ ADE) = m ($\angle$ B) (corresponding angles) (2)

, $\because \overleftrightarrow{DE} // \overleftrightarrow{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ AED) = m ($\angle$ C) (corresponding angles) (3)

From (1) , (2) and (3) : $\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

17 In Δ ABC : $\because$ AB = AC

, $\overrightarrow{AE}$ bisects $\angle$ BAC

$\therefore$ BE = $\frac{1}{2}$ BC (Q.E.D. 1)

, $\overline{AE} \perp \overline{BC}$

$\therefore$ $\overleftrightarrow{AE}$ is the axis of symmetry of $\overline{BC}$

, $\because$ D $\in \overleftrightarrow{AE}$

$\therefore$ BD = CD (Q.E.D. 2)

18 In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC $\quad$ $\therefore$ $2x - 1 = 7$

$\therefore$ $2x = 8$ $\quad$ $\therefore$ $x = 4$ (First req.)

$\therefore$ BC = 9 – 4 = 5 cm.

, $\because$ AB = AC = 7 cm.

$\therefore$ The perimeter of Δ ABC = 7 + 7 + 5 = 19 cm. (Second req.)

19 In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC $\quad$ $\therefore$ $2x - 1 = x + 3$

$\therefore$ $2x - x = 3 + 1$

$\therefore$ $x = 4$ $\quad$ $\therefore$ AB = AC = 4 + 3 = 7 cm.

, BC = 9 – 4 = 5 cm.

$\therefore$ The perimeter of Δ ABC = 7 + 7 + 5 = 19 cm. (The req.)

20 $\because$ The sum of measures of the angles of the triangle = 180°

$\therefore$ $2x + 3x - 10° + x + 40° = 180°$

$\therefore$ $6x + 30° = 180°$

$\therefore$ $6x = 150°$ $\quad$ $\therefore$ $x = \frac{150°}{6} = 25°$

$\therefore$ m ($\angle$ A) = 2 × 25° = 50°

, m ($\angle$ B) = 25° + 40° = 65°

, m ($\angle$ C) = 3 × 25° – 10° = 65°

$\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

21 In Δ ABC : $\because$ D is the midpoint of $\overline{AC}$

$\therefore$ BD is a median from the vertex of the right angle

$\therefore$ BD = $\frac{1}{2}$ AC (1)

In Δ BDE : $\because$ m ($\angle$ BDE) = 90°

, m ($\angle$ E) = 30° $\quad$ $\therefore$ BD = $\frac{1}{2}$ BE (2)

From (1) and (2) : $\therefore$ AC = BE (Q.E.D.)

22 In $\Delta\Delta$ ADC and BDC :

$\left\{ \begin{array}{l} \text{AC = BC} \\ \overline{\text{CD}} \text{ is a common side} \\ \text{m} (\angle \text{ ACD}) = \text{m} (\angle \text{ BCD}) \end{array} \right.$

$\therefore$ Δ ADC $\equiv$ Δ BDC and we deduce that :

AD = BD

, $\because$ ED = BD $\quad$ $\therefore$ AD = ED

$\therefore$ m ($\angle$ E) = m ($\angle$ EAD) (Q.E.D.)

23 $\because$ m ($\angle$ BAD) = 90° , m ($\angle$ BAE) = 30°

$\therefore$ m ($\angle$ DAF) = 60°

In Δ AFD :

$\because$ m ($\angle$ AFD) = 90° , m ($\angle$ DAF) = 60°

$\therefore$ m ($\angle$ ADF) = 180° – (90° + 60°) = 30°

$\therefore$ AD = 2 AF = 8 cm.

$\therefore$ The area of the square = 8 × 8 = 64 cm^2. (The req.)

24 In Δ ABC : $\because$ D is the midpoint of $\overline{BC}$

$\therefore$ $\overline{AD}$ is a median

, $\because$ AM = 2 MD

$\therefore$ M is the intersection point of the medians of Δ ABC

, $\because$ M $\in \overline{CE}$ $\quad$ $\therefore$ $\overline{CE}$ is a median in Δ ABC

$\therefore$ EM = $\frac{1}{3}$ EC = $\frac{1}{3}$ × 12 = 4 cm. (The req.)

[illegible]

First Answers of multiple choice questions

1 (c) **2** (d) **3** (c) **4** (a) **5** (d)

6 (b) **7** (b) **8** (a) **9** (d) **10** (b)

11 (d) **12** (b)

Second Answers of complete questions

1 opposite to a side greater in length than that opposite to the other angle.

2 the angle of the greater measure

3 $\overline{AB}$ 4 $\overline{BC}$

5 $>$ 6 $]4 , 14[$

7 the perpendicular line segment

8 the hypotenuse 9 AC 10 A + X

Answers of essay questions

1 In Δ ABC :

$\because$ m ($\angle$ C) = 180° – (80° + 40°) = 60°

$\therefore$ m ($\angle$ B) > m ($\angle$ C) > m ($\angle$ A)

$\therefore$ AC > AB > BC (The req.)

2 In Δ ABC : $\because$ AB < BC < AC

$\therefore$ m ($\angle$ C) < m ($\angle$ A) < m ($\angle$ B) (The req.)

3 In Δ ABC : $\because$ AC = AB

$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC) (1)

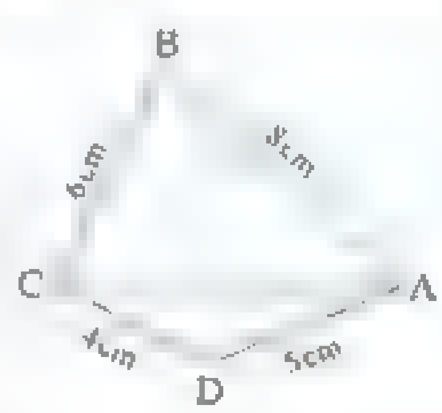
In Δ BCD : $\because$ BD > CD

$\therefore$ m ($\angle$ BCD) > m ($\angle$ CBD) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ ACD) > m ($\angle$ ABD) (Q.E.D.)

4



Const : Draw $\overline{AC}$

Proof : In Δ ABC :

$\because$ AB > BC

$\therefore$ m ($\angle$ BCA) > m ($\angle$ BAC) (1)

In Δ ADC : $\because$ AD > CD

$\therefore$ m ($\angle$ DCA) > m ($\angle$ DAC) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ BCD) > m ($\angle$ BAD) (Q.E.D.)

5 In Δ ABC : $\because$ AB < AC

$\therefore$ m ($\angle$ ACB) < m ($\angle$ ABC)

, $\because$ $\overrightarrow{CM}$ bisects $\angle$ ACB , $\overrightarrow{BM}$ bisects $\angle$ ABC

$\therefore$ m ($\angle$ MCB) = $\frac{1}{2}$ m ($\angle$ ACB)

, m ($\angle$ MBC) = $\frac{1}{2}$ m ($\angle$ ABC)

m ($\angle$ MCB) < m ($\angle$ MBC)

$\therefore$ BM < CM (Q.E.D.)

6 In Δ ABC : $\because$ AB > BC

$\therefore$ m ($\angle$ ACB) > m ($\angle$ BAC) (1)

, $\because$ $\overleftrightarrow{XY}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ XYA) = m ($\angle$ ACB)

(corresponding angles) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ XYA) > m ($\angle$ BAC)

$\therefore$ AX > XY (Q.E.D.)

7 $\because$ $\overleftrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ B) = m ($\angle$ BAD) = 40° (alternate angles)

In Δ ABC :

$\therefore$ m ($\angle$ C) = 180° – (80° + 40°) = 60°

$\therefore$ m ($\angle$ C) > m ($\angle$ B)

$\therefore$ AB > AC (Q.E.D.)

8 In Δ AXY :

$\because$ m ($\angle$ AXY) = m ($\angle$ AYX)

$\therefore$ AY = AX (1)

, $\because$ AC > AB (2)

Subtracting (1) from (2) :

$\therefore$ AC – AY > AB – AX

$\therefore$ YC > XB (Q.E.D.)

9 In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ B) = 65°

$\therefore$ m ($\angle$ BAC) = 180° – 65° × 2 = 50°

, $\because$ $\angle$ BAC is an exterior angle of Δ ACD

$\therefore$ m ($\angle$ ADC) = 50° – 20° = 30°

$\therefore$ In Δ ADC : m ($\angle$ ADC) > m ($\angle$ ACD)

$\therefore$ AC > AD

, $\because$ AB = AC

$\therefore$ AB > AD (Q.E.D.)

10 In Δ ABD : $\because$ BD = AD

$\therefore$ m ($\angle$ BAD) = m ($\angle$ B)

$\therefore$ m ($\angle$ BAD) + m ($\angle$ DAC) > m ($\angle$ B)

$\therefore$ m ($\angle$ BAC) > m ($\angle$ B)

$\therefore$ BC > AC (Q.E.D.)

11 In Δ ABC : $\because$ AB = AC

$\therefore m(\angle B) = m(\angle C)$ (1)

, $\because D \in \overline{BC}$

$\therefore \angle$ ADC is an exterior angle of Δ ADB

$\therefore m(\angle ADC) > m(\angle B)$ (2)

From (1) and (2) : $\therefore m(\angle ADC) > m(\angle C)$

$\therefore AC > AD$ (Q.E.D.)

12 In Δ ABD : $\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AD}$

$\therefore XY = \frac{1}{2} BD$ (1)

In Δ DBC : $\because m(\angle BDC) = 90°$

, E is the midpoint of $\overline{BC}$

$\therefore \overline{DE}$ is a median from the vertex of the right angle

$\therefore DE = \frac{1}{2} BC$ (2)

, $\because \overline{BC}$ is a hypotenuse of Δ BDC

$\therefore BD < BC$

$\therefore \frac{1}{2} BD < \frac{1}{2} BC$ (3)

From (1) , (2) and (3) :

$\therefore XY < DE$ (Q.E.D.)

13 $\because \angle$ ADC is an exterior angle of Δ ABD

$\therefore m(\angle ADC) > m(\angle BAD)$ (1)

, $\because \overrightarrow{AD}$ bisects $\angle$ BAC

$\therefore m(\angle BAD) = m(\angle DAC)$ (2)

From (1) and (2) :

$\therefore m(\angle ADC) > m(\angle DAC)$

$\therefore AC > DC$ (Q.E.D.)

14 In Δ ACE :

$\because AE = EC$

$\therefore m(\angle ECA) = m(\angle CAE) = 40°$

$\therefore m(\angle AEC) = 180° - (40° + 40°) = 100°$

$\therefore m(\angle AEC) > m(\angle ACE)$

$\therefore AC > AE$ (Q.E.D. 1)

In Δ ACD :

$\because \overline{AE}$ is a median , $AE = \frac{1}{2} CD$

$\therefore m(\angle DAC) = 90°$

$\therefore$ In Δ ABC : $\overline{BC}$ is a hypotenuse

$\therefore BC > AC$ (Q.E.D. 2)

Answers of the school book model examinations on Geometry

Model 1

1

1 the hypotenuse 2 5 cm. , 9 cm.
3 a side greater in length than that opposite to the other angle.
4 the angle at this vertex is right 5 equilateral

2

1 c	2 a	3 b
4 b	5 a	6 d

3

[a] >

[b] ∵ Δ DBC is equilateral

∴ m (∠ DBC) = 60°

In Δ ABC : ∵ AB = AC

∴ m (∠ ABC) = m (∠ ACB) = $\frac{180° - 50°}{2}$ = 65°

∴ m (∠ ABD) = 60° + 65° = 125° (The req.)

[c] ∵ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

∴ m (∠ ACB) = m (∠ DAC) = 50° (alternate angles)

∴ In Δ ABC : m (∠ B) = 180° – (70° + 50°) = 60°

∴ m (∠ BAC) > m (∠ B)

∴ BC > AC (Q.E.D.)

4

[a] Theoretical.

[b] In Δ ABC : ∵ AB = AC

∴ m (∠ ABC) = m (∠ ACB)

∴ $\frac{1}{2}$ m (∠ ABC) = $\frac{1}{2}$ m (∠ ACB)

∴ m (∠ DBC) = m (∠ DCB)

∴ Δ DBC is isosceles. (Q.E.D.)

5

[a] ∵ $\overline{AC}$ is the longest side.

∴ ∠ B is the greatest angle in measure.

, ∵ $\overline{AB}$ is the shortest side.

∴ ∠ C is the smallest angle in measure.

∴ The descending order of measures of the angles is m (∠ B) , m (∠ A) and m (∠ C) (The req.)

[b] In Δ ABC : ∵ AB > BC

∴ m (∠ ACB) > m (∠ BAC) (1)

, ∵ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal.

∴ m (∠ XYA) = m (∠ ACB)

(corresponding angles) (2)

From (1) and (2) : ∴ m (∠ XYA) > m (∠ BAC)

∴ AX > XY (Q.E.D.)

Model 2

1

1 d	2 a	3 b
4 b	5 d	6 d

2

1 isosceles. 2 is less than
3 XY 4 $\frac{1}{2}$
5 is perpendicular to it.

3

[a] ∵ $\overline{AB}$ is the longest side.

∴ ∠ C is the greatest angle in measure.

, ∵ $\overline{CB}$ is the shortest side.

∴ ∠ A is the smallest angle in measure.

∴ The ascending order of measures of the angles is m (∠ A) , m (∠ B) and m (∠ C) (The req.)

[b] In Δ ABC : ∵ m (∠ B) = 90°

, ∵ D is the midpoint of $\overline{AC}$

, ∵ E is the midpoint of $\overline{BC}$

∴ $\overline{BD}$, $\overline{AE}$ are two medians in Δ ABC

∴ M is the intersection point of the medians of Δ ABC

∴ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 9 = 4.5 cm.

, BM = $\frac{2}{3}$ BD = $\frac{2}{3}$ × 4.5 = 3 cm.

, ∵ m (∠ C) = 30°

∴ AB = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 9 = 4.5 cm. (The req.)

4

[a] In Δ ABC : ∵ D is the midpoint of $\overline{AC}$

∴ $\overline{BD}$ is a median.

, ∵ m (∠ ABC) = 90° ∴ BD = $\frac{1}{2}$ AC (1)

In Δ BDE : ∵ m (∠ BDE) = 90° , m (∠ E) = 30°

∴ BD = $\frac{1}{2}$ BE (2)

From (1) and (2) : ∴ AC = BE (Q.E.D.)

[b] $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle ACB) = m(\angle CAD) = 30°$ (alternate angles)

$\therefore$ In $\triangle ABC$:

$m(\angle B) = 180° - (70° + 30°) = 80°$

$\therefore m(\angle B) > m(\angle BAC)$

$\therefore AC > BC$ (Q.E.D.)

[a] a side greater in length than that opposite to the other angle.

[b] $\because \overrightarrow{AB}$ bisects $\angle YAZ$

$\therefore m(\angle YAB) = m(\angle BAZ)$ (1)

, $\because \overline{AB} \parallel \overline{XY}$, $\overleftrightarrow{AY}$ is a transversal.

$\therefore m(\angle BAY) = m(\angle AYX)$ (alternate angles) (2)

, $\because \overline{AB} \parallel \overline{XY}$, $\overleftrightarrow{ZX}$ is a transversal.

$\therefore m(\angle X) = m(\angle BAZ)$ (corresponding angles) (3)

From (1) , (2) and (3) : $\therefore m(\angle AYX) = m(\angle X)$

$\therefore m(\angle AYX) + m(\angle AYZ) > m(\angle X)$

$\therefore m(\angle ZYX) > m(\angle X)$

$\therefore XZ > YZ$ (Q.E.D.)

1 1 : 2

2 half the length of the hypotenuse

3 congruent

4 >

5 bisects it , perpendicular to the base.

1 b   2 a   3 d   4 a   5 a

$\because m(\angle B) = 90°$, $m(\angle C) = 30°$

$\therefore AB = \frac{1}{2} \times AC$

$\therefore AC = 10$ cm

[a] AC , AB , BC

[b] 1 40°   2 $\overline{AB}$

1 ✔   2 ✘   3 ✘   4 ✔   5 ✔

Answers of the schools examinations on Geometry

1 Cairo

1

[1] (b) [2] (c) [3] (a) [4] (d) [5] (b)

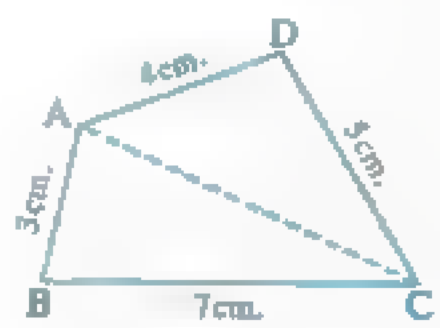
2

[1] 10 cm [2] equilateral

[3] 45° [4] 4

3

[a] Const. :



Draw $\overline{AC}$

Proof : In Δ ABC :

$\because$ BC > AB

$\therefore$ m ($\angle$ BAC) > m ($\angle$ BCA) (1)

In Δ ACD : $\because$ CD > AD

$\therefore$ m ($\angle$ CAD) > m ($\angle$ ACD) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ BAD) > m ($\angle$ BCD) (Q.E.D.)

[b] In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

In $\Delta\Delta$ ABX and ACY :

$$\begin{cases} m(\angle B) = m(\angle C) \\ AB = AC \\ BX = CY \end{cases}$$

$\therefore$ Δ ABX $\equiv$ Δ ACY , then we deduce that :

AX = AY (Q.E.D.)

4

[a] In Δ ABC : $\because$ m ($\angle$ ABC) = 90°

, $\overline{BD}$ is a median

$\therefore$ BD = $\frac{1}{2}$ AC (1)

In Δ ACE : $\because$ X is the midpoint of $\overline{AE}$

, Y is the midpoint of $\overline{CE}$

$\therefore$ YX = $\frac{1}{2}$ AC (2)

From (1) and (2) :

$\therefore$ BD = YX (Q.E.D.)

[b] In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC $\therefore$ $2x - 1 = x + 3$ $\therefore$ $x = 4$

$\therefore$ AB = 2 × 4 – 1 = 7 cm.

, AC = 4 + 3 = 7 cm.

, BC = 9 – 4 = 5 cm.

$\therefore$ The perimeter of Δ ABC = 7 + 7 + 5 = 19 cm.

(The req.)

5

[a] In Δ ABC : $\because$ $\overline{CD}$ and $\overline{BE}$ are two medians

$\therefore$ $\overline{BE} \cap \overline{CD} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore$ MD = $\frac{1}{3}$ CD = $\frac{1}{3}$ × 9 = 3 cm.

, ME = $\frac{1}{2}$ BM = $\frac{1}{2}$ × 4 = 2 cm.

, AE = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm.

, AD = $\frac{1}{2}$ AB = $\frac{1}{2}$ × 6 = 3 cm.

$\therefore$ The perimeter of the figure ADME = 3 + 2 + 5 + 3 = 13 cm. (The req.)

[b] Assuming that ABC is a triangle

$\therefore$ AB < AC + BC

Adding AB to both sides :

$\therefore$ 2 AB < AC + BC + AB

$\therefore$ AB < $\frac{1}{2}$ the perimeter of Δ ABC

$\therefore$ The length of any side in the triangle is less than the half of the perimeter of the triangle

(Q.E.D.)

2 Cairo

1

[1] (d) [2] (b) [3] (b) [4] (b) [5] (b)

2

[1] 6 cm. [2] <

[3] half the length of the hypotenuse. [4] 110°

3

[a] In Δ XYZ : $\because$ XY < XZ < YZ

$\therefore$ m ($\angle$ Z) < m ($\angle$ Y) < m ($\angle$ X) (The req.)

[b] $\because$ B $\in \overleftrightarrow{CD}$

$\therefore$ m ($\angle$ ABC) = 180° – 150° = 30°

, $\because$ C $\in \overleftrightarrow{AE}$

$\therefore$ m ($\angle$ ACB) = 180° – 105° = 75°

$\therefore$ m ($\angle$ ACB) > m ($\angle$ ABC)

$\therefore$ AB > AC (Q.E.D.)

[a] In Δ ABC : ∵ AB = BC

∴ m (∠ ACB) = m (∠ BAC) = $\frac{180° - 90°}{2}$ = 45°

, ∵ Δ BCD is an equilateral triangle.

∴ m (∠ BCD) = 60°

∴ m (∠ ACD) = 45° + 60° = 105° (The req.)

[b] ∵ $\overrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

∴ m (∠ ACB) = m (∠ CAD) = 40° (alternate angles)

∴ In Δ ABC :

∴ m (∠ BAC) = 180° – (40° + 70°) = 70°

∴ m (∠ BAC) = m (∠ B)

∴ AC = BC

∴ Δ ABC is an isosceles triangle. (Q.E.D.)

[a] In Δ ABC : ∵ m (∠ ABC) = 90°

, $\overline{BD}$ is a median

∴ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm.

, In Δ BDE : ∵ m (∠ BDE) = 90° , m (∠ E) = 30°

BE = 2 BD = 2 × 5 = 10 cm. (The req.)

[b] In Δ ABC : ∵ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

∴ DE = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 14 = 7 cm.

, ∵ $\overline{CD}$ and $\overline{BE}$ are two medians

, $\overline{CD}$ ⋂ $\overline{BE}$ = {M}

∴ M is the point of intersection of the medians

∴ MD = $\frac{1}{3}$ CD = $\frac{1}{3}$ × 15 = 5 cm.

, ME = $\frac{1}{2}$ BM = $\frac{1}{2}$ × 6 = 3 cm.

The perimeter of Δ MDE = 7 + 5 + 3 = 15 cm. (The req.)

1 (c) 2 (d) 3 (b) 4 (a) 5 (a)

1 the hypotenuse
2 congruent
3 210°
4 120°

[a] In Δ ABC : ∵ AB = AC

∴ m (∠ B) = m (∠ C) (1)

, ∵ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

∴ m (∠ AXY) = m (∠ B) (corresponding angles) (2)

, ∵ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

∴ m (∠ AYX) = m (∠ C) (corresponding angles) (3)

From (1) , (2) and (3) :

∴ m (∠ AXY) = m (∠ AYX)

∴ AX = AY

∴ Δ AXY is an isosceles triangle (Q.E.D.)

[b] ∵ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

∴ DE = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 12 = 6 cm.

, ∵ $\overline{CD}$, $\overline{BE}$ are two medians intersecting at M

MD = $\frac{1}{2}$ CM = $\frac{1}{2}$ × 6 = 3 cm.

, ME = $\frac{1}{2}$ BM = $\frac{1}{2}$ × 8 = 4 cm.

∴ The perimeter of Δ MDE = 6 + 3 + 4 = 13 cm (The req.)

[a] In Δ ABC : ∵ AB < BC < AC

∴ m (∠ C) < m (∠ A) < m (∠ B) (The req.)

[b] In Δ ABC : ∵ AB = BC

∴ m (∠ BAC) = m (∠ BCA) = $\frac{180° - 40°}{2}$ = 70°

, ∵ Δ ACD is an equilateral triangle

m (∠ CAD) = 60°

∴ m (∠ BAD) = 70° + 60° = 130° (The req.)

[a] ∵ $\overrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

∴ m (∠ C) = m (∠ CAD) = 30° (alternate angles)

In Δ ABC : ∵ AC = BC

∴ m (∠ BAC) = m (∠ B) = $\frac{180° - 30°}{2}$ = 75° (The req.)

[b] In ΔABC : $\because AB > BC$

$\therefore m(\angle C) > m(\angle A)$

, $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle AYX) = m(\angle C)$ (corresponding angles)

$m(\angle AYX) > m(\angle A)$

$\therefore$ In ΔAXY : $AX > XY$ (Q.E.D.)

[illegible]

1

$\boxed{1}$ (d) $\boxed{2}$ (a) $\boxed{3}$ (c) $\boxed{4}$ (b) $\boxed{5}$ (b)

2

$\boxed{1}$ $\overline{AB}$

$\boxed{2}$ bisects the base and is perpendicular to it

$\boxed{3}$ the greater angle in measure $\boxed{4}$ half

3

[a] $\because$ E is the midpoint of $\overline{AC}$

, D is the midpoint of $\overline{BC}$

$\therefore ED = \frac{1}{2} AB = \frac{1}{2} \times 9 = 4.5$ cm.

, $\because \overline{AD}$ and $\overline{BE}$ are two medians

, $\overline{AD} \cap \overline{BE} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore MD = \frac{1}{3} AD = \frac{1}{3} \times 12 = 4$ cm.

, $ME = \frac{1}{2} MB = \frac{1}{2} \times 6 = 3$ cm.

$\therefore$ The perimeter of $\Delta EMD = 4.5 + 4 + 3 = 11.5$ cm. (The req.)

[b] $\because Z \in \overleftrightarrow{XN}$

$\therefore m(\angle XZY) = 180° - 110° = 70°$

In ΔXYZ : $\because \angle XYM$ is an exterior angle

$\therefore m(\angle X) = 125° - 70° = 55°$

$\therefore m(\angle XZY) > (\angle X)$ $\therefore XY > ZY$ (Q.E.D.)

4

[a] In ΔXYZ $\because$ M is the midpoint of $\overline{XY}$

, N is the midpoint of $\overline{YZ}$

$\therefore MN = \frac{1}{2} XZ = \frac{1}{2} \times 16 = 8$ cm.

In ΔMYN which is right-angled at Y

$\because \overline{YL}$ is a median

$\therefore YL = \frac{1}{2} MN = \frac{1}{2} \times 8 = 4$ cm. (The req.)

[b] $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{BD}$ is a transversal to them

$\therefore m(\angle BDE) = m(\angle CBD)$ (alternate angles)

, $\because \overrightarrow{BD}$ bisects $\angle ABC$

$\therefore m(\angle DBE) = m(\angle CBD)$

$\therefore m(\angle DBE) = m(\angle BDE)$

$\therefore BE = DE$

$\therefore \Delta EBD$ is an isosceles triangle. (Q.E.D.)

5

[a] In ΔCDE : $\because EC = DC$

$\therefore m(\angle D) = m(\angle E) = 30°$

$\therefore m(\angle ACB) = 30° + 30° = 60°$ (exterior angle of ΔCDE)

$\therefore$ In ΔABC : $m(\angle ABC) = 180° - (60° + 60°) = 60°$

$\therefore m(\angle A) = m(\angle ACB) = m(\angle ABC) = 60°$

$\therefore \Delta ABC$ is an equilateral triangle (Q.E.D.)

[b] In ΔABC : $\because m(\angle ABC) = 90°$

, $\overline{BE}$ is a median $\therefore BE = \frac{1}{2} AC$

, $\because BE = DE$ $\therefore DE = \frac{1}{2} AC$

, $\because \overline{DE}$ is a median in ΔADC

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

Alexandria

1

$\boxed{1}$ (b) $\boxed{2}$ (c) $\boxed{3}$ (a) $\boxed{4}$ (b) $\boxed{5}$ (b)

2

$\boxed{1}$ zero $\boxed{2}$ 120°

$\boxed{3}$ the hypotenuse $\boxed{4}$ an isosceles

3

[a] In ΔABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 50°}{2} = 65°$

, $\because \Delta DBC$ is an equilateral triangle

$\therefore m(\angle CBD) = 60°$

$\therefore m(\angle ABD) = 65° + 60° = 125°$ (The req.)

[b] In ΔABC : $\because AB < BC < CA$

$\therefore m(\angle C) < m(\angle A) < m(\angle B)$ (The req.)

[a] $\because \Delta ABC$ is right-angled at B , $m(\angle C) = 30°$

$\therefore AB = \frac{1}{2} AC$

, $\because \overline{BD}$ is a median

$\therefore DB = \frac{1}{2} AC$

$\therefore AB = DB$ (Q.E.D.)

[b] $\because \Delta ABC$ is right-angled at B

, $m(\angle ACB) = 30°$

$\therefore AC = 2\,AB = 2 \times 5 = 10$ cm.

, in ΔACD : $\because \overline{DE}$ is a median , $DE = \frac{1}{2} AC$

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

---

[a] In ΔBCE : $\because EB = EC$

$\therefore m(\angle B) = m(\angle C)$ (1)

, $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{BD}$ is a transversal to them

$\therefore m(\angle D) = m(\angle B)$ (alternate angles) (2)

, $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore m(\angle A) = m(\angle C)$ (alternate angles) (3)

From (1) , (2) and (3) : $\therefore m(\angle A) = m(\angle D)$

$\therefore EA = ED$ (Q.E.D.)

[b] $\because \overrightarrow{SR} \parallel \overline{LM}$, $\overleftrightarrow{SM}$ is a transversal to them

$\therefore m(\angle M) = m(\angle RSM) = 30°$ (alternate angles)

$\therefore$ In ΔLSM :

$m(\angle LSM) = 180° - (50° + 30°) = 100°$

$\therefore m(\angle LSM) > m(\angle M)$

$\therefore ML > SL$ (Q.E.D.)

El-Kalyoubia

1

1 (c) 2 (c) 3 (b) 4 (b) 5 (c)

2

1 $\overline{BC}$ 2 90° 3 2 4 zero

---

[a] $\because \angle ADB$ is an exterior angle of ΔACD

$\therefore m(\angle ADB) > m(\angle C)$

, in ΔABC : $\because AB = AC$

$\therefore m(\angle B) = m(\angle C)$

$\therefore$ In ΔABD : $m(\angle ADB) > m(\angle B)$

$\therefore AB > AD$ (Q.E.D.)

[b] In ΔABD : $\because AD = BD$

$\therefore m(\angle BAD) = m(\angle B) = 35°$

$\therefore m(\angle ADC) = 35° + 35° = 70°$

(an exterior angle of ΔABD)

In ΔACD : $\because AC = AD$

$\therefore m(\angle C) = m(\angle ADC) = 70°$

In ΔABC :

$\therefore m(\angle BAC) = 180° - (35° + 70°) = 75°$ (The req.)

---

[a] In ΔABC :

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2} AC = \frac{1}{2} \times 22 = 11$ cm.

In ΔXBY : $\because m(\angle XBY) = 90°$

, $\overline{BD}$ is a median

$\therefore BD = \frac{1}{2} XY = \frac{1}{2} \times 11 = 5.5$ cm. (The req.)

[b] In ΔABC : $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6$ cm.

, $\because \overline{CD}$ and $\overline{BE}$ are two medians

, $\overline{CD} \cap \overline{BE} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore DM = \frac{1}{2} MC = \frac{1}{2} \times 8 = 4$ cm.

, $EM = \frac{1}{2} MB = \frac{1}{2} \times 6 = 3$ cm.

$\therefore$ The perimeter of ΔMDE

$= 6 + 4 + 3 = 13$ cm. (The req.)

---

[a] $\overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore m(\angle B) + m(\angle BAD) = 180°$ (interior angles on the same side of the transversal)

$\therefore m(\angle B) = 180° - (70° + 35°) = 75°$

In ΔABC : $\therefore m(\angle B) > m(\angle BAC)$

$\therefore AC > BC$ (Q.E.D.)

[b] $\because \overrightarrow{BE} // \overline{AC}$, $\overleftrightarrow{BC}$ is a transversal to them

$\therefore$ m ($\angle$ C) = m ($\angle$ CBE) (alternate angles)

, $\because \overrightarrow{BE} // \overline{AC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ A) = m ($\angle$ DBE) (corresponding angles)

, $\because$ m ($\angle$ CBE) = m ($\angle$ DBE)

$\therefore$ m ($\angle$ C) = m ($\angle$ A) $\therefore$ AB = BC

$\therefore \Delta$ ABC is an isosceles triangle (Q.E.D.)

7 El-Sharkia

1

1 (d) 2 (b) 3 (a) 4 (c) 5 (a)

2

1 2 , 8 2 1 : 2

3 is perpendicular to it. 4 6 cm.

3

[a] In Δ ABC : $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ BC = 2 DE = 2 × 6 = 12 cm.

, $\because \overline{BE}$ and $\overline{CD}$ are two medians

, $\overline{BE} \cap \overline{CD} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore$ MB = $\frac{2}{3}$ BE = $\frac{2}{3}$ × 9 = 6 cm.

, MC = 2 MD = 2 × 4 = 8 cm.

$\therefore$ The perimeter of Δ BMC = 12 + 6 + 8 = 26 cm. (The req.)

[b] In Δ ABD : $\because$ AD > AB

$\therefore$ m ($\angle$ ABD) > m ($\angle$ ADB) (1)

, in Δ BCD : $\because$ CD > CB

$\therefore$ m ($\angle$ CBD) > m ($\angle$ CDB) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ ABC) > m ($\angle$ ADC) (Q.E.D.)

4

[a] In Δ ABC : $\because$ AC = AB

$\therefore$ m ($\angle$ ACB) = m ($\angle$ B) = $\frac{180° - 50°}{2}$ = 65°

, $\because \Delta$ ACD is an equilateral triangle

$\therefore$ m ($\angle$ ACD) = 60° (1)

$\therefore$ m ($\angle$ BCD) = 65° + 60° = 125° (The req.)

[b] $\because \overline{AD} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ ACB) = m ($\angle$ CAD) = 32° (alternate angles)

$\therefore$ m ($\angle$ BAC) > m ($\angle$ ACB)

$\therefore$ BC > AB (Q.E.D.)

5

[a] $\because \Delta$ ABC is right-angled at B

, m ($\angle$ ACB) = 30°

$\therefore$ AC = 2 AB = 2 × 5 = 10 cm.

$\therefore$ In Δ ADC :

$\because \overline{DE}$ is a median , DE = $\frac{1}{2}$ AC

$\therefore$ m ($\angle$ ADC) = 90° (Q.E.D.)

[b] In Δ ABC : m ($\angle$ BAC) = 180° – (30° + 70°) = 80°

, $\because \overrightarrow{AD}$ bisects $\angle$ BAC

$\therefore$ m ($\angle$ CAD) = $\frac{1}{2}$ m ($\angle$ BAC) = $\frac{1}{2}$ × 80° = 40°

In Δ ACD :

$\therefore$ m ($\angle$ ADC) = 180° – (70° + 40°) = 70°

$\therefore$ m ($\angle$ C) = m ($\angle$ ADC)

$\therefore$ AD = AC (Q.E.D.)

8 El-Dakahlia

1

1 (a) 2 (b) 3 (c) 4 (a) 5 (c)

2

1 the hypotenuse

2 bisects 3 BC 4 2 , 10

3

[a] In Δ ABC : $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ BC = 2 DE = 2 × 5 = 10 cm.

, $\because$ M is the intersection point of the medians

$\therefore$ MB = 2 ME = 2 × 3 = 6 cm.

, MC = 2 MD = 2 × 4 = 8 cm.

$\therefore$ The perimeter of Δ MBC = 10 + 6 + 8 = 24 cm. (The req.)

[b] In Δ ABC : $\because$ BC < AC < AB

$\therefore$ m ($\angle$ A) < m ($\angle$ B) < m ($\angle$ C) (The req.)

4

[a] In Δ ACD : $\because$ X is the midpoint of $\overline{AD}$

, Y is the midpoint of $\overline{CD}$

$\therefore$ AC = 2 XY = 2 × 6 = 12 cm.

, in Δ ABC :

$\because$ m ($\angle$ ABC) = 90° , $\overline{BE}$ is a median

$\therefore$ BE = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 12 = 6 cm. (The req.)

[b] $\because$ $\overleftrightarrow{XL}$ // $\overline{YZ}$, $\overleftrightarrow{XZ}$ is a transversal to them

$\therefore$ m ($\angle$ Z) = m ($\angle$ LXZ) = 50° (alternate angles)

In Δ XYZ :

$\therefore$ m ($\angle$ YXZ) = 180° − (50° + 80°) = 50°

$\therefore$ m ($\angle$ Z) = m ($\angle$ YXZ)

$\therefore$ XY = YZ (Q.E.D.)

5

[a] In Δ ABC : $\because$ AB = AC , $\overline{AD} \perp \overline{BC}$

$\therefore$ BD = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 10 = 5 cm. (First req.)

In Δ ABD : $\because$ m ($\angle$ ADB) = 90°

$\therefore$ $(AD)^2 = (AB)^2 - (BD)^2 = 169 - 25 = 144$

$\therefore$ AD = $\sqrt{144}$ = 12 cm.

$\therefore$ The area of Δ ABC = $\frac{1}{2}$ × 10 × 12 = 60 cm^2.

(Second req.)

[b] In Δ ABC : $\because$ AB > AC

$\therefore$ m ($\angle$ ACB) > m ($\angle$ ABC)

, $\because$ $\overrightarrow{BM}$ bisects $\angle$ ABC

, $\overrightarrow{CM}$ bisects $\angle$ ACB

$\therefore$ $\frac{1}{2}$ m ($\angle$ACB) > $\frac{1}{2}$ m ($\angle$ ABC)

$\therefore$ In Δ BMC : m ($\angle$ MCB) > m ($\angle$ MBC)

$\therefore$ MB > MC (Q.E.D.)

9 Suez

1

1 7 2 2 , 8 3 120°

4 bisects the base , is perpendicular to it.

2

1 (a) 2 (b) 3 (b) 4 (a) 5 (b)

3

[a] In Δ ABC : $\because$ m ($\angle$ A) = 180° − (40° + 80°) = 60°

$\therefore$ m ($\angle$ B) < m ($\angle$ A) < m ($\angle$ C)

$\therefore$ AC < BC < AB (The req.)

[b] In Δ ABC :

$\because$ X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

$\therefore$ XY = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 12 = 6 cm.

, $\because$ $\overline{CX}$ and $\overline{BY}$ are two medians

, $\overline{CX} \cap \overline{BY}$ = {M}

$\therefore$ M is the point of intersection of the medians

$\therefore$ MX = $\frac{1}{2}$ MC = $\frac{1}{2}$ × 10 = 5 cm.

, MY = $\frac{1}{2}$ MB = $\frac{1}{2}$ × 4 = 2 cm.

$\therefore$ The perimeter of Δ MXY = 6 + 5 + 2 = 13 cm.

(The req.)

4

[a] In Δ ABC : $\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AC = 2 AB = 2 × 5 = 10 cm. (First req.)

In Δ ADC : $\because$ m ($\angle$ ADC) = 90°

, $\overline{DE}$ is a median

$\therefore$ DE = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm. (Second req.)

[b] $\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ ACB) = m ($\angle$ CAD) = 30°

(alternate angles)

$\therefore$ In Δ ABC : m ($\angle$ BAC) > m ($\angle$ ACB)

$\therefore$ BC > AB (Q.E.D.)

5

[a] In Δ ABD : $\because$ AD > AB

$\therefore$ m ($\angle$ ABD) > m ($\angle$ ADB) (1)

In Δ BCD : $\because$ DC > BC

$\therefore$ m ($\angle$ CBD) > m ($\angle$ CDB) (2)

Adding (1) and (2) :

m ($\angle$ ABC) > m ($\angle$ ADC) (Q.E.D.)

[b] In Δ XYZ : $\because$ ZX = XY = YZ

$\therefore$ m ($\angle$ YXZ) = m ($\angle$ XZY) = m ($\angle$ Y) = 60°

(First req.)

In Δ XZL : $\because$ LZ = ZX

$\therefore$ m ($\angle$ LXZ) = m ($\angle$ XLZ) = $\frac{180° - 90°}{2}$ = 45°

$\therefore$ m ($\angle$ LXY) = 60° + 45° = 105° (Second req.)

10 El-Beheira

1

1 (c) 2 (b) 3 (d) 4 (a) 5 (b)

2

1 $\overline{DF}$ 2 one 3 12 4 half

3

[a] In Δ ABC : ∵ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

∴ AC = 2 XY = 2 × 5 = 10 cm.

, ∵ $\overline{AY}$ and $\overline{CX}$ are two medians

, $\overline{AY} \cap \overline{CX} = \{M\}$

∴ M is the point of intersection of the medians

∴ AM = 2 YM = 2 × 3 = 6 cm.

∴ The perimeter of Δ MAC = 10 + 8 + 6 = 24 cm. (The req.)

[b] ∵ ∠ ADC is an exterior angle of Δ ABD

∵ m (∠ ADC) = 30° + 40° = 70°

In Δ ACD : ∵ AC = AD

∴ m (∠ C) = m (∠ ADC) = 70°

In Δ ABC :

∴ m (∠ BAC) = 180° – (70° + 40°) = 70°

∴ m (∠ C) = m (∠ BAC)

∴ AB = CB (Q.E.D.)

4

[a] In Δ AMB : MA + MB > AB (1)

, in Δ BMC : MB + MC > BC (2)

, in Δ CMA : MC + MA > AC (3)

Adding (1) , (2) and (3) :

∴ 2 MA + 2 MB + 2 MC > AB + BC + AC

∴ 2 (MA + MB + MC) > The perimeter of the triangle ABC

∴ MA + MB + MC > $\frac{1}{2}$ the perimeter of the triangle ABC (Q.E.D.)

[b] In Δ XYZ : ∵ m (∠ Z) = 180° – (40° + 80°) = 60°

∴ m (∠ X) < m (∠ Z) < m (∠ Y)

∴ YZ < XY < XZ (The req.)

5

[a] In Δ ABC : m (∠ A) = 180° – (90° + 30°) = 60°

In Δ BCD : ∵ BD = CD

∴ m (∠ DBC) = m (∠ C) = 30°

∴ m (∠ ABD) = 90° – 30° = 60°

In Δ ABD : m (∠ ADB) = 180° – (60° + 60°) = 60°

∴ m (∠ A) = m (∠ ABD) = m (∠ ADB) = 60°

∴ Δ ABD is an equilateral triangle (Q.E.D.)

[b] ∵ AB = AC ∴ A ∈ the axis of $\overline{BC}$

, ∵ EB = EC

∴ E ∈ the axis of $\overline{BC}$

∴ $\overleftrightarrow{AE}$ is the axis of $\overline{BC}$ (Q.E.D. 1)

, ∵ D ∈ $\overleftrightarrow{AE}$

∴ BD = DC (Q.E.D. 2)

11 Beni Suef

1

1 (c) 2 (c) 3 (c) 4 (c) 5 (a)

2

1 $\frac{1}{2}$ 2 congruent , isosceles

3 one 4 $\overline{BC}$

3

[a] In Δ ABC : ∵ AB = BC

∴ m (∠ BAC) = m (∠ C) = $\frac{180° - 90°}{2}$ = 45°

, ∵ Δ ABD is an equilateral triangle

∴ m (∠ BAD) = 60°

∴ m (∠ CAD) = 45° + 60° = 105° (The req.)

[b] In Δ ABC : ∵ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

∴ BC = 2 DE = 2 × 4 = 8 cm.

, ∵ $\overline{CD}$ and $\overline{BE}$ are two medians

, $\overline{CD} \cap \overline{BE} = \{M\}$

∴ M is the point of intersection of the medians

∴ MC = 2 MD = 2 × 3 = 6 cm.

, MB = $\frac{2}{3}$ BE = $\frac{2}{3}$ × 6 = 4 cm.

∴ The perimeter of Δ BMC = 8 + 6 + 4 = 18 cm. (The req.)

4

[a] $\because \overrightarrow{DE} \parallel \overline{BA}$, $\overleftrightarrow{AD}$ is a transversal to them

$\therefore m(\angle A) = m(\angle D) = 60°$ (alternate angles)

In ΔABC : $m(\angle ACB) = 180° - (60° + 60°) = 60°$

$\therefore m(\angle A) = m(\angle B) = m(\angle ACB)$

$\therefore \Delta ABC$ is an equilateral triangle (Q.E.D.)

[b] In ΔABC : $\because AB = AC$ and $\overline{AD} \perp \overline{BC}$

$\therefore DB = CD = 4$ cm.

, $m(\angle BAC) = 2\, m(\angle BAD) = 2 \times 30° = 60°$

(The req.)

5

[a] In ΔABC : $\because BC > BA$

$\therefore m(\angle BAC) > m(\angle ACB)$ (1)

, in ΔACD : $\because DA = DC$

$\therefore m(\angle CAD) = m(\angle ACD)$ (2)

Adding (1) and (2) :

$\therefore m(\angle BAD) > m(\angle BCD)$ (Q.E.D.)

[b] $\because m(\angle ABC) = 180° - 150° = 30°$

, $m(\angle ACB) = 180° - 110° = 70°$

$\therefore$ In ΔABC : $m(\angle A) = 180° - (30° + 70°) = 80°$

$\because m(\angle A) > m(\angle ACB)$

$\therefore CB > AB$ (Q.E.D.)

12 Aswan

1

1 (d) 2 (c) 3 (a) 4 (d) 5 (c)

2

1 is perpendicular to it. 2 10

3 the hypotenuse 4 $m(\angle C)$

3

[a] $\because \overrightarrow{AD} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore m(\angle B) = m(\angle BAD) = 50°$ (alternate angles)

In ΔABC : $\because AB = BC$

$\therefore m(\angle BAC) = m(\angle C) = \frac{180° - 50°}{2} = 65°$

$\therefore m(\angle DAC) = 50° + 65° = 115°$ (The req.)

[b] In ΔABC : $\because m(\angle B) = 90°$, $m(\angle ACB) = 30°$

$\therefore AB = \frac{1}{2} AC$

, $\because AB = DE$ $\therefore DE = \frac{1}{2} AC$

, $\because \overline{DE}$ is a median in ΔADC

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

4

[a] In ΔABC :

$\because$ Y is the midpoint of $\overline{CB}$, X is the midpoint of $\overline{AB}$

$\therefore AC = 2\, XY = 2 \times 5 = 10$ cm.

, $\because \overline{AY}$ and $\overline{CX}$ are two medians

, $\overline{AY} \cap \overline{CX} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore MA = 2\, MY = 2 \times 3 = 6$ cm.

, $MC = \frac{2}{3} CX = \frac{2}{3} \times 12 = 8$ cm.

$\therefore$ The perimeter of $\Delta AMC = 10 + 6 + 8 = 24$ cm.

(The req.)

[b] In ΔABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\therefore m(\angle ABD) = m(\angle ACE)$

(supplementary of two equal angles)

In $\Delta\Delta$ ABD , ACE :

$$\begin{cases} AB = AC \\ m(\angle ABD) = m(\angle ACE) \\ BD = EC \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACE$, then we deduce that :

$AD = AE$ (Q.E.D.)

5

[a] $\because XZ < XY < YZ$

$\therefore m(\angle Y) < m(\angle Z) < m(\angle X)$ (The req.)

[b] In ΔABC : $\because AC = AB$

$\therefore m(\angle ACB) = m(\angle ABC)$ (1)

, in ΔBCD : $\because BD > CD$

$\therefore m(\angle BCD) > m(\angle CBD)$ (2)

Adding (1) and (2) :

$\therefore m(\angle ACD) > m(\angle ABD)$ (Q.E.D.)